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QCD Corrections to the Top Decay Mode $t \to \tilde{t}\chi^0$

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Abstract

In supersymmetric theories, the top quark can decay into its scalar partner plus a neutralino, with an appreciable rate. We calculate the $\mathcal{O}(\alpha_s)$ QCD corrections to this decay mode in the minimal supersymmetric extension of the Standard Model. These corrections can be either positive or negative and increase logarithmically with the gluino mass. For gluino masses below 1 TeV, they are at most of the order of twenty percent and therefore, well under control.

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1. Introduction

It is well–known that in supersymmetric theories (SUSY), the heavy top quark can decay at tree–level into its lightest scalar partner and a light neutralino state [1, 2]. This possibility is intimately related to the large value of the top mass m_t: the mixing between the left– and right–handed scalar partners of the top quark, $\tilde{t}_L$ and $\tilde{t}_R$, is proportional to m_t; after diagonalization of the mass matrix, the lightest stop mass eigenstate $\tilde{t}$ can be much lighter than the top quark and all the scalar partners of the light quarks [1].

The experimental bounds [3] on the lightest stop and neutralino masses leave a large room for this decay to occur. Moreover, light stop squarks and chargino states [and hence the light neutralino χ_1^0 which is the lightest supersymmetric particle] might explain the large deviation of the experimentally measured value of the ratio $R_b = \Gamma(Z \rightarrow b\bar{b})/\Gamma(Z \rightarrow \text{hadrons})$ from the Standard Model prediction [4]. In fact, in the area of the MSSM parameter space which provides the best fit [4] to the electroweak precision data, all four neutralinos are light enough [while the charginos are heavier than the present experimental bounds [3] from LEP1.5] for the top quark to decay into the lightest stop squark plus one of the four neutralino states.

In the minimal supersymmetric extension of the Standard Model (MSSM), the decay $t \rightarrow \tilde{t}\chi_1^0$ could have a sizeable branching fraction[1] when it is allowed kinematically [2]. In a large area of the SUSY parameter space, this decay can have a signature similar to that of the standard decay $t \rightarrow bW^+ \rightarrow bl^+\nu$ or b+2jets [5]. Indeed, the stop could mainly decay into the lightest chargino and a bottom quark, and the chargino will subsequently decay into a neutralino and a virtual W boson which then goes into $l^+\nu$ or 2–jets. This gives the same topology as the $t \rightarrow bW^+$ channel, except for the large amount of missing energy in the SUSY decay mode due to the escaping neutralinos, which leads to a softer transverse momentum spectrum of the charged leptons. Such decays could have an impact on the top quark studies at the Tevatron [6], altering for instance the reported production cross section values[2] since these events would not all pass the experimental cuts designed for the standard $t \rightarrow bW^+$ decay.

A detailed study of the $t \rightarrow \tilde{t}\chi_1^0$ decay mode is therefore mandatory. To have a more reliable prediction for the decay branching ratio, the QCD corrections have to be taken into account. The one–loop QCD corrections to the two other decay modes of the top quark in supersymmetric theories, $t \rightarrow bW^+$ and $t \rightarrow H^+b$, can be found in Refs.[8] and [9] respectively. In this article, we present the $\mathcal{O}(\alpha_s)$ QCD corrections to the top decay into a stop squark and a neutralino. These results can be easily generalized to the possible decay into a light sbottom squark and a chargino state.

[1]The present agreement between the top quark production cross section at the Tevatron [6] and the Standard Model prediction still allows appreciable $t \rightarrow \tilde{t}\chi_1^0$ decay rates. A detailed analysis of the area of the parameter space where these decays are allowed or disfavored by Tevatron data is beyond the scope of the present paper.

[2]For such light stop squarks, the pair production $pp \rightarrow \tilde{t}\tilde{t}^*$ is possible at the Tevatron and will also contribute to the event samples; for a recent discussion of this possibility see for instance Ref.[7].

2. Born Approximation

In the MSSM, the decay width $\Gamma(t \to \tilde{t}_i \chi_j^0)$ [where $i = 1,2$ are for the stop and j = 1–4 for the neutralino states] will depend on five parameters: the left– and right–handed scalar stop masses [which in general are taken to be equal], the Higgs–higgsino mass parameter μ, the soft–SUSY breaking trilinear coupling A_t, the ratio of the vacuum expectation values of the two Higgs doublet MSSM fields $\tan\beta$ [which fix the stop masses and the mixing angle] as well as the wino mass parameter M_2 [which together with μ and $\tan\beta$ fixes the neutralino masses]; the gluino mass is related to the parameter $M_2, m_{\tilde{g}} \sim 3.5 M_2$, when the gaugino masses and the three coupling constants of SU(3)×SU(2)×U(1) are unified at the Grand Unification scale.

In the Born approximation, the partial widths for the decays $t \to \tilde{t}_i \chi_j^0$ [we will drop for simplicity the indices for the neutralino masses, $m_\chi \equiv m_{\chi_j^0}$] are given by

$$\Gamma(t \to \tilde{t}_i \chi_j^0) = \frac{\alpha}{8\, m_t^3} \left\{ a_L^i\, a_R^i\, (4\, m_t\, m_\chi \epsilon_\chi) + ({a_L^i}^2 + {a_R^i}^2)\, (m_t^2 - m_{\tilde{t}_i}^2 + m_\chi^2) \right\} \lambda^{1/2}(m_t^2, m_\chi^2, m_{\tilde{t}_i}^2) \quad (1)$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2\,(xy + xz + yz)$ is the usual two–body phase space function and ϵ_χ is the sign of the eigenvalue of the neutralino χ; the couplings $a_{L,R}^i$ which form the Born $\chi t \tilde{t}_i$ vertex

$$\Gamma_0^i = ie \left[a_L^i P_L + a_R^i P_R \right] \quad (2)$$

with $P_{L,R}$ the chirality projectors $P_{L,R} = (1 \mp \gamma_5)/2$, are given by

$$\left\{ \begin{array}{c} a_L^1 \\ a_L^2 \end{array} \right\} = b\, m_t \left\{ \begin{array}{c} s_\theta \\ c_\theta \end{array} \right\} + c_L \left\{ \begin{array}{c} c_\theta \\ -s_\theta \end{array} \right\} , \qquad \left\{ \begin{array}{c} a_R^1 \\ a_R^2 \end{array} \right\} = b\, m_t \left\{ \begin{array}{c} c_\theta \\ -s_\theta \end{array} \right\} + c_R \left\{ \begin{array}{c} s_\theta \\ c_\theta \end{array} \right\}$$

$$\begin{aligned} b &= \frac{1}{\sqrt{2}\, M_W \sin\beta s_W}\, N_{j4} \\ c_L &= \sqrt{2} \left[\frac{2}{3}\, N'_{j1} + \left(\frac{1}{2} - \frac{2}{3}\, s_W^2 \right) \frac{1}{c_W s_W}\, N'_{j2} \right] \\ c_R &= -\sqrt{2} \left[\frac{2}{3}\, N'_{j1} - \frac{2}{3} \frac{s_W}{c_W}\, N'_{j2} \right] \end{aligned} \quad (3)$$

Here θ is the stop mixing angle [which can be expressed in terms of the Higgs–higgsino SUSY mass parameter μ, $\tan\beta$ and the soft–SUSY breaking trilinear coupling A_t] with $s_\theta = \sin\theta$, $c_\theta = \cos\theta$ etc.; $s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W$. N is the diagonalizing matrix for the neutralino states [10] and

$$N'_{j1} = c_W N_{j1} + s_W N_{j2} \quad , \quad N'_{j2} = -s_W N_{j1} + c_W N_{j2} \quad (4)$$

The masses of the two stop squarks and their mixing angle are determined by diagonalizing the following mass matrix:

$$\mathcal{M}_{\tilde{t}}^2 = \left(\begin{array}{cc} M_{\tilde{t}_L}^2 + m_t^2 + \cos 2\beta (\frac{1}{2} - \frac{2}{3} s_W^2)\, M_Z^2 & m_t\, M_{LR} \\ m_t\, M_{LR} & M_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3} \cos 2\beta\; s_W^2\, M_Z^2 \end{array} \right) \quad (5)$$

where M_{LR} in the off–diagonal term reads $M_{LR} = A_t - \mu \cot\beta$.

Assuming that the MSSM charged Higgs boson, H^+, is heavier than the top quark so that the decay channel $t \to H^+ b$ is kinematically closed, the branching ratios for the $t \to \tilde{t}_2 \chi_j^0$ decay modes [$\tilde{t}_2$ is the lightest stop squark in the notation introduced above] are shown in Fig. 1 for $\tan\beta = 1.6$, a value favored by theories with b–τ Yukawa coupling unification. The top quark mass is fixed to $m_t = 180$ GeV. In Fig.1a, we fix the wino mass to $M_2 = \mu = 150, 200$ and 250 GeV and plot the branching ratio for the decay into the stop and the lightest neutralinos as a function of the $\tilde{t}_2$ mass [obtained by varying the parameter A_t]. In Fig.1b, M_2 is kept fixed at $M_2 = 400$ GeV, and two values have been chosen for μ, -100 and 150 GeV. Whenever the decay in more than one of the neutralinos is possible, the branching ratio has been summed. As can be seen, it can reach the level of several ten percent, and can even dominate over the standard $t \to bW^+$ decay mode.

In Fig.1c, we choose the parameter μ to be negative and equal to the wino mass $M_2 = -\mu = 50$ GeV; this area of the MSSM parameter space, together with a rather light right–handed stop squark, $m_{\tilde{t}} \lesssim 100$ GeV, provides the best MSSM fit to the high–precision electroweak data, and enhances the ratio $R_b = \Gamma(Z \to b\bar{b})/\Gamma(Z \to \text{hadrons})$ to the level where it is almost consistent with its experimental value [4]. The branching ratios of the four decays $t \to \tilde{t} + \chi_j^0$ with j =1–4 [the masses of the neutralinos can be read off from the thresholds, and the lightest chargino in this scenario has a mass $m_{\chi_1^+} \sim 70$ GeV] are shown together with their sum. They are also large, and can dominate over the $t \to bW^+$ branching ratio.

3. QCD corrections

The QCD corrections to the decay width, eq.(1), consist of virtual corrections Figs.2a–d, and real corrections with an additional gluon emitted off the initial or final state, Fig.2e. The $\mathcal{O}(\alpha_s)$ virtual contributions can be split into gluon and gluino exchange in the t–$\tilde{t}$–χ vertex, and the top and stop wave function renormalizations. The renormalization of the t–$\tilde{t}$–χ coupling is achieved by renormalizing the top quark mass and the stop mixing angle. We will use the dimensional reduction scheme [11] which preserves supersymmetry[3] to regularize the ultraviolet divergencies and we introduce a fictitious gluon mass λ to regularize the infrared divergencies. The sum of the virtual corrections obtained by taking the interference between the Born and QCD corrected amplitudes, and the real correction, obtained by squaring the sum of the two diagrams in Fig.2e, is finite as it should be.

3.1 Virtual Corrections

The QCD corrections to the t–$\tilde{t}_i$–χ vertex, eq.(2), can be written as

$$\delta\Gamma^i = ie\,\frac{\alpha_s}{3\pi} \sum_{j=g,\tilde{g},mix,ct} \left[G^i_{j,L} P_L + G^i_{j,R} P_R\right] \tag{6}$$

[3]Because the $\chi\tilde{t}t$ coupling is renormalized by QCD, the dimensional regularization and dimensional reduction schemes give slightly different results.

where G^i_g, $G^i_{\tilde{g}}$, G^i_{mix} and G^i_{ct} denote the gluon and gluino exchanges in the vertex, and the mixing and counterterm contributions, respectively.

The gluon exchange contributions [Fig.2a] can be written as

$$G^i_{g,L} = a^i_L\, F^i_1 + a^i_R\, F^i_2 \quad , \quad G^i_{g,R} = a^i_R\, F^i_1 + a^i_L\, F^i_2 \tag{7}$$

with the form factors $F^i_{1,2}$ given by

$$\begin{aligned} F^i_1 &= B_0(m^2_{\tilde{t}_i}, \lambda, m_{\tilde{t}_i}) + 2\, m^2_t\, C_0 - 2\, m^2_{\tilde{t}_i}\, (C_{11} - C_{12}) + 2\, m^2_\chi\, C_{11} \\ F^i_2 &= -2\, m_t\, m_\chi\, (C_0 + C_{11}) \end{aligned} \tag{8}$$

where the two and three–point Passarino–Veltman functions, B_0 and $C_{..} \equiv C_{..}(m^2_t, m^2_{\tilde{t}_i}, m^2_\chi, m^2_t, \lambda^2, m^2_{\tilde{t}_i})$ can be found in Ref.[12].

The gluino exchange contributions [Fig2b], are given by

$$\begin{aligned} G^1_{\tilde{g},L} &= a^1_L(s_{2\theta}F^{11}_4 - c_{2\theta}F^{11}_5 - F^{11}_5) + a^1_R(s_{2\theta}F^{11}_1 - c_{2\theta}F^{11}_2 + F^{11}_3) \\ &+ a^2_L(s_{2\theta}F^{12}_5 + c_{2\theta}F^{12}_4 - F^{12}_7) + a^2_R(s_{2\theta}F^{12}_2 + c_{2\theta}F^{12}_1 - F^{12}_6) \\ G^1_{\tilde{g},R} &= a^1_L(s_{2\theta}F^{11}_1 + c_{2\theta}F^{11}_2 + F^{11}_3) + a^1_R(s_{2\theta}F^{11}_4 + c_{2\theta}F^{11}_5 - F^{11}_5) \\ &+ a^2_L(-s_{2\theta}F^{12}_2 + c_{2\theta}F^{12}_1 + F^{12}_6) + a^2_R(-s_{2\theta}F^{12}_5 + c_{2\theta}F^{12}_4 + F^{12}_7) \\ G^2_{\tilde{g},L} &= a^1_L(s_{2\theta}F^{21}_5 + c_{2\theta}F^{21}_4 + F^{21}_7) + a^1_R(s_{2\theta}F^{21}_2 + c_{2\theta}F^{21}_1 + F^{21}_6) \\ &+ a^2_L(-s_{2\theta}F^{22}_4 + c_{2\theta}F^{22}_5 - F^{22}_5) + a^2_R(-s_{2\theta}F^{22}_1 + c_{2\theta}F^{22}_2 + F^{22}_3) \\ G^2_{\tilde{g},R} &= a^1_L(-s_{2\theta}F^{21}_2 + c_{2\theta}F^{21}_1 - F^{21}_6) + a^1_R(-s_{2\theta}F^{21}_5 + c_{2\theta}F^{21}_4 - F^{21}_7) \\ &+ a^2_L(-s_{2\theta}F^{22}_1 - c_{2\theta}F^{22}_2 + F^{22}_3) + a^2_R(-s_{2\theta}F^{22}_4 - c_{2\theta}F^{22}_5 - F^{22}_5) \end{aligned} \tag{9}$$

where in terms of the functions $C^{ik}_{..} = C^{ik}_{..}(m^2_t, m^2_{\tilde{t}_i}, m_\chi, m^2_{\tilde{t}_k}, m^2_{\tilde{g}}, m^2_t)$ [k denotes the virtual (summed over) and i the outgoing squarks], the form factors F^{ik} are given by

$$\begin{aligned} F^{ik}_1 &= (m^2_{\tilde{t}_k} + m^2_t)\, C^{ik}_0 + 2\, m^2_t\, C^{ik}_{11} + (m^2_\chi - 2\, m^2_t)\, C^{ik}_{12} + B_0(m^2_{\tilde{t}_i}, m_{\tilde{g}}, m_t) \\ F^{ik}_2 &= m_{\tilde{g}}\, m_t\, (C^{ik}_0 - C^{ik}_{11} + C^{ik}_{12}) \\ F^{ik}_3 &= m_{\tilde{g}}\, m_t\, (-C^{ik}_0 - C^{ik}_{11} + C^{ik}_{12}) \\ F^{ik}_4 &= m_t\, m_\chi\, (C^{ik}_0 + C^{ik}_{11} + C^{ik}_{12}) \\ F^{ik}_5 &= m_{\tilde{g}}\, m_\chi\, (C^{ik}_0 + C^{ik}_{12}) \\ F^{ik}_6 &= (m^2_{\tilde{t}_k} - m^2_t)\, C^{ik}_0 + m^2_\chi C^{ik}_{12} + B_0(m^2_{\tilde{t}_i}, m_{\tilde{g}}, m_t) \\ F^{ik}_7 &= m_t\, m_\chi(-C^{ik}_0 - C^{ik}_{11} + C^{ik}_{12}) \end{aligned} \tag{10}$$

The mixing contribution is due to the diagrams of Fig.2c where the top–gluino exchange and the quartic scalar interaction contributions switch the Born $\chi t\tilde{t}_1$ vertex to the $\mathcal{O}(\alpha_s)$ $\chi t\tilde{t}_2$ one and *vice versa*; it is given by

$$G^i_{mix,L} = \frac{(-1)^i\, (\delta_{1i}\, a^2_L + \delta_{2i}\, a^1_L)}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \left[4 m_t\, m_{\tilde{g}}\, c_{2\theta}\, B_0(m^2_{\tilde{t}_i}, m_t, m_{\tilde{g}}) + c_{2\theta} s_{2\theta} (A_0(m^2_{\tilde{t}_2}) - A_0(m^2_{\tilde{t}_1}))\right]$$

$$G^i_{mix,R} = \frac{(-1)^i\,(\delta_{1i}\,a_R^2 + \delta_{2i}\,a_R^1)}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \left[4m_t\, m_{\tilde{g}}\, c_{2\theta}\, B_0(m^2_{\tilde{t}_i}, m_t, m_{\tilde{g}}) + c_{2\theta} s_{2\theta}(A_0(m^2_{\tilde{t}_2}) - A_0(m^2_{\tilde{t}_1}))\right] \tag{11}$$

3.2 Counterterms

The counterterm contributions in eq.(6) are due to the top and stop wave function renormalizations [Fig.2d] as well as the renormalization of the top quark mass m_t and the mixing angle θ, which appear in the Born coupling,

$$\begin{aligned} G^{1,2}_{ct,L} &= \frac{1}{2} a_L^{1,2} \left(\delta Z^t_L + \delta Z_{\tilde{t}_{1,2}}\right) + b\,\{s_\theta, c_\theta\}\,\delta m_t + b\, m_t\,\{c_\theta, -s_\theta\}\,\delta\theta - c_L\,\{s_\theta, c_\theta\}\,\delta\theta \\ G^{1,2}_{ct,R} &= \frac{1}{2} a_R^{1,2} \left(\delta Z^t_R + \delta Z_{\tilde{t}_{1,2}}\right) + b\,\{c_\theta, -s_\theta\}\,\delta m_t - b\, m_t\,\{s_\theta, c_\theta\}\,\delta\theta + c_R\,\{c_\theta, -s_\theta\}\,\delta\theta \end{aligned} \tag{12}$$

In the on–shell scheme, the quark and squark masses are defined as the poles of the propagators and the wave–function renormalization constants follow from the residues at the poles; the corresponding counterterms are given by

$$\begin{aligned} \frac{\delta m_t}{m_t} &= \frac{1}{2}\left[\Sigma^t_R(m_t^2) + \Sigma^t_L(m_t^2)\right] + \Sigma^t_S(m_t^2) \\ \delta Z^t_L &= -\Sigma^t_L(m_t^2) - m_t^2\left[\Sigma^{t\,\prime}_L(m_t^2) + \Sigma^{t\,\prime}_R(m_t^2) + 2\,\Sigma^{t\,\prime}_S(m_t^2)\right] \\ \delta Z^t_R &= -\Sigma^t_R(m_t^2) - m_t^2\left[\Sigma^{t\,\prime}_L(m_t^2) + \Sigma^{t\,\prime}_R(m_t^2) + 2\,\Sigma^{t\,\prime}_S(m_t^2)\right] \\ \delta Z_{\tilde{t}_i} &= -\left(\Sigma^{ii}_{\tilde{t}}\right)'(m^2_{\tilde{t}_i}) \end{aligned} \tag{13}$$

where the self–energies Σ and their derivatives Σ', up to a factor $\alpha_s/3\pi$ which has been factorized out, are given in the dimensional reduction scheme by

$$\begin{aligned} \Sigma^t_L(k^2) &= -\Big[2\,B_1(k^2, m_t, \lambda) + (1+c_{2\theta})B_1(k^2, m_{\tilde{g}}, m_{\tilde{t}_1}) + (1-c_{2\theta})B_1(k^2, m_{\tilde{g}}, m_{\tilde{t}_2})\Big] \\ \Sigma^t_R(k^2) &= -\Big[2\,B_1(k^2, m_t, \lambda) + (1-c_{2\theta})B_1(k^2, m_{\tilde{g}}, m_{\tilde{t}_1}) + (1+c_{2\theta})B_1(k^2, m_{\tilde{g}}, m_{\tilde{t}_2})\Big] \\ \Sigma^t_S(k^2) &= -\Big[4\,B_0(k^2, m_t, \lambda) + \frac{m_{\tilde{g}}}{m_t}\, s_{2\theta}\,(B_0(k^2, m_{\tilde{g}}, m_{\tilde{t}_1}) - B_0(k^2, m_{\tilde{g}}, m_{\tilde{t}_2}))\Big] \\ (\Sigma^{ii}_{\tilde{t}})'(k^2) &= -2\Big[-2\,B_1(k^2, m_{\tilde{t}_i}, \lambda) - 2\,k^2\,B_1'(k^2, m_{\tilde{t}_i}, \lambda) + (m_t^2 + m_{\tilde{g}}^2 - k^2)\,B_0'(k^2, m_t, m_{\tilde{g}}) \\ &\quad - B_0(k^2, m_t, m_{\tilde{g}}) + (-1)^i\,2\,s_{2\theta}\,m_t\,m_{\tilde{g}} B_0'(k^2, m_t, m_{\tilde{g}})\Big] \end{aligned} \tag{14}$$

Finally, the mixing angle counterterm is chosen[4] in such a way that it cancels exactly the mixing contributions G^i_{mix} of eq.(11) [following Ref.[13]],

$$\delta\theta = \frac{1}{m^2_{\tilde{t}_1} - m^2_{\tilde{t}_2}} \left[4m_t\, m_{\tilde{g}}\, c_{2\theta}\, B_0(m^2_{\tilde{t}_i}, m_t, m_{\tilde{g}}) + c_{2\theta} s_{2\theta}(A_0(m^2_{\tilde{t}_2}) - A_0(m^2_{\tilde{t}_1}))\right] \tag{15}$$

[4]Had we chosen the $\overline{\rm MS}$ scheme, i.e. subtracting only the poles and the related constants in eq.(10), we would have been left with contributions which increase linearly with the gluino mass.

In practice, this reduces to simply discarding the contributions of the diagrams Fig.2c.

The complete virtual corrections to the $t \to \tilde{t}_i \chi$ decay width will be then given by

$$\Gamma^V(t \to \tilde{t}_i \chi_j^0) = \frac{\alpha}{3\, m_t^3} \frac{\alpha_s}{4\pi} \,\mathrm{Re}\, \Big\{ (m_t^2 - m_{\tilde{t}_i}^2 + m_\chi^2)(a_L^i\, G_L^i + a_R^i\, G_R^i) + 2\, m_t\, m_\chi \epsilon_\chi\, (a_L^i\, G_R^i + a_R^i\, G_L^i) \Big\} \lambda^{1/2}(m_t^2, m_\chi^2, m_{\tilde{t}_i}^2) \qquad (16)$$

These corrections are infrared divergent; the divergencies are cancelled after adding the real corrections.

3.3 Real Corrections

These corrections are obtained by adding the amplitudes of the diagrams Fig.2e with real gluon emission and squaring the resulting amplitude; the corrected decay width is then

$$\begin{aligned}\Gamma_R^i = \frac{\alpha}{3m_t} \frac{\alpha_s}{\pi} \Big\{ & 8\, a_L^i\, a_R^i\, m_t\, m_\chi \epsilon_\chi\, [\, (m_\chi^2 - m_{\tilde{t}_i}^2 - m_t^2)\, I_{01} - m_{\tilde{t}_i}^2\, I_{11} - m_t^2\, I_{00} - I_0 - I_1] \\ & + ({a_L^i}^2 + {a_R^i}^2)\, [\, 2\, (m_{\tilde{t}_i}^2 - m_\chi^2 - m_t^2)\, (m_t^2\, I_{00} + m_{\tilde{t}_i}^2\, I_{11} + I_0 + I_1) \\ & +2\, (\, (m_\chi^2 - m_{\tilde{t}_i}^2)^2 - m_t^4)\, I_{01} + I + I_0^1] \Big\}\end{aligned} \qquad (17)$$

where the phase space integrals $I(m_t, m_{\tilde{t}_i}, m_\chi) \equiv I$ are given by [14]

$$\begin{aligned}
I_{00} &= \frac{1}{4\, m_t^4}\Big[\kappa \log\Big(\frac{\kappa^2}{\lambda\, m_t\, m_{\tilde{t}_i}\, m_\chi}\Big) - \kappa - (m_{\tilde{t}_i}^2 - m_\chi^2) \log\Big(\frac{\beta_1}{\beta_2}\Big) - m_t^2 \log(\beta_0)\Big] \\
I_{11} &= \frac{1}{4\, m_{\tilde{t}_i}^2\, m_t^2}\Big[\kappa \log\Big(\frac{\kappa^2}{\lambda\, m_t\, m_{\tilde{t}_i}\, m_\chi}\Big) - \kappa - (m_t^2 - m_\chi^2) \log\Big(\frac{\beta_0}{\beta_2}\Big) - m_{\tilde{t}_i}^2 \log(\beta_1)\Big] \\
I_{01} &= \frac{1}{4\, m_t^2}\Big[- 2 \log\Big(\frac{\lambda\, m_t\, m_{\tilde{t}_i}\, m_\chi}{\kappa^2}\Big) \log(\beta_2) + 2 \log^2(\beta_2) - \log^2(\beta_0) - \log^2(\beta_1) \\
&\quad +2\, \mathrm{Li}\,(1 - \beta_2^2) - \mathrm{Li}\,(1 - \beta_0^2) - \mathrm{Li}\,(1 - \beta_1^2)\Big] \\
I &= \frac{1}{4\, m_t^2}\Big[\frac{\kappa}{2}(m_t^2 + m_{\tilde{t}_i}^2 + m_\chi^2) + 2\, m_t^2\, m_{\tilde{t}_i}^2 \log(\beta_2) + 2\, m_t^2\, m_\chi^2 \log(\beta_1) + 2\, m_{\tilde{t}_i}^2\, m_\chi^2 \log(\beta_0)\Big] \\
I_0 &= \frac{1}{4\, m_t^2}\Big[- 2\, m_{\tilde{t}_i}^2 \log(\beta_2) - 2\, m_\chi^2 \log(\beta_1) - \kappa\Big] \\
I_1 &= \frac{1}{4\, m_t^2}\Big[- 2\, m_t^2 \log(\beta_2) - 2\, m_\chi^2 \log(\beta_0) - \kappa\Big] \\
I_0^1 &= \frac{1}{4\, m_t^2}\Big[m_{\tilde{t}_i}^4 \log(\beta_2) - m_\chi^2\, (2\, m_t^2 - 2\, m_{\tilde{t}_i}^2 + m_\chi^2) \log(\beta_1) - \frac{\kappa}{4}\, (m_{\tilde{t}_i}^2 - 3\, m_t^2 + 5\, m_\chi^2)\Big] \qquad (18)
\end{aligned}$$

with $\kappa = \lambda^{1/2}(m_t^2, m_{\tilde{t}_i}, m_\chi)$ and

$$\beta_0 = \frac{m_t^2 - m_{\tilde{t}_i}^2 - m_\chi^2 + \kappa}{2\, m_{\tilde{t}_i}\, m_\chi}, \quad \beta_1 = \frac{m_t^2 - m_{\tilde{t}_i}^2 + m_\chi^2 - \kappa}{2\, m_t\, m_\chi}, \quad \beta_2 = \frac{m_t^2 + m_{\tilde{t}_i}^2 - m_\chi^2 - \kappa}{2\, m_t\, m_{\tilde{t}_i}} \qquad (19)$$

4. Results and Discussions

The $\mathcal{O}(\alpha_s)$ partial widths of the decays $t \to \tilde{t}_2\chi$ normalized to their Born expressions, are shown in Figs. 3 as a function of the lightest stop mass for the scenarios discussed in section 2. For the numerical analysis, we choose $m_t = 180$ GeV for the top quark mass and for the strong coupling constant we take the value $\alpha_s = 0.118$. In Fig. 3a, the corrections are shown for $M_2 = \mu = 150$ (solid), 200 (dashed) and 250 GeV (dotted); they are negative for relatively small values of the gluino mass [solid line, $m_{\tilde{g}} \sim 530$ GeV] and $m_{\tilde{t}_2} \gtrsim 80$ GeV and positive for larger values of $m_{\tilde{g}}$ [dotted and dashed lines, $m_{\tilde{g}} \sim 900$ and 700 GeV]. The QCD correction varies between -20 and +20%. In Fig. 3b M_2 is again fixed at 400 GeV and $\mu = -100$ (solid) and 150 GeV (dashed). The QCD corrections are again of the order of $\pm 20\%$.

The QCD corrections to the top decays into the lightest stop squark and one of the four neutralinos, $t \to \tilde{t}\chi_j^0$, in the scenario where $M_2 = -\mu = 50$ GeV are shown in Fig. 3c. Here again, the corrections are well under control being less than 15% [in absolute value] in all cases. As in Figs.1a–c, the sum was computed if more than one neutralino contributes.

For $M_2 = 400$ GeV and hence for large gluino mass, $m_{\tilde{g}} \sim 1.4$ TeV, the QCD corrections are negative and increase in absolute value, reaching the level of 17% for light stop squarks. This means that the gluino does not decouple for large $m_{\tilde{g}}$. There is a logarithmic dependence of the correction on the gluino mass which is visualized in Fig. 4, where the correction is plotted against the gluino mass for $\mu = -100$ GeV and $m_{\tilde{t}} = 50$ GeV. This logarithmic behaviour is due to the wave function renormalization and is a consequence of the breakdown of supersymmetry as discussed in Ref. [15], where a similiar feature appears in the decay of a squark into a massless quark plus a photino.

5. Conclusion

We have calculated the $\mathcal{O}(\alpha_s)$ QCD corrections to the top decay into the lightest stop and a neutralino in the minimal supersymmetric extension of the Standard Model. These corrections can be either positive or negative and increase logarithmically with the gluino mass. For gluino masses below 1 TeV, they are at most of the order of ten percent and therefore, well under control.

In the case where the top quark is lighter than its scalar partners, the reverse process $\tilde{t} \to t\chi_0$ might occur. This decay, together with the decay $\tilde{t} \to b\chi^+$, would be the dominant decay mode of stop squarks if they are lighter than $m_t + m_{\tilde{g}}$. The QCD corrections to these processes are under investigation [16].

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Figure Captions

Fig. 1: Branching ratios for top decays into the lightest stop and neutralinos as a function of the stop mass in the Born approximation; $\tan\beta$ is fixed to $\tan\beta = 1.6$ and the top quark mass to $m_t = 180$ GeV. (a): $M_2 = \mu = 150$ (solid line), 200 (dashed line) and 250 GeV (dotted line); (b): $M_2 = 400$ GeV and $\mu = -100$ GeV (solid line) and $M_2 = 400$ GeV and $\mu = 150$ GeV (dashed line); (c): $M_2 = -\mu = 50$ GeV the thin solid, dashed, dotted and dashed–dotted lines correspond respectively to the branching ratios of $t \to \tilde{t}_2 \chi_j^0$ with $j = 1$–4 (the neutralinos are ordered with increasing mass), the thick solid line corresponds to the sum over all contributing neutralinos.

Fig. 2 : Feynman diagrams relevant for the $\mathcal{O}(\alpha_s)$ QCD corrections to the decay of the top quark into a stop squark and a neutralino.

Fig. 3: Relative size of the $\mathcal{O}(\alpha_s)$ QCD corrections to the decay rate $t \to \tilde{t}\chi$ as a function of the lightest stop squark mass. We use the same inputs as in Figs.1.

Fig. 4: The size of the QCD corrections as a function of the gluino mass $m_{\tilde{g}}$ with $M_2 = 400$ GeV, $\mu = -100$ GeV and $m_{\tilde{t}_2} \sim 50$ GeV.