KA-TP-2-1995

ELECTROWEAK PRECISION OBSERVABLES - AN INDIRECT ACCESS TO THE TOP QUARK *

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ABSTRACT

The role of the top quark in the Standard Model predictions for electroweak precision observables is reviewed and the implications of the experimental data for the indirect determination of the top mass range is discussed.

1. Electroweak precision observables

The top quark completes the third generation of fermions and makes the Standard Model anomaly free. The experimental observation of the top has been announced recently by the CDF collaboration [1] with mass $m_t = 176 \pm 8 \pm 10$ GeV and by the D0 collaboration [2] with $m_t = 199^{+19}_{-21} \pm 22$ GeV.

On the other hand, indirect information about the top quark is obtained from electroweak precision observables. The possibility of performing precision tests of the electroweak theory is based on the formulation of the Standard Model as a renormalizable quantum field theory preserving its predictive power beyond tree level calculations. With the experimental accuracy in the investigation of the fermion-gauge boson interactions being sensitive to the loop induced quantum effects, especially the heavy fermion sector of the Standard Model is probed.

Before one can make predictions from the theory, a set of independent parameters has to be determined from experiment. All the practical schemes make use of the same physical input quantities $\alpha,\ G_\mu,\ M_Z,\ m_f,\ M_H$ for fixing the free parameters of the SM. In terms of these the set of precisely measureable quantities M_W and $\Gamma_Z,\ \Gamma_f, A_{FB},\ A_{LR},\ A_\tau^{pol}, \cdots$ at the Z resonance can be calculated as predictions depending on m_t and M_H, together with the strong coupling α_s.

The spectrum of the vector bosons $\gamma, W^\pm, Z$ with masses [3,4]

$$M_W = 80.23 \pm 0.18\,\mathrm{GeV}, \qquad M_Z = 91.1888 \pm 0.0044\,\mathrm{GeV} \tag{1}$$

is reconciled with the SU(2)×U(1) local gauge symmetry with the help of the Higgs mechanism. For a general structure of the scalar sector, the electroweak mixing

*talk at the 138. Heraeus Seminar on Heavy Quarks Physics, Bad Honnef, Germany, December 1994

angle is related to the vector boson masses by

$$s_\theta^2 \equiv \sin^2\theta = 1 - \frac{M_W^2}{\rho M_Z^2} = 1 - \frac{M_W^2}{M_Z^2} + \frac{M_W^2}{M_Z^2}\Delta\rho \equiv s_W^2 + c_W^2\Delta\rho \tag{2}$$

where the ρ-parameter $\rho = (1-\Delta\rho)^{-1}$ is an additional free parameter. In models with scalar doublets only, in particular in the minimal model, one has the tree level relation $\rho = 1$. Loop effects, however, induce a deviation $\Delta\rho \neq 0$.

The Standard Model prediction for $\Delta\rho$ from radiative corrections is dominated by the (t,b) doublet contribution [5], in 1-loop and neglecting m_b:

$$\Delta\rho \;=\; \frac{\Sigma^{ZZ}(0)}{M_Z^2} - \frac{\Sigma^{WW}(0)}{M_W^2} \;\simeq\; \frac{3G_\mu m_t^2}{8\pi^2\sqrt{2}} \;\equiv\; 3x_t\,. \tag{3}$$

This large contribution constitutes the leading shift for the electroweak mixing angle when inserted into Eq. (2).
Another large loop effect in the electroweak parameters is due to the fermionic content of the subtracted photon vacuum polarization [6]

$$\Delta\alpha = \Pi^\gamma_{ferm}(0) - \mathrm{Re}\,\Pi^\gamma_{ferm}(M_Z^2) = 0.0593 \pm 0.0007 \tag{4}$$

as a recent update confirming essentially the previous result [7] but with a sightly smaller error. It corresponds to a QED induced shift in the electromagnetic fine structure constant yielding an effective fine structure constant at the Z mass scale:

$$\alpha(M_Z^2) \;=\; \frac{\alpha}{1-\Delta\alpha} \;=\; \frac{1}{128.9 \pm 0.1}\,.$$

2. The vector boson masses

The correlation between the masses M_W, M_Z of the vector bosons in terms of the Fermi constant G_μ reads in 1-loop order of the Standard Model [8]:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\,\alpha}{2 s_W^2 M_W^2}\left[1 + \Delta r(\alpha, M_W, M_Z, M_H, m_t)\right]\,. \tag{5}$$

The 1-loop correction Δr can be written in the following way

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\,\Delta\rho + (\Delta r)_{remainder}\,. \tag{6}$$

in order to separate the leading fermionic contributions $\Delta\alpha$ and $\Delta\rho$. All other terms are collected in $(\Delta r)_{remainder}$, the typical size of which is of the order ~ 0.01.

The presence of large terms in Δr requires the consideration of higher than 1-loop effects. The modification of Eq. (5) according to

$$1+\Delta r \;\rightarrow\; \frac{1}{(1-\Delta\alpha)\cdot(1+\frac{c_W^2}{s_W^2}\Delta\overline{\rho}) \;-\; (\Delta r)_{remainder}} \equiv \frac{1}{1-\Delta r} \tag{7}$$

with

$$\Delta\overline{\rho} = 3\, x_t \cdot \left[1 + x_t\, \rho^{(2)}) + \delta\rho_{\,QCD}\right] \tag{8}$$

accommodates, besides the leading log resummation of $\Delta\alpha$, the resummation of the leading top contribution [9] in terms of $\Delta\overline{\rho}$ which contains also irreducible higher order parts: the electroweak 2-loop contribution $\rho^{(2)}(M_H/m_t)$ [10], and the QCD correction $\delta\rho_{\,QCD}$ up to $O(\alpha\alpha_s^2)$ [11,12]. The complete $O(\alpha\alpha_s)$ corrections to the self energies beyond the m_t^2 approximation are available from perturbative calculations [13] and by means of dispersion relations [14] (see also [15]).

The quantity Δr in Eq. (7)

$$\Delta r \;=\; 1 \;-\; \frac{\pi\alpha}{\sqrt{2}G_\mu}\, \frac{1}{M_W^2\left(1-\frac{M_W^2}{M_Z^2}\right)}\,.$$

is experimentally determined by M_Z and M_W. Theoretically, it is computed from M_Z, G_μ, α after specifying the masses M_H, m_t. The theoretical prediction for Δr is displayed in Figure 1. For comparison with data, the experimental 1σ limits from the direct measurements of M_Z at LEP and M_W in $p\bar{p}$ are indicated, fully consistent with the recent direct top mass determination and with the standard theory.

The quantity s_W^2 resp. the ratio M_W/M_Z can indirectly be measured in deep-inelastic neutrino scattering, yielding the present world average [16] consistent with the direct vector boson mass measurements:

$$s_W^2 = 0.2256 \pm 0.0047\,.$$

3. Z boson observables

Effective Z boson couplings: The predictions for the various Z widths and asymmetries can conveniently be calculated in terms of effective coupling constants. They follow from the set of 1-loop diagrams without virtual photons. These weak corrections can be expressed in terms of fermion-dependent overall normalizations ρ_f and effective mixing angles s_f^2 in the NC vertices [17]:

$$\left(\sqrt{2}G_\mu M_Z^2 \rho_f\right)^{1/2}\left[(I_3^f - 2Q_f s_f^2)\gamma_\nu - I_3^f\gamma_\nu\gamma_5\right] = \left(\sqrt{2}G_\mu M_Z^2\right)^{1/2}\,[g_V^f\,\gamma_\nu - g_A^f\,\gamma_\nu\gamma_5]\,. \tag{9}$$

ρ_f and s_f^2 contain universal parts (i.e. independent of the fermion species) and non-universal parts which explicitly depend on the type of the external fermions. In

Figure 1: Δr as a function of the top mass for $M_H = 60$ and 1000 GeV. 1σ bounds from M_Z and s_W^2: horizontal band from $p\bar{p}$, $\bullet$ from νN.

their leading terms they are given by

$$\rho_f = \frac{1}{1-\Delta\overline{\rho}} + \cdots, \quad s_f^2 = s_W^2 + c_W^2\,\Delta\overline{\rho} + \cdots \tag{10}$$

with $\Delta\overline{\rho}$ from Eq. (8).

For the b quark, also the non-universal parts have a strong dependence on m_t resulting from virtual top quarks in the vertex corrections. The difference between the d and b couplings can be parametrized in the following way

$$\rho_b = \rho_d(1+\tau)^2, \quad s_b^2 = s_d^2(1+\tau)^{-1} \tag{11}$$

with the quantity $\tau = \Delta\tau^{(1)} + \Delta\tau^{(2)} + \Delta\tau^{(\alpha_s)}$ calculated perturbatively, at the present level comprising: the complete 1-loop order term [18] $\Delta\tau^{(1)} = -2x_t + \cdots$; the electroweak 2-loop contribution of $O(G_\mu^2 m_t^4)$ [10,19]

$$\Delta\tau^{(2)} = -2\,x_t^2\,\tau^{(2)}, \tag{12}$$

where $\tau^{(2)}$ is a function of M_H/m_t with $\tau^{(2)} = 9 - \pi^2/3$ for $M_H \ll m_t$; the QCD corrections to the leading term of $O(\alpha_s G_\mu m_t^2)$ [20]

$$\Delta\tau^{(\alpha_s)} = 2\,x_t \cdot \frac{\alpha_s}{\pi} \cdot \frac{\pi^2}{3}. \tag{13}$$

Asymmetries and mixing angles: The effective mixing angles are of particular interest since they determine the on-resonance asymmetries via the combinations

$$A_f = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}\,. \tag{14}$$

Measurements of the asymmetries hence are measurements of the ratios

$$g_V^f/g_A^f = 1 - 2Q_f s_f^2 \tag{15}$$

or the effective mixing angles, respectively.

Z width and partial widths: The total Z width Γ_Z can be calculated essentially as the sum over the fermionic partial decay widths (other decay channels are not significant). Expressed in terms of the effective coupling constants they read up to 2nd order in the (light) fermion masses:

$$\Gamma_f \;=\; \Gamma_0 \left[(g_V^f)^2 + (g_A^f)^2\left(1 - \frac{6m_f^2}{M_Z^2}\right)\right]\cdot(1 + Q_f^2\frac{3\alpha}{4\pi}) \;+\; \Delta\Gamma_{QCD}^f$$

with

$$\Gamma_0 \;=\; N_C^f\,\frac{\sqrt{2}G_\mu M_Z^3}{12\pi}, \quad N_C^f = 1 \text{ (leptons)}, \;\; = 3 \text{ (quarks)}.$$

The QCD correction for the light quarks [21] with $m_q \simeq 0$ is given by

$$\Delta\Gamma_{QCD}^f \;=\; \Gamma_0\left[(g_V^f)^2 + (g_A^f)^2\right]\cdot\left[\frac{\alpha_s}{\pi} + 1.41\left(\frac{\alpha_s}{\pi}\right)^2 - 12.8\left(\frac{\alpha_s}{\pi}\right)^3\right]. \tag{16}$$

For b quarks the QCD corrections are different due to finite b mass terms and to top quark dependent 2-loop diagrams for the axial part. They are calculated up to third order in the vector and up to second order in the axial part [22].

Implications of precision data: In table 1 the Standard Model predictions for Z pole observables are put together. The first error corresponds to the variation with m_t, M_H in the range allowed by M_W and Δr (Fig. 1), the second error is the hadronic uncertainty from $\alpha_s = 0.123 \pm 0.006$ measured by QCD observables at the Z. The recent combined LEP results on the Z resonance parameters [3], under the assumption of lepton universality, are also shown in table 1, together with s_e^2 from the left-right asymmetry at the SLC [23]. The direct information on m_t is not included.

The Z observables are more constraining to the top mass than M_W, as can be seen from table 1. Assuming the validity of the Standard Model a global fit to all electroweak LEP results yields an indirect determination of the parameters m_t, α_s as follows: [3]

$$m_t = 173^{+12+18}_{-13-20}\,\text{GeV}, \quad \alpha_s = 0.126 \pm 0.005 \pm 0.002 \tag{17}$$

Table 1: LEP/SLC results and Standard Model predictions for the Z parameters.

observable	LEP/SLC 1994	Standard Model range
M_Z (GeV)	91.1888 ± 0.0044	input
Γ_Z (GeV)	2.4974 ± 0.0038	$2.4922 \pm 0.0075 \pm 0.0033$
σ_0^{had} (nb)	41.49 ± 0.12	$41.45 \pm 0.03 \pm 0.04$
Γ_{had}/Γ_e	20.795 ± 0.040	$20.772 \pm 0.028 \pm 0.038$
Γ_{inv} (MeV)	499.8 ± 3.5	500.8 ± 1.3
Γ_b/Γ_{had}	0.2202 ± 0.0020	0.2158 ± 0.0013
ρ_ℓ	1.0047 ± 0.0022	1.0038 ± 0.0026
s_ℓ^2	0.2321 ± 0.0004	0.2324 ± 0.0012
$s_e^2(A_{LR})$	0.2292 ± 0.0010 (SLC result)	0.2324 ± 0.0012

with $M_H = 300$ GeV for the central value. The second error is from the variation of M_H between 60 GeV and 1 TeV. The fit result includes the uncertainties of the Standard Model calculations to be discussed in the next section.

Including the information on neutrino scattering and M_W modifies the fit result only marginally [3]:

$$m_t = 171^{+11+18}_{-12-21}\,\mathrm{GeV}, \quad \alpha_s = 0.126 \pm 0.005 \pm 0.002\,. \tag{18}$$

Incorporating also the SLC result on A_{LR} yields [3]

$$m_t = 178^{+11+18}_{-11-19}\,\mathrm{GeV}, \quad \alpha_s = 0.125 \pm 0.005 \pm 0.002\,. \tag{19}$$

A simultaneous fit to m_t and M_H from all low and high energy data but for constrained $\alpha_s = 0.118 \pm 0.007$ yields a slightly lower range [24] $m_t = 153 \pm 15$ GeV. For larger values of α_s the result is very close to the one in Eq. (18) [24].

4. Status of the Standard Model predictions

For a discussion of the theoretical reliability of the Standard Model predictions one has to consider various sources of uncertainties:

The error of the hadronic contribution to $\alpha(M_Z^2)$, Eq. (4), leads to $\delta M_W = 13$ MeV in the W mass prediction, and $\delta \sin^2\theta = 0.0002$ common to all of the mixing angles, which matches with the future experimental precision.

The uncertainties from the QCD contributions, besides the 3 MeV in the hadronic Z width, can essentially be traced back to those in the top quark loops for the ρ-parameter. They can be combined into the following net effects [15] $\delta(\Delta\rho) \simeq 2 \cdot 10^{-4}$, $\delta s_\ell^2 \simeq 1 \cdot 10^{-4}$ for $m_t = 150$ GeV and somewhat larger for heavier top.

The size of unknown higher order contributions can be estimated by different arrangements of non-leading higher order terms and investigations of the scheme dependence. Detailed studies by use of different computer codes, based on on-shell and $\overline{MS}$ calculations, for the Z resonance observables have shown differences around 0.1%, in particular, $\delta s_\ell^2 = 1-2\cdot 10^{-4}$. A comprehensive documentation is meanwhile available [25].

5. Conclusions

The agreement of the experimental high and low energy precision data with the Standard Model predictions has shown that the Standard Model works as a fully fledged quantum field theory. A great success of the Standard Model is the directly measured top mass range which coincides in an impressive way with the indirect determination from loop effects in precision data.

The steadily increasing accuracy of the data starts to exhibit also sensitivity to the Higgs mass [3,24], although still marginally ($M_H < 1$ TeV at 95% C.L.) for the current situation.

Not understood at present are the deviations from the theoretical expectation observed in the measurement of A_{LR} and R_b, in particular if the CDF top mass range. Whether they they might be first hints for non-standard physics makes the future investigations particularly exciting.

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