# Search for the Higgs Bosons of the NMSSM at Linear Colliders

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**Abstract**

We show that at least one of the Higgs bosons of the Next to Minimal Supersymmetric Standard Model can be detected at future Linear Colliders of 500, 1000 and 2000 GeV c.m. energies.

## 1. Introduction

The Next to Minimal Supersymmetric Standard Model (NMSSM) [1–3] is a minimal extension to the Minimal Supersymmetric Standard Model [4]. The NMSSM provides the most economic solution of the so called $\mu$-problem of the MSSM by introducing an additional Higgs singlet superfield $\mathcal{N} = (N, \psi_N, F_N)$ with a Higgs singlet $N$, a higgsino singlet $\psi_N$ and an auxiliary field $F_N$. Together with the two Higgs doublets superfields $\mathcal{H}_{1,2} = (H_{1,2}, \psi_{1,2}, F_{1,2})$ the superpotential of the NMSSM is given by

$$W = \lambda \mathcal{H}_1^T \epsilon \mathcal{H}_2 \mathcal{N} - \frac{1}{3} k \mathcal{N}^3 \tag{1}$$

The soft breaking part of the Higgs sector is given by

$$V_{soft} = -\lambda A_\lambda H_1^T \epsilon H_2 N - \frac{1}{3} k A_k N^3 + h.c. \tag{2}$$

where $A_\lambda$ and $A_k$ are soft breaking mass parameters.

$H_1, H_2$ and $N$ develop vacuum expectation values $v_1, v_2$ and $x$ respectively. The NMSSM contains three scalar Higgs bosons $S_1, S_2$ and $S_3$ with masses $m_{S_1} \leq m_{S_2} \leq m_{S_3}$, two pseudoscalar Higgs bosons $P_1$ and $P_2$ with masses $m_{P_1} \leq m_{P_2}$ and a charged one $H^+$ with mass $m_C$.

The Higgs sector has 6 free parameters $\lambda, k, \tan\beta = v_1/v_2, x, A_\lambda$ and $A_k$.

## 2. Constraints on parameters

A remarkable result of the MSSM is the tree level bound of $m_{S_1} \leq m_Z \cos 2\beta$. This is due to the fact that all quartic terms have gauge coupling constants. In case of the NMSSM there is a quartic term with the coupling constant $\lambda$. It turns out that the upper bound of $\lambda$ may be relevant for $m_{S_1}$. An effective way of determining this bound is RG-analysis [1–8].

The one loop RG-equation of $\lambda$ is coupled with that of $k$ and $h_t$, the Yukawa coupling constant of the top quark (neglecting other quarks) and is given by

$$\begin{aligned}
\frac{d\lambda}{dt} &= \frac{1}{8\pi^2}\left(k^2+2\lambda^2+\frac{3}{2}h_t^2+\frac{3}{2}g_2^2-\frac{1}{2}g_1^2\right)\lambda \\
\frac{dk}{dt} &= \frac{3}{8\pi^2}\left(k^2+\lambda^2\right)k \\
\frac{dh_t}{dt} &= \frac{1}{8\pi^2}\left(\frac{1}{2}\lambda^2+3h_t^2-\frac{8}{3}g_3^2-\frac{3}{2}g_2^2-\frac{13}{18}g_1^2\right)h_t
\end{aligned} \tag{3}$$

where $t = \ln\mu$ and $\mu$ being the renormalization scale. By demanding no Landau pole up to the GUT scale one can determine from eq. (4) the upper bound of $\lambda$ and $k$ and the lower bound of $\tan\beta$ at the electroweak scale. We plot our results in Fig. 1 (Fig. 2) for $m_t = 175$ GeV (190 GeV). They show that $\lambda_{max}$ decreases with increasing $k$*. The lower bound of $\tan\beta$ is about 1.24 for $m_t = 175$GeV and 2.6 for $m_t = 190$GeV. For $\tan\beta \gtrsim 3$ $\lambda_{max}$ is almost independent on $\tan\beta$. The upper bound of $k$ is about 0.7.

## 3. Mass upper bounds

The tree level bound of $m_{S_1}$ is given by [3]

$$m_{S_1}^2 \leq m_Z^2\left(\cos^2 2\beta+\frac{2\lambda^2}{g_1^2+g_2^2}\sin^2 2\beta\right) = m^2_{S_1^{max}} \tag{4}$$

The upper bound of $m_{S_2}$ and $m_{S_3}$ can be expressed in terms of $m_{S_1^{max}}$ and $m_{S_1}$

$$\begin{aligned}
m_{S_2}^2 &\leq m^2_{S_2^{max}} = \frac{m^2_{S_1^{max}}-R_1^2 m_{S_1}^2}{1-R_1^2} \\
m_{S_3}^2 &\leq m^2_{S_3^{max}} = \frac{m^2_{S_1^{max}}-(R_1^2+R_2^2) m_{S_1}^2}{1-(R_1^2+R_2^2)}
\end{aligned} \tag{5}$$

where $R_i = U_{i1}\cos\beta + U_{i2}\sin\beta$ and $U_{ij}$ is the $3\times 3$ orthogonal matrix which diagonalizes the scalar mass matrix.

$R_1$ and $R_2$ satisfy the unitarity condition $0 \leq R_1^2 + r_2^2 \leq 1$. The tree level upper bound (4) yields $m_{S_1} \leq m_Z$ for $\lambda^2 \leq (g_1^2+g_2^2)/2 = (0.52)^2$ and $m_{S_1} \leq \sqrt{2/(g_1^2+g_2^2)}\lambda m_Z = 1.92\lambda m_Z$ for $\lambda^2 > (0.52)^2$. Using $\lambda_{max} = 0.64 - 0.74$ from section 2 the tree level relation yields $m_{S_1} \leq 113\text{GeV} - 131\text{GeV}$.

As in the case of the MSSM the contributions of radiative corrections may change this result considerably. Several groups calculated higher order contributions to the mass matrices using the one loop effective potential and determined the corrected upper bound [8–11].

*We obtain $\lambda_{max} = 0.64 - 0.74$ for $m_t = 175 - 190$ GeV.

The result in our notation [8] is given by

$$m_{S_1}^2 \leq m_Z^2 \left(\cos^2 2\beta + \frac{2\lambda^2}{g_1^2+g_2^2}\sin^2 2\beta\right) + \alpha\cos^2\beta + \beta\sin 2\beta + \gamma\sin^2\beta \tag{6}$$

with $(A_T = -A_t + \lambda x \cot\beta)$

$$\begin{aligned}
\alpha &= -\frac{1}{16\pi^2}\left(\frac{\lambda x A_T}{v_1}\right)^2\left(\frac{m_t}{m_{\tilde{t}}}\right)^4 \\
\beta &= \frac{3}{8\pi}\lambda x A_T\left(\frac{m_t^2}{m_{\tilde{t}}v_1}\right)^2\left(1+\frac{A_t A_T}{6m_{\tilde{t}}^2}\right) \\
\gamma &= \frac{3}{8\pi}\left(\frac{m_t^2}{v_1^2}\right)^2\left[2\ln\frac{m_{\tilde{t}}^2}{m_t^2} - \frac{2A_tA_T}{m_{\tilde{t}}^2} - \frac{A_t^2A_T^2}{6m_{\tilde{t}}^4}\right]
\end{aligned} \tag{7}$$

In this result only top and stop contributions were taken into account. We numerical calculated $m_{S_1^{max}}$ in the region $175\text{GeV} \leq m_t \leq 190\text{GeV}$, $250\text{GeV} \leq x, A_\lambda, A_t, m_{\tilde{t}} \leq 1000\text{GeV}$ and $2 \leq \tan\beta \leq 20$ and obtained [8]

$$120 \leq m_{S_1^{max}} \leq 156\text{GeV} \tag{8}$$

## 4. Production of scalar Higgs bosons at $e^+e^-$ Colliders

The upper bound $m_{S_1} \leq 120 - 156\text{GeV}$ suggests that the accessible area of the parameter space at LEP1 with $\sqrt{s} = m_Z$ might be very small. Actually we showed that the existing LEP1 data do not exclude the existence of $S_1$ with $m_{S_1} = 0\text{GeV}$ [12].

For colliders with $\sqrt{s} = 500, 1000$ or $2000\text{GeV}$ the situation is different. In this case the production cross section of one $S_i$ via the Higgsstrahlung $e^+e^- \to ZS_i$ with real $Z$ and $S_i$ is always possible as the collider energy is larger than $E_T = 212 - 248\text{GeV}$. $E_T$ is a kind of threshold energy and is an important quantity of a model.

In this case it is possible to derive a lower bound for the production cross section $\sigma_i$ of $S_i$ as a function of the collision energy only. This lower bound would give information about how far the model could be tested.

In order to derive the lower bound of $\sigma_i$ we consider the production cross sections of $S_1, S_2, S_3$ via the Higgsstrahlungs process, denoted by $\sigma_1, \sigma_2, \sigma_3$, which can be expressed in terms of the standard model Higgs production cross section $\sigma_{SM}$ and $R_1$ and $R_2$ defined in section 2:

$$\begin{aligned}
\sigma_1(m_{S_1}) &= R_1^2\sigma_{SM}(m_{S_1}) \\
\sigma_2(m_{S_2}) &= R_2^2\sigma_{SM}(m_{S_2}) \\
\sigma_3(m_{S_3}) &= (1-R_1^2-R_2^2)\sigma_{SM}(m_{S_3})
\end{aligned} \tag{9}$$

A useful observation is that $\sigma_i(m_{S_i}^{max}) \leq \sigma_i(m_{S_i})$ which allows one to derive parameter independent lower limits on $\sigma_i$ as we will see in the following.

First we determine at a fixed set of $m_{S_1}, R_1, R_2$ the cross sections $\sigma_1(R_1, R_2, m_{S_1})$, $\sigma_2(R_1, R_2, m_{S_2^{max}})$ and $\sigma_3(R_1, R_2, m_{S_3^{max}})$. Then we keep $R_1$ and $R_2$ fixed, but vary $m_{S_1}$ from its minimum to its maximum value and determine the quantity $\sigma(R_1, R_2)$ defined by

$$\sigma(R_1, R_2) = \min_{0 \leq m_{S_1} \leq m_{S_1^{max}}} [\max(\sigma_1, \sigma_2, \sigma_3)] \tag{10}$$

where $\sigma_1 = \sigma_1(R_1, R_2, m_{S_1})$, $\sigma_2 = \sigma_2(R_1, R_2, m_{S_2^{max}})$ and $\sigma_3 = \sigma_3(R_1, R_2, m_{S_3^{max}})$. As a last step we vary $R_1^2$ and $R_2^2$ from 0 to 1 with $R_1^2 + R_2^2 \leq 1$ and plot $\sigma(R_1, R_2)$ in the $R_1$-$R_2$-plane. It is plausible that $\sigma(R_1, R_2) = 0$ for $\sqrt{s} < E_T = m_z + m_{S_1^{max}} = 212 - 248$GeV in the entire $R_1$-$R_2$-plane. This is the case for LEP2 with $\sqrt{s} \leq 205$ GeV. Therefore this method does not give any results for LEP2. For $\sqrt{s} > E_T$ $\sigma(R_1, R_2)$ never vanishes and the minimum of $\sigma(R_1, R_2)$ is a parameter independent lower limit of one of the $\sigma_i$. This minimum is thus a characteristic quantity of the model.

In Fig. 3 we plotted $\sigma(R_1, R_2)$ for $\sqrt{s} = 500$ GeV and $m_{S_1^{max}} = 145$ GeV. The minimum is about 16 fb. When the discovery limit is about 30 events, one would need a luminosity of about 25 fb, which is a realistic one. Fig. 4 (Fig. 5) shows $\sigma(R_1, R_2)$ for $\sqrt{s} = 1000(2000)$ GeV with minimum cross section of 4 fb (1 fb). Fig. 6 shows the minimum of $\sigma(R_1, R_2)$ as a function of $\sqrt{s}$ and $m_{S_1^{max}}$ as a parameter. We see that the effect of $m_{S_1^{max}}$ on $\sigma_{min}$ is very big around $\sqrt{s} = 300$ GeV, but rather small for $\sqrt{s} \geq 500$ GeV.

Fig. 7 shows the tree level cross sections $\sigma_1, \sigma_2$ and $\sigma_3$ for an exemplary set of parameters with the contributions from (i) the Higgsstrahlungsprocess $e^+e^- \to ZS_i \to b\bar{b}S_i$, (ii) the process where $S_i$ is radiated off from $b$ or $\bar{b}$ and (iii) $e^+e^- \to Z \to P_jS_i \to b\bar{b}S_i$, where $P_j (j = 1, 2)$ is a pseudoscalar Higgs boson.

Fig. 8 shows the same as Fig. 7, but with one loop contributions via the effective potential. The higher order contribution is rather important for the energy region around 150 GeV and decreases with $\sqrt{s}$. In this parameter region the dominant production is that of $S_2$ at $\sqrt{s} = 500$ GeV and is about 13 fb.

We conclude that the Higgs sector of the NMSSM can most probably be tested conclusively at the future linear $e^+e^-$-colliders with 500, 1000 or 2000 GeV c.m. energies.

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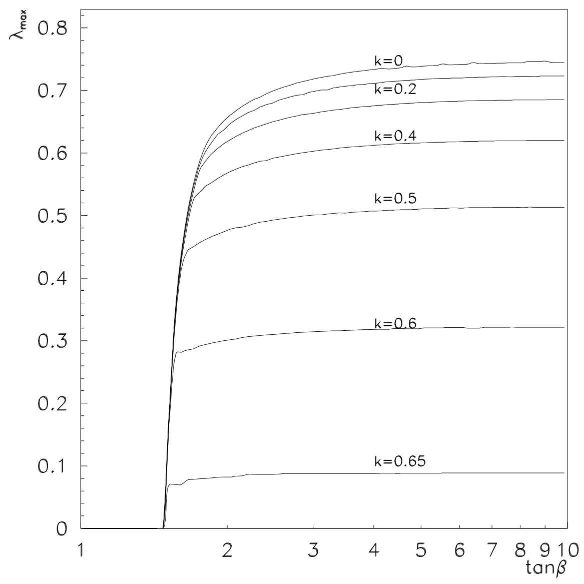


Figure 1: Upper bound for $\lambda$ as a funtion of $\tan\beta$ for $m_{top} = 175$ GeV.

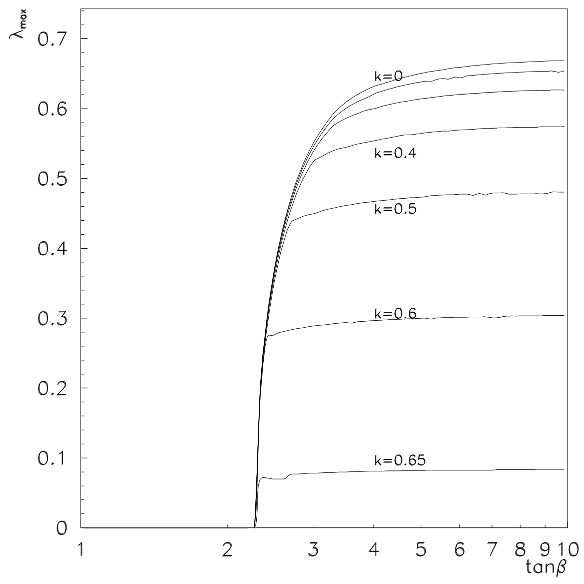


Figure 2: Upper bound for $\lambda$ as a funtion of $\tan\beta$ for $m_{top} = 190$ GeV.

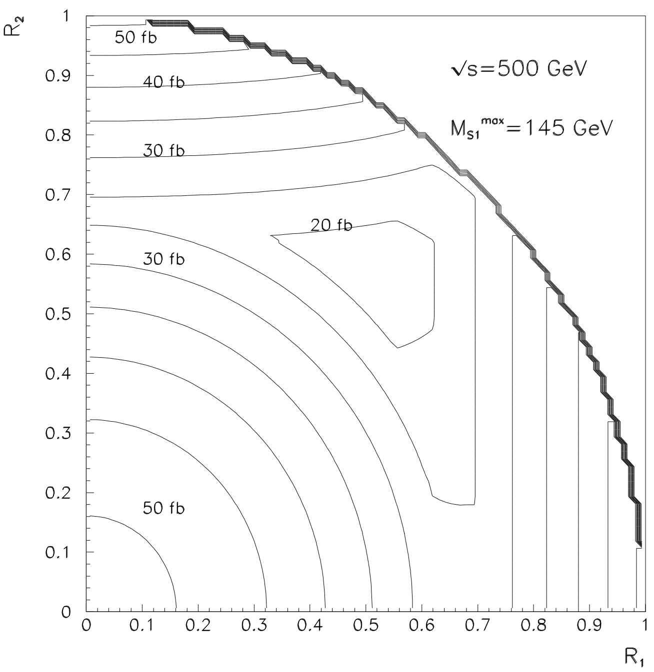


Figure 3: $\sigma(R_1, R_2)$ as definded in the text for $\sqrt{s} = 500$ GeV.

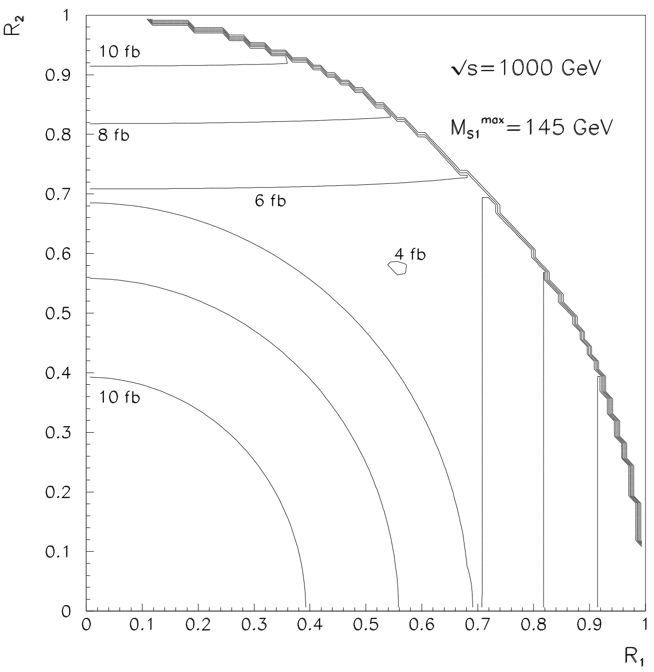


Figure 4: $\sigma(R_1, R_2)$ for $\sqrt{s} = 1000$ GeV.

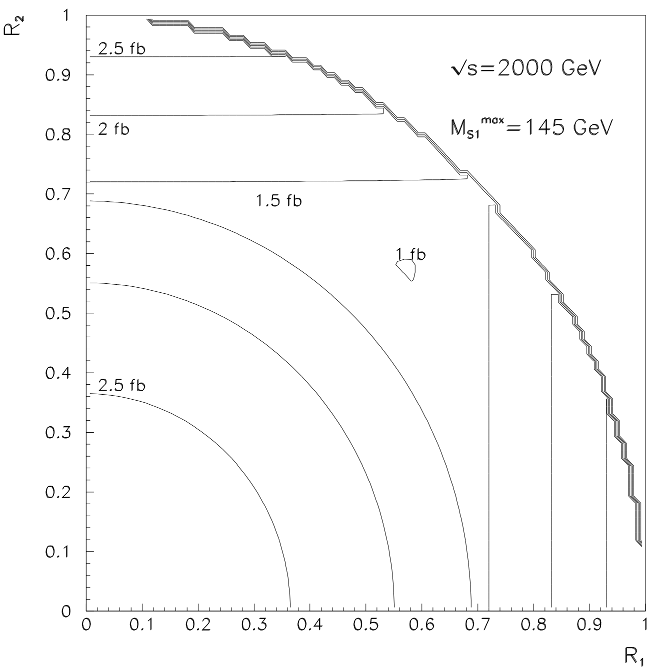


Figure 5: $\sigma(R_1, R_2)$ for $\sqrt{s} = 2000$ GeV.

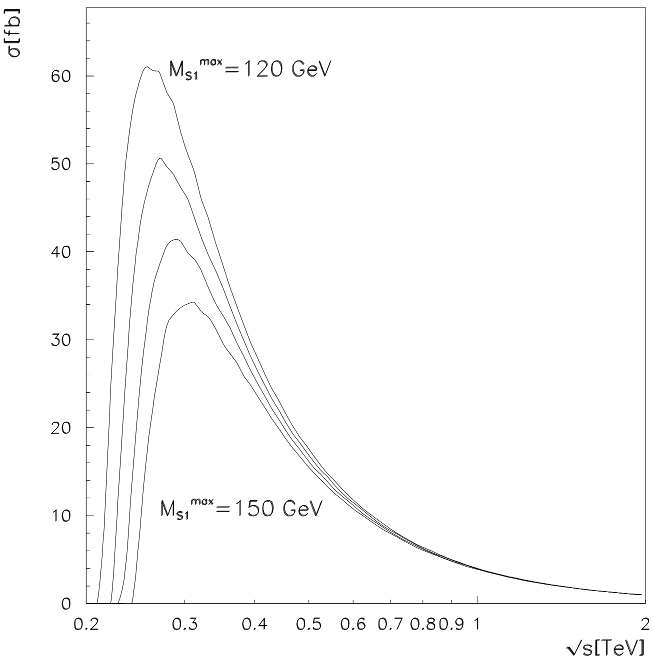


Figure 6: Minimal value of $\sigma(R_1, R_2)$ as a function of $\sqrt{s}$ for various values of $m_{S_1^{max}}$

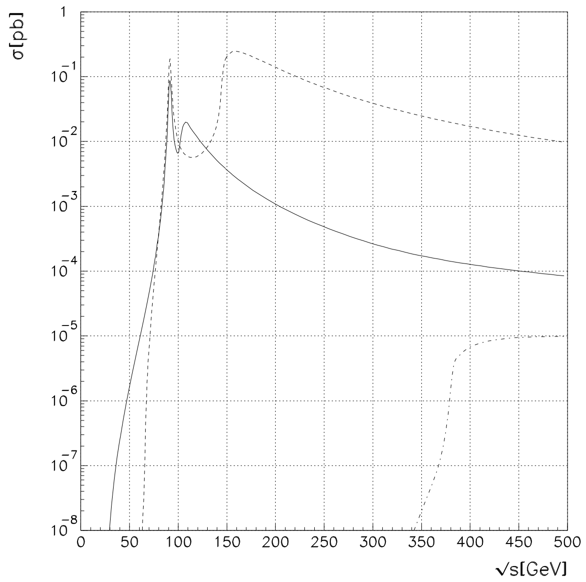


Figure 7: Cross section for $e^+e^- \to Zb\bar{b}$ for $A_\lambda = 220$ GeV, $A_k = 160$ GeV, $x = 1000$ GeV, $\tan\beta = 2, k = 0.04, \lambda = 0.12$. Masses and mixing angles have calculated from the tree level protential.

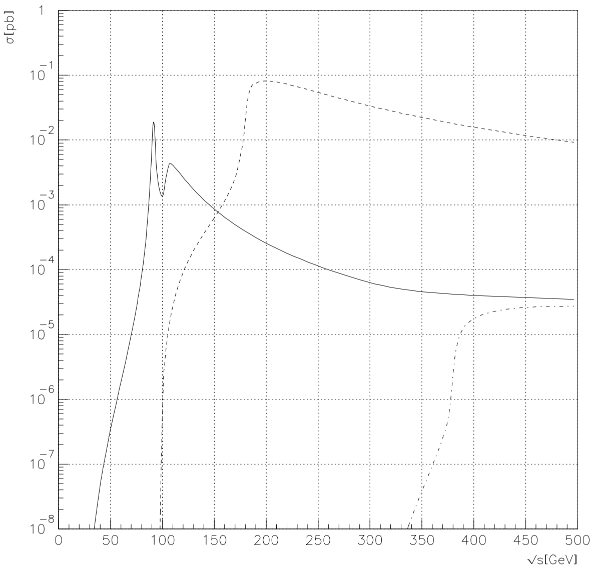


Figure 8: The same as above, but with masses and mixing angles obtained from the one loop effective potential. The top mass is 175 GeV, $m_{\tilde{t}_l} = m_{\tilde{t}_R} = 1$ TeV and $A_t = 0$ GeV.