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ELECTROWEAK RADIATIVE CORRECTIONS TO RESONANT CHARGED GAUGE BOSON PRODUCTION

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Abstract

The electroweak $\mathcal{O}(\alpha)$ contribution to the resonant single W production in a general 4-fermion process is discussed with particular emphasis on a gauge invariant decomposition into a QED-like and weak part. The cross section in the vicinity of the resonance can be represented in terms of a convolution of a 'hard' Breit-Wigner-cross section, comprising the (m_t, M_H)-dependent weak 1-loop corrections, with a universal radiator function. The numerical impact of the various contributions on the W line shape are discussed, together with the concepts of s-dependent and constant width approach. Analytic formulae for the W decay width are also provided including the 1-loop electroweak and QCD corrections.

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1 Introduction

Future experiments at LEP and the Tevatron will access sectors of the Minimal Standard Model (MSM) [1] yet unchallenged: the Yang-Mills structure of gauge boson self couplings and mass generation by the concept of spontaneous symmetry breaking [2]. With LEP II operating above the threshold for W pair production, for the first time a precise direct measurement of the triple gauge boson coupling $(\gamma, Z)W^+W^-$ can be performed, allowing to test the non-Abelian structure of the MSM [3]. Moreover, our current knowledge of the W boson mass (world average value [4])

$$M_W = 80.33 \pm 0.15 \text{ GeV}$$

will be improved up to an uncertainty in the range of 30-50 MeV at LEP II [5] and 20-30 MeV at the Tevatron upgrade [6].

Thus, in order to meet the precision of these future experiments the knowledge of the observed cross sections beyond leading order perturbation theory is crucial.

The W pair production cross section in the limit of stable W bosons beyond leading order is already known [7], but not sufficient at CM energies only a few W boson decay widths above the threshold. In the course of the calculation of the corrections to the realistic scenario at LEP II with the subsequent decay of the W bosons into fermions: $e^+e^- \to W^+W^- \to 4f$ the following problems arise:

1. the production and decay of W bosons in the vicinity of the threshold, where two energetically strongly varying phenomena occur: the resonant cross section at $\sqrt{s_\pm} = M_W$ ($s_\pm$: invariant masses of the outgoing fermion pairs) and its increase at the threshold $\sqrt{s} = 2M_W$;
2. the consistent treatment of unstable charged gauge bosons within perturbation theory, which involves infra-red singular interactions with real and virtual photons.

At present, there exists no complete calculation of the electroweak $\mathcal{O}(\alpha)$ contribution to the off-shell W pair production cross section: explicit results have been derived only for parts of the photonic corrections. An overview on the present knowledge of the off-shell W pair production beyond leading order and the concessions to the consistency of the theory in order to gain it is given in [7].

The idea of this paper is to contribute to the description of charged unstable gauge bosons beyond leading order perturbation theory by studying the second problem separately and discussing the electroweak $\mathcal{O}(\alpha)$ contribution to the resonant single W production in a 4-fermion process: $ii' \to W^+ \to ff'$. It appears as part of the t-channel W pair production process and its better understanding can show a way to an improved description of the off-shell W pair production. Moreover, it represents the W production process via the Drell-Yan-mechanism at the Tevatron and thus, in view of the future improved W mass measurement at hadron colliders, requires a careful treatment beyond lowest order in perturbation theory.

The discussion of the electroweak radiative corrections to the W production in the vicinity of the resonance is guided by the successful treatment of the Z line shape beyond leading order [8], which has been precisely measured at LEP I and SLC [9]. In contrary to the Z resonance the electroweak radiative corrections to the resonant W production can not be naturally subdivided into a gauge invariant photonic and non-photonic part. A separated treatment is motivated by the following reasons:

- Usually, the photon contribution depends on cuts imposed on the photon phase space and thus is dependent on the experimental setup.
- The enhancement of the fine structure constant α due to large logarithms $\log(s/m^2)$ arising in connection with infra-red (IR) and collinear singularities requires either the consideration of higher orders in perturbation theory or the performance of a suitable resummation procedure.
- The interesting model-specific contributions are contained in the non-photonic sector.

Therefore, in analogy to the description of the Z resonance, we seek a consistent gauge invariant representation of the resonant W production cross section of the inclusive process $ii' \to W^+ \to ff'X$ with X =photons as a convolution integral of the following form [10]:

$$\sigma(s) = \frac{1}{s} \int_{s_0=4m_f^2}^{s} ds' \, G(z) \, \sigma_w(s') \; . \tag{1.1}$$

The shift of the invariant mass squared $s' = zs$ of the final state fermions is due to initial state photon emission, which is described by the universal radiator function $G(z)$. The latter also takes into account the possibility of multiple soft photon emission. The model dependent 'hard' cross section $\sigma_w(s)$ has a Breit-Wigner form. In next-to-leading order perturbation theory $\sigma_w(s)$ comprises the weak (m_t, M_H)- dependent $\mathcal{O}(\alpha)$ contribution.

The paper is organised as follows:

In Sec. 3, after recalling the Born-cross section and the tree level W width (Sec. 2), we concentrate on the gauge invariant separation of the electroweak $\mathcal{O}(\alpha)$ contribution to the W production into a QED-like and (modified) weak contribution. Our starting point is a thorough perturbative treatment of the 1-loop corrections to the lowest order matrix element. For checking the cancellation of the unphysical gauge parameter dependence the calculation is performed in R_ξ-gauge. The application of the procedure developed in [11] in order to extract a gauge invariant multiplicative factor to the Born-cross section from the IR-singular photon contribution leads to QED-like form factors describing the initial, final state and interference contribution, separately U(1) gauge invariant. In the resonance region, the remaining interference term can be absorbed into a modified weak contribution, which then also factorises. After performing an equivalent discussion of the electroweak $\mathcal{O}(\alpha)$ contribution to the partial W width ($\to$ App. B), the numerator of the Breit-Wigner can be represented as a product of W partial widths describing the W production and decay, respectively. At the end of Sec. 3, after a detailed discussion of the QED-form factors and the modified weak contribution, we present the cross section including the electroweak radiative corrections to the W production in the vicinity of the resonance in terms of the convolution integral given by Eq. 1.1. After a brief summary (Sec. 4) we provide numerical results for the various contributions in Eq. 1.1 accompanied by a numerical discussion of the W decay width including 1-loop electroweak corrections and QCD corrections (Sec. 5).

In App. A, we discuss the aspect of gauge invariance in the description of an unstable charged gauge boson beyond leading order from a more fundamental point of view. The problem of a consistent description of an unstable particle together with a definition of mass and width, which meets the requirement of gauge invariance order by order in perturbation theory, already had to be solved in the context of the precision measurements at the Z resonance. There, two approaches have been discussed: the S-Matrix theory inspired ansatz and the quantum field theoretical approach, yielding a description with constant and s-dependent width, respectively. The resulting prescriptions derived for the Z resonance need to be tested with regard to consistency and applicability to the W resonance, facing the additional difficulty of having IR-singular interactions of the W boson with virtual or real photons. At the end of App. A the corresponding prescriptions for the case of a charged vector boson resonance will be provided, especially, a transformation will be derived, which connects both descriptions and enables the consideration of an s-dependent W width in Eq. 1.1 in an easy way. In the remaining appendices the explicit expressions for the electroweak $\mathcal{O}(\alpha)$ contribution to the W production and W width are provided and some details of the calculation are shown.

2 W production and W width in leading order

The decay width of a W boson into quarks or leptons in leading order perturbation theory, which is graphical represented by the decay process in Fig. 1 (with $q^2 = M_W^2$), is given by [12]

$$\begin{aligned}
\Gamma^{(0)}_{W\to ff'} &= \frac{\alpha M_W}{12 s_w^2}\, N_c^f\, |V_{ff'}|^2\, \frac{1}{M_W^2}\, \sqrt{(M_W^2-(m_f+m_{f'})^2)(M_W^2-(m_f-m_{f'})^2)} \times \\
&\quad \left[1 - \frac{m_f^2+m_{f'}^2}{2M_W^2} - \frac{(m_f^2-m_{f'}^2)^2}{2M_W^4}\right], \qquad (2.1)
\end{aligned}$$

where α and s_w denote the fine structure constant and the sine of the Weinberg-angle, respectively. The quark mixing is taken into account by the Kobayashi-Maskawa-matrix elements V_{ij} [13] with $V_{ij} = \delta_{ij}$ for leptons. N_c^f denotes the colour factor with $N_c^{f=l,q} = 1, 3$. By using the leading order relation for the Fermi-constant G_μ (measured in the μ-decay)

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2} G_\mu s_w^2} \qquad (2.2)$$

the partial W width in the limit of massless decay products turns to

$$\overline{\Gamma}^{(0)}_{W\to ff'} = \frac{\sqrt{2} G_\mu M_W^3}{12\pi}\, N_c^f\, |V_{ff'}|^2\,. \qquad (2.3)$$

This G_μ-representation has the advantage to being independent of s_w. The total width results from the summation of the partial decay widths into all fermionic final states compatible with energy momentum conservation

$$\Gamma_W^{(0)} = \sum_{(f,f')} \Gamma^{(0)}_{W\to ff'}\,. \qquad (2.4)$$

The production of a W boson in a 4-fermion process in leading order perturbation theory is graphical represented by the Feynman-diagram shown in Fig. 1. We choose the Mandelstam variables

$$\begin{aligned}
s &= q^2 = (p_f+p_{f'})^2 = (p_i+p_{i'})^2 \\
t &= (p_f-p_i)^2 = (p_{f'}-p_{i'})^2 = -\frac{s}{2}(1-\cos\theta) \\
u &= (p_f-p_{i'})^2 = (p_{f'}-p_i)^2\,. \qquad (2.5)
\end{aligned}$$

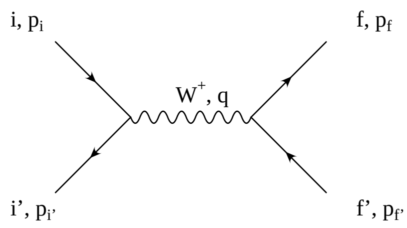


Figure 1: W production in the 4-fermion process at leading order

θ denotes the scattering angle of the outgoing fermion f with respect to $\vec{p}_i$. The differential cross section for this two-particle scattering process can be written as follows:

$$\frac{d\sigma}{dt} = \frac{1}{16\pi s^2}\overline{\sum}|\mathcal{M}|^2(s,t) \tag{2.6}$$

with the matrix element squared and averaged (summed) over the initial (and final) state spin and colour degrees of freedom. With the momentum assignment of Fig. 1 the Born-matrix element of the W production in the limit of massless external fermions yields as follows:

$$\mathcal{M}^{(0)} = i\frac{\pi\alpha}{2s_w^2}V_{ii'}V_{ff'}\ \frac{\overline{u}_f(p_f,s_f)\gamma_\mu(1-\gamma_5)v_{f'}(p_{f'},s_{f'})\overline{v}_{i'}(p_{i'},s_{i'})\gamma^\mu(1-\gamma_5)u_i(p_i,s_i)}{s-M_W^2}\ . \tag{2.7}$$

In the vicinity of the resonance the Dyson-resummed propagator has to be used (Eq. A.3), so that the differential Born-cross section of the resonant W production has Breit-Wigner-form

$$\frac{d\sigma^{(0)}(s,t)}{dt} = \frac{\pi\alpha^2}{s_w^4\, s^2}\ |V_{ii'}|^2\,|V_{ff'}|^2\,\frac{N_c^f}{N_c^i}\,[\frac{1}{2};\frac{1}{4}]\ \frac{(s+t)^2}{[(s-M_W^2)^2+M_W^2(\Gamma_W^{(0)})^2]}\ . \tag{2.8}$$

The square bracket takes into account, that for the case of incoming leptons the spin average yields only a factor 1/2, since the neutrino is a purely left-handed particle, whereas the average over quark spins leads to a factor 1/4. After performing the integration over the Mandelstam variable t ($-s < t < 0$) the total cross section of the resonant W production in leading order perturbation theory yields

$$\sigma^{(0)}(s) = \frac{\pi\alpha^2}{3\,s_w^4}\ |V_{ii'}|^2\,|V_{ff'}|^2\,\frac{N_c^f}{N_c^i}\,[\frac{1}{2};\frac{1}{4}]\ \frac{s}{[(s-M_W^2)^2+M_W^2(\Gamma_W^{(0)})^2]}\ , \tag{2.9}$$

which in G_μ-representation is given by

$$\overline{\sigma}^{(0)}(s) = \frac{2G_\mu^2\,M_W^4}{3\pi}\ |V_{ii'}|^2\,|V_{ff'}|^2\,\frac{N_c^f}{N_c^i}\,[\frac{1}{2};\frac{1}{4}]\ \frac{s}{[(s-M_W^2)^2+M_W^2(\overline{\Gamma}_W^{(0)})^2]}\ . \tag{2.10}$$

3 Electroweak radiative corrections in $\mathcal{O}(\alpha)$ to the W production

As motivated in the introduction, our aim is to provide a consistent description of the W resonance beyond lowest order perturbation theory in form of a convolution integral given by

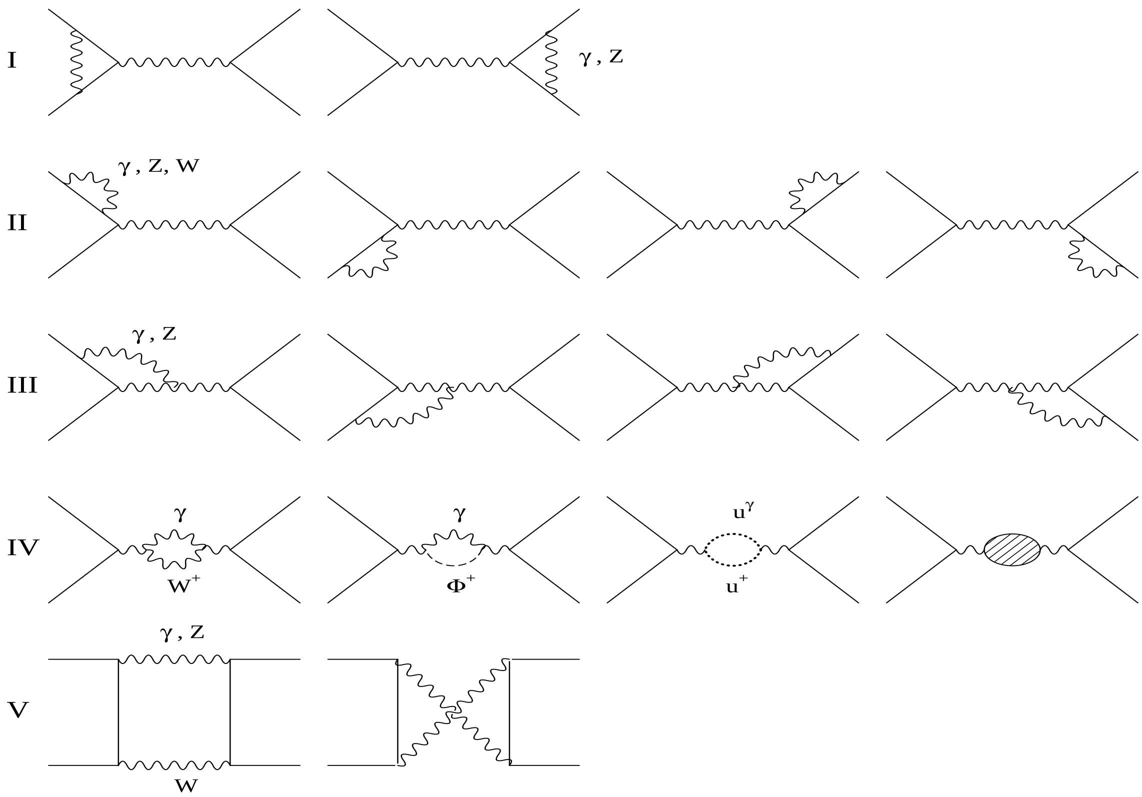


Figure 2: 1-loop corrections to the W production in the 4-fermion process (Φ^+: Higgs-ghost, u^+, u^γ: Faddeev-Popov-ghosts; the non-photonic contribution to the W self energy is symbolised by the shaded loop; an explicit representation can be found in [22], e.g.)

Eq. 1.1. To this end, a gauge invariant separation of the electroweak radiative corrections under consideration into a QED-like and weak contribution is required.

The starting point is a perturbative treatment of the W production in the 4-fermion process in $\mathcal{O}(\alpha^3)$. The electroweak $\mathcal{O}(\alpha)$ contributions under consideration are schematically represented by the Feynman-diagrams depicted in Fig. 2 and Fig. 3. The virtual electroweak contribution, shown in Fig. 2, consists of vertex corrections due to photon and Z boson exchange (diagram I,III), self energy insertions to the external fermions (diagram II), the WZ and $W\gamma$ box diagrams (diagram V) and the W self energy contribution (diagram IV). Since the calculation is performed in R_ξ-gauge, the latter also involves Higgs- and Faddeev-Popov-ghosts. After renormalisation (here we work in the on-shell scheme [14]) the virtual contribution can be described by means of a gauge parameter ($\xi_i, i = \gamma, Z, W$) independent, UV-finite, but IR-singular, form factor $\hat{F}_{virt.}(s,t)$ ($\hat{}$ denotes renormalised quantities) multiplying the Born-cross section given by Eq. 2.8. When taking into account the real soft photon emission (photon momentum $|\vec{k}| < \Delta E \ll \sqrt{s}$), shown in Fig. 3, which can also be done in form of a multiplicative IR-singular factor $F^s_{BR}(s,t)$, the IR-singularities cancel as expected [15]. Finally, the W production in $\mathcal{O}(\alpha^3)$ in a 4-fermion process can be described by

$$\frac{d\sigma^{(0+1)}(s,t)}{dt} = \frac{d\sigma^{(0)}(s,t)}{dt} \left[1 + 2\,\mathcal{R}e\,\hat{F}_{virt.}(s,t) + F^s_{BR}(s,t)\right], \tag{3.1}$$

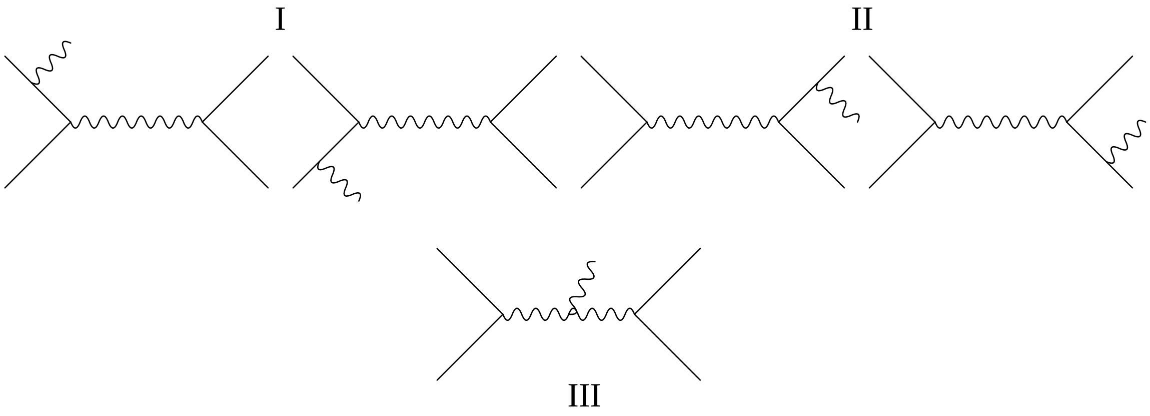


Figure 3: Real photon contribution in $\mathcal{O}(\alpha)$ to the W production in the 4-fermion process

where the explicit expressions for the contributions to $\hat{F}_{virt.}(s,t)$ and $F^s_{BR}(s,t)$ of Eq. 3.2 and 3.21, resp., are provided in App. D. For the special choice $\xi_i = 1$ the electroweak 1-loop corrections described by $\hat{F}_{virt.}(s,t)$ can also be found in [14]. The remaining photon phase space integration over the hard photon region is done in App. E.

In the following, we concentrate on the virtual electroweak contribution and discuss the photon contribution F_γ separately from the non-photonic pure weak contribution F_{weak}

$$\hat{F}_{virt.}(s,t) = (F_\gamma + F_{weak})(s,t) \, . \tag{3.2}$$

The virtual photon contribution comprises all Feynman-diagrams in Fig. 2 involving a photon, where the photonic correction to the W self energy is explicitly represented by the first three diagrams of the subset IV. In contrary to the Z production, these Feynman-diagrams do not build a gauge invariant subset and thus $F_\gamma(s,t)$ and $F_{weak}(s,t)$ are UV-divergent and gauge parameter dependent.

Since, finally, we are only interested in the cross section in the vicinity of the W resonance, we have a closer look on the resonance structure of the different contributions to the virtual corrections depicted in Fig. 2. It turns out, that the WZ-box diagrams can be neglected as a non-resonant contribution of higher order, so that in the vicinity of the W resonance the pure weak contribution in next-to-leading order evaluated at $s = M_W^2$

$$F_{weak}(M_W^2) = (F^i_{weak} + F^f_{weak})(M_W^2) \, . \tag{3.3}$$

is determined according to the prescription given in App. A (Eq. A.13). The resulting form factors $F^{(i;f)}_{weak}(M_W^2)$ describe the non-photonic 1-loop corrections to the W production and decay, respectively, and are explicitly given by Eq. D.27.

Far more involved is the calculation of the photonic form factor $F_\gamma(s,t)$: the non-factorisable $W\gamma$-box diagram is a resonant contribution and has to be considered at the required level of accuracy, the arising IR-singularities have to cancel and logarithms of the form $\log(s - M_W^2)$, which diverge for $s \to M_W^2$ (on-shell singularities), needed to be regularised in a gauge invariant way, when approaching the resonance region. In order to obtain a separation of the 1-loop corrections into a QED-like and weak contribution, we first extract gauge invariant form factors, so-called YFS-form factors $\tilde{F}^a_{YFS}(s)$, from the IR-singular Feynman-diagrams I,II and V (Fig. 2), so that the virtual photon contribution can be written as follows:

$$F_\gamma(s,t) = \sum_{\substack{a=initial,final,\\ interf.}} \tilde{F}^a_{YFS}(s) + F_\gamma^{finite}(s,t)\ . \tag{3.4}$$

These YFS-form factors together with the real photon contribution build IR-finite gauge invariant form factors $F^a_{QED}(s,t)$, which are independent from the internal structure of the W production and thus can be interpreted as a QED-like correction. For that, the bremsstrahlung contribution, shown in Fig. 3, needs also to be represented by a separately conserved initial and final state current, which cannot be easily obtained due to the γW^+W^--coupling in diagram III. The sum of the remaining IR-finite contribution $F_\gamma^{finite}(s,t)$, a part of the QED-form factor describing the interference of initial and final state bremsstrahlung $F_{QED}^{interf.}$ and the pure weak part $F^{i,f}_{weak}$ represents a form factor $\tilde{F}^{i,f}_{weak}$, which is independent of the external fermions and thus can be interpreted as a modified weak contribution. For the sake of clearness, the characteristics of the electroweak corrections in $\mathcal{O}(\alpha)$ are summarised and the different steps, which lead to a description of the W resonance given by Eq. 1.1, are schematically presented in Tab. 1.

<table>
<tr><td>$F^s_{BR}(s,t)$</td><td colspan="4">$\hat{F}_{virt.}(s,t)$</td></tr>
<tr><td>$F^s_{BR}(s,t)$</td><td colspan="3">$F_\gamma(s,t)$</td><td>$F_{weak}(M_W^2)$</td></tr>
<tr><td>Fig. 3: I, II, III</td><td colspan="2">Fig. 2: $\underbrace{I\ II}_{UV,\xi_i,IR}$ $\underbrace{V}_{\xi_i,IR,os}$</td><td>$\underbrace{III\ IV}_{UV,\xi_i,os}$</td><td>$\underbrace{I\ldots IV}_{UV,\xi_i}$</td></tr>
<tr><td>$\underbrace{F_{BR}^{initial}(s)}_{IR,os} + \underbrace{F_{BR}^{final}(s)}_{IR}$
$+\underbrace{F_{BR}^{interf.}(s,t)}_{IR,os}$</td><td>$\underbrace{\tilde{F}_{YFS}^{initial}(s) + \tilde{F}_{YFS}^{final}(s)}_{IR}$
$+\underbrace{\tilde{F}_{YFS}^{interf.}(s,t)}_{IR,os}$</td><td>$\underbrace{F_{I,II,V}^{finite}(s,t)}_{UV,\xi_i,(subtr.)}$</td><td>$\underbrace{F^\gamma_{III,IV}(s)}_{UV,\xi_i,(subtr.)}$</td><td>$\underbrace{F^{i,f}_{weak}(M_W^2)}_{UV,\xi_i}$</td></tr>
<tr><td colspan="2">$(\underbrace{F_{QED}^{initial}}_{os} + F_{QED}^{final})(s)$ $+$ $\underbrace{F_{QED}^{interf.}(s,t)}_{F_{QED}^{interf.}\vert_{log.}+\delta_{v+s}^{interf.}}$</td><td colspan="2">$\underbrace{F_\gamma^{finite}(s,t)}_{UV,\xi_i}$</td><td>$\underbrace{F^{i,f}_{weak}(M_W^2)}_{UV,\xi_i}$</td></tr>
<tr><td>$(\underbrace{F_{QED}^{initial}}_{os} + F_{QED}^{final})(s)$
$+F_{QED}^{interf.}\vert_{log.}(s,t)$</td><td colspan="4">$(\tilde{F}^i_{weak} + \tilde{F}^f_{weak})(M_W^2)$</td></tr>
</table>

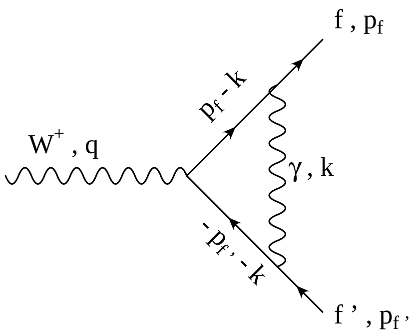
Table 1: Scheme to the extraction of a QED-form factor to the W production (UV, ξ_i, IR, os denote the UV-divergence, ξ_i-dependence, IR-singularity and on-shell singularity, resp.; ($subtr.$) is refered to a prescription concerning the on-shell singularities, which will be given in detail in Sec. 3.1

In the following, this briefly outlined method to find a gauge invariant separation into QED-like and weak part, where even the $\mathcal{O}(\alpha)$ contribution to the W production and decay process are separately represented by gauge invariant form factors, is going to be performed in detail.

3.1 The definition of a QED-form factor to the W production

In the context of a general treatment of IR-singularities occurring in QED, Yennie, Frautschi and Suura (YFS) [11] gave a prescription how to separate these singularities as a multiplicative gauge invariant factor to the Born-cross section. The basis of the perturbative treatment à la YFS is the observation that the singularities arise only in connection with soft photons emitted by external particles. The cross section of this soft photon radiation (virtual or real) can be described as the Born-cross section and factors, which only depend on the four momenta of the external particles and not on the internal structure of the process under consideration. This enables the treatment of soft photon radiation, especially the demonstration of the cancellation of the IR-singularities, to all orders in perturbation theory. In the following, the YFS-method will be applied to the photonic 1-loop contributions to the W production and later on also to the W width. By the example of the photon exchange between the final state fermions the extraction of the YFS-form factor $F_{YFS}(s,t)$ from the diagrams I,II and V in Fig. 2 will be illustrated. The IR- and UV-singularities arising in the course of the calculation are made mathematically well-defined by introducing a fictitious photon mass λ and by dimensional regularisation [16], respectively. The external fermions are considered in the massless approximation unless they occur in singular logarithms of the form $\log(s/m^2)$, where a finite fermion mass has been retained. The explicit expressions for the IR-singular and -finite parts of the diagrams under consideration can be found in App. D.1.

The application of the Feynman-rules of the electroweak MSM leads to the following expression for the photonic final state correction described by diagram I:



$$: i\Lambda_\mu^{I,f} = ig_w\gamma_\mu(1-\gamma_5)\,[F_{I,f}^{IR}(s)+F_{I,f}^{finite}(s)] \qquad \left[g_w=\frac{e}{2\sqrt{2}s_w}\right] \tag{3.5}$$

$$i\Lambda_\mu^{I,f}|_{\xi_i=1} = (-g_w)\,4\pi\alpha\,Q_fQ_{f'}\int_D \frac{\gamma_\alpha[\not p_f-\not k]\gamma_\mu(1-\gamma_5)[\not p_{f'}+\not k]\gamma^\alpha}{\underbrace{[k^2-\lambda^2]}_{D_\lambda}\underbrace{[k^2-2(kp_f)]}_{D_f}\underbrace{[k^2+2(kp_{f'})]}_{D_{f'}}}\,. \tag{3.6}$$

Following the prescription given by YFS, the numerator of the IR-singular Feynman-integral in Eq. 3.6 sandwiched in between the spinors describing the final state fermions can be written as follows:

$$\begin{aligned}\text{numerator} &= \overline{u}(p_f)\gamma_\alpha[\not p_f-\not k]\gamma_\mu(1-\gamma_5)[\not p_{f'}+\not k]\gamma^\alpha v(p_{f'})\\ &= \overline{u}(p_f)[2p_{f\alpha}-\gamma_\alpha\not k]\gamma_\mu(1-\gamma_5)[2p^\alpha_{f'}+\not k\gamma^\alpha]v(p_{f'})\\ &= \overline{u}(p_f)\gamma_\mu(1-\gamma_5)v(p_{f'})(2p_f-k)(2p_{f'}+k)+\text{terms}\propto\sigma^{\beta\alpha}k_\beta\,,\end{aligned} \tag{3.7}$$

where the following relations have been used:

$$\not{k}\gamma_\mu = k_\mu I + \frac{1}{2}[\not{k},\gamma_\mu] = k_\mu I - i\sigma_{\nu\mu}k^\nu$$

and

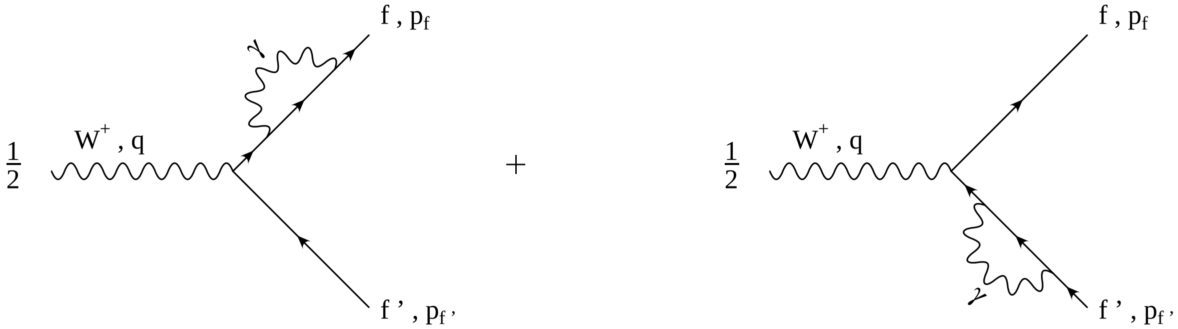
$$\overline{u}(p_f)\ \not{p}_f = m_f\overline{u}(p_f) = 0\ ,\ \ \not{p}_{f'}v(p_{f'}) = -m_{f'}v(p_{f'}) = 0\ .$$

The first term in Eq. 3.7 leads to the IR-singular contribution of diagram I, which will be part of the YFS-form factor

$$F^{IR}_{I,f}(s) = (i4\pi\alpha)Q_fQ_{f'}\int_D \frac{(2p_f-k)(2p_{f'}+k)}{D_\lambda D_f D_{f'}}\ , \tag{3.8}$$

whereas the IR-finite 'magnetic' part contributes to $F^{finite}_\gamma(s,t)$ in Eq. 3.4.

The application of this procedure to the photonic self energy insertions to the external final state fermions and to the photonic box diagrams leads to the following IR-singular form factors:

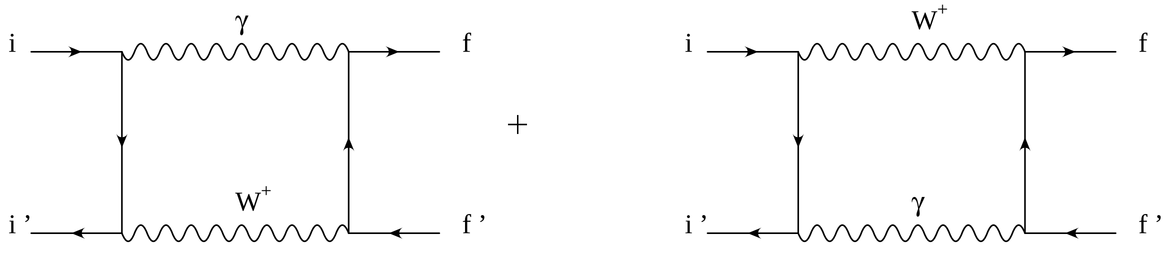


$$:\ i\Lambda^{II,f}_\mu = ig_w\gamma_\mu(1-\gamma_5)\ [F^{IR}_{II,f}(s) + F^{finite}_{II,f}(s)] \tag{3.9}$$

with

$$F^{IR}_{II,f}(s) = \frac{1}{2}(i4\pi\alpha)\left\{Q_f^2\int_D \frac{(2p_f-k)^2}{D_\lambda D_f^2} + Q_{f'}^2\int_D \frac{(2p_{f'}+k)^2}{D_\lambda D_{f'}^2}\right\} \tag{3.10}$$

and



$$:\ iB^t(s,t) = i\mathcal{M}^{(0)}(s)\ [F^{IR}_{V,t} + F^{finite}_{V,t}](s,t) \tag{3.11}$$

with

$$F^{IR}_{V,t}(s,t) = -(i4\pi\alpha)\left\{Q_iQ_f\int_D \frac{(2p_f-k)(2p_i-k)}{D_\lambda D_f D_i} + Q_{i'}Q_{f'}\int_D \frac{(2p_{f'}+k)(2p_{i'}+k)}{D_\lambda D_{f'} D_{i'}}\right\}\ . \tag{3.12}$$

The form factors describing the initial state vertex corrections $F^{IR,finite}_{(I,II),i}(s)$ can be derived from the final state ones by the substitution

$$(Q_f, Q_{f'}, m_f, m_{f'}) \to (Q_i, Q_{i'}, m_i, m_{i'})\ , \tag{3.13}$$

which in the following will be abbreviated by $(f,f') \to (i,i')$. The u-channel form factors $F_{V,u}^{IR,finite}(s,u)$ describing the crossed box diagrams in Fig. 2 follow from $F_{V,t}^{IR,finite}(s,t)$ by the substitutions

$$(i,f),(i',f') \to (i,f'),(i',f) \text{ and } t \to u \tag{3.14}$$

and, additionally, by multiplying with a global minus sign. The Born-matrix element $\mathcal{M}^{(0)}$ is given by Eq. 2.7.

These IR-singular form factors are extracted from the virtual photon contribution in such a way, that their sum has a structure similar to that of the amplitude describing real (soft) photon radiation

$$F_{YFS}(s,t) = \frac{1}{2}(i4\pi\alpha)\int_D \frac{1}{D_\lambda}\times \left[\overbrace{\frac{Q_i(2p_i-k)_\rho}{k^2-2kp_i} + \frac{Q_{i'}(2p_{i'}+k)_\rho}{k^2+2kp_{i'}} - \frac{Q_f(2p_f-k)_\rho}{k^2-2kp_f} - \frac{Q_{f'}(2p_{f'}+k)_\rho}{k^2+2kp_{f'}}}^{k^\rho \mathcal{J}_\rho=0}\right]^2 . \tag{3.15}$$

Thus, the $U(1)$-gauge invariance of the YFS-form factor is guaranteed by the existence of a conserved current. The initial and final state contribution to the YFS-form factor, however, distinguished by the corresponding charge quantum numbers ($Q_i, Q_{i'}$ and $Q_f, Q_{f'}$) are *not* separately gauge invariant. Therefore, a 'zero' will be added, so that the YFS-form factor can be written as a sum of two separately conserved $U(1)$-currents, which describe the virtual photonic correction to the W production and decay process, respectively.

$$\begin{aligned} F_{YFS}(s,t) &= \frac{1}{2}(i4\pi\alpha)\int_D \frac{1}{D_\lambda}\times \\ &\left[\overbrace{\frac{Q_i(2p_i-k)_\rho}{D_i} + \frac{Q_{i'}(2p_{i'}+k)_\rho}{D_{i'}} - \frac{1}{2}\frac{(Q_i-Q_{i'})(2q-k)_\rho}{(k^2-2kq)} + \frac{1}{2}\frac{(Q_i-Q_{i'})(2q+k)_\rho}{(k^2+2kq)}}^{k^\rho \mathcal{J}_\rho^{initial}=0}\right. \\ &+ \left.\overbrace{\frac{1}{2}\frac{(Q_f-Q_{f'})(2q-k)_\rho}{(k^2-2kq)} - \frac{1}{2}\frac{(Q_f-Q_{f'})(2q+k)_\rho}{(k^2+2kq)} - \frac{Q_f(2p_f-k)_\rho}{D_f} - \frac{Q_{f'}(2p_{f'}+k)_\rho}{D_{f'}}}^{k^\rho \mathcal{J}_\rho^{final}=0}\right]^2 \\ &=: F_{YFS}^{initial}(s) + F_{YFS}^{final}(s) + F_{YFS}^{interf.}(s,t) . \end{aligned} \tag{3.16}$$

This, at the first sight, arbitrary extension will receive its justification from the structure of the real photon contribution and its interpretation in the course of the corresponding discussion of the photon contribution to the W width (App. B). The explicit expressions for the gauge invariant form factors after the evaluation of the loop integral in Eq. 3.16 can be found in App. D.1. Before we deal with the real photon contribution a closer inspection of the occurring mass singularities $\log(s/m^2)$ and logarithms of the form $\log(s-M_W^2)$ is needed. Since the occurrence of those singularities is a pure QED phenomenon, they build a gauge invariant subset

$$[F_{YFS}^{initial,final,interf.}+F_{I,(f,i)}^{finite}+F_{V,(t,u)}^{finite}+F_{III}]_{mass-sing.}=\frac{\alpha}{4\pi}\sum_{k=i,i',f,f'}Q_k^2\frac{1}{2}\log\left(\frac{s}{m_k^2}\right) \qquad (3.17)$$

and ($\beta_{int.}$ of Eq. 3.37)

$$[F_{V,(t,u)}^{finite}+F_{III}+F_{IV}]_{on-shell-sing.}=\frac{1}{2}\,\beta_{int.}(s,t)\ \log\left(\frac{M_W^2}{|s-M_W^2|}\right)\ , \qquad (3.18)$$

which can be assigned to the initial state, final state and interference YFS-form factors according to their structure and under the maintenance of gauge invariance. It has to be mentioned, that the sum of the IR-finite photon contributions which are not included in the YFS-form factors develops a further QED-specific term

$$\frac{\alpha}{4\pi}\sum_{k=i,i',f,f'}Q_k^2\ ,$$

which thus can be absorbed in a modified YFS-form factor, as well. Finally, the resulting modified YFS-form factors in Eq. 3.4 are connected to the original ones (Eq. 3.16) as follows:

$$\begin{aligned}\tilde{F}_{YFS}^{(initial;final)} &= F_{YFS}^{(initial;final)}-[F_{YFS}^{(initial,final)}]_{mass-sing.}\\ &+\frac{\alpha}{4\pi}\sum_{k=(i,i');(f,f')}Q_k^2\left[\frac{1}{2}\log\left(\frac{s}{m_k^2}\right)-1\right]\\ \tilde{F}_{YFS}^{interf.} &= F_{YFS}^{interf.}-[F_{YFS}^{interf.}]_{mass-sing.}+\frac{1}{2}\,\beta_{int.}(s,t)\ \log\left(\frac{M_W^2}{|s-M_W^2|}\right)\ . \qquad (3.19)\end{aligned}$$

It is this modification which guarantees, that the inclusive cross section including the hard final state photons fulfills the KLN-theorem [17] and that the occurrence of the on-shell singularities is restricted to the initial state contribution.

The last step to extract a QED-like form factor from the electroweak radiative corrections to the W production is to find a gauge invariant separation of the real photon radiation into initial and final state contribution. It turns out, that diagram III in Fig. 3 can be divided into one part, which develops the propagator structure of a initial state contribution and another one, which can be assigned to the final state [18]

$$\begin{aligned}\text{diagram III}\quad &: \quad \frac{1}{[q^2-M_W^2][(q-k)^2-M_W^2]}\\ &= \underbrace{\frac{1}{[(q-k)^2-M_W^2][2kq]}}_{\leftarrow\text{initial state}}-\underbrace{\frac{1}{[q^2-M_W^2][2kq]}}_{\rightarrow\text{final state}}\ . \qquad (3.20)\end{aligned}$$

Using this separation the contribution of the real soft photons shown in Fig. 3 can be described by a multiplicative factor being composed of separately conserved initial and final state $U(1)$-currents

$$F_{BR}^s(s,t)\ =\ (-4\pi\alpha)\int_{|\vec{k}|<\Delta E}\frac{d^3k}{2(2\pi)^3k^0}\overbrace{\left|\frac{s-M_W^2}{(s-M_W^2-2kq)}\left[\frac{Q_ip_i^\rho}{kp_i}-\frac{Q_{i'}p_{i'}^\rho}{kp_{i'}}-\frac{(Q_i-Q_{i'})q^\rho}{kq}\right]\right.}^{k_\rho\mathcal{J}^\rho_{initial}=0}$$

$$
+ \quad \overbrace{\frac{(Q_f - Q_{f'})q^\rho}{kq} - \frac{Q_f p_f^\rho}{kp_f} + \frac{Q_{f'} p_{f'}^\rho}{kp_{f'}}}^{k_\rho \mathcal{J}^\rho_{final}=0}\Bigg|^2
$$

$$
=: \quad F_{BR}^{initial}(s) + F_{BR}^{final}(s) + F_{BR}^{interf.}(s,t)\ . \tag{3.21}
$$

There the impact of a photon radiated by a initial state fermion to the W propagator has been taken into account, as well. The explicit expression for the gauge invariant form factors $F_{BR}^a(s,t)$ after performing the photon phase space integration in the soft photon limit can be found in App. D.3.

Finally, the QED-like form factors of the W production, which correspond to the QED-form factors describing the next-to-leading order photonic corrections to the Z production, are determined by the YFS- and bremsstrahlung form factors derived above as follows:

$$
F_{QED}^a = 2\, \mathcal{R}e\ \tilde{F}_{YFS}^a + F_{BR}^a \quad \text{with} \quad \text{a} = \text{initial, final, interf.}\,. \tag{3.22}
$$

Up to now, we only considered the radiation of soft photons, since they develop IR-singularities, which have to be cancelled in the sum of the real and virtual contribution. In the following, it will be shown that in Eq. 3.22 this cancellation works. Moreover, the radiation of hard photons will be considered by performing the integration over the remaining photon phase space: $k^0_{min} = \Delta E$ up to $k^0_{max} = M_W/2$ as it is described in App. E. Since we are interested in the cross section of the W production in the vicinity of the resonance, those terms, which would vanish for $s \to M_W^2$, have been neglected. Furthermore, the W width will be introduced in order to cope with the arising on-shell singular logarithms by the replacement

$$
s - M_W^2 \xrightarrow{R} \Delta_W = s - M_W^2 + i M_W \Gamma_W^{(0+1)}\ ,
$$

which can be done without spoiling the $U(1)$-current conservation as it can easily verified by Eq. 3.21. The replacement of $\sigma^{(0)}(s)$ with $\tilde{\sigma}^{(0)}(s)$ (Eq. 2.9 with $\Gamma_W^{(0)} \to \Gamma_W^{(0+1)}$) in the vicinity of the resonance follows the prescription developed in App. A.

The initial state QED-form factor:

The gauge invariant QED-like contribution to the total cross section in $\mathcal{O}(\alpha^3)$ in the vicinity of the W resonance, which has been extracted from the virtual and real (soft) photonic initial state correction to the W production in the 4-fermion process, yields (Eqs. D.43,D.10 with Eq. 3.13 and $Q_i - Q_{i'} = 1$)

$$
\begin{aligned}
\sigma_{i,v+s}^{(0+1)}(s) &= \tilde{\sigma}^{(0)}(s)\ (1 + F_{QED}^{initial}(s)) \\
&= \tilde{\sigma}^{(0)}(s)\ \left\{1 + \beta_i(s)\left[\log\left(\frac{2\Delta E}{\sqrt{s}}\left|\frac{\Delta_W}{\Delta_W - 2\sqrt{s}\Delta E}\right|\right) + \delta_p(s)\right] + 2\delta^i_{v+s}(s)\right\}
\end{aligned} \tag{3.23}
$$

with

$$
\beta_i(s) = \frac{\alpha}{\pi}\left[Q_i^2\left(\log\left(\frac{s}{m_i^2}\right) - 1\right) + Q_{i'}^2\left(\log\left(\frac{s}{m_{i'}^2}\right) - 1\right) - 1\right]\ , \tag{3.24}
$$

$$\delta^i_{v+s}(s) \;=\; \frac{\alpha}{4\pi}\left\{Q_i^2\left[\frac{3}{2}\log\left(\frac{s}{m_i^2}\right)+\frac{\pi^2}{3}-2\right]+Q_{i'}^2[i\to i']+3+\frac{\pi^2}{12}\right\} \tag{3.25}$$

and the phase-shift of the resonance

$$\delta_p(s)=\frac{(s-M_W^2)}{M_W\Gamma_W^{(0+1)}}\left[\arctan\left(\frac{s-M_W^2}{M_W\Gamma_W^{(0+1)}}\right)+\arctan\left(\frac{2\sqrt{s}\Delta E-s+M_W^2}{M_W\Gamma_W^{(0+1)}}\right)\right] . \tag{3.26}$$

This represents the main contribution to the entire electroweak 1-loop corrections due to the occurrence of large logarithms, for instance $\log\left(\frac{s}{m_e^2}\right)\approx 24$ for $s=M_W^2$. In the case of the Z resonance a procedure has been developed how to cope with those large contributions [10]. The achieved description of the initial state photon contribution by the QED-form factor given by Eq. 3.23 enables now its application also to the W resonance. For that purpose, the phase space integration over the hard photons will be rewritten in accordance with Eq. E.12 by using $z=1-k=1-\frac{2k^0}{q^0}$ as follows:

$$\begin{aligned}
\sigma^{(1)}_{i,hard}(s) \;&=\; \tilde\sigma^{(0)}(s)\int_\epsilon^1 dk\,\frac{|\Delta_W|^2(1-k)}{|\Delta_W-sk|^2}\,\left\{\beta_i(s)\frac{1}{k}+\frac{1}{2}\beta_i(s)\,(k-2)+\frac{\alpha}{\pi}\frac{k}{6}\right\}\\
&=\; \int_0^1 dz\,\theta(1-z-\epsilon)\,\tilde\sigma^{(0)}(sz)\left\{\frac{\beta_i(s)}{1-z}+\tilde\delta_h(s)\right\}
\end{aligned} \tag{3.27}$$

with $\epsilon=\frac{2\Delta E}{\sqrt{s}}$ and $\tilde\delta_h$ is given by

$$\tilde\delta_h(s)=\frac{\alpha}{\pi}\frac{1-z}{6}-\frac{1}{2}\,(1+z)\,\beta_i(s)\ . \tag{3.28}$$

As it can easily be verified, the term $\propto 1/(1-z)$ of Eq. 3.27 cancels the ΔE-dependence of the soft QED-form factor. Thus, the cut-off parameter ΔE can be chosen to be so small that it can be neglected in Eq. 3.23 as compared to the W width. As a consequence, the initial state bremsstrahlung to the W resonance can also be written in form of a convolution integral

$$\begin{aligned}
\sigma^{(0+1)}_{i,s+h}(s) \;&=\; \sigma^{(0+1)}_{i,v+s}+\sigma^{(1)}_{i,hard}\\
&=\; \int_0^1 dz\,G^{(0+1)}(z)\,\tilde\sigma^{(0)}(sz)
\end{aligned} \tag{3.29}$$

with the radiator function at 1-loop level

$$\begin{aligned}
G^{(0+1)}(z) \;&=\; \delta(1-z)\\
&+\; \delta(1-z)\left[\beta_i(s)\,\log(\epsilon)+2\delta^i_{v+s}(s)\right]\\
&+\; \theta(1-z-\epsilon)\,\left[\frac{\beta_i(s)}{1-z}+\tilde\delta_h(s)\right]\ .
\end{aligned} \tag{3.30}$$

This representation enables the consideration of the remaining electroweak 1-loop corrections and the effect of an s-dependent width in a simple way. After performing the summation of the logarithms connected to the soft photons to all orders in perturbation theory (*soft photon exponentiation*) the convolution integral in Eq. 3.29 reads as follows:

$$\sigma_{i,exp.}(s)=\int_0^1 dz\,G(z)\,\tilde\sigma^{(0)}(sz)\ , \tag{3.31}$$

with the radiator function in the exponentiated version

$$G(z) = \beta_i \; (1-z)^{\beta_i - 1} \; (1 + 2\delta^i_{v+s}) + \tilde{\delta}_h \; . \tag{3.32}$$

The calculation of the initial state bremsstrahlung at 2-loop level in the case of the Z resonance [10], either performed explicitly or by using the structure function method, has shown, that the soft photon exponentiation together with the remaining 1-loop contributions of the virtual and hard photons represents the main part of the initial state bremsstrahlung. A renormalisation group analysis [19] confirms the method of the summation of the leading logarithms arising in connection with the emission of soft photons ($\rightarrow$ Eq. 3.32).

The *final state* QED-form factor:

The gauge invariant QED-form factor describing the soft photons radiated by the final state fermions is given by (Eqs. D.10,D.44 and $Q_f - Q_{f'} = 1$)

$$F^{final}_{QED}(s) = \beta_f(s) \log\left(\frac{2\Delta E}{\sqrt{s}}\right) + 2 \; \delta^f_{v+s}(s) \; , \tag{3.33}$$

where $\beta_f(s)$ and $\delta^f_{v+s}(s)$ again can be derived from the corresponding initial state expressions (Eqs. 3.24,3.25) by applying the substitutions $(i,i') \rightarrow (f,f')$. After taking into account the radiation of hard photons the so-defined soft photon contribution to the resonant W production cross section fulfills the KLN-theorem [17] provided that no constraints on the invariant mass of the final state fermion pair will be imposed: the mass singularities cancel out and finally a QED-form factor δ^f_{QED} remains multiplying the inclusive total Born cross section

$$\sigma^{(0+1)}_{f,s+h}(s) = \tilde{\sigma}^{(0)}(s) \; (1 + \delta^f_{QED}) \; , \tag{3.34}$$

which has the following form:

$$\delta^f_{QED} = \frac{\alpha}{\pi} \; \left[\frac{3}{8} \; (Q_f^2 + Q_{f'}^2) + \frac{7}{3} + \frac{\pi^2}{24}\right] \overset{f,f'=\nu,l}{\approx} \; 0.0072 \; . \tag{3.35}$$

Thus, as in the Z resonance case, this small effect of the final state bremsstrahlung can be taken into account by attaching a multiplicative factor to the convolution integral in Eq. 3.31

$$\tilde{\sigma}^{(0)}(s) \; \rightarrow \; \tilde{\sigma}^{(0)}(s) \; (1 + \delta^f_{QED}) \; .$$

The interference contribution:

The interference of initial and final state soft bremsstrahlung leads to the following QED-form factor (Eqs. D.11,D.45 with $Q_i - Q_{i'} = Q_f - Q_{f'} = 1$):

$$F^{interf.}_{QED}(s,t) = \underbrace{\beta_{int.}(s,t) \log\left(\frac{2\Delta E}{\sqrt{s}} \frac{M_W^2}{|\Delta_W - 2\sqrt{s}\Delta E|}\right)}_{\rightarrow 0 \text{ for } s=M_W^2, \Delta E \gg \frac{\Gamma_W}{2}} +2 \; \delta^{interf.}_{v+s}(s,t) \tag{3.36}$$

with

$$\beta_{int.}(s,t) = \frac{\alpha}{\pi}\left[(Q_iQ_f + Q_{i'}Q_{f'})\log\left(\frac{t^2}{s^2}\right) - (Q_iQ_{f'} + Q_{i'}Q_f)\log\left(\frac{u^2}{s^2}\right) + 2\right] \tag{3.37}$$

and

$$\begin{aligned}\delta_{v+s}^{interf.}(s,t) &= \frac{\alpha}{4\pi}\left\{(Q_iQ_f + Q_{i'}Q_{f'})\left[-\frac{1}{4}\log^2\left(\frac{t^2}{s^2}\right) - 2\mathrm{Sp}(1+\frac{s}{t}) + \frac{1}{2}\log\left(\frac{t^2}{s^2}\right)\right]\right.\\ &- \left.(Q_iQ_{f'} + Q_{i'}Q_f)[t \to u] - 6 - \frac{7}{6}\pi^2\right\}. \end{aligned} \tag{3.38}$$

Sp(z) denotes the Spence-function described in [20]. The integration over the scattering angle of the remnant of the IR-singular logarithm in Eq. 3.36

$$\begin{aligned}\sigma_{interf.}^{(1)}(s)|_{\log.} &= \int_{-1}^{1} d\cos\theta\, \frac{d\tilde{\sigma}^{(0)}(s,t)}{d\cos\theta}\, \beta_{int.}(s,t)\, \log\left(\frac{2\Delta E}{\sqrt{s}}\frac{M_W^2}{|\Delta_W - 2\sqrt{s}\Delta E|}\right)\\ &= \tilde{\sigma}^{(0)}(s)\, \frac{\alpha}{\pi}\left(-\frac{1}{3}\right)\, [5(Q_iQ_f + Q_{i'}Q_{f'}) + 4(Q_{i'}Q_f + Q_{f'}Q_i)] \times\\ &\quad \log\left(\frac{2\Delta E}{\sqrt{s}}\frac{M_W^2}{|\Delta_W - 2\sqrt{s}\Delta E|}\right) \end{aligned} \tag{3.39}$$

leads to a contribution, which will be completely compensated by the hard photon contribution $\sigma_h^{interf.}(s)$ in Eq. E.17 evaluated at $s = M_W^2$. The remaining factor $\delta_{v+s}^{interf.}(s,t)$ together with the IR-finite parts of the box diagrams $F_{V,(t,u)}^{finite}(s,t)$ (Eq. D.12), where on-shell and mass singularities have been subtracted according to Eq. 3.19, are independent of the charge quantum numbers characterising the external fermions

$$(\delta_{v+s}^{interf.} + F_{V,(t,u)}^{finite})(s = M_W^2) = -\frac{\alpha}{4\pi}\left[\Delta_{M_W} + 8 + \frac{5}{6}\pi^2\right], \tag{3.40}$$

and, thus, can be absorbed into a modified weak contribution to the differential Born-cross section. This compensation of the non-factorisable $t(u)$-dependent remnants of the photonic box diagram by $\delta_{v+s}^{interf.}$ is essential to the factorisation of the numerator of the resonant cross section into partial W widths describing the W production and decay, respectively.

3.2 The modified weak 1-loop correction to the W production

The IR-finite rest of the virtual photon contribution $F_\gamma^{finite}(s,t)$ of Eq. 3.4 consists of the remnants of the YFS-prescription $F_{rem}^\gamma(s,t)$ and the IR-finite Feynman-diagrams III and IV

$$F_\gamma^{finite}(s,t) = F_{rem.}^\gamma(s,t) + (F_{III,f}^\gamma + F_{III,i}^\gamma + F_{IV}^\gamma)(s)|_{subtr.} \tag{3.41}$$

with

$$F_{rem.}^\gamma(s,t) = \left\{\sum_{j=I,II}(F_{j,f}^{finite} + F_{j,i}^{finite})(s) + (F_{V,t}^{finite} + F_{V,u}^{finite})(s,t)\right\}_{subtr.}, \tag{3.42}$$

where $|_{subtr.}$ reminds of the subtraction of the mass and on-shell singularities described by Eq. 3.19. After taking into account the remaining part of the interference QED-form factor

$\delta_{v+s}^{interf.}(s,t)$ from Eq. 3.38, as it has already been discussed in Sec. 3.1, these photon contributions can be absorbed into a modified weak contribution to the W resonance

$$\tilde{F}_{weak}(s=M_W^2) = (F_{weak}^i + F_{weak}^f + F_\gamma^{finite} + \delta_{v+s}^{interf.})(s=M_W^2) \;, \tag{3.43}$$

where $F_{weak}^{i,f}$ denote the pure weak contributions given by Eq. D.27. With this UV-finite and ξ_i-independent form factor the separation of the electroweak corrections to the W resonance aimed for is completed.

Finally, it remains to check, whether $\tilde{F}_{weak}(M_W^2)$ can be represented as a sum of the modified weak corrections to the W width: $\delta\tilde{\Gamma}_{weak}^f$ and $\delta\tilde{\Gamma}_{weak}^i$. According to Eq. B.11 this is equivalent to the verification of the identity

$$(F_\gamma^{finite} + \delta_{v+s}^{interf.})(s=M_W^2) \equiv 2\;\delta\Gamma_{rem.}^\gamma$$

with $\delta\Gamma_{rem.}^\gamma$ given by Eq. B.10. In fact, by performing its explicit calculation this identity is proven to be true and $\tilde{F}_{weak}(M_W^2)$ can be written as follows:

$$\begin{aligned}\tilde{F}_{weak}(M_W^2) &= \underbrace{F_{weak}^i(M_W^2) + \delta\Gamma_{rem.}^\gamma}_{=:\,\tilde{F}_{weak}^i(M_W^2)} + \underbrace{F_{weak}^f(M_W^2) + \delta\Gamma_{rem.}^\gamma}_{=:\,\tilde{F}_{weak}^f(M_W^2)} \\ &\equiv \delta\tilde{\Gamma}_{weak}^i + \delta\tilde{\Gamma}_{weak}^f \;. \end{aligned} \tag{3.44}$$

By using this result and by following the prescription derived in App. A the W production cross section in the vicinity of the resonance including (modified) weak 1-loop corrections has Breit-Wigner-form

$$\sigma_w(s) = \frac{6\pi}{M_W^2}\,\frac{(5-N_c^i)}{N_c^{i2}}\,\frac{s\;\tilde{\Gamma}_{W\to ff'}^{(0+1)}\;\tilde{\Gamma}_{W\to ii'}^{(0+1)}}{[(s-M_W^2)^2 + M_W^2\;(\Gamma_W^{(0+1)})^2]} \;, \tag{3.45}$$

where $\tilde{\Gamma}$ denotes the QED-subtracted W width defined by Eq. B.12.

4 Summary

In order to match the requirements of future precision experiments at LEP II and the Tevatron the corresponding cross sections for resonant W production have to be calculated beyond leading order perturbation theory. Having in mind the successful treatment of the electroweak $\mathcal{O}(\alpha)$ contribution to the Z resonance [8], we strove for the analogous description of the resonant W production in a 4-fermion process at the required level of accuracy. After a thorough perturbative discussion of the electroweak $\mathcal{O}(\alpha)$ contribution to the W production we succeeded in extracting a gauge invariant QED-like form factor from the photon contribution. We showed, that, when approaching the W resonance, the occurrence of on-shell singularities is restricted to the initial state contribution and can be 'regularised' by introducing the W width as a physical cut-off parameter in a gauge invariant way. The similar structure of the resulting initial state QED-form factor to that of the Z resonance allowed us to apply the same technique to cope with the enhancement of the electroweak coupling by large mass singular logarithms (*soft photon exponentiation*). By separating the electroweak 1-loop corrections to the W width into QED and weak contribution, too, it turned out, that the (modified) weak corrections to the resonant W production cross section also factorises into QED-subtracted partial W widths.

In summary, we achieved a representation of the electroweak radiative corrections to the W production cross section in the vicinity of the resonance, which is in analogy to deep inelastic hadronic scattering a convolution of a process specific 'hard' cross section $\sigma_w(s)$ (Eq. 3.45) with an universal radiator function $G(z)$ (Eq. 3.32) describing the initial state photon contribution, where the possibility of multiple soft photon emission has been taken into account

$$\boxed{\sigma(s) = \int_0^1 dz\, G(z)\, \sigma_w(sz)\, (1+\delta^f_{QED})}\,. \tag{4.1}$$

δ^f_{QED}, defined by Eq. 3.35, denotes the final state photon contribution, which is free of large mass singular logarithms. As a result of the comparative discussion of the S-matrix inspired ansatz and the perturbative approach a transformation of the parameter of the resonance (Eq. A.27) connects between the two descriptions.

5 Numerical discussion

In the following the numerical relevance of the different contributions to the electroweak radiative corrections and their impact on the line shape of the W resonance will be discussed. For the numerical evaluation the following set of parameters has been used [9],[13]:

$\alpha = 1/137.0359895$	$G_\mu = 1.16639 \cdot 10^{-5}$ GeV^{-2}
$\alpha_s = 0.123$	$M_Z = 91.1884$ GeV
$m_d = m_u = 0.0468$ GeV	$m_c = 1.55$ GeV
$m_s = 0.17$ GeV	$m_b = 4.7$ GeV
$\lvert V_{ud}\rvert = 0.975$	$\lvert V_{cs}\rvert = 0.974$
$\lvert V_{tb}\rvert = 0.999$	$\lvert V_{us}\rvert = \lvert V_{cd}\rvert = 0.222$
$\lvert V_{cb}\rvert = \lvert V_{ts}\rvert = 0.044$	$\lvert V_{ub}\rvert = \lvert V_{td}\rvert = 0.007$

The masses of the light quarks are effective quark masses in the sense, that they reproduce the correct hadronic vacuum polarisation given by the dispersion integral calculated in [21] and have no further physical meaning. Using this set of input parameters the W boson mass is determined via the relation

$$M_W^2 = \frac{M_Z^2}{2}\left[1+\sqrt{1-\frac{4\pi\alpha}{\sqrt{2}G_\mu}\frac{1}{M_Z^2}\frac{1}{1-\Delta r}}\right] \tag{5.1}$$

as a function of the not precisely known or even unknown parameters of the MSM: m_t and M_H. A detailed description of Δr, which comprises the radiative corrections to the muon decay, can be found in [22],[23].

The W width is an important ingredient of the description of the resonant W boson production. The numerical results for the W width at leading order $\overline{\Gamma}_W^{(0)}$ (Eq. 2.4) and at next-to-leading order $\overline{\Gamma}_W^{(0+1)}$ (Eq. 5.2) are summarised in Tab. 2. Besides the electroweak $\mathcal{O}(\alpha)$ contribution calculated in App. B, the latter contains also the contribution of virtual and real gluons, so that

$\overline{\Gamma}_W^{(0+1)}$ yields in G_μ-representation (Eq. 2.3) as follows:

$$\overline{\Gamma}_W^{(0+1)} = \sum_{(ff'),f\neq t} \overline{\Gamma}^{(0)}_{W\to ff'} \, (1 + 2\mathcal{R}e\, \delta\tilde{\Gamma}^f_{weak} - \Delta r + \delta^f_{QED} + \frac{N_c^f - 1}{2}\, \delta_{QCD}) \,, \tag{5.2}$$

where the modified weak correction and the QED-form factor are given by Eq. B.11 and Eq. 3.35, respectively. The QCD corrections are derived in the limit of massless decay products [24]

$$\delta_{QCD} = \frac{\alpha_s}{\pi} \left[1 + 1.40932 \, \left(\frac{\alpha_s}{\pi}\right) - 12.76706 \, \left(\frac{\alpha_s}{\pi}\right)^2 \right] \,, \tag{5.3}$$

which for our case represents a sufficient approximation. In the course of the calculation of the W width the Kobayashi-Maskawa-mixing has been neglected, but the final result has been multiplied with the square of the corresponding physical matrix element V_{ij}. From a numerical point of view, this procedure does not significantly differ from a consideration of the Kobayashi-Maskawa-matrix in the renormalisation procedure as it has been pointed out in [25]. In order to illustrate the variation of M_W and $\overline{\Gamma}_W^{(0+1)}$ with the electroweak input parameters, they are given in Tab. 2 for different values of m_t and M_H. The ratio $\overline{\Gamma}_W^{(0+1)}/M_W$ illustrates the very weak dependence of the W width on m_t and M_H: due to the cancellation of large leading (quadratic) m_t-dependent contributions in $\delta\tilde{\Gamma}_{weak}$ and Δr only a logarithmic dependence on m_t (and M_H) survives and thus the variation of $\overline{\Gamma}_W^{(0+1)}$ is mainly a consequence of the variation of M_W. Our result obtained for the W width in next-to-leading order is in very good agreement with the total W width derived in [12]: relative deviation $\leq 0.005\%$.

In the subsequent discussion of the line shape of the W resonance the top quark mass and the W boson mass are chosen to be the central values of their current world average ([26] and [4], resp.)

$$m_t = 175 \pm 9 \text{ GeV}$$

$$M_W = 80.33 \pm 0.15 \text{ GeV} \,.$$

Using these input parameters the Higgs-boson mass and the total W width yield

$$M_H = 273 \text{ GeV} \;\Rightarrow\; \overline{\Gamma}_W^{(0)} = 2.0406 \text{ GeV} \quad \text{and} \quad \overline{\Gamma}_W^{(0+1)} = 2.0887 \text{ GeV}$$

compared to the measured value of Γ_W [13]

$$\Gamma_W = 2.08 \pm 0.07 \text{ GeV} \,.$$

The 'hard' cross section $\sigma_w(s)$

The effect of the (modified) weak 1-loop correction described by Eq. 3.45 to the W line shape is shown in Fig. 4 for the example of a pure leptonic process: $\nu_e e^+ \to \nu_\mu \mu^+$. There is no noticeable impact on the location of the maximum of the resonant cross section $\overline{\sigma}_w(s)$ (in G_μ-representation)

$$s_{max} = M_W^2 \, \sqrt{1 + \gamma^2} \,, \tag{5.4}$$

M_H [GeV]	60	300	1000
$m_t = 165$ GeV			
M_W [GeV]	80.3648	80.2618	80.1647
$\overline{\Gamma}_W^{(0)}$ [GeV]	2.0433	2.0354	2.0280
$\overline{\Gamma}_W^{(0+1)}$ [GeV]	2.0911	2.0834	2.0759
$\overline{\Gamma}_W^{(0+1)}/M_W$	0.0260	0.0260	0.0259
$m_t = 175$ GeV			
	80.4275	80.3228	80.2244
	2.0481	2.0401	2.0326
	2.0960	2.0882	2.0806
	0.0261	0.0260	0.0259
$m_t = 185$ GeV			
	80.4927	80.3861	80.2862
	2.0531	2.0449	2.0373
	2.1012	2.0932	2.0854
	0.0261	0.0260	0.0260

Table 2: The total W width (and M_W) in G_μ-representation including the described radiative corrections

where the abbreviation $\gamma = \frac{\overline{\Gamma}_W^{(0+1)}}{M_W}$ has been used, due to the smallness of γ in the above equation ($\Delta s_{max} = 0.6$ MeV). The maximum of the cross section, however,

$$\overline{\sigma}_{w,max} = \frac{6\pi}{M_W^2} \, \frac{5 - N_c^i}{N_c^{i2}} \, \frac{\tilde{\overline{\Gamma}}{}^{(0+1)}_{W\to ff'} \, \tilde{\overline{\Gamma}}{}^{(0+1)}_{W\to ii'}}{(\overline{\Gamma}_W^{(0+1)})^2} \, (1 + \frac{1}{4}\gamma^2) \tag{5.5}$$

is reduced as compared to the peak value in leading order perturbation theory $\overline{\sigma}^{(0)}_{max}$ ($= \overline{\sigma}_{w,max}$ with $\Gamma^{(0+1)} \to \Gamma^{(0)}$). For the case of the leptonic process this reduction yields

$$\overline{\sigma}_{w,max} = 0.9347 \, \overline{\sigma}^{(0)}_{max}$$

and is mainly due to the QCD correction to the total W width given by Eq. 5.3. Thus, when considering the W production process $\nu_e e^+ \to u\overline{d}$ the reduction of the maximum cross section only amounts to

$$\overline{\sigma}_{w,max} = 0.9729 \, \overline{\sigma}^{(0)}_{max} \, ,$$

since now the QED-subtracted partial W width $\tilde{\overline{\Gamma}}{}^{(0+1)}_{W\to u\overline{d}}$ of Eq. 5.5 also includes the QCD contribution. Tab. 3 shows the negligible small dependence of the peak value $\overline{\sigma}_{w,max}$ on the top quark and Higgs-boson mass due to the aforementioned cancellation of leading (quadratic) m_t-dependent contributions in the partial W width calculated in the G_μ-representation.

The further discussion is dedicated to the QED-like contribution, especially to the initial state photon radiation. The final state QED contribution described by δ^f_{QED} of Eq. 3.35 has a

m_t [GeV]	M_H [GeV]	$\overline{\Gamma}_W^{(0+1)}$ [GeV]	$\overline{\sigma}_{w,max}$ [nb]
166	124.19	2.0886	52.5449
175	273.32	2.0887	52.5451
184	549.30	2.0888	52.5452

Table 3: The W width $\overline{\Gamma}_W^{(0+1)}$ and the peak value $\overline{\sigma}_{w,max}$ for different top quark masses. Besides the top quark mass the W boson mass $M_W = 80.33$ GeV has been used as an input parameter, so that the Higgs-boson mass is determined by Eq. 5.1.

tiny effect on the peak value: $\delta_{QED}^{f=\mu} \sim 0.0072$ for leptons and $\delta_{QED}^{f=u} \sim 0.0069$ for quarks, but has no impact on the peak position of the resonant cross section. The leftovers of the interference contribution have already been absorbed in the 'hard' cross section as it has been described in Sec. 3.1.

The *initial state* bremsstrahlung

The initial state bremsstrahlung, described by Eq. 3.23 (soft photons) together with Eq. E.15 (hard photons), does not only carry the main contribution to the reduction of the peak value, but is also responsible for the distortion of the line shape, especially for the shift in the peak position. The main effect to the reduction of the maximum can roughly be estimated by the factor

$$1 - \beta_{i=e}(M_W^2)\ \log\left(\frac{M_W}{\overline{\Gamma}_W^{(0+1)}}\right) = 0.81$$

with $\beta_{i=e}(M_W^2)$ given by Eq. 3.24. For comparison, the corresponding factor for the case of the Z resonance is given by [22]

$$1 - 4\frac{\alpha}{\pi}\log\left(\frac{M_Z}{m_e}\right)\log\left(\frac{M_Z}{\Gamma_Z}\right) = 0.6\ .$$

The effect is much smaller, when the soft photon is emitted by quarks

$$1 - \beta_{i=u}(M_W^2)\ \log\left(\frac{M_W}{\overline{\Gamma}_W^{(0+1)}}\right) = 0.94\ ,$$

where the numerical evaluation has been performed by using the effective quark masses. They have no physical meaning, but in a realistic hadronic scattering process they are rather included in the parton distribution as parts of the interacting hadrons, with which the parton cross section has to be convoluted in order to obtain an observable cross section [27].

In Fig. 5 the impact of the initial state bremsstrahlung to the W line shape in a pure leptonic process $\nu_e e^+ \to \nu_\mu \mu^+$ is shown. The shift of the peak position due to the energy loss in the resonant W propagator in $\mathcal{O}(\alpha)$ amounts to

$$\Delta M_W = +53 \text{ MeV}\ ,$$

which reduces to

$$\Delta M_W = +42 \text{ MeV}$$

after performing soft photon exponentiation as it is described by Eq. 3.31. This shows, that the calculation performed in $\mathcal{O}(\alpha)$ overestimates the W boson mass by 11 MeV. Due to the different charge structure for the case of quarks in the initial state only a shift of the peak position by $\Delta M_W = +14$ MeV can be observed, which still amounts to $\Delta M_W = +13$ MeV after the resummation of the soft photon contribution. Since these soft photons represent the main contribution to the resonant W production, we expect no significant contribution from hard photons at 2-loop level, which has been confirmed by an explicit 2-loop calculation in the case of the Z resonance [10].

In summary, the electroweak $\mathcal{O}(\alpha)$ contribution to the resonant W production develops the same characteristics as the corresponding corrections to the Z line shape. Fig. 6 shows the total cross section of the W production in the vicinity of the resonance as it is described by the convolution integral of Eq. 4.1, where the s-dependence of the W width has been considered by applying the transformations of Eq. A.27. The main impact of the discussed radiative corrections on the W line shape can be summarised as follows:

- The peak position s_{max} of the resonant cross section (Eq. 5.4) is shifted about +42 MeV (Z : +96 MeV) (constant W width) and suffers an additional shift about −27 MeV (Z : −34 MeV), when assuming an s-dependent width.

- The peak value of the resonant cross section is reduced by a factor 0.82 (Z :∼0.6) with respect to $\overline{\sigma}^{(0)}_{max}$.

For comparison, the corresponding values in case of the Z resonance are also provided [22] (in brackets).

Acknowledgments

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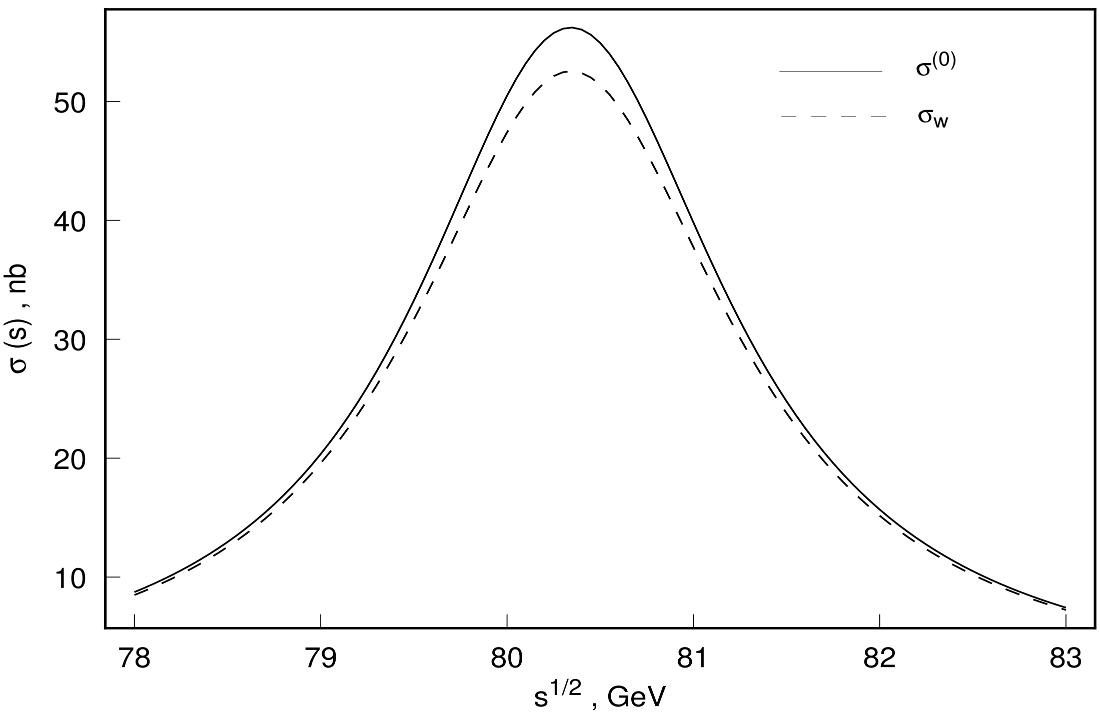


Figure 4: The 'hard' cross section $\overline{\sigma}_w(s)$ of Eq. 3.45 compared to the Born-cross section for $\nu_e e^+ \to \nu_\mu \mu^+$

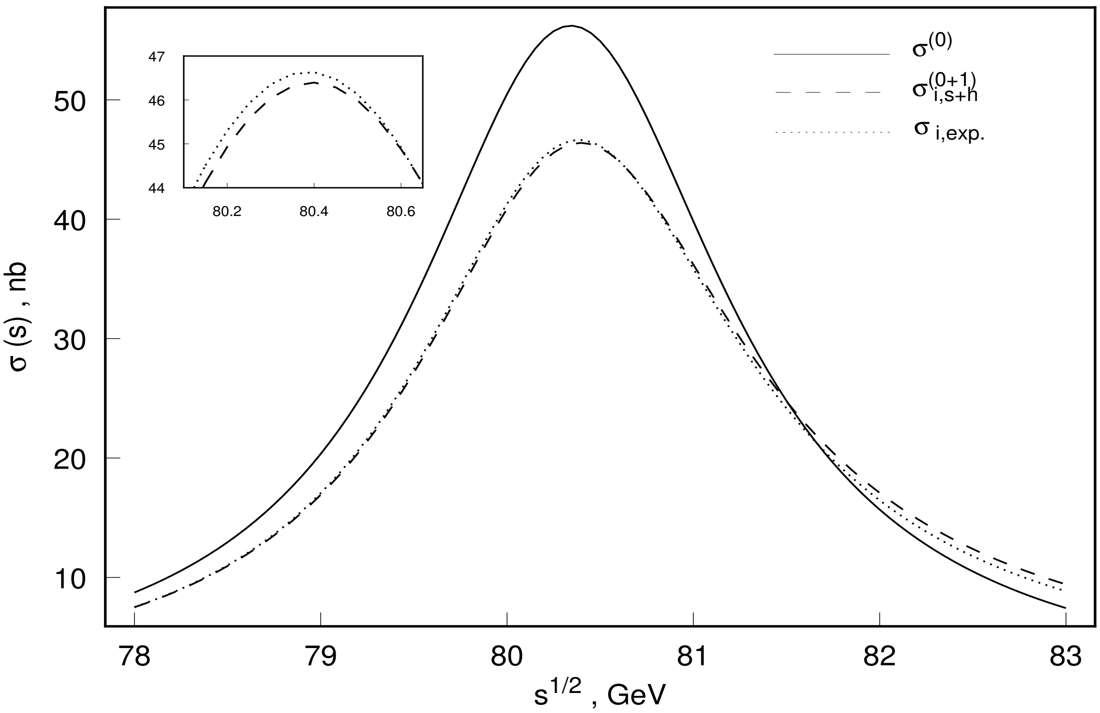


Figure 5: The effect of initial state bremsstrahlung in $\mathcal{O}(\alpha)$ described by $\sigma_{i,s+h}^{(0+1)}(s)$ of Eq. 3.29 and after soft photon exponentiation (Eq. 3.31)

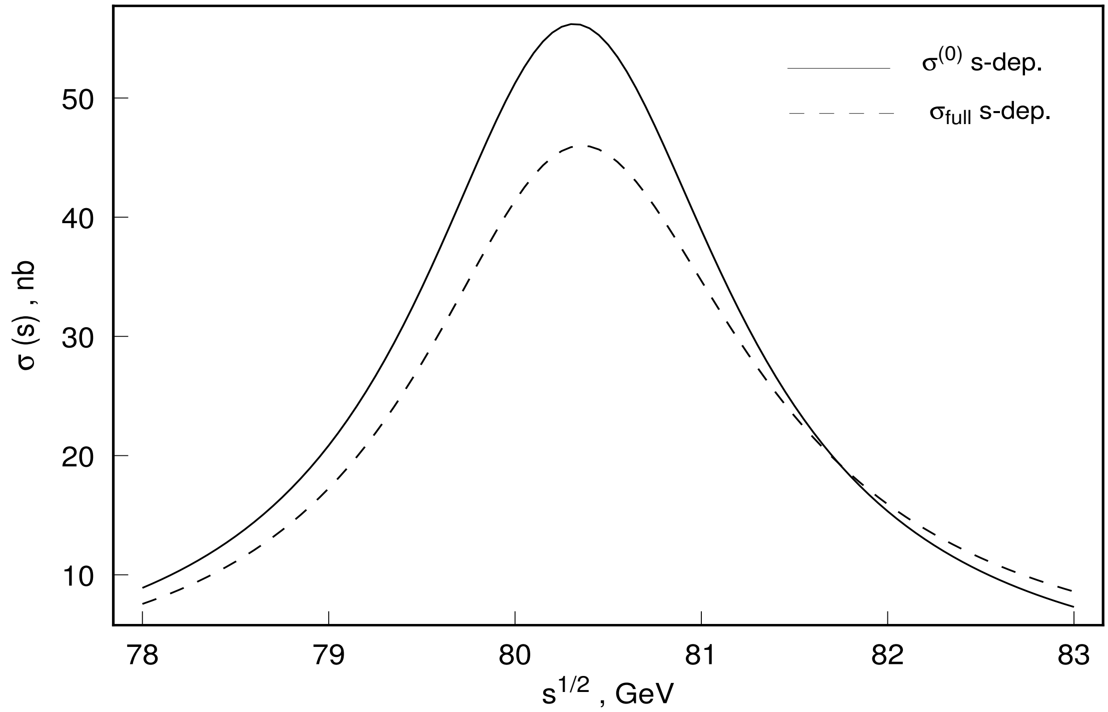


Figure 6: The W production cross section in the vicinity of the resonance including the discussed electroweak radiative corrections (Eq. 4.1)

Appendix

A Unstable particles and gauge invariance

In S-matrix theory an unstable particle, experimentally seen as a resonance during the interaction of stable particles, can be easily described when neglecting all singularities besides a single complex pole close to the real energy axes with negative imaginary part [28]. Therefore the S-matrix is approximately of the form of a Breit-Wigner resonance

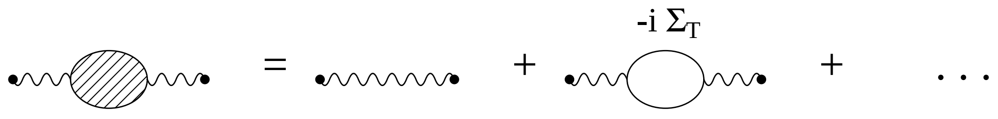
$$\mathcal{M}(s) = \frac{R}{s - M_c^2} + F(s) \ , \tag{A.1}$$

where $F(s)$ is an analytic function with no poles. The residue R of the complex pole M_c^2 can be interpreted as a product of coupling constants, with which the unstable particle couples to the external particles [28]. The resonance in the scattering amplitude arises in the vicinity of $s = \mathcal{Re}(M_c^2)$, the physical mass of the unstable particle, and the width of the resonance is given by $\mathcal{Im}(M_c^2)$:

$$M_c^2 = M_{phys.}^2 - iM_{phys.}\Gamma \quad \text{or e.g.} \quad M_c^2 = (M_{phys.} - i\frac{\Gamma}{2})^2 \ . \tag{A.2}$$

The S-Matrix given by Eq. A.1 is gauge invariant in the physical region (s real and $s > 0$) and thus -via analytic continuation- also in the complex energy plane, which enables its application in a gauge theory. The fact that the complex pole M_c, its residue R and the non-resonant part $F(s)$ are separately gauge invariant has been used to find a gauge invariant description of the Z resonance at the required level of accuracy [29],[30].

From the quantum field theory's point of view a resonance in the scattering amplitude is caused by a pole in the propagator of an unstable particle. In the vicinity of the resonance the resummed propagator has to be used, which is a formal summation of a geometric series with the 1PI self energy of the unstable particle as argument (Dyson-resummation) [31]:



$$D^{\mu\nu} = \frac{-ig^{\mu\nu}}{s - M_0^2 + \Sigma_T(s)} = \frac{-ig^{\mu\nu}}{s - M_0^2 + i\epsilon} \left[1 + \left(\frac{-\Sigma_T(s)}{s - M_0^2 + i\epsilon} \right) + \ldots \right] . \tag{A.3}$$

M_0 denotes the unrenormalised bare mass and $\Sigma_T(s)$ is the transverse part of the 1PI self energy. Since the external particles are considered to be massless as long as no singularities occur, the longitudinal part of the propagator does not contribute and will not be discussed. Veltman [32] showed that the S-matrix constructed by using the Dyson-resummed propagator and assuming only transitions between stable particles obeys the principles of unitarity, renormalisability and causality. Thus, the field theoretical description of gauge boson resonances is given by the following amplitude, after performing a renormalisation procedure:

$$\mathcal{M}(s) = \frac{\hat{V}_i(s)\,\hat{V}_f(s)}{s - M_R^2 + \hat{\Sigma}_T(s)} + B(s) . \tag{A.4}$$

$\hat{V}_{i,f}(s)$ denote the renormalised vertices, describing the production and decay of the unstable particle, M_R denotes the renormalised mass and $\hat{\Sigma}_T(s)$ the renormalised self energy. $B(s)$ comprises the non-resonant contributions, e.g. box diagrams.

The S-matrix theory inspired construction of a gauge invariant amplitude using a Laurent expansion of Eq. A.4 around the complex pole and afterwards performing a consistent evaluation of the parameters of the resonance in the coupling constant g results in a description with constant width. Choosing the field theoretical ansatz and carrying out a consistent treatment of the inverse of the propagator in Eq. A.4 can lead to a scattering amplitude with s-dependent width [33]. Analysing the Z line shape in the S-matrix theory approach yields a Z boson mass which is about 34 MeV larger, at $\mathcal{O}(g^2)$ accuracy, than the corresponding value obtained in an s-dependent width prescription. Since these two descriptions are connected by a transformation of the line shape parameters [34] there are equivalent and, thus, the difference in the Z boson mass has no physical meaning.

The future precise measurement of the W boson mass at LEP II and at an upgrade of the Tevatron raises the same questions for a charged gauge boson resonance. In the following, the applicability of the prescriptions, derived in the context of the Z resonance, to a charged vector boson resonance will be studied.

$\mathcal{M}_{ii' \to ff'}(s)$ with constant width

After carrying out the systematic treatment developed by Veltman [29], which can be directly applied to the W resonance, a gauge invariant scattering amplitude and a definition of mass and width to the required level of accuracy can be given:

- $\mathcal{O}(g^0)$ accuracy:

At 1-loop level the physical mass M_W is connected to the renormalised mass as follows:

$$\mathcal{R}e(M_c^2) = M_W^2 = M_R^2 - \mathcal{R}e\hat{\Sigma}_T(M_R^2, g^2) \;, \tag{A.5}$$

which yields the equality of physical and renormalised mass when using the on-shell renormalisation condition $\mathcal{R}e\hat{\Sigma}_T(M_R^2, g^2) = 0$ in order to determine the mass renormalisation constant $\delta M_W^2 \equiv M_0^2 - M_R^2$. In leading order perturbation theory the W width corresponds to the imaginary part of the 1-loop corrected renormalised W self energy

$$M_W\,\Gamma_W^{(0)} = \mathcal{I}m\hat{\Sigma}_T(M_W^2, g^2) \;. \tag{A.6}$$

Thus, the W resonance is described by

$$\mathcal{M}^{(0)}(s) = \frac{\mathcal{R}(g^2)}{s - M_W^2 + iM_W\,\Gamma_W^{(0)}} + \mathcal{O}(g^2) \tag{A.7}$$

with

$$\mathcal{R}(g^2) = V_i(g)V_f(g) \;.$$

- $\underline{\mathcal{O}(g^2)\text{ accuracy}}$:
In next-to-leading order Eq. A.5 turns to

$$\begin{aligned} M_W^2 &= M_R^2 - (1 - \mathcal{R}e\hat{\Pi}_T(M_R^2, g^2))\,\mathcal{R}e\hat{\Sigma}_T(M_R^2, g^2) \\ &- \mathcal{R}e\hat{\Sigma}_T(M_R^2, g^4) - \mathcal{I}m\hat{\Sigma}_T(M_R^2, g^2)\,\mathcal{I}m\hat{\Pi}_T(M_R^2, g^2) \;, \end{aligned} \tag{A.8}$$

where the following abbreviation has been used:

$$\hat{\Pi}_T(s) \equiv \frac{\partial\hat{\Sigma}_T(s)}{\partial s} \;.$$

Taking the renormalised mass as the zero of the real part of the inverse propagator, which corresponds to the field theoretical definition of a stable particle's mass, this reduces to

$$M_W^2 = M_R^2 - \mathcal{I}m\hat{\Sigma}_T(M_R^2, g^2)\,\mathcal{I}m\hat{\Pi}_T(M_R^2, g^2) \;. \tag{A.9}$$

Thus, one obtains a shifted renormalised mass with respect to the physical mass. By considering a renormalisation condition, however, which reads at 2-loop level as follows:

$$\mathcal{R}e\hat{\Sigma}_T(M_R^2, g^4) + \mathcal{I}m\hat{\Sigma}_T(M_R^2, g^2)\,\mathcal{I}m\hat{\Pi}_T(M_R^2, g^2) = 0 \;, \tag{A.10}$$

the equality of physical and renormalised mass is recovered [30]. Then the W width in next-to-leading order yields

$$M_W\Gamma_W^{(0+1)} = (1 - \mathcal{R}e\hat{\Pi}_T(M_W^2, g^2))\,\mathcal{I}m\hat{\Sigma}_T(M_W^2, g^2) + \mathcal{I}m\hat{\Sigma}_T(M_W^2, g^4) \;. \tag{A.11}$$

The calculation of $\Gamma_W^{(0+1)}$ in the MSM and for $\xi_i = 1$ can be found in [12] and will be additionally performed in App. B for R_ξ-gauge and in the limit of massless decay products. Finally, a gauge invariant description of the W resonance can be given, which completely takes into account the electroweak radiative corrections up to order $\mathcal{O}(g^2)$

$$\mathcal{M}^{(0+1)}(s) = \frac{\mathcal{R}(g^2) + \mathcal{R}(M_W^2, g^4)}{s - M_W^2 + iM_W\,\Gamma_W^{(0+1)}} + \mathcal{O}(g^4) \tag{A.12}$$

with the residue in next-to-leading order

$$\mathcal{R}(M_W^2, g^4) = \hat{V}_i(M_W^2, g^3)V_f(g) + V_i(g)\hat{V}_f(M_W^2, g^3) - V_i(g)V_f(g)\,\hat{\Pi}_T(M_W^2, g^2)\;. \quad \text{(A.13)}$$

The $\hat{V}_{i,f}(M_W^2, g^3)$ denote the renormalised vertices including 1-loop corrections to the production and decay of a W boson, respectively.

$\mathcal{M}_{ii' \to ff'}(s)$ with s-dependent width

Next we present the results obtained by using the field theoretical ansatz and we discuss the equivalency of both approaches also for a charged gauge boson resonance. The latter cannot be readily expected in the case of a W resonance, since the existence of a transformation given by Bardin et al. [34] for the case of the Z resonance is due to the linear s-dependence of the imaginary part of the Z self energy. Therefore a careful study of the s-dependence of the W self energy is required. After evaluating the real part of the W self energy in Eq. A.3 (after renormalisation) around $s = M_R^2$ and using the on-shell renormalisation condition $\mathcal{R}e\hat{\Sigma}_T(M_R^2) = 0$ the W propagator is given by

$$D_W^{\mu\nu} = -ig^{\mu\nu}\,\frac{1 - \mathcal{R}e\hat{\Pi}_T(M_R^2)}{s - M_R^2 + i\mathcal{I}m\hat{\Sigma}_T(s)\,(1 - \mathcal{R}e\hat{\Pi}_T(M_R^2))}\;. \quad \text{(A.14)}$$

Thus, following the argumentation of Wetzel [33] in the vicinity of the resonance the residue of the complex pole in Eq. A.4 in next-to-leading order is given by

$$R^{(0+1)}(M_W^2) = \hat{V}_i(M_W^2, g^3)V_f(g) + V_i(g)\hat{V}_f(M_W^2, g^3) + V_i(g)V_f(g)\,(1 - \mathcal{R}e\hat{\Pi}_T(M_W^2, g^2))\;, \quad \text{(A.15)}$$

where $M_R = M_W$ has been used. Since the inverse W propagator is of order g^2 in the vicinity of the resonance the complete $\mathcal{O}(g^4)$-contribution to the denominator has to be taken into account. Thus, after using the definition of the W width given by Eq. A.11, the following definition for the s-dependent W width can be given:

$$\begin{aligned} \text{denominator} &= s - M_W^2 + iM_W\Gamma_W^{(0+1)} + i\mathcal{I}m[\hat{\Sigma}_T(s, g^2) - \hat{\Sigma}_T(M_W^2, g^2)] \\ &=: s - M_W^2 + iM_W\Gamma_W^{(0+1)}(s)\;. \end{aligned} \quad \text{(A.16)}$$

In contrary to the Z boson, where the imaginary part of the derivative of the 1PI Z self energy develops gauge dependent contributions only when [35]

$$\xi_W \leq \left(\frac{M_Z}{2M_W}\right)^2\;,$$

the corresponding quantity in the W boson case $\mathcal{I}m\hat{\Pi}_T(M_W^2, g^2)$ is gauge parameter dependent for each gauge parameter $\xi_i \neq 1$ (Eq. D.18). This is due to the existence of Feynman-diagrams involving photons, which couple to the W boson via the triple gauge boson coupling. The 1-loop contributions to the W self energy are shown in Fig. 7 for R_ξ-gauge. However, when the Dyson-resummed contribution

$$\mathcal{I}m[\hat{\Sigma}_T(s, g^2) - \hat{\Sigma}_T(M_W^2, g^2)] \overset{s \to M_W^2}{=} (s - M_W^2)\,\mathcal{I}m\hat{\Pi}_T(M_W^2, g^2)$$

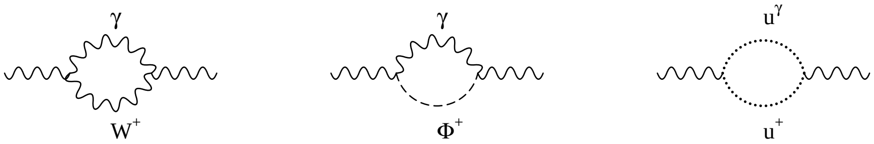


Figure 7: Feynman-diagrams for the photonic 1-loop correction to the W self energy (the dashed and dotted lines denote a charged Higgs-ghost $\Phi^{\pm}$ and the Faddeev-Popov-ghosts $u^{\pm}$ or u^{γ}, resp.)

is treated perturbatively in order to cancel the gauge parameter dependent contributions to the imaginary part of the 1PI vertex corrections in $R^{(0+1)}(M_W^2)$, the Breit-Wigner resonance formula with constant width from Eq. A.12 in combination with the renormalisation condition of order $\mathcal{O}(g^4)$ given by Eq. A.10 is recovered.

In order to obtain the physical description of the W resonance with s-dependent width, the following approximation of the s-dependence of the photon contribution to the imaginary part of the W self energy shown in Fig. 7 is useful ($I(s)$: Eq. D.17):

$$\begin{aligned} \mathcal{I}m\hat{\Sigma}_T^{\gamma}(s) &= (s-M_W^2)\,\theta(s-M_W^2)\,I(s) \\ &\stackrel{R}{=} (s-M_W^2)\,\theta(s-M_W^2)\,I(M_W^2) := (s-M_W^2)\,\mathcal{I}m\hat{\Pi}_T^{\gamma}(M_W^2)\,. \end{aligned} \tag{A.17}$$

Since the derivative $\mathcal{I}m\hat{\Pi}_T^{\gamma}(M_W^2)$ does not exist in a strict mathematical sense due to the threshold at $s=M_W^2$, the above equation has to be understood as a definition. The fermion contribution to $\mathcal{I}m\hat{\Sigma}_T(s)$, however, is linear in s in the case of massless fermions, so that the s-dependence can be extracted as follows:

$$\mathcal{I}m\hat{\Sigma}_T(s,g^2) = \frac{s}{M_W^2}\,\mathcal{I}m\hat{\Sigma}_T(M_W^2,g^2) + (s-M_W^2)\,\mathcal{I}m\hat{\Pi}_T^{\gamma}(M_W^2,g^2)\,. \tag{A.18}$$

By using this s-dependence in the W propagator given by Eq. A.14 and after undoing the resummation of $\mathcal{I}m\hat{\Pi}_T^{\gamma}(M_W^2,g^2)$ the W propagator turns out to be as follows:

$$D_W^{\mu\nu} = -ig^{\mu\nu}\,\frac{1-\mathcal{R}e\hat{\Pi}_T(M_W^2,g^2)-i\mathcal{I}m\hat{\Pi}_T^{\gamma}(M_W^2,g^2)+\mathcal{O}(g^4)}{s-M_W^2+i\frac{s}{M_W^2}\mathcal{I}m\hat{\Sigma}_T(M_W^2)\,(1-\mathcal{R}e\hat{\Pi}_T(M_W^2,g^2)-i\mathcal{I}m\hat{\Pi}_T^{\gamma}(M_W^2,g^2))+\mathcal{O}(g^6)}\,, \tag{A.19}$$

where the validity of Eq. A.18 at least up to order g^4 has been assumed. In summary, the scattering amplitude constructed with the help of this propagator and a subsequent consistent evaluation in the coupling constant of the numerator and the denominator, which results in a gauge invariant description of a resonant produced W boson at the required level of accuracy, will be given:

- <u>$\mathcal{O}(g^0)$ accuracy</u>:

$$\mathcal{M}^{(0)}(s) = \frac{R^{(0)}}{s-M_W^2+i\frac{s}{M_W}\,\Gamma_W^{(0)}} + \mathcal{O}(g^2) \tag{A.20}$$

with

$$R^{(0)} = V_i(g)V_f(g)$$

and the definition of the W width given by Eq. A.6.

- $\underline{\mathcal{O}(g^2) \text{ accuracy}}$:
  By considering the following renormalisation condition:

$$\mathcal{R}e\hat{\Sigma}_T(M_W^2, g^4) + \mathcal{I}m\hat{\Sigma}_T(M_W^2, g^2)\ \mathcal{I}m\hat{\Pi}_T^{\gamma}(M_W^2, g^2) = 0\ , \tag{A.21}$$

which differs from Eq. A.10 by

$$M_W \Gamma_W^{(0)}\ \mathcal{I}m\hat{\Pi}_T^{ferm.}(M_W^2, g^2) = (\Gamma_W^{(0)})^2 \tag{A.22}$$

the scattering amplitude is given by:

$$\mathcal{M}^{(0+1)}(s) = \frac{R^{(0+1)}(M_W^2)}{s - M_W^2 + i\frac{s}{M_W}\ \Gamma_W^{(0+1)}} + \mathcal{O}(g^4) \tag{A.23}$$

with

$$\begin{aligned} R^{(0+1)}(M_W^2) &= V_i(g)V_f(g) + \hat{V}_i(M_W^2, g^3)V_f(g) + V_i(g)\hat{V}_f(M_W^2, g^3) \\ &- V_i(g)V_f(g)\ [\mathcal{R}e\hat{\Pi}_T(M_W^2, g^2) + i\mathcal{I}m\hat{\Pi}_T^{\gamma}(M_W^2, g^2)]\ . \end{aligned} \tag{A.24}$$

The next-to-leading order W width $\Gamma_W^{(0+1)}$ is again defined by Eq. A.11. $R^{(0+1)}(M_W^2)$ differs from $\mathcal{R}(M_W^2, g^4)$ of Eq. A.13 concerning their imaginary parts by

$$V_i(g)V_f(g)\ \mathcal{I}m\hat{\Pi}_T^{ferm.}(M_W^2, g^2) = V_i(g)V_f(g)\ \frac{\Gamma_W^{(0)}}{M_W}\ . \tag{A.25}$$

It remains to check whether both descriptions are equivalent. For that purpose Eq. A.23 will be rewritten as follows (with $\gamma = \frac{\Gamma_W^{(0+1)}}{M_W}$):

$$\begin{aligned} \mathcal{M}^{(0+1)}(s) &= \frac{R^{(0+1)}(M_W^2)}{s\ (1 + i\gamma) - M_W^2} \\ &= \frac{R^{(0+1)}(M_W^2)\ \frac{(1-i\gamma)}{(1+\gamma^2)}}{s - M_W^2\ (1 - \gamma^2) + iM_W^2(1 - \gamma^2)\gamma} \\ &=: \frac{\overline{R}^{(0+1)}(M_W^2)}{s - \overline{M}_W^2 + i\overline{M}_W\ \overline{\Gamma}_W^{(0+1)}}\ . \end{aligned} \tag{A.26}$$

The evaluation of the numerator and denominator of the above equation up to the order required for a $\mathcal{O}(g^2)$ accuracy easily verifies, that exactly those terms arise, in which the s-dependent width description differs from the constant width amplitude given by the Eqs. A.22, A.25. Thus, a transformation of the parameters of the resonance: residue, position of the pole ($\rightarrow$ mass) and width can be given, which connects both descriptions

$$\begin{aligned} R^{(0+1)}(M_W^2) &\rightarrow \overline{R}^{(0+1)}(M_W^2) = R^{(0+1)}(M_W^2)\frac{(1 - i\gamma)}{(1 + \gamma^2)} \\ M_W &\rightarrow \overline{M}_W = M_W\ (1 + \gamma^2)^{-\frac{1}{2}} \\ \Gamma_W^{(0+1)} &\rightarrow \overline{\Gamma}_W^{(0+1)} = \Gamma_W^{(0+1)}\ (1 + \gamma^2)^{-\frac{1}{2}}\ . \end{aligned} \tag{A.27}$$

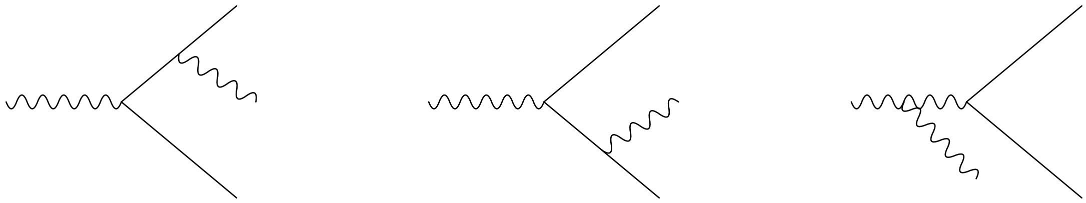


Figure 8: Real photon corrections in $\mathcal{O}(\alpha)$ to the partial W width

Consequently, the W boson mass in the description with s-dependent width is about ~ 27 MeV smaller as compared to the constant width approximation. With the help of these transformations the effect of an s-dependent width can be easily studied without the necessity to deal with the - with regard to the s-dependence - complicated scattering amplitude from Eq. A.23, especially when a convolution integral as it is given by Eq. 1.1 has to be calculated.

In recent publications, either in connection with the W pair production at LEP II [36] or with the radiative W production at the Tevatron [37], several approaches to consider an s-dependent width in the W propagator in a gauge invariant way have been discussed. The prescription given by Baur et al. [37] results from taking into account the imaginary part of the virtual fermionic correction to the γWW-vertex. We checked, that applying the transformation we derived (Eq. A.27) in order to consider an s-dependent width yields the same modification of the bremsstrahlung amplitude as presented in [37].

B The partial W width in $\mathcal{O}(\alpha^2)$

The partial W width in $\mathcal{O}(\alpha^2)$ can be written as follows:

$$\Gamma^{(0+1)}_{W\to ff'} = \Gamma^{(0)}_{W\to ff'}\;(1 + 2\,\mathcal{R}e\;\delta\hat{\Gamma}_{virt.} + \delta\Gamma_{BR})\;, \tag{B.1}$$

where $\Gamma^{(0)}_{W\to ff'}$ denotes the partial width in leading order given by Eq. 2.1. $\delta\hat{\Gamma}_{virt.}$ and $\delta\Gamma_{BR}$ represent the virtual and real contributions, resp., calculated in R_ξ gauge and in the limit of massless decay products. The discussion of the electroweak $\mathcal{O}(\alpha)$ contribution to the W width performed in Feynman 't Hooft gauge and under consideration of massive decay products can also be found in [12]. In the following we concentrate on the gauge invariant separation into a QED-like and weak part.

The Feynman-diagrams representing real photon emission described by

$$\delta\Gamma_{BR} = \delta\Gamma^{s}_{BR} + \delta\Gamma^{h}_{BR}$$

are shown in Fig. 8. The soft $\delta\Gamma^{s}_{BR}$ and hard $\delta\Gamma^{h}_{BR}$ bremsstrahlung contribution can both be described by the same form factors we already have derived for the final state photon emission in the W production process evaluated at $s = M_W^2$: $\delta\Gamma^{s}_{BR} = F^{final}_{BR}(M_W^2)$ given by Eq. D.44 and $\delta\Gamma^{h}_{BR}$ is defined by Eq. E.22.

$\delta\hat{\Gamma}_{virt.}$ comprises the renormalised vertex correction (diagram I,II,III in Fig. 9 and the counter term given by Eq. C.1) and the wave function renormalisation for the W boson (diagram IV in Fig. 9 together with Eq. C.2). Again, we discuss the photon and pure weak contribution separately

$$\delta\hat{\Gamma}_{virt.} = F^{f}_{weak}(M_W^2) + F^{f}_{\gamma}(M_W^2)\;. \tag{B.2}$$

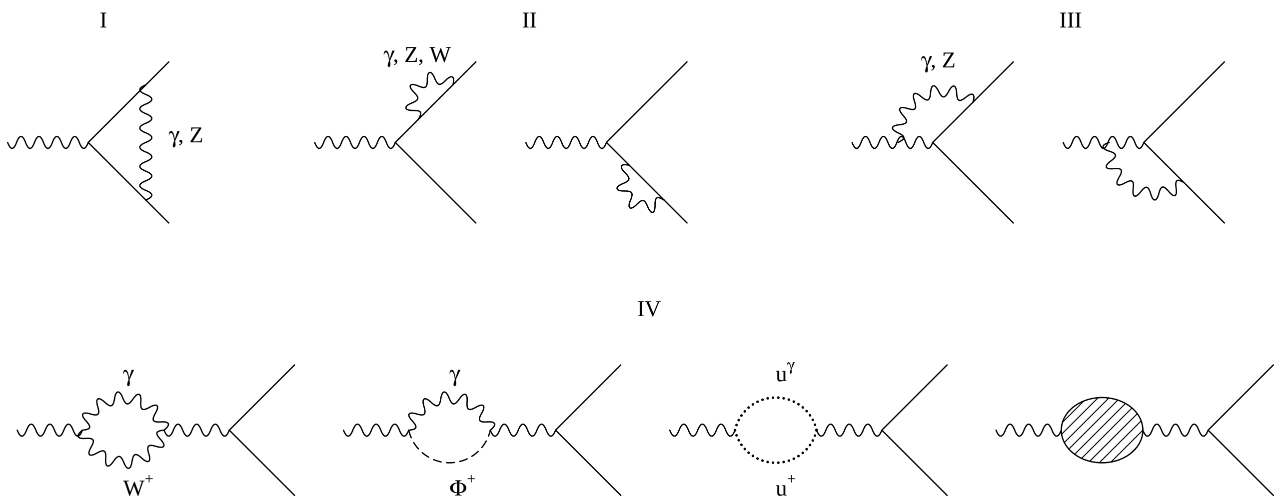


Figure 9: Electroweak 1-loop corrections in $\mathcal{O}(\alpha)$ to the partial W width (again, the non-photonic corrections to the W self energy are symbolised by the shaded loop)

The pure weak contribution can be described by the same form factor $F^f_{weak}(M_W^2)$ of Eq. D.27, which has been derived from the weak corrections to the W decay process of the resonant W production in the 4-fermion process. In contrary, the structure of the virtual photon contribution $F^f_\gamma(M_W^2)$ differs from that of the W resonance and requires a separate discussion. For a W boson being on-shell all photonic 1-loop corrections in Fig. 9 develop IR-singularities. Thus, in order to gain a gauge invariant separation into a QED-like δ^f_{QED} and a (modified) weak part $\delta\tilde{\Gamma}^f_{weak}$

$$\Gamma^{(0+1)}_{W\to ff'} = \Gamma^{(0)}_{W\to ff'}\ (1 + 2\mathcal{R}e\ \delta\tilde{\Gamma}^f_{weak} + \delta^f_{QED})\ , \tag{B.3}$$

the diagrams III and IV have also to be considered by the YFS-procedure. The application of the prescription given in Sec. 3.1 to these diagrams

$$\text{diagram III}\ \ :\ i\Lambda^{III,f}_\mu = ig_w\gamma_\mu(1-\gamma_5)\ [F^{IR}_{III,f} + F^{finite}_{III,f}](s = M_W^2) \tag{B.4}$$

with

$$F^{IR}_{III,f}(s = M_W^2) = (i4\pi\alpha)\left\{Q_f\int_D \frac{(2p_f - k)(k - 2q)}{D_\lambda D_f(k^2 - 2kq)} + Q_{f'}\int_D \frac{(2p_{f'} + k)(k + 2q)}{D_\lambda D_{f'}(k^2 + 2kq)}\right\} \tag{B.5}$$

and

$$\text{diagram IV}\ \ :\ i\Lambda^{IV}_\mu = ig_w\gamma_\mu(1-\gamma_5)\ \frac{1}{2}\ [F^{IR}_{IV} + F^{finite}_{IV}](s = M_W^2) \tag{B.6}$$

with

$$F^{IR}_{IV}(s = M_W^2) = (i4\pi\alpha)\int_D \frac{(2q - k)^2}{D_\lambda(k^2 - 2kq)^2} \tag{B.7}$$

together with the IR-singular parts extracted from the diagrams I,II (Eqs. 3.8,3.10 evaluated at $s = M_W^2$) yield a gauge invariant YFS-form factor multiplying the tree level W width, which is the same as for the final state photon contribution to the W production

$$F^{final}_{YFS}(s = M_W^2) = \frac{1}{2}(i4\pi\alpha)\int_D \frac{1}{D_\lambda}\times$$

$$\left[\overbrace{\frac{Q_f(2p_f-k)_\rho}{(k^2-2kp_f)}+\frac{Q_{f'}(2p_{f'}+k)_\rho}{(k^2+2kp_{f'})}-\frac{1}{2}\frac{(Q_f-Q_{f'})(2q-k)_\rho}{(k^2-2kq)}+\frac{1}{2}\frac{(Q_f-Q_{f'})(2q+k)_\rho}{(k^2+2kq)}}^{k^\rho \mathcal{J}_\rho^{final}=0}\right]^2 \tag{B.8}$$

The only difference is, that the ad-hoc addition of a 'zero' in Eq. 3.16 can now be traced back to the IR-singular contributions of diagrams involving the γWW-coupling, when the W boson is considered to be on-shell. The explicit expressions for $F^{IR}_{III,f}(M_W^2), F^{IR}_{IV}(M_W^2)$ and the corresponding IR-finite parts are given by the Eqs. D.22-D.26. Consequently, the QED-like form factor to the W width from Eq. B.3

$$\begin{aligned}\delta^f_{QED} &= F^{final}_{QED}(s=M_W^2)+\delta\Gamma^h_{BR} \\ &= \frac{\alpha}{\pi}\left[\frac{3}{8}\,(Q_f^2+Q_{f'}^2)+\frac{7}{3}+\frac{\pi^2}{24}\right]\end{aligned} \tag{B.9}$$

is the same as for the final state QED contribution to the W resonance given by Eq. 3.34. This result can be compared with the 'QED-factor' for a leptonic W decay given in [18]

$$\delta_Q=\frac{\alpha}{\pi}\left[\frac{77}{24}-\frac{\pi^2}{3}\right]\ ,$$

which has been derived by considering from the photonic virtual contribution only the mass singular logarithms being gauge invariant by themselves.

The IR-finite remnants of the YFS-prescription in the case of the W width yield

$$\begin{aligned}\delta\Gamma^\gamma_{rem.} &= \sum_{j=I,II,III} F^{finite}_{j,f}\,|_{subtr.}\,(M_W^2)+\frac{1}{2}F^{finite}_{IV}(M_W^2)-\frac{1}{2}\frac{\alpha}{4\pi}(2+\frac{3}{2}\pi^2) \\ &= \frac{1}{2}\frac{\alpha}{4\pi}\left\{\frac{25}{3}\,\Delta_{M_W}+\frac{68}{9}-\frac{3}{2}\pi^2+(\xi_W-1)\alpha_W\right\}-\frac{1}{2}\delta Z_2^{W,\gamma}\ ,\end{aligned} \tag{B.10}$$

which can be absorbed in a modified weak contribution

$$\delta\tilde{\Gamma}^f_{weak}=F^f_{weak}(M_W^2)+\delta\Gamma^\gamma_{rem.}\ . \tag{B.11}$$

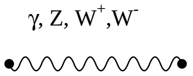
This completes the gauge invariant separation of the electroweak corrections in $\mathcal{O}(\alpha)$ to the partial W width due to Eq. B.3. Finally, a QED-subtracted partial W width can be defined

$$\tilde{\Gamma}^{(0+1)}_{W\to ff'}=\Gamma^{(0)}_{W\to ff'}\,(1+2\,\mathcal{R}e\,\delta\tilde{\Gamma}^f_{weak})\ , \tag{B.12}$$

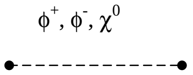
which will appear in the residue of the Breit-Wigner-form of the resonant W production cross section.

C Feynman-rules

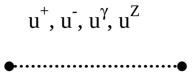
In the following the Feynman-rules, which differ from the ones in Feynman-'t Hooft-gauge ($\xi_i = 1$) are explicitly given. The remaining Feynman-rules can be found in [14].



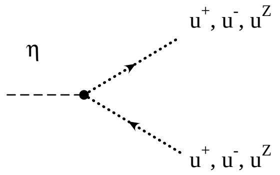
$$: \quad \frac{-i}{q^2 - M_V^2 + i\epsilon} \left(g^{\mu\nu} + \frac{(\xi_V - 1)\, q^\mu q^\nu}{q^2 - \xi_V\ M_V^2} \right)$$



$$: \quad \frac{i}{q^2 - \xi_{(W,Z)}\ M_{(W,Z)}^2 + i\epsilon}$$

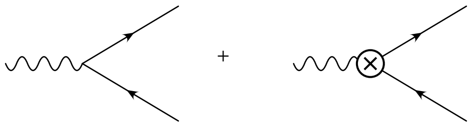


$$: \quad \frac{i}{q^2 - \xi_{(W,Z,\gamma)}\ M_{(W,Z,\gamma)}^2 + i\epsilon}$$



$$: \quad -i\, e\, \frac{M_W}{2\, s_w}\, [\xi_W; \frac{\xi_Z}{c_w^2}]$$

As it has already been pointed out, a renormalisation procedure needed to be performed in order to cope with the arising UV-divergences. Thus, after the multiplicative renormalisation of the $SU(2)$ gauge coupling constant and the gauge boson field W_μ^a, the Wff'-vertex counter term yields as follows [14]:



$$: \quad \frac{ie}{2\sqrt{2}s_w}\gamma_\mu(1-\gamma_5)(1 + \delta Z_1^W - \delta Z_2^W) \qquad \text{(C.1)}$$

and the renormalised W self energy is defined by

$$\hat{\Sigma}_T^W(s) = \Sigma_T^W(s) + (s - M_W^2)\, \delta Z_2^W - \delta M_W^2\ . \qquad \text{(C.2)}$$

The renormalisation constants determined in the on-shell renormalisation scheme are given by [14], [22]

$$\begin{aligned}
\delta Z_1^W &= -\Pi^\gamma(0) - \frac{3 - 2s_w^2}{s_w c_w}\, \frac{\Sigma_T^{\gamma Z}(0)}{M_Z^2} + \frac{c_w^2}{s_w^2}\left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right] \\
\delta Z_2^W &= -\Pi^\gamma(0) - 2\frac{c_w}{s_w}\, \frac{\Sigma_T^{\gamma Z}(0)}{M_Z^2} + \frac{c_w^2}{s_w^2}\left[\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right] \qquad \text{(C.3)}
\end{aligned}$$

with

$$\delta M_{(W,Z)}^2 = \mathcal{R}e\Sigma_T^{(W,Z)}(s = M_{(W,Z)}^2)\ . \qquad \text{(C.4)}$$

Π^γ, $\Sigma_T^{\gamma Z}$ denote the photon vacuum polarisation and the photon-Z-mixing, respectively.

It should be mentioned, that we do not perform an 'explicit' wave function renormalisation for the external fermions, but rather take into account the modification due to their self interaction by the consideration of the 1-loop contributions shown in Fig. 2 (diagram II). Therefore no renormalisation constant for the fermion doublet δZ_L occurs in the counter term for the Wff'-vertex.

D The form factors

In the following we provide the explicit expressions for the different contributions to the form factor describing the virtual electroweak $\mathcal{O}(\alpha)$ contribution to the W production process $\hat{F}_{virt.}(s,t)$ given by Eq. 3.2. They are calculated in R_ξ-gauge, where, following [35], the ξ_i-dependent parts are expressed in terms of the functions $\alpha_i, v_{ij}, \eta_{ij}$. The latter are described in App. F, where the explicit expressions for the IR- and/or on-shell singular scalar 2-,3- and 4-point integrals B_0, C_0, D_0 can be found, too. In order to regularise the arising IR-singularities a fictive photon mass λ has been used. After dimensional regularisation the UV-divergences have been extracted in form of the following singular terms:

$$\Delta_s \equiv \Delta - \log\left(\frac{s}{\mu^2}\right) \quad \text{and} \quad \Delta_m \equiv \Delta - \log\left(\frac{m^2}{\mu^2}\right)$$

with $\Delta = \dfrac{2}{4-D} - \gamma_E + \log 4\pi$ (γ_E: Euler constant).

D.1 The form factor describing the photonic 1-loop corrections

The photonic form factor $F_\gamma(s,t)$ of Eq. 3.2 is composed as follows:

$$F_\gamma(s,t) = \sum_{j=I,II,III} F_j^\gamma(s) \underbrace{-\frac{\Sigma_T^{W,\gamma}(s) - \mathcal{R}e\Sigma_T^{W,\gamma}(M_W^2)}{s-M_W^2} - \delta Z_2^{W,\gamma}}_{=:F_{IV}^\gamma(s)} + F_V^\gamma(s,t) \tag{D.1}$$

with

$$\begin{aligned} F_j^\gamma(s) &= (F_{j,f}^\gamma + F_{j,i}^\gamma)(s) \\ F_V^\gamma(s,t) &= (F_V^t + F_V^u)(s,t)\ . \end{aligned} \tag{D.2}$$

In the following the explicit expressions for the different contributions to the photonic form factor will be provided, starting with the final state photonic vertex corrections. By applying the substitution $(f,f') \to (i,i')$ the corresponding intial state contribution can be easily derived.

diagram I:

$$\begin{aligned} F_{I,f}^\gamma(s) &= \frac{\alpha}{4\pi} Q_f Q_{f'} \Big\{ -2s\, C_0(s,m_f,m_{f'},\lambda) + 2B_0(p_f^2,\lambda,m_f) + 2B_0(p_{f'}^2,\lambda,m_{f'}) \\ &- 3B_0(s,m_f,m_{f'}) - 2 + (\xi_\gamma - 1)\alpha_\gamma \Big\}\ . \end{aligned} \tag{D.3}$$

Performing the loop-integration of Eq. 3.8 the IR-singular contribution is given by

$$\begin{aligned}
F^{IR}_{I,f}(s) &= \frac{\alpha}{4\pi} Q_f Q_{f'} \left\{ -2s\, C_0(s, m_f, m_{f'}, \lambda) + B_0(p_f^2, \lambda, m_f) + B_0(p_{f'}^2, \lambda, m_{f'}) \right. \\
&- \left. B_0(s, m_f, m_{f'}) \right\} \\
&= \frac{\alpha}{4\pi} Q_f Q_{f'} \left\{ \Delta_s + 2\log\left(\frac{s}{m_f m_{f'}}\right) + 2\log\left(\frac{s}{m_f m_{f'}}\right) \log\left(\frac{\lambda^2}{s}\right) + 2 \right. \\
&+ \left. \frac{1}{2}\log^2\left(\frac{s}{m_f^2}\right) + \frac{1}{2}\log^2\left(\frac{s}{m_{f'}^2}\right) + \frac{4}{3}\pi^2 + i\pi[2\log\left(\frac{s}{\lambda^2}\right) - 1] \right\} .
\end{aligned} \tag{D.4}$$

<u>**diagram II**</u>:

$$\begin{aligned}
F^{\gamma}_{II,f}(s) &= -\frac{1}{2}\frac{\alpha}{4\pi} \left\{ Q_f^2 \left[\Delta_s + 3\log\left(\frac{s}{m_f^2}\right) + 4 + 2\ \log\left(\frac{\lambda^2}{s}\right) + (\xi_\gamma - 1)\alpha_\gamma \right] \right. \\
&+ \left. Q_{f'}^2\ [f \to f'] \right\} .
\end{aligned} \tag{D.5}$$

Computing the 1-loop integral of Eq. 3.10 leads to

$$F^{IR}_{II,f}(s) = -\frac{1}{2}\frac{\alpha}{4\pi} \left\{ Q_f^2 \left[\Delta_s + 3\log\left(\frac{s}{m_f^2}\right) + 4 + 2\ \log\left(\frac{\lambda^2}{s}\right) \right] + Q_{f'}^2\ [f \to f'] \right\} . \tag{D.6}$$

<u>**diagram V**</u>:

$$\begin{aligned}
F^t_V(s,t) &= \frac{\alpha}{4\pi} \left\{ Q_i Q_f \left[-2t(s - M_W^2)\ D_0(s, t, m_i, m_f, M_W, \lambda) + \frac{(s - M_W^2)}{(s+t)^2}\ f_{V,t}(s,t) \right] \right. \\
&+ \left. Q_{i'} Q_{f'}\ [(i,f) \to (i', f')] \right\} .
\end{aligned} \tag{D.7}$$

In order to provide a complete representation of the 1-loop corrections also the non-resonant contribution $f_{V,t}(s,t)$, which is negligible in the vicinity of the resonance, will be explicitly given

$$\begin{aligned}
f_{V,t}(s,t) &= 2(s+t)\ [B_0(s, \lambda, M_W) - B_0(t, m_i, m_f)] - t(2t + s + M_W^2)\ C_0(1) \\
&+ ((s+t)^2 + t^2 - sM_W^2)\ [C_0(3) + C_0(4)] + t(s + M_W^2 + 2t)\ \left[(s - M_W^2) D_0 \right. \\
&- \left. C_0(2) \right] + (s+t)^2\ [(\xi_W - 1)\eta_{W\gamma}(s) + (\gamma \leftrightarrow W)] .
\end{aligned} \tag{D.8}$$

From Eq. 3.12 the t-channel box contribution to the IR singular YFS-form factor is given by

$$\begin{aligned}
F^{IR}_{V,t}(s,t) &= \frac{\alpha}{4\pi} \left\{ Q_i Q_f\ \left[-2t\ C_0(2) - B_0(t, m_i, m_f) + B_0(p_f^2, \lambda, m_f) + B_0(p_i^2, \lambda, m_i) \right] \right. \\
&+ \left. Q_{i'} Q_{f'}\ [(i,f) \to (i', f')] \right\} \\
&= \frac{\alpha}{4\pi} \left\{ Q_i Q_f \left[\log\left(\frac{t^2}{m_f^2 m_i^2}\right) \log\left(\frac{\lambda^2}{s}\right) + \frac{1}{2}\log^2\left(\frac{s}{m_f^2}\right) + \frac{1}{2}\log^2\left(\frac{s}{m_i^2}\right) \right. \right. \\
&- \left. \frac{1}{4}\log^2\left(\frac{t^2}{s^2}\right) + \frac{\pi^2}{3} + \Delta_s + \frac{1}{2}\log\left(\frac{t^2}{s^2}\right) + 2\log\left(\frac{s}{m_f m_i}\right) + 2 + i\pi \right] \\
&+ \left. Q_{i'} Q_{f'} [(f,i) \to (f', i')] \right\} .
\end{aligned} \tag{D.9}$$

The application of the substitution described by Eq. 3.14 leads to the corresponding u-channel form factors.

From these IR-singular photonic 1-loop contributions the following gauge invariant YFS-form factors of Eq. 3.16 have been extracted:

$$
\begin{aligned}
F^{final}_{YFS}(s) &= (F^{IR}_{I,f} + F^{IR}_{II,f})(s) \\
&+ Q_f(Q_f - Q_{f'}) \left[\log\left(\frac{s}{m_f}\right)\log\left(\frac{\lambda^2}{s}\right) + \frac{1}{2}\log^2\left(\frac{s}{m_f^2}\right) + \log\left(\frac{s}{m_f^2}\right)\right. \\
&+ \left. \Delta_s + 3 - \frac{1}{2}(1 - \frac{3}{2}\pi^2)\right] - Q_{f'}(Q_f - Q_{f'})[f \to f'] \\
&- (Q_f - Q_{f'})^2 \left.\left[\log\left(\frac{\lambda^2}{s}\right) + \frac{1}{2}\Delta_s + \frac{1}{2}\right]\right\} \qquad \text{(D.10)} \\
F^{interf.}_{YFS}(s,t) &= (F^{IR}_{V,t} + F^{IR}_{V,u})(s,t) \\
&- Q_i(Q_f - Q_{f'}) \left[\log\left(\frac{s}{m_i^2}\right)\log\left(\frac{\lambda^2}{s}\right) + \frac{1}{2}\log^2\left(\frac{s}{m_i^2}\right) + \log\left(\frac{s}{m_i^2}\right)\right. \\
&+ \left. \Delta_s + 3 - \frac{1}{2}(1 - \frac{3}{2}\pi^2)\right] + Q_{i'}(Q_f - Q_{f'})\,[i \to i'] \\
&- Q_f(Q_i - Q_{i'})\,[i \to f] + Q_{f'}(Q_i - Q_{i'})\,[i \to f'] \\
&+ (Q_i - Q_{i'})(Q_f - Q_{f'}) \left.\left[2\log\left(\frac{\lambda^2}{s}\right) + \Delta_s + 1\right]\right\} . \qquad \text{(D.11)}
\end{aligned}
$$

The IR-finite remainders of the YFS-prescription are determined by

$$
\begin{aligned}
F^{finite}_{j,f} &= F^{\gamma}_{j,f} - F^{IR}_{j,f} \\
F^{finite}_{V,(t,u)} &= F^{(t,u)}_{V} - F^{IR}_{V,(t,u)} . \qquad \text{(D.12)}
\end{aligned}
$$

The remaining photonic Feynman-diagrams shown in Fig. 2 are IR-finite and, thus, are not considered by the YFS-prescription, but develop on-shell singularities in the vicinity of the W resonance. In detail, they are described by the following form factors:

diagram III:

$$
\begin{aligned}
F^{\gamma}_{III,f}(s) &= \frac{\alpha}{4\pi}\left\{Q_f\left[2\,C_0(s,m_f,\lambda,M_W) + 2B_0(p_f^2,\lambda,m_f) + (2 + \frac{M_W^2}{s})B_0(p_f^2,m_{f'},M_W)\right.\right. \\
&- \left.(1 + \frac{M_W^2}{s})B_0(s,\lambda,M_W) + \frac{1}{2}[(\xi_W - 1)(v_{W\gamma}(s) + \alpha_W) + (\gamma \leftrightarrow W)]\right] \\
&- \left. Q_{f'}[f \to f']\right\} \\
&= \frac{\alpha}{4\pi}\left\{Q_f\left[3\Delta_s + 2\log\left(\frac{s}{m_f^2}\right) + 3 + 2\log\left(\frac{s}{m_f^2}\right)\log\left(\frac{|\Delta_W|}{M_W^2}\right) - \frac{\pi^2}{3}\right.\right. \\
&+ \left.\left. f_{III,f}(s) + \frac{1}{2}[(\xi_W - 1)(v_{W\gamma}(s) + \alpha_W) + (\gamma \leftrightarrow W)]\right] - Q_{f'}[f \to f']\right\} , \qquad \text{(D.13)}
\end{aligned}
$$

where $f_{III,f}(s)$ can be neglected in the resonance region ($w = \frac{M_W^2}{s}$):

$$
f_{III,f}(s) = \frac{\alpha}{4\pi}Q_f\left\{(1-w)\ \left[1 + (1+w)\log\left(\frac{|\Delta_W|}{M_W^2}\right) - 2\log\left(\frac{s}{m_f^2}\right)\log\left(\frac{|\Delta_W|}{M_W^2}\right) + \frac{\pi^2}{3}\right]\right.
$$

$$- \quad 2w\mathrm{Sp}(1-w) - w\log^2(w) + \log(w) - i\pi\theta(s-M_W^2)[1-w^2+2w\log\left(\frac{s}{m_f^2}\right)]\Big\} \; . \tag{D.14}$$

diagram IV:
The renormalised W self energy contribution is described by $F^{\gamma}_{IV}(s)$ of Eq. D.1, where $\delta Z_2^{W,\gamma}$ denotes the photon contribution to the wave function renormalisation of the W boson given by Eq. C.3. The photon contribution to the W self energy reads

$$\begin{aligned}
\Sigma_T^{W,\gamma}(s) &= (-\frac{\alpha}{4\pi})\Big\{\frac{7}{3}M_W^2\Delta_{M_W} + \frac{5}{3}M_W^2 + \frac{2}{9}s + 4sB_0(s,\lambda,M_W) \\
&+ \frac{4}{3}(s-M_W^2)B_1(s,\lambda,M_W) \\
&- (s-M_W^2)\left[(\xi_W-1)(v_{W\gamma}(s) + \frac{1}{2}(s-M_W^2)\eta_{W\gamma}(s)) + (\gamma \leftrightarrow W)\right]\Big\} \; .
\end{aligned} \tag{D.15}$$

In App. A also the imaginary part of $\Sigma_T^{W,\gamma}(s)$ has been carefully studied

$$\mathcal{I}m\Sigma_T^{W,\gamma}(s) = (s-M_W^2)\;\theta(s-M_W^2)\;I(s) \tag{D.16}$$

with

$$\begin{aligned}
I(s) &= \frac{\alpha}{4\pi}\left\{-4\pi\left[1+\frac{1}{6}(1-\frac{M_W^2}{s})^2\right]\right. \\
&+ \left.\mathcal{I}m\left[(\xi_W-1)(v_{W\gamma}(s) + \frac{1}{2}(s-M_W^2)\eta_{W\gamma}(s)) + (\gamma \leftrightarrow W)\right]\right\} \; ,
\end{aligned} \tag{D.17}$$

where $v_{W\gamma}(s)$ and $\eta_{W\gamma}(s)$ (Eq. F.20) develop imaginary parts, when [35]

$$s \geq M_W^2 \;,\; (\sqrt{\xi_{\gamma,W}}+1)^2\;M_W^2 \;,\; 4\,\xi_{\gamma,W}\,M_W^2 \; . \tag{D.18}$$

Using Eq. D.15 the form factor $F^{\gamma}_{IV}(s)$ defined by Eq. D.1 yields

$$\begin{aligned}
F^{\gamma}_{IV}(s) &= \frac{\alpha}{4\pi}\left\{\frac{10}{3}\,\Delta_{M_W} + \frac{68}{9} - 4\log\left(\frac{|\Delta_W|}{M_W^2}\right) - [(\xi_W-1)v_{W\gamma}(s) + (\gamma \leftrightarrow W)]\right. \\
&+ \left. f_{IV}(s)\right\} - \delta Z_2^{W,\gamma} \; ,
\end{aligned} \tag{D.19}$$

where

$$\begin{aligned}
f_{IV}(s) &= (1-w)\left[\frac{2}{3}(1-w)\;\log\left(\frac{|\Delta_W|}{M_W^2}\right) - \frac{2}{3} - [(\xi_W-1)\frac{1}{2s}\eta_{W\gamma}(s) + (\gamma \leftrightarrow W)]\right] \\
&+ i\pi\theta(s-M_W^2)\;\left[\frac{2}{3}\frac{\Delta_W^2}{s^2} - 4\right]
\end{aligned} \tag{D.20}$$

again describes a contribution, which vanishes for $s = M_W^2$. Due to Eq. C.3 the photon contribution to the renormalisation constant $\delta Z_2^W = \delta Z_2^{W,\gamma} + \delta Z_2^{W,weak}$ is determined by δM_W^2

$$\delta Z_2^{W,\gamma} = \frac{\alpha}{4\pi}\left(\frac{c_w}{s_w}\right)^2\;\left[\frac{19}{3}\,\Delta_{M_W} + \frac{89}{9}\right] \; . \tag{D.21}$$

In the course of the extraction of a gauge invariant YFS-form factor from the photonic 1-loop corrections to the W width the IR-singular Feynman-diagrams III and IV of Fig. 5 also needed to be considered. In the following, we provide the explicit expressions for the complete form factor $F^{\gamma}_{j,f}(M_W^2)$ and the IR-singular part $F^{IR}_{j,f}$ extracted according to the YFS-prescription, now evaluated at $s = M_W^2$.

diagram III:

$$\begin{aligned} F^{\gamma}_{III,f}(M_W^2) &= \frac{\alpha}{4\pi}\left\{Q_f\left[3\Delta_{M_W} + 4\log\left(\frac{M_W}{m_f}\right) + 2\log^2\left(\frac{M_W}{m_f}\right) + 3\right.\right.\\ &+ 4\log\left(\frac{M_W}{m_f}\right)\log\left(\frac{\lambda}{M_W}\right)\\ &+ \left.\left.\frac{1}{2}[(\xi_W - 1)(v_{W\gamma}(M_W^2) + \alpha_W) + (\gamma \leftrightarrow W)]\right] - Q_{f'}[f \to f']\right\} \end{aligned} \tag{D.22}$$

Performing the loop-integration in Eq. B.5 leads to

$$\begin{aligned} F^{IR}_{III,f}(M_W^2) &= \frac{\alpha}{4\pi}\left\{Q_f\left[\Delta_{M_W} + 2\log\left(\frac{M_W}{m_f}\right) + 2\log^2\left(\frac{M_W}{m_f}\right) + 3\right.\right.\\ &+ \left.\left.4\log\left(\frac{M_W}{m_f}\right)\log\left(\frac{\lambda}{M_W}\right)\right] - Q_{f'}[f \to f']\right\}. \end{aligned} \tag{D.23}$$

diagram IV:

$$\begin{aligned} F^{\gamma}_{IV}(M_W^2) &= -\lim_{s\to M_W^2}\frac{\Sigma_T^{W,\gamma}(s) - \mathcal{R}e\Sigma_T^{W,\gamma}(M_W^2)}{s - M_W^2} - \delta Z_2^{W,\gamma}\\ &= -\left.\frac{\partial\Sigma_T^{W,\gamma}(s)}{\partial s}\right|_{s=M_W^2} - \delta Z_2^{W,\gamma} \end{aligned} \tag{D.24}$$

Using Eq. D.15 $F^{\gamma}_{IV}(M_W^2)$ is given by

$$F^{\gamma}_{IV}(M_W^2) = \frac{\alpha}{4\pi}\left\{\frac{10}{3}\,\Delta_{M_W} + \frac{32}{9} - 4\log\left(\frac{\lambda}{M_W}\right) - [(\xi_W - 1)v_{W\gamma}(M_W^2) + (\gamma \leftrightarrow W)]\right\} - \delta Z_2^{W,\gamma}. \tag{D.25}$$

The explicit expression for Eq. B.7 reads

$$F^{IR}_{IV}(M_W^2) = -\frac{\alpha}{4\pi}\left\{\Delta_{M_W} + 4 + 4\log\left(\frac{\lambda}{M_W}\right)\right\}. \tag{D.26}$$

D.2 The form factors describing the pure weak 1-loop corrections

The pure weak form factor $F_{weak}(s = M_W^2)$ is given by Eq. 3.3 with the final state contribution

$$F^f_{weak}(M_W^2) = \sum_{j=I,II,III} F^{weak}_{j,f}(M_W^2) + \delta Z_1^W - \delta Z_2^W \underbrace{-\frac{1}{2}\frac{\partial\Sigma_T^{W,weak}(s)}{\partial s}|_{s=M_W^2} - \frac{1}{2}\delta Z_2^{W,weak}}_{=:F^{weak}_{IV,f}(M_W^2)}. \tag{D.27}$$

Performing the substitution $(f, f') \to (i, i')$ yields the corresponding initial state form factor $F^i_{weak}(M_W^2)$. In the following we provide the explicit expressions for the different contributions in Eq. D.27.

diagram I(Z-boson exchange):

$$\begin{aligned} F_{I,f}^{weak}(s) = & \frac{\alpha}{4\pi}(v_f+a_f)(v_{f'}+a_{f'})\left\{\Delta_{M_Z}-(2z+3)\log(z)-2z-4\right. \\ + & 2(1+z)^2\left[\log(z)\log\left(\frac{z+1}{z}\right)-\mathrm{Sp}(-\frac{1}{z})\right]-i\pi[2z+3+2(1+z)^2\log\left(\frac{1+z}{z}\right)] \\ + & \left.(\xi_Z-1)\alpha_Z\right\} \end{aligned} \tag{D.28}$$

with $z=\frac{M_Z^2}{s}$ and the couplings $v_f=(I_3^f-2s_w^2Q_f)/(2s_wc_w)$, $a_f=I_3^f/(2s_wc_w)$.

diagram II(Z- and W-boson exchange):

$$F_{II,f}^{weak}(s)=F_{II,f}^{Z}(s)+F_{II,f}^{W}(s) \tag{D.29}$$

with

$$F_{II,f}^{Z}(s)=\frac{1}{2}\frac{\alpha}{4\pi}[(v_f+a_f)^2+(v_{f'}+a_{f'})^2]\left\{-\Delta_{M_Z}+\frac{1}{2}-(\xi_Z-1)\alpha_Z\right\} \tag{D.30}$$

$$F_{II,f}^{W}(s)=\frac{1}{2}\frac{\alpha}{4\pi}\frac{1}{s_w^2}\left\{-\Delta_{M_W}+\frac{1}{2}-(\xi_W-1)\alpha_W\right\}\ . \tag{D.31}$$

diagram III(Z-boson exchange):

$$\begin{aligned} F_{III,f}^{weak}(s) = & \frac{\alpha}{4\pi}\frac{c_w}{s_w}(v_f+a_f-v_{f'}-a_{f'})\left\{\frac{1}{2}(4+w+z)(\Delta_{M_Z}+\Delta_{M_W})+(w-z)\log\left(\frac{M_Z}{M_W}\right)\right. \\ - & (w+z+1)B_0(s,M_Z,M_W)+2s(z+w+wz)\,C_0(s,m_{(f,f')}=0,M_W,M_Z) \\ + & \left.4+w+z+\frac{1}{2}[(\xi_W-1)[v_{WZ}(s)+\alpha_W]+(W\to Z)]\right\}\ . \end{aligned} \tag{D.32}$$

The scalar 3-point integral C_0 evaluated at $s=M_W^2$ yields as follows:

$$C_0(s=M_W^2,0,M_W,M_Z)=-\frac{1}{M_W^2}\ \log\left(\frac{x_1}{x_1-1}\right)\ \log\left(\frac{x_2}{x_2-1}\right) \tag{D.33}$$

with

$$x_{1,2}=\frac{M_Z^2}{2M_W^2}\ \left(1\pm i\sqrt{\frac{4M_W^2}{M_Z^2}-1}\right)\ .$$

vertex counter part:

The explicit expression for the counter part to the Wff'-vertex (Eq. C.1) reads as follows:

$$\delta Z_1^W-\delta Z_2^W=\frac{\alpha}{4\pi}\frac{1}{s_w^2}\ (-2\Delta_{M_W}-(\xi_W-1)v_W(0))\ . \tag{D.34}$$

diagram IV:

The contribution of the renormalised W self energy to the weak form factor $F_{IV,f}^{weak}(M_W^2)$ of Eq. D.27 is determined by

$$\begin{aligned} \delta Z_2^{W,weak} = & \frac{\alpha}{4\pi}\left\{-\frac{4}{3}\sum_f Q_f^2\Delta_{m_f}+(3-4\left(\frac{c_w}{s_w}\right)^2)\Delta_{M_W}+\frac{2}{3}-\frac{2}{s_w^2}(\xi_W-1)v_W(0)\right\} \\ + & \left(\frac{c_w}{s_w}\right)^2\left[\frac{\mathcal{R}e\Sigma_T^Z(M_Z^2)}{M_Z^2}-\frac{\mathcal{R}e\Sigma_T^{W,weak}(M_W^2)}{M_W^2}\right] \end{aligned} \tag{D.35}$$

and the derivative of $\Sigma_T^{W,weak}$ given by Eq. D.37, D.40. The ξ_i-dependence of the Z self energy and the weak 1-loop correction to the W self energy reads as follows ($(v,\eta)_{i,j} \overset{i=j}{\equiv} (v,\eta)_i$):

$$
\begin{aligned}
\Sigma_T^Z(s) &= \Sigma_T^Z(s)|_{\xi_i=1} \\
&+ \frac{\alpha}{4\pi} 2\,\frac{c_w^2}{s_w^2}(s-M_Z^2)\,(\xi_W-1)\,[v_W(s)+\frac{1}{2}(s-M_Z^2)\eta_W(s)] \qquad \text{(D.36)} \\
\Sigma_T^{W,weak}(s) &= \Sigma_T^{W,weak}(s)|_{\xi_i=1} \\
&+ \frac{\alpha}{4\pi}\frac{c_w^2}{s_w^2}(s-M_W^2)\,[(\xi_W-1)[v_{WZ}(s)+\frac{1}{2}(s-M_W^2)\eta_{WZ}(s)]+(W\leftrightarrow Z)]\,, \qquad \text{(D.37)}
\end{aligned}
$$

so that, finally, the ξ_i-dependent part of the weak form factor yields

$$
F_{weak}^f(M_W^2) = F_{weak}^f(M_W^2)|_{\xi_i=1} - \frac{1}{2}\frac{\alpha}{4\pi}\,(\xi_W-1)\,\alpha_W\,, \qquad \text{(D.38)}
$$

which cancels the ξ_W-dependence of the IR-finite photonic correction $\delta\Gamma_{rem.}^{\gamma}$ from Eq. 3.44.

For the sake of completeness the explicit expressions for the Z self energy and the non-photonic contribution to the W self energy in Feynman-'t Hooft gauge will also be provided, although they are already given in [14]:

$$
\begin{aligned}
\Sigma_T^Z(s)|_{\xi_i=1} &= \frac{\alpha}{4\pi}\left\{\sum_{f\neq\nu}\frac{4}{3}N_c^f\left[(v_f^2+a_f^2)\,(s\Delta_{m_f}+(2m_f^2+s)\,F(s,m_f,m_f)-\frac{s}{3})\right.\right. \\
&- \left. a_f^2 6m_f^2\,(\Delta_{m_f}+F(s,m_f,m_f))\right]+\sum_{f=\nu}\frac{8}{3}a_f^2 s\,\left[\Delta-\log\left(\frac{s}{\mu^2}\right)+\frac{5}{3}\right] \\
&+ \frac{1}{c_w^2\,s_w^2}\,\left[(-(12c_w^4-4c_w^2+1)\,B_2^0+2(-2sc_w^4-2M_W^2c_w^2+M_W^2)\,B_0)(s,M_W,M_W)\right. \\
&+ (6c_w^4-2c_w^2+\frac{1}{2})\,A_0(M_W)-\frac{2}{3}\,s\,c_w^4 \\
&+ \left.\left.(-B_2^0+M_Z^2\,B_0)(s,M_Z,M_\eta)+\frac{1}{4}\,(A_0(M_\eta)+A_0(M_Z))\right]\right\} \qquad \text{(D.39)}
\end{aligned}
$$

and

$$
\begin{aligned}
\Sigma_T^{W,weak}(s)|_{\xi_i=1} &= \frac{\alpha}{4\pi}\frac{1}{s_w^2}\left\{\frac{1}{3}\sum_{f=e,\mu,\tau}\left[(s-\frac{3}{2}m_f^2)\,\Delta_{m_f}+(s-\frac{m_f^2}{2}-\frac{m_f^4}{2s})\,F(s,0,m_f)+\frac{2}{3}s-\frac{m_f^2}{2}\right]\right. \\
&+ \sum_{(q_+,q_-)}N_c^f\left[(2B_2^0+\frac{1}{2}(s-m_+^2-m_-^2)\,B_0)(s,m_+,m_-)\right. \\
&- \left.\frac{1}{2}(A_0(m_+)+A_0(m_-))\right] \\
&+ [-(8c_w^2+1)\,B_2^0+(s_w^4M_Z^2-c_w^2(4s+M_W^2+M_Z^2))\,B_0](s,M_Z,M_W) \\
&+ (2c_w^2+\frac{1}{4})A_0(M_Z)+(-c_w^2+\frac{7}{2})A_0(M_W)-2M_W^2+2c_w^2(M_W^2-\frac{1}{3}s) \\
&+ \left.[-B_2^0+M_W^2B_0](s,M_W,M_\eta)+\frac{1}{4}A_0(M_\eta)\right\} \qquad \text{(D.40)}
\end{aligned}
$$

with

$$
\begin{aligned}
B_2^0(s,m_1,m_2) &= \frac{1}{3}\,\left[(m_1^2\,B_0 + \frac{1}{2}(s+m_1^2-m_2^2)B_1)(s,m_1,m_2)\right.\\
&+ \left.\frac{1}{2}\,A_0(m_2) + \frac{m_1^2+m_2^2}{2} - \frac{s}{6}\right] \qquad \text{(D.41)}\\
A_0(m) &= m^2\;\;(\Delta_m+1)\;\;. \qquad \text{(D.42)}
\end{aligned}
$$

The function $F(s,m_1,m_2)$ can be found in [14].

D.3 The form factor describing the soft photon radiation

Performing the photon phase space integration in Eq. 3.21 leads to the following gauge invariant form factors in the soft photon limit:

$$
\begin{aligned}
F_{BR}^{initial}(s) &= \frac{\alpha}{4\pi}\left\{Q_iQ_{i'}\left[8\log\left(\frac{s}{m_{i'}m_i}\right)\;[\mathcal{L}_W+\delta_p(s)] - \log^2\left(\frac{s}{m_i^2}\right) - \log^2\left(\frac{s}{m_{i'}^2}\right) - \frac{4}{3}\pi^2\right]\right.\\
&- 2\,Q_i^2\left[2[\mathcal{L}_W+\delta_p(s)] - \log\left(\frac{s}{m_i^2}\right)\right] - 2\,Q_{i'}^2\;[i\to i']\\
&+ 2\,Q_i(Q_i-Q_{i'})\left[2\log\left(\frac{s}{m_i^2}\right)\;[\mathcal{L}_W+\delta_p(s)] - \frac{1}{2}\log^2\left(\frac{s}{m_i^2}\right) - \frac{\pi^2}{3}\right]\\
&- \left.2\,Q_{i'}(Q_i-Q_{i'})\;[i\to i'] - 4\;(Q_i-Q_{i'})^2\,[\mathcal{L}_W+\delta_p(s)-1]\right\} \qquad \text{(D.43)}
\end{aligned}
$$

with

$$
\mathcal{L}_W \equiv \log\left(\frac{2\Delta E}{\lambda}\left|\frac{\Delta_W}{\Delta_W - 2\sqrt{s}\Delta E}\right|\right)
$$

and δ_p from Eq. 3.26,

$$
F_{BR}^{final}(s) = F_{BR}^{initial}(s) \quad \text{with} \quad [(i,i');\mathcal{L}_W,\delta_p] \to [(f,f');\log\left(\frac{2\Delta E}{\lambda}\right);0] \qquad \text{(D.44)}
$$

and

$$
\begin{aligned}
F_{BR}^{interf.}(s,t) &= \frac{\alpha}{4\pi}\left\{Q_iQ_f\left[4\log\left(\frac{t^2}{m_f^2m_i^2}\right)\;\mathcal{L}_W - \log^2\left(\frac{s}{m_f^2}\right) - \log^2\left(\frac{s}{m_i^2}\right)\right.\right.\\
&- \left.4\mathrm{Sp}\left(1+\frac{s}{t}\right) - \frac{4}{3}\pi^2\right]\\
&+ 2\,Q_{i'}Q_{f'}\;[(f,i)\to(f',i')] - 2\,Q_{i'}Q_f\;[(i,t)\to(i',u)]\\
&- 2\,Q_iQ_{f'}\;[(f,t)\to(f',u)]\\
&- 2\,Q_i(Q_f-Q_{f'})\left[2\log\left(\frac{s}{m_i^2}\right)\;\mathcal{L}_W - \frac{1}{2}\log^2\left(\frac{s}{m_i^2}\right) - \frac{\pi^2}{3}\right]\\
&+ 2\,Q_{i'}(Q_f-Q_{f'})\;[i\to i'] - 2\,Q_f(Q_i-Q_{i'})\;[i\to f]\\
&+ \left.2\,Q_{f'}(Q_i-Q_{i'})\;[i\to f'] + 8\;(Q_i-Q_{i'})(Q_f-Q_{f'})\,[\mathcal{L}_W-1]\right\}\;. \qquad \text{(D.45)}
\end{aligned}
$$

E The hard photon contribution

The differential cross section for the process $i(p_i)i'(p_{i'}) \to f(p_f)f'(p_{f'})\gamma(k)$ reads in the CMS as follows (with $s = (q^0)^2$ and q^0 denotes the CM energy):

$$d\sigma_h = \frac{1}{2s}\frac{1}{(2\pi)^5}\,\frac{d^3p_f\,d^3p_{f'}\,d^3k}{8p_f^0 p_{f'}^0 k^0}\;\delta(p_i + p_{i'} - p_f - p_{f'} - k)\,\overline{\sum}|\mathcal{M}_{BR}|^2\,, \tag{E.1}$$

where the matrix element $\mathcal{M}_{BR}$ results from the application of the MSM Feynman-rules to the bremsstrahlung diagrams shown in Fig. 3 (now without any restriction on the photon momentum k; $\Delta_W = s - M_W^2$)

$$\begin{aligned}\mathcal{M}_{BR} &= i\frac{\pi\alpha}{2s_w^2}\,\sqrt{4\pi\alpha}\,\frac{1}{\Delta_W}\,\Big\{\overline{u}_f\,G^{\rho}_{\mu,f}\,(1-\gamma_5)\,v_{f'}\,\overline{v}_{i'}\gamma^{\mu}(1-\gamma_5)u_i \\ &- \frac{\Delta_W}{\Delta_W - 2kq}\,[\overline{u}_f\,\gamma_\mu(1-\gamma_5)\,v_{f'}\,\overline{v}_{i'}\,G_i^{\mu\rho}(1-\gamma_5)\,u_i]\Big\}\;\epsilon^*_\rho(k)\,,\end{aligned} \tag{E.2}$$

where ϵ_ρ denotes the photon polarisation vector and

$$\begin{aligned}G_f^{\mu\rho} &= Q_f\,\frac{(p_f^\rho + \gamma^\rho\;\not{k}/2)\,\gamma^\mu}{kp_f} - Q_{f'}\,\frac{\gamma^\mu\,(p_{f'}^\rho + \not{k}\,\gamma^\rho/2)}{kp_{f'}} - \frac{\gamma^\mu\,q^\rho + k^\mu\,\gamma^\rho - g^{\mu\rho}\;\not{k}}{kq} \\ G_i^{\mu\rho} &= Q_i\,\frac{\gamma^\mu\,(p_i^\rho - \not{k}\,\gamma^\rho/2)}{kp_i} - Q_{i'}\,\frac{(p_{i'}^\rho - \gamma^\rho\;\not{k}/2)\,\gamma^\mu}{kp_{i'}} - \frac{\gamma^\mu\,q^\rho - k^\mu\,\gamma^\rho + g^{\mu\rho}\;\not{k}}{kq}\,.\end{aligned} \tag{E.3}$$

The initial and final state currents are separately conserved: $k_\rho\,G_f^{\mu\rho} = (Q_f - Q_{f'} - 1)\gamma^\mu = 0$ and $k_\rho\,G_i^{\mu\rho} = (Q_i - Q_{i'} - 1)\gamma^\mu = 0$. At first, the Lorentz-invariant 3-particle phase space

$$I = \int\,\frac{d^3p_f\,d^3p_{f'}\,d^3k}{8p_f^0\,p_{f'}^0\,k^0}\;\delta(p_i + p_{i'} - p_f - p_{f'} - k) \tag{E.4}$$

will be thoroughly discussed. Under consideration of the energy momentum conservation described by the δ-function the phase space integration will be rewritten, so that only the photon phase space integration survives in order to gain the photon spectra describing hard photon radiation. We follow the procedure suggested in [39],[38] and choose the following coordinate system: the momenta $\vec{p}_i$ and $\vec{k}$ are in the (1,3)-plane, with the photon momentum along the third axis.

The spatial part of the δ-function constraints the momenta in such a way, that in the CMS ($\vec{q} = \vec{p}_i + \vec{p}_{i'} = 0$) the relation $|\vec{p}_f| = |\vec{p}_{f'} + \vec{k}| = p_f^0$ holds and the phase space integral can be written as follows

$$I = 2\pi\,\int_{\Delta E}^{\omega}\frac{|\vec{k}|\,k^0\,dk^0}{2\,k^0}\int_{-1}^{1}dx\int_{p_a}^{p_b}\frac{|\vec{p}_{f'}|\,p_{f'}^0\,dp_{f'}^0}{2\,p_{f'}^0}\int_0^{2\pi}d\Phi\int_{-1}^{1}\frac{dz}{2\,p_f^0}\delta(p_i^0 + p_{i'}^0 - p_{f'}^0 - k^0 - p_f^0) \tag{E.5}$$

with $x = \cos\angle(\vec{k},\vec{p_i}), z = \cos\angle(\vec{k},\vec{p_{f'}})$ and Φ denotes the azimuthal angle of $p_{f'}$ with respect to the (1,3)-plane. Since the soft photon contribution has already been discussed separately the lower bound of photon phase space integration can be chosen to be $|\vec{k}| = \Delta E$ and no IR singularities occur. Using

$$\delta(f(x)) = \frac{\delta(x - x_0)}{|f'(x)|_{x=x_0}}\,, \tag{E.6}$$

where $f(x)$ is an arbitrary function with $f(x_0)=0$ (here: $f(z)=p_f^0$)

$$\delta(p_i^0+p_{i'}^0-p_{f'}^0-k^0-p_f^0)=\left|\frac{p_f^0}{|\vec{k}|\,|\vec{p}_{f'}|}\right|_{z=z_0}\delta(z-z_0)$$

with

$$2|\vec{k}|\,|\vec{p}_{f'}|\,z_0=(q^0-k^0-p_{f'}^0)^2-(k^0)^2-(p_{f'}^0)^2+m_{f'}^2-m_f^2\;,$$

the phase space integral $I(s)$ can be written as follows:

$$I=\pi\int_{\Delta E}^{\omega}\frac{dk^0}{2}\int_{-1}^{1}dx\int_{p_a}^{p_b}\frac{dp_{f'}^0}{2}\int_0^{2\pi}d\Phi\;. \tag{E.7}$$

The requirement $-1\le z_0\le 1$ leads to the following limits on the $p_{f'}^0$-integration:

$$p_{a,b} \;=\; \frac{(q^0-k^0)\,\kappa\,\pm\,k^0\,\sqrt{(\kappa-2m_{f'}^2)^2-4\,m_{f'}^2\,m_f^2}}{2\,(\kappa-m_{f'}^2+m_f^2)} \tag{E.8}$$

$$\omega \;=\; \frac{(q^0)^2-(m_f+m_{f'})^2}{2\,q^0} \tag{E.9}$$

with

$$\kappa=q^0\,(q^0-2k^0)+m_{f'}^2-m_f^2\;.$$

Finally, after introducing a new variable y

$$p_{f'}^0=\frac{\kappa}{2\,(q^0-k^0)}+\frac{k^0\,p_i^0}{q^0}\,y\;,$$

the starting point for obtaining the hard photon spectra is reached (with $p_i^0=q^0/2$)

$$\sigma_h(s)=\frac{1}{16s}\frac{1}{(2\pi)^4}\int_{\Delta E}^{\omega}\frac{dk^0\,k^0}{2}\int_{-1}^{1}dx\int_{y_a}^{y_b}dy\int_0^{2\pi}d\Phi\,\overline{\sum}|\mathcal{M}_{BR}|^2\;. \tag{E.10}$$

The computation of the spin averaged squared matrix element leads to the initial state, final state and interference contributions depending only on the scalar products of the involved four momenta, which have to be expressed in terms of the integration variables, e.g.

$$p_{i'}\,p_{f'}=p_{i'}^0\,p_{f'}^0+|\vec{p}_i|\,|\vec{p}_{f'}|\,\cos\varphi \tag{E.11}$$

with

$$\cos\varphi=(xz+\sqrt{1-x^2}\,\sqrt{1-z^2}\,\cos\Phi)|_{z=z_0}\;.$$

Finally, the performance of all integrations up to the one over the photon energy yields the following hard photon spectra (with $k=2k^0/q^0$ and $k_m=2\Delta E/q^0$):

$$\sigma_h^{initial}(s) \;=\; \tilde{\sigma}^{(0)}(s)\int_{k_m}^{1}dk\left|\frac{\Delta_W}{\Delta_W-sk}\right|^2\frac{1-k}{2k}\left\{\beta_i(s)\,[1+(1-k)^2]+\frac{\alpha}{\pi}\frac{k^2}{3}\right\} \tag{E.12}$$

$$\begin{aligned}\sigma_h^{final}(s) \;&=\; \tilde{\sigma}^{(0)}(s)\int_{k_m}^{1}\frac{dk}{2k}\left\{\beta_f(s)\,[1+(1-k)^2]+\frac{\alpha}{\pi}\,\frac{k^2}{3}\right.\\ &+\;\left.\frac{\alpha}{\pi}(Q_f^2+Q_{f'}^2)\,[1+(1-k)^2]\,\log(1-k)]\right\}\end{aligned} \tag{E.13}$$

$$\sigma_h^{interf.}(s) \;=\; \tilde{\sigma}^{(0)}(s)\frac{\alpha}{\pi}\int_{k_m}^{1}\frac{dk}{k}\left[\frac{\Delta_W}{\Delta_W - sk} + \frac{\Delta_W^*}{\Delta_W^* - sk}\right]\frac{5}{12}\left\{3k - k^2 - 2\right\} \qquad \text{(E.14)}$$

The final state hard photon spectrum $\sigma_h^{final}(s)$ coincides with the result obtained in [18]. From the photon spectra the total cross sections describing hard photon radiation can be obtained

$$\begin{aligned}\sigma_h^{initial}(s) \;&=\; \tilde{\sigma}^{(0)}(s)\,\beta_i(s)\left\{\log\left(\frac{|\Delta_W - 2\sqrt{s}\Delta E|}{2\sqrt{s}\Delta E}\right)\right. \\ &+\; \left.\frac{s - M_W^2}{M_W\Gamma_W^{(0+1)}}\left[\arctan\left(\frac{M_W}{\Gamma_W^{(0+1)}}\right) - \arctan\left(\frac{2\sqrt{s}\Delta E - s + M_W^2}{M_W\Gamma_W^{(0+1)}}\right)\right]\right\} \qquad \text{(E.15)}\end{aligned}$$

$$\begin{aligned}\sigma_h^{final}(s) \;&=\; \tilde{\sigma}^{(0)}(s)\left\{\beta_f(s)\,\log\left(\frac{\sqrt{s}}{2\Delta E}\right)\right. \\ &+\; \left.\frac{\alpha}{\pi}\left[Q_f^2\left(-\frac{3}{4}\log\left(\frac{s}{m_f^2}\right) - \frac{\pi^2}{6} + \frac{11}{8}\right) + Q_{f'}^2\,(f \to f') + \frac{5}{6}\right]\right\} \qquad \text{(E.16)}\end{aligned}$$

$$\sigma_h^{interf.}(s) \;=\; \tilde{\sigma}^{(0)}(s)\,\frac{\alpha}{\pi}\frac{1}{3}\left[5(Q_iQ_f + Q_{i'}Q_{f'}) + 4(Q_{i'}Q_f + Q_{f'}Q_i)\right]\,\log\left(\frac{2\Delta E\sqrt{s}}{|\,\Delta_W - 2\sqrt{s}\Delta E\,|}\right)\,. \qquad \text{(E.17)}$$

Since we are interested on the contribution in the vicinity of the W resonance terms $\propto (s - M_W^2)$ and $\propto \Delta E$ have been neglected.

The parametrisation of the 3-particle phase space in the course of the computation of hard bremsstrahlung for the case of the W width is less complicated, since the orientation of the dreibein made of the three outgoing momenta can be freely chosen: the solid angle Ω determines the orientation of the photon momentum and Φ describes the rotation of the $(\vec{p}_f, \vec{p}_{f'})$-system around $\vec{k}$. Thus, the hard photon contribution to the partial W width (in the CMS of the W boson with $q^2 = M_W^2$)

$$d\Gamma_{W\to ff'}^{h} = \frac{1}{2M_W}\frac{1}{(2\pi)^5}\,\frac{d^3p_f\,d^3p_{f'}\,d^3k}{8p_f^0p_{f'}^0k^0}\,\delta(q - p_f - p_{f'} - k)\,\overline{\sum}|\mathcal{M}_{BR}^{final}|^2\,, \qquad \text{(E.18)}$$

turns into [38]

$$\Gamma_{W\to ff'}^{h} = \frac{1}{2M_W}\frac{1}{256\pi^5}\int_{\Delta E}^{\omega}dk^0\int_0^{4\pi}d\Omega\int_0^{2\pi}d\Phi\int_{x_-}^{x^+}dx\overline{\sum}|\mathcal{M}_{BR}^{final}|^2\,, \qquad \text{(E.19)}$$

where ω is given by Eq. E.9 and the substitution $p_{f,f'} = \pm x + (M_W - k^0)/2)$ has been performed. The limits on the x-integration $x_\pm$ are given by

$$x_\pm = \frac{1}{2\tilde{M}}\left\{\frac{m_f^2 - m_{f'}^2}{2M_W}(M_W - k^0) \pm k^0\sqrt{(\tilde{M} - \frac{(m_f + m_{f'})^2}{2M_W})(\tilde{M} - \frac{(m_f - m_{f'})^2}{2M_W})}\right\} \qquad \text{(E.20)}$$

with $\tilde{M} = M_W/2 - k^0$. The matrix element $\mathcal{M}_{BR}^{final}$ reads as follows (η^μ: polarisation vector of the W boson):

$$\mathcal{M}_{BR}^{final} = i\,\frac{\sqrt{(2)}\pi\alpha}{s_w}\,\overline{u}_f\,G_{\mu,f}^{\rho}\,(1-\gamma_5)\,v_{f'}\,\eta^\mu(q)\,\epsilon_\rho^*(k) \qquad \text{(E.21)}$$

with $G^{\rho}_{\mu,f}$ given by Eq. E.3, which leads to the same hard photon spectrum as for the case of finale state bremsstrahlung in the 4-fermion process ($\rightarrow$ Eq. E.13)

$$
\begin{aligned}
\Gamma^{h}_{W\rightarrow ff'} &= \Gamma^{(0)}_{W\rightarrow ff'} \int_{k_m}^{1} \frac{dk}{2k} \left\{\beta_f(M_W^2)\,[1+(1-k)^2]\right. \\
&+ \left.\frac{\alpha}{\pi}\,\frac{k^2}{3} + \frac{\alpha}{\pi}(Q_f^2+Q_{f'}^2)\,[1+(1-k)^2]\,\log(1-k)]\right\} \\
&=: \Gamma^{(0)}_{W\rightarrow ff'}\,\delta\Gamma^{h}_{BR}\,. \qquad\qquad (E.22)
\end{aligned}
$$

Thus, the factor $\delta\Gamma^{h}_{BR}$ coincides with the one, which multiplies the Born-cross section in Eq. E.16 evaluated at $s = M_W^2$.

F Integrals

In the following, the explicit expressions for some special cases of scalar 2-, 3- and 4-point integrals and of photon phase space integrals will be provided, which have been derived in course of the calculation of the photonic corrections usually developing IR and/or on-shell singularities. The dimensional regularisation enables the extraction of the UV-divergence occurring in the scalar and vectorial 2-point integrals $B_{0,1}$ ($\int_D \equiv \mu^{4-D} \int \frac{d^Dk}{(2\pi)^D}$)

$$
\frac{i}{16\pi^2}(B_0; p_\mu\, B_1)(p^2,m_1,m_2) = \mu^{4-D} \int \frac{d^Dk}{(2\pi)^D}\,\frac{(1;k_\mu)}{[k^2-m_1^2]\,[(k+p)^2-m_2^2]}\,, \qquad (F.1)
$$

so that they can be written as follows [22]:

$$
\begin{aligned}
B_0(p^2,m_1,m_2) &= \Delta - \int_0^1 dx\,\log\frac{x^2p^2 - x(p^2+m_1^2-m_2^2)+m_1^2-i\epsilon}{\mu^2} \qquad (F.2)\\
B_1(p^2,m_1,m_2) &= \frac{1}{2p^2}\,[m_1^2(\Delta_{m_1}+1) - m_2^2(\Delta_{m_2}+1) \\
&\quad +(m_2^2-m_1^2-p^2)\,B_0(p^2,m_1,m_2)]\,. \qquad (F.3)
\end{aligned}
$$

The following results for the scalar integrals have been used [22], [40]:

$$
B_0(p^2,\lambda=0,m) = \Delta_m + 2 + \left(\frac{m^2}{p^2}-1\right)\,\log\left(1-\frac{p^2}{m^2}-i\epsilon\right) \qquad (F.4)
$$

$$
\left.\frac{\partial B_0(p^2,\lambda,m)}{\partial p^2}\right|_{p^2=m^2} = -\frac{1}{m^2}\,\left[\log\left(\frac{\lambda}{m}\right)+1\right] \qquad (F.5)
$$

$$
\begin{aligned}
C_0(s,m_f,m_{f'},\lambda) &= \int_{D=4} \frac{1}{[k^2-\lambda^2]\,[(k+p_{f'})^2-m_{f'}^2]\,[(k-p_f)^2-m_f^2]} \\
= -\frac{1}{s}&\left[\log\left(\frac{s}{m_fm_{f'}}\right)\log\left(\frac{\lambda^2}{s}\right) + \frac{1}{4}\log^2\left(\frac{s}{m_f^2}\right) + \frac{1}{4}\log^2\left(\frac{s}{m_{f'}^2}\right) + \frac{2}{3}\pi^2 - i\pi\log\left(\frac{\lambda^2}{s}\right)\right]
\end{aligned}
$$

(F.6)

$$C_0(s, M_W, m_f, \lambda) = \int_{D=4} \frac{1}{[k^2-\lambda^2]\,[(k-p_f)^2-m_f^2]\,[(k-q)^2-M_W^2]}$$
$$= \frac{1}{s}\left[\log\left(\frac{s}{m_f^2}\right)\log\left(1-\frac{s}{M_W^2}-i\epsilon\right) - \mathrm{Sp}(1-\frac{M_W^2}{s}) - \frac{1}{2}\log^2\left(\frac{M_W^2}{s}\right) - \frac{\pi^2}{6}\right] \tag{F.7}$$

$$C_0(s=M_W^2, M_W, m_f, \lambda) = \frac{1}{M_W^2}\left[2\log\left(\frac{M_W}{m_f}\right)\log\left(\frac{\lambda}{M_W}\right) + \log^2\left(\frac{M_W}{m_f}\right)\right] \tag{F.8}$$

$$\begin{aligned} C_0(1) \equiv C_0(t, m_f, m_i, M_W) &= \int_{D=4}\frac{1}{[(k-p_f)^2-m_f^2]\,[(k-p_i)^2-m_i^2]\,[(k-q)^2-M_W^2]} \\ &= -\frac{1}{t}\left[\mathrm{Sp}(1+\frac{t+i\epsilon}{M_W^2}) - \frac{\pi^2}{6}\right] \end{aligned} \tag{F.9}$$

$$C_0(3;4) \equiv C_0(s, M_W, (m_f; m_i), \lambda) \tag{F.10}$$

$$C_0(2) \equiv C_0(t, m_f, m_i, \lambda) = \int_{D=4}\frac{1}{[k^2-\lambda^2]\,[(k-p_f)^2-m_f^2]\,[(k-p_i)^2-m_i^2]}$$
$$= -\frac{1}{2t}\left[\log\left(\frac{t^2}{m_f^2 m_i^2}\right)\log\left(\frac{\lambda^2}{s}\right) - \frac{1}{4}\log^2\left(\frac{t^2}{s^2}\right) + \frac{1}{2}\log^2\left(\frac{s}{m_f^2}\right) + \frac{1}{2}\log^2\left(\frac{s}{m_i^2}\right) + \frac{\pi^2}{3}\right] \tag{F.11}$$

$$D_0(s, t, m_f, m_i, M_W, \lambda) = \int_{D=4}\frac{1}{[k^2-\lambda^2]\,[(k-p_f)^2-m_f^2]\,[(k-p_i)^2-m_i^2]\,[(k-q)^2-M_W^2]}$$
$$= -\frac{1}{t}\,\frac{1}{s-M_W^2}\left[\log\left(\frac{t^2}{m_f^2 m_i^2}\right)\log\left(\frac{M_W\lambda}{M_W^2-s-i\epsilon}\right) + \log^2\left(\frac{m_f}{M_W}\right) + \log^2\left(\frac{m_i}{M_W}\right)\right.$$
$$\left. + \mathrm{Sp}(1+\frac{M_W^2}{t+i\epsilon}) + \frac{\pi^2}{3}\right] \tag{F.12}$$

In addition, the following soft photon phase space integrations have been performed ($\int_k \equiv \int \frac{d^3k}{2(2\pi)^3\,k^0}$ and $\Delta_W = s - M_W^2$ is considered to be complex):

$$\int_k \frac{\Delta_W\,|2p_ip_j|}{(\Delta_W - 2k^0q^0)(kp_i)(kp_j)} \overset{p_i\neq p_j}{=} \frac{1}{2(2\pi)^2}\left\{2\log\left(\frac{(2p_ip_j)^2}{p_i^2p_j^2}\right)\log\left(\frac{2\Delta E}{\lambda}\frac{\Delta_W}{\Delta_W - 2\sqrt{s}\Delta E}\right) - I_x\right\} \tag{F.13}$$

with

$$\begin{aligned} I_x &= \log\left(\frac{(2p_ip_j)^2}{p_i^2p_j^2}\right)\log\left(\frac{s}{|2p_ip_j|}\right) + \frac{1}{2}\log^2\left(\frac{p_i^2}{|2p_ip_j|}\right) + \frac{1}{2}\log^2\left(\frac{p_j^2}{|2p_ip_j|}\right) \\ &+ 2\mathrm{Sp}(1-\frac{s}{2p_ip_j}) + \log^2\left(\frac{s}{|2p_ip_j|}\right) + \left\{\frac{2\pi^2}{3};\frac{\pi^2}{3}\right\}\ , \end{aligned} \tag{F.14}$$

where the second term in the curly bracket has to be used, when one of the momenta p_i, p_j is equal to the CM momentum q.

$$\int_k \frac{\Delta_W\, p^2}{(\Delta_W - 2k^0 q^0)(kp)^2} \quad = \quad \frac{1}{2(2\pi)^2}\left\{2\log\left(\frac{2\Delta E}{\lambda}\frac{\Delta_W}{\Delta_W - 2\sqrt{s}\Delta E}\right) - \tilde{I}_x\right\} \tag{F.15}$$

with

$$\tilde{I}_x = \log\left(\frac{s}{p^2}\right) + \{0;2\}\ , \tag{F.16}$$

where again the second term in the curly bracket has to be taken, when $p \equiv q$ holds.

$$\int_k \frac{2p_ip_j}{(kp_i)(kp_j)} \quad \stackrel{p_i \neq p_j}{=} \quad \frac{1}{2(2\pi)^2}\left\{2\log\left(\frac{(2p_ip_j)^2}{m_i^2 m_j^2}\right)\log\left(\frac{2\Delta E}{\lambda}\right) - I_x\right\} \tag{F.17}$$

$$\int_k \frac{p^2}{(kp)^2} \quad = \quad \frac{1}{2(2\pi)^2}\left\{2\log\left(\frac{2\Delta E}{\lambda}\right) - \tilde{I}_x\right\}\ . \tag{F.18}$$

Finally, the functions α_i, v_{ij} and η_{ij} used in order to describe the ξ_i-dependence of the form factors are defined as follows [35]:

$$v_{ij}(q^2) \equiv \alpha_i - 2\beta_{ij}(q^2) - q^2\eta_{ij}(q^2) \tag{F.19}$$

with

$$\begin{aligned}
\frac{i}{16\pi^2}\alpha_i &= \int_D \frac{1}{[k^2 - m_i^2]\,[k^2 - \xi_i m_i^2]} \\
\frac{i}{16\pi^2}\beta_{ij}(q^2) &= t^{\mu\nu}\int_D \frac{k_\mu k_\nu - g_{\mu\nu} m_j^2}{[k^2 - m_i^2]\,[k^2 - \xi_i m_i^2]\,[(k+q)^2 - m_j^2]} \\
\frac{i}{16\pi^2}\eta_{ij}(q^2) &= t^{\mu\nu}\int_D \frac{1}{[k^2 - m_i^2]\,[k^2 - \xi_i m_i^2]\,[(k+q)^2 - m_j^2]}\left[2g_{\mu\nu} + \frac{(\xi_j - 1)k_\mu k_\nu}{[(k+q)^2 - \xi_j m_j^2]}\right]\ ,
\end{aligned} \tag{F.20}$$

where the abbreviations $t^{\mu\nu} = (g^{\mu\nu} - q^\mu q^\nu/q^2)/(D-1)$ and $\int_D \equiv \mu^{4-D}\int \frac{d^Dk}{(2\pi)^D}$ have been used.

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