Lorentz invariance, vacuum energy, and cosmology

Frans R. Klinkhamer

Institute for Theoretical Physics, University of Karlsruhe (TH)

Email: frans.klinkhamer@physik.uni-karlsruhe.de

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[1] FRK & M. Risse, PRD 77, 117901 (2008), arXiv:0806.4351 [hep-ph].

[2] FRK & G.E. Volovik, PRD 77, 085015 (2008), arXiv:0711.3170 [gr-qc].

[3a] FRK & G.E. Volovik, arXiv:0806.2805 [gr-qc].

[3b] FRK, arXiv:0803.0281 [gr-qc].

[3c] FRK & G.E. Volovik, arXiv:0807.3896 [gr-qc].

Introduction

0. INTRODUCTION

Thanks to observational cosmology, there are now three cosmological constant problems instead of one:

1. why is $|\rho_{\text{vac}}| \ll (E_{\text{Planck}})^4 \approx (10^{28}\ \text{eV})^4$?
2. why is $\rho_{\text{vac}} \neq 0$?
3. why is $\rho_{\text{vac}} \sim \rho_{\text{matter}}\big|_{\text{now}} \approx (10^{-3}\ \text{eV})^4$?

Clearly, we first need to get a handle on problem No. 1.
Only then can we start worrying about Nos. 2 and 3.

This talk gives an overview of ongoing work with Grisha Volovik.

The main focus is on the vacuum energy density, but let's start with an important fact of life.

Lorentz invariance

1. LORENTZ INVARIANCE (LI)

Lorentz violation (LV): *difficult to bound something that is unknown.* Hence, the need for simple concrete models.

Consider a LV deformation of quantum electrodynamics (QED):

$$S_{\text{modQED}} = S_{\text{modM}} + S_{\text{standD}}\,, \tag{1}$$

with a **modified-Maxwell term** and a standard Dirac term for a spin–$\frac{1}{2}$ particle with charge e and mass M:

$$S_{\text{modM}} = \int_{\mathbb{R}^4} \mathrm{d}^4x \left(-\tfrac{1}{4}\left(\eta^{\mu\rho}\eta^{\nu\sigma} + \kappa^{\mu\nu\rho\sigma}\right) F_{\mu\nu}(x)\, F_{\rho\sigma}(x)\right), \tag{2a}$$

$$S_{\text{standD}} = \int_{\mathbb{R}^4} \mathrm{d}^4x\, \overline{\psi}(x)\Big(\gamma^\mu\left(\mathrm{i}\,\partial_\mu - eA_\mu(x)\right) - M\Big)\psi(x)\,. \tag{2b}$$

Theory is gauge-invariant, CPT–even, and power-counting renormalizable.

Lorentz invariance

$\kappa^{\mu\nu\rho\sigma}$ is a constant background tensor with the same symmetries as the Riemann curvature tensor and a double trace condition $\kappa^{\mu\nu}{}_{\mu\nu} = 0$ $\Rightarrow 20 - 1 = 19$ real and dimensionless components.

As 10 birefringent parameters are already constrained at the 10^{-32} level, [Kostelecky & Mewes, hep-ph/0205211], theory can be restricted to the **nonbirefringent sector**:

$$\kappa^{\mu\nu\rho\sigma} = \tfrac{1}{2}\Big(\eta^{\mu\rho}\,\widetilde{\kappa}^{\nu\sigma} - \eta^{\mu\sigma}\,\widetilde{\kappa}^{\nu\rho} - \eta^{\nu\rho}\,\widetilde{\kappa}^{\mu\sigma} + \eta^{\nu\sigma}\,\widetilde{\kappa}^{\mu\rho}\Big), \tag{3}$$

for a symmetric and traceless matrix $\widetilde{\kappa}^{\mu\nu}$ with $10 - 1 = 9$ components.

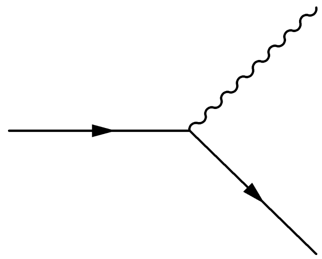
These 9 parameters $\widetilde{\kappa}^{\mu\nu}$ can be grouped as follows:

1 parity-even isotropic parameter $\widetilde{\kappa}_{\text{tr}}$;
3 parity-odd parameters in an antisymmetric traceless 3×3 matrix $(\widetilde{\kappa}_{\text{o}+})^{mn}$;
5 parity-even parameters in a symmetric traceless 3×3 matrix $(\widetilde{\kappa}_{\text{e}-})^{mn}$.

Ultra-high-energy cosmic rays (UHECRs) give tight bounds on $\widetilde{\kappa}^{\mu\nu}$.
Skip all the details.

Lorentz invariance

From the <u>absence</u> of "vacuum Cherenkov radiation" in 29 selected UHECR events (with accurately measured primary energies and directions), the following $\mathbf{2\sigma}$ **bounds** have been obtained [1]:



$$(ij) \in \{(23),(31),(12)\} \quad : \quad \left|(\widetilde{\kappa}_{\mathrm{o}+})^{(ij)}\right| < 2 \times 10^{-18}\,, \tag{4a}$$

$$(kl) \in \{(11),(12),(13),(22),(23)\} \quad : \quad \left|(\widetilde{\kappa}_{\mathrm{e}-})^{(kl)}\right| < 4 \times 10^{-18}\,, \tag{4b}$$

$$\widetilde{\kappa}_{\mathrm{tr}} < 1.4 \times 10^{-19}\,. \tag{4c}$$

Current laboratory bounds for these $\widetilde{\kappa}$ range between 10^{-7} and 10^{-16}.

A brief review: FRK, arXiv:0807.2147.

Conclusion 1: *LI of electromagnetic sector verified to high precision.*

Vacuum energy

2. VACUUM ENERGY

Cosmology suggests a nonzero cosmological constant $\Lambda > 0$ or gravitating vacuum energy density $\rho_\mathsf{V} = -P_\mathsf{V} > 0$.

But what is the theory?

First the statics, simple picture based on three assumptions [2]:

1. perfect quantum vacuum is a **Lorentz-invariant state**;
2. perfect quantum vacuum is a **self-sustained medium** at zero external pressure;
3. perfect quantum vacuum is characterized by a new type of **conserved charge** q, which is constant over spacetime.

Analog of q in cond-mat: the Larkin–Pikin effect (1969) in magnetic phase transitions of crystals.

Vacuum energy

Thermodynamics and charge conservation ($Q = qV =$ const) give a Gibbs–Duhem relation (in this context, first discussed by Volovik):

$$\widetilde{\epsilon}_{\text{V}}(q_0) \equiv \left[\epsilon(q) - q\, \frac{d\epsilon(q)}{dq} \right]_{q=q_0} = -P_{\text{ext}} = 0\,, \tag{5}$$

for energy density $\epsilon(q)$ in the action and equilibrium value q_0 of the vacuum variable q.

Three remarks:

1. effective energy density $\widetilde{\epsilon}_{\text{V}}(q)$ is zero by cancelation of two terms which can each be of order E^4_{Planck};
2. $\widetilde{\epsilon}_{\text{V}}$ gravitates and ϵ not $\Rightarrow$ $\rho_{\text{V}} = \widetilde{\epsilon}_{\text{V}}(q_0)$;
3. explicit models known which give this result for ρ_{V} (see later).

Vacuum energy

All in all, we have in a perfect LI quantum vacuum

$$\Lambda \equiv \rho_V \stackrel{①}{=} \widetilde{\epsilon}_V \stackrel{②}{=} -P_V \stackrel{③}{=} -P_{ext} = 0\,, \tag{6}$$

with step ① from thermodynamics, step ② from LI, and step ③ from pressure equilibrium.

In the presence of thermal matter (i.e., a non-LI state), pressure equilibrium gives

$$P_V + P_M = P_{ext} = 0\,, \tag{7}$$

so that $P_V = -P_M$ and

$$\rho_V = -P_V = P_M = w_M\,\rho_M\,. \tag{8}$$

Conclusion 2:

$$\Lambda\Big|_{\text{perfect quantum vacuum in equilibrium}} = 0\,, \tag{9a}$$

$$\Lambda\Big|_{\text{imperfect quantum vacuum in equilibrium}} \propto \rho_M\,. \tag{9b}$$

3. COSMOLOGY

Up till now, equilibrium. But what about Hubble's expanding universe?

Difficult to say as this concerns the exchange of energy between the deep vacuum (e.g., the variable q) and the low-energy degrees of freedom (e.g., standard model and general relativity).

Two different approaches have been followed up till now:

[3a]: how, starting far away from equilibrium in a very early phase of universe, the vacuum may reach its equilibrium state.

[3b]: how, starting from an equilibrium state at late times, a universe can arise which resembles our present universe.

Early cosmology

3.1 Equilibrium approach for an early start

Now, vacuum variable q represented by a four-form field F.

The action [3a] has a general term $\epsilon(F)$ and Newton's constant G_N replaced by a gravitational coupling parameter $G = G(F)$:

$$S[A,g,\psi] = -\int_{\mathbb{R}^4} d^4x \sqrt{|g|}\left(\frac{R}{16\pi G(F)} + \epsilon(F) + \mathcal{L}^\mathrm{M}(\psi)\right), \quad \text{(10a)}$$

$$F^2 \equiv -\frac{1}{24} F_{\kappa\lambda\mu\nu} F^{\kappa\lambda\mu\nu}, \quad F_{\kappa\lambda\mu\nu} \equiv \nabla_{[\kappa} A_{\lambda\mu\nu]}, \quad \text{(10b)}$$

$$F_{\kappa\lambda\mu\nu} = F\sqrt{|g|}\, e_{\kappa\lambda\mu\nu}, \quad F^{\kappa\lambda\mu\nu} = F\, e^{\kappa\lambda\mu\nu}/\sqrt{|g|}, \quad \text{(10c)}$$

where ψ stands for a generic low-energy matter field ($\hbar = c = 1$).

Early cosmology

Variational principle gives generalized Maxwell equations:

$$\nabla_\mu \left(\sqrt{|g|}\, \frac{F^{\kappa\lambda\mu\nu}}{F} \left(\frac{d\epsilon(F)}{dF} + \frac{R}{16\pi}\, \frac{dG^{-1}(F)}{dF} \right) \right) = 0\,, \tag{11}$$

and generalized Einstein equations:

$$\frac{1}{8\pi G(F)} \left(R_{\mu\nu} - \frac{1}{2}\, R\, g_{\mu\nu} \right) + \frac{1}{16\pi}\, F\, \frac{dG^{-1}(F)}{dF}\, R\, g_{\mu\nu}$$
$$+ \frac{1}{8\pi} \Big(\nabla_\mu \nabla_\nu\, G^{-1}(F) - g_{\mu\nu}\, \Box\, G^{-1}(F) \Big) - \widetilde{\epsilon}(F) g_{\mu\nu} + T^{\mathsf{M}}_{\mu\nu} = 0\,, \tag{12}$$

with effective vacuum energy density

$$\widetilde{\epsilon}(F) \equiv \epsilon(F) - F\, \frac{d\epsilon(F)}{dF}\,, \tag{13}$$

whose form has been argued on thermodynamic grounds in Part 2.

Early cosmology

Generalized Maxwell equations solved by

$$\frac{d\epsilon(F)}{dF} + \frac{R}{16\pi}\frac{dG^{-1}(F)}{dF} = \mu\,, \tag{14}$$

with an integration constant (chemical potential) μ.

Reduced generalized Einstein equations:

$$\frac{1}{8\pi G(F)}\Big(R_{\mu\nu} - \tfrac{1}{2}\,R\,g_{\mu\nu}\Big) + \frac{1}{8\pi}\Big(\nabla_\mu\nabla_\nu\,G^{-1}(F) - g_{\mu\nu}\,\Box\,G^{-1}(F)\Big)$$
$$-\Big(\epsilon(F) - \mu\,F\Big)\,g_{\mu\nu} + T^{\mathsf{M}}_{\mu\nu} = 0\,. \tag{15}$$

These two equations need to be solved simultaneously.

Early cosmology

Flat ($k = 0$) FRW universe with expansion factor $a(t)$, homogeneous perfect-fluid matter, and homogeneous four-form amplitude $F(t)$.

With Hubble parameter $H(t) \equiv (da/dt)/a$, we then have:

$$\frac{3}{8\pi}\,\frac{dG^{-1}}{dF}\left(\frac{dH}{dt} + 2H^2\right) = \frac{d\rho_{\text{V}}}{dF}\,, \qquad \text{(16a)}$$

$$H^2 + HG\,\frac{dG^{-1}}{dt} = \frac{8\pi}{3}\,G\,\rho_{\text{tot}}\,, \qquad \text{(16b)}$$

$$\frac{d\rho_{\text{M}}}{dt} + 3H\left(P_{\text{M}} + \rho_{\text{M}}\right) = 0\,, \qquad \text{(16c)}$$

for

$$\rho_{\text{tot}} \equiv \rho_{\text{V}} + \rho_{\text{M}}\,, \qquad P_{\text{tot}} \equiv P_{\text{V}} + P_{\text{M}}\,, \qquad \text{(17a)}$$

$$\rho_{\text{V}}(F) = \epsilon(F) - \mu F\,, \qquad P_{\text{V}}(F) = -\rho_{\text{V}}(F)\,. \qquad \text{(17b)}$$

Early cosmology

Introduce dimensionless variables:

$$F = f\,F_0\,, \qquad \rho_{\mathsf{V,M}} = r_{\mathsf{V,M}}/\chi\,, \tag{18a}$$

$$\mu = u/(\chi\,F_0)\,, \qquad G^{-1}(F) = k(f)\,|F_0|\,, \tag{18b}$$

$$H = h/\sqrt{\chi\,|F_0|}\,, \qquad t = \tau\,\sqrt{\chi\,|F_0|}\,, \tag{18c}$$

using the vacuum compressibility [2]

$$\chi(F_0) \equiv \left(F^2\, d^2\epsilon(F)/dF^2\,\Big|_{F=F_0} \right)^{-1}\,. \tag{19}$$

Simple *Ansätze* suffice for the moment:

$$r_{\mathsf{V}}(f) = \frac{1}{2}\left(-f^2 + \frac{1}{3}f^4\right) - uf\,\Big|_{u=u_0=-1/3}\,, \qquad k(f) = s\,f\,, \tag{20}$$

with f taken positive ($f \sim 1$) and a dimensionless parameter $s > 0$. Note that $r_{\mathsf{V}}(1) = 0$ for equilibrium value u_0 of the chemical potential.

Early cosmology

Absorbing the numerical constant s in τ and h^{-1}, the final ODEs are:

$$\frac{dh}{d\tau} + 2h^2 = \frac{dr_\mathsf{V}}{df}\,, \tag{21a}$$

$$h\,\frac{df}{d\tau} + f h^2 = r_\mathsf{V} + r_\mathsf{M}\,, \tag{21b}$$

$$\frac{dr_\mathsf{M}}{d\tau} + 3h\left(1 + w_\mathsf{M}\right) r_\mathsf{M} = 0\,, \tag{21c}$$

with $r_\mathsf{V}(f)$ from (20) and equation-of-state parameter $w_\mathsf{M} \equiv P_\mathsf{M}/\rho_\mathsf{M}$.

ODEs solved, first, numerically and, then, analytically for $\tau \to \infty$.

Early cosmology

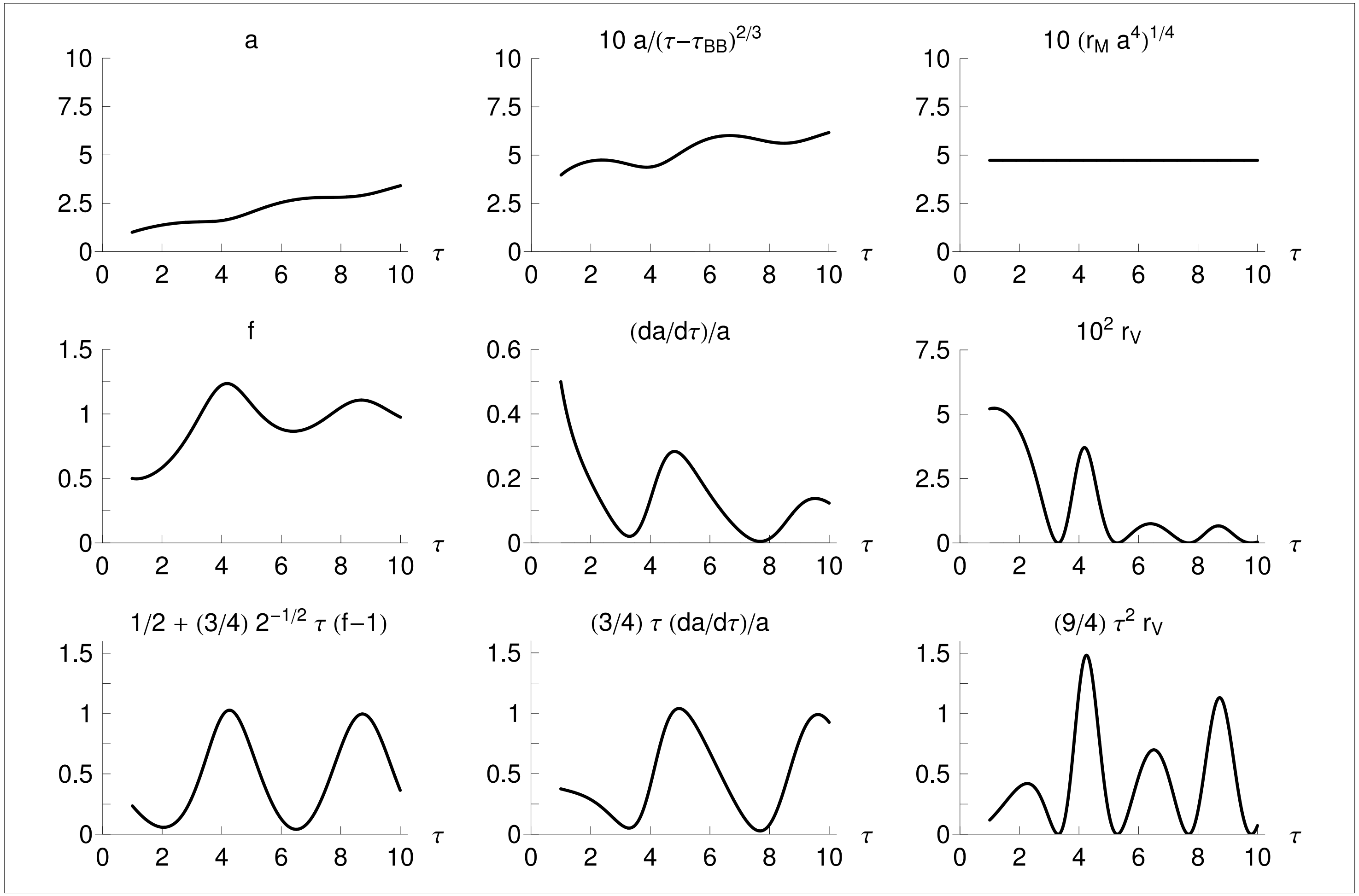


Figure 1: Flat FRW universe with ultrarelativistic matter ($w_{\mathrm{M}} = 1/3$) and boundary conditions $(a, h, f, r_{\mathrm{M}})|_{\tau=1} = (1,\ 1/2,\ 1/2,\ 1/20)$.

Early cosmology

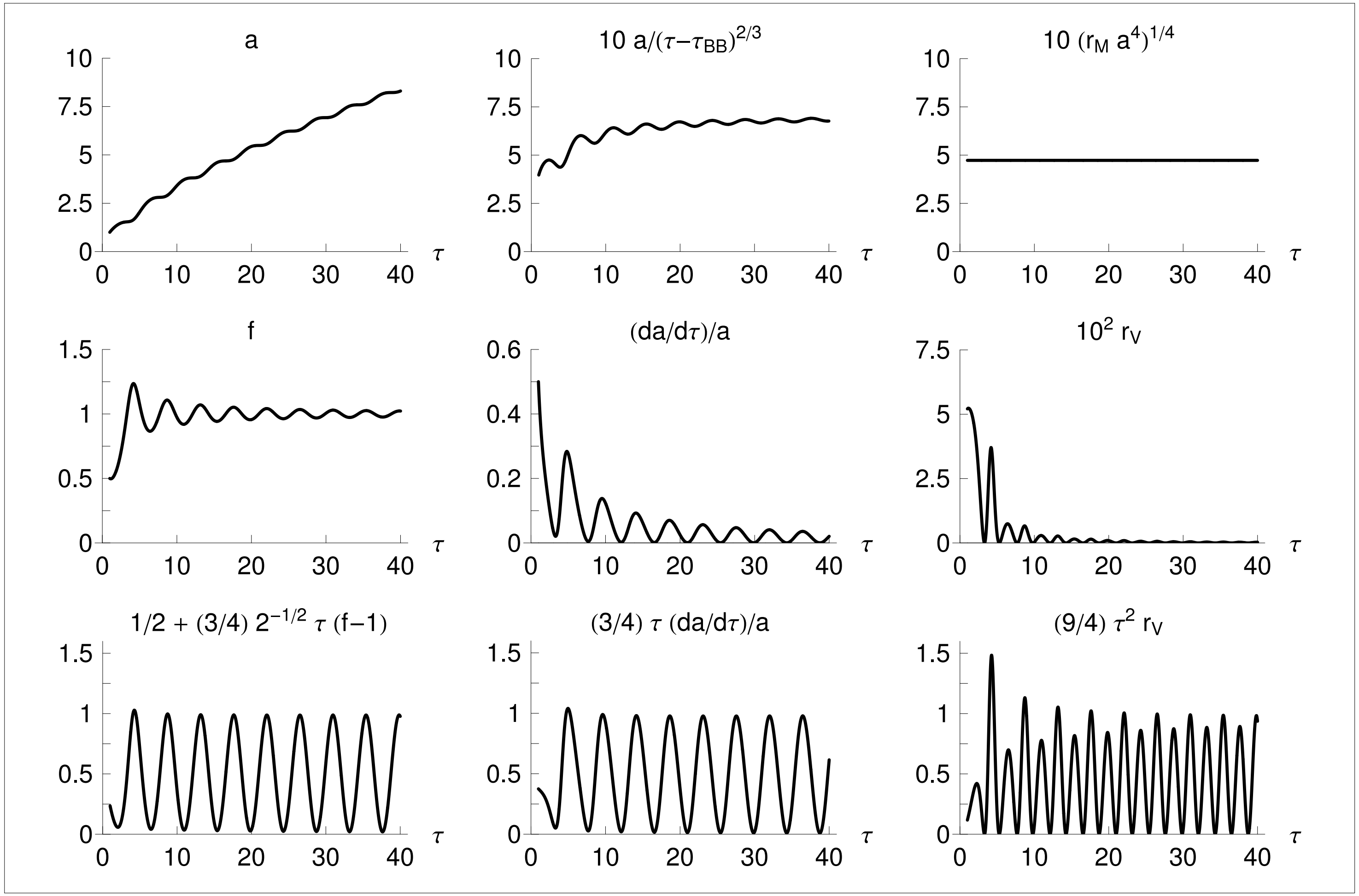


Figure 2: Same as Fig. 1 but over a longer time.

Early cosmology

Asymptotic behavior:

$$f \sim 1 + \frac{2}{3}\, d_{\mathsf{M}}\, \frac{\omega}{\tau}\, \sin\omega\tau\,, \tag{22a}$$

$$h \sim \frac{2}{3}\, \frac{1}{\tau}\, \Big(1 - d_{\mathsf{M}}\, \cos\omega\tau\Big)\,, \tag{22b}$$

$$r_{\mathsf{V}} \sim \frac{4}{9}\, d_{\mathsf{M}}^2\, \frac{1}{\tau^2}\, \sin^2\omega\tau\,, \tag{22c}$$

with $\omega = \sqrt{2}$ and damping factor

$$d_{\mathsf{M}} = 1 + \delta_{w_{\mathsf{M}},0}\, \Big(\sqrt{1 - (9/4)\, r_{\mathsf{M}\infty}} - 1\Big)\,, \tag{23a}$$

for coefficient $r_{\mathsf{M}\infty}$ of the asymptotic matter energy density $r_{\mathsf{M}} \sim r_{\mathsf{M}\infty}/\tau^2$.

Early cosmology

Effective CDM-like behavior:

Oscillating vacuum energy density $\rho_{\text{V}}(t)$ and oscillating gravitational coupling parameter $G(t)$ conspire to give the same Hubble expansion as pressureless matter (e.g., CDM) in a standard FRW universe with fixed gravitational coupling constant $G = G_{\text{N}}$.

Perhaps this type of oscillating vacuum energy density constitutes a part of the cold dark matter (CDM) in the standard FRW framework?

An outstanding task is to establish the clustering properties of the oscillating vacuum energy density.

Early cosmology

Extrapolation to large times:

For a *standard* flat FRW universe, the *total* energy density is always equal to the critical density $\rho_{\text{c}} \equiv 3H^2/(8\pi G_{\text{N}})$.

But, here, the gravitational coupling parameter is variable, $G = G(t)$, and there are rapid oscillations, so that, for example, $\langle H \rangle^2 \neq \langle H^2 \rangle$.

Result for $w_{\text{M}} > 0$:

$$\lim_{t\to\infty} \frac{\langle \rho_{\text{V}} \rangle}{3\langle H \rangle^2 / \big(8\pi \langle G \rangle\big)} = \frac{1}{2}\,, \tag{24}$$

which is of order 1 but not exactly equal to 1.

Early cosmology

Gravitational coupling $G(t)$ relaxes to an equilibrium value:

$$G^{-1}(t) \sim G_\infty^{-1} \left[1 + c_0 \, \frac{t_{\mathsf{UV}}}{t} \, \sin\left(\frac{t}{t_{\mathsf{UV}}}\right)\right] , \qquad (25)$$

with c_0 a constant of order unity,
G_∞ a gravitational constant (presumably $G_\infty \approx G_{\mathsf{N}}$), and
$t_{\mathsf{UV}} = \sqrt{\chi |F_0|/2}$ an ultraviolet timescale (presumably $t_{\mathsf{UV}} \approx 10^{-43}$ s).

Very different from previous suggestions for the dynamics of $G(t)$, including Dirac's original suggestion $G \propto 1/t$.

For the present universe and the solar system in it, this gravitational coupling parameter would have minuscule oscillations. Combined with the Planck-scale mass of the F degree of freedom, this would suggest that all solar-system experimental bounds are satisfied.

Early cosmology

Summary of the main results:

(i) a mechanism of **vacuum-energy decay**, which, starting from a "natural" Planck-scale value at very early times, leads to the correct order of magnitude for the present cosmological constant;

(ii) the realization that a substantial part of the **inferred CDM** may come from an oscillating vacuum energy density;

(iii) the important role of **oscillations of the vacuum variable F**, which drive the vacuum energy density oscillations responsible for results (i)–(ii).

Conclusion 3a: *The dynamic vacuum variable q (here, F) allows for the vacuum energy density to relax to its equilibrium value, $\rho_V = 0$, corresponding to Minkowski spacetime.*.

Late cosmology

3.2 Equilibrium boundary conditions at late times

Consider this question:

is it possible at all to relate equilibrium boundary conditions for $\rho_{\mathsf{V}}(0)$ *to an expanding universe which matches the observations, even if we are free to choose the type of vacuum-energy dynamics,* $d\rho_{\mathsf{V}}/dt \neq 0$*?*

One possible answer goes as follows. That is, in mathematical terms, we present an "existence proof" for this type of universe [3b].

Now, standard gravity with $G = G_{\mathsf{N}}$.

Late cosmology

Take a closed FRW universe with two equilibrium conditions at $t_{\text{eq}} \equiv 0$.

The first condition may be called the Friedmann condition,

$$(8\pi G_{\text{N}}/3)\left(\rho_{\text{V}}(t_{\text{eq}}) + \rho_{\text{M}}(t_{\text{eq}})\right) = a(t_{\text{eq}})^{-2}\,, \tag{26}$$

and the second a Gibbs–Duhem-type condition [Volovik, gr-qc/0405012],

$$\rho_{\text{V}}(t_{\text{eq}}) = \tfrac{1}{2}\,(1 + 3\,w_{\text{M}})\,\rho_{\text{M}}(t_{\text{eq}})\,, \tag{27}$$

for constant $w_{\text{M}} \equiv P_{\text{M}}/\rho_{\text{M}}$.

These two conditions allow for a time-independent solution of the gravitational field equations: the static Einstein universe (1917).

Late cosmology

Indeed, the Einstein equations reduce to the Friedmann equation (overdot standing for d/dt)

$$\left(\dot{a}/a\right)^2 = (8\pi G_\mathsf{N}/3)\left(\rho_\mathsf{V} + \rho_\mathsf{M}\right) - 1/a^2\,, \tag{28}$$

and the energy-conservation equation,

$$\left(\dot{\rho}_\mathsf{V} + \dot{\rho}_\mathsf{M}\right) = -3\,(\dot{a}/a)\,(1 + w_\mathsf{M})\,\rho_\mathsf{M}\,. \tag{29}$$

But, for a nonstatic solution, a nonvanishing vacuum-energy equation is needed, for which we take

$$\dot{\rho}_\mathsf{V} = \gamma\,\Gamma_\mathsf{VM}\;\rho_\mathsf{M}\,, \tag{30}$$

where Γ_VM is a new fundamental decay constant and $\gamma = \gamma[a(t)/a_\mathsf{eq}]$ a dimensionless functional with appropriate normalization.

Late cosmology

Specific *Ansatz* for this vacuum dynamics functional:

$$\gamma[\alpha(t)] = \big((c_1)^6 + 1\big)$$

$$\times \left(\frac{(1-\alpha)^6}{(c_1)^6 + (1-\alpha)^6}\, \alpha^2 \sin(c_2 \pi \alpha) + \frac{\alpha^7}{(c_1)^6 + \alpha^6} \left(\frac{(c_3)^{1/3}}{(c_3)^{1/3} + |1-\alpha|^{1/3}} \right)^4 \right), \tag{31}$$

with $\alpha(t) \equiv a(t)/a_{\text{eq}}$, coefficients c_n, and normalization $\gamma[1] = 1$.

Note that (31) is O(a^3) for $a \to 0$ and nonzero for $a/a_{\text{eq}} = 1$.

Late cosmology

Boundary conditions and constants (in units with $8\pi G_{\mathsf{N}}/3 = c = 1$):

$$\begin{pmatrix} a(0) \\ \rho_{\mathsf{M}}(0) \\ \rho_{\mathsf{V}}(0) \\ w_{\mathsf{M}} \\ \Gamma_{\mathsf{VM}} \\ c_1 \\ c_2 \\ c_3 \end{pmatrix} = \begin{pmatrix} 10 \\ 2/300 \\ 1/300 \\ 0 \\ 50 \\ 1/5 \\ 9/4 \\ 1/15 \end{pmatrix} . \tag{32}$$

Numerical solution of the three coupled ODEs (28)–(30) with these boundary conditions is given in Fig. 3.

Late cosmology

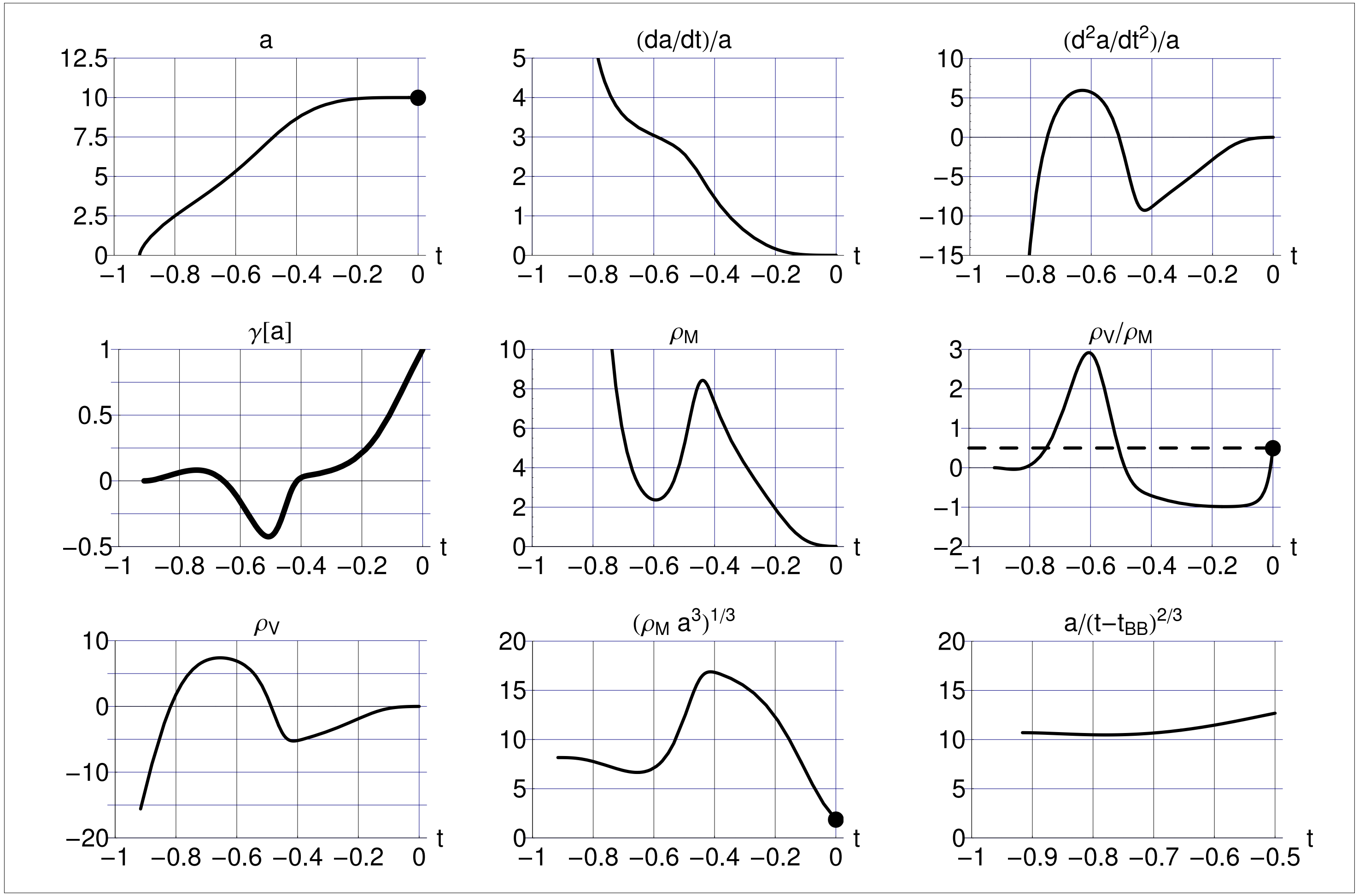


Figure 3: Closed FRW universe with pressureless matter ($w_\mathrm{M} = 0$), dynamic vacuum energy ($w_\mathrm{V} = -1$), and equilibrium boundary conditions (32).

Late cosmology

"Present universe" with density ratio $\rho_V/\rho_M \approx 2.75$ (WMAP–5yr mean value for $h = 0.70$) would approximately correspond to

$$\begin{pmatrix} t \\ t - t_{BB} \\ \dot{a}/a \\ \rho_V/\rho_M \\ \Omega_V + \Omega_M \end{pmatrix} = \begin{pmatrix} -0.584 \\ 0.332 \\ 2.985 \\ 2.750 \\ 1.004 \end{pmatrix}, \qquad (33)$$

where $\Omega_X \equiv \rho_X/(\dot{a}/a)^2$.

Identifying the calculated value $\dot{a}/a = 2.985$ with the measured value $H_0 \approx 0.70/(9.78\ \text{Gyr})$, the present age of the model universe becomes:

$$\tau_0 \approx 13.85\ (0.70/h)\ \text{Gyr}\,. \qquad (34)$$

<u>Far from trivial</u> that reasonable values for ρ_{V0}/ρ_{M0}, $\Omega_{V0} + \Omega_{M0}$, and τ_0 can be produced at all in our approach.

Late cosmology

Main features of this closed model universe:

1. **Gibbs–Duhem-type boundary condition** at $t = t_{\text{eq}}$ with $\rho_{\text{V}}(t_{\text{eq}}) = \frac{1}{2}\,\rho_{\text{M}}(t_{\text{eq}})$ for $w_{\text{M}} = 0$;
2. **finite** $|\boldsymbol{\rho_{\text{V}}(t)}|$ within a factor 10^4 from the value set at $t = t_{\text{eq}}$;
3. **Big Bang phase** with $a(t) \propto (t - t_{\text{BB}})^{2/3}$ for $w_{\text{M}} = 0$;
4. an **accelerating universe** for "present times," with $\rho_{\text{V}}/\rho_{\text{M}}$ of order 1 and an approximately flat 3–geometry.

Conclusion 3b: *An "existence proof" has been given for a universe with both equilibrium boundary conditions and a Big Bang.*

Conclusion

4. CONCLUSION

The **dynamics of the quantum vacuum** is a new topic in physics waiting for input from:

- theory (e.g., emergent-symmetry approach inspired by cond-mat);
- experiment (e.g., observational cosmology).