HIGGS MASS PREDICTION

W. HOLLIK
Institut für Theoretische Physik, Universität Karlsruhe,
D-76128 Karlsruhe, Germany

In this talk the Higgs boson effects in electroweak precision observables are reviewed and the possibility of indirect information on the Higgs mass from electroweak radiative corrections and precision data is discussed.

1 Introduction

By the present high precision experiments stringent tests on the standard model of electroweak and strong interactions are imposed. Impressive achievements have been made in the determination of the Z boson parameters [1], the W mass [2], and the confirmation of the top quark at the Tevatron [3,4] with mass $m_t = 175 \pm 6$ GeV, but direct experimental evidence for the Higgs boson is still lacking.

Also a sizeable amount of theoretical work has contributed over the last few years to a steadily rising improvement of the standard model predictions (for a review see ref. [5]). The availability of both highly accurate measurements and theoretical predictions provides tests of the quantum structure of the standard model thereby probing the empirically yet unknown Higgs particle via its contribution to the electroweak radiative corrections.

2 Theoretical basis

2.1 Radiative corrections

The possibility of performing precision tests is based on the formulation of the standard model as a renormalizable quantum field theory preserving its predictive power beyond tree level calculations. With the experimental accuracy being sensitive to the loop induced quantum effects, also the Higgs sector of the standard model is probed. The higher order terms induce the sensitivity of electroweak observables to the top and Higgs mass m_t, M_H and to the strong coupling constant α_s, which are not present at the tree level.

Before one can make predictions from the theory, a set of independent parameters has to be taken from experiment. For practical calculations the physical input quantities α, G_μ, M_Z, m_f, M_H; α_s are commonly used for fixing the free parameters of the standard model. Differences between various schemes are formally of higher order than the one under consideration. The study of the scheme dependence of the perturbative results, after improvement by resumming the leading terms, allows us to estimate the missing higher order contributions.

Two fermion induced large loop effects in electroweak observables deserve a special discussion:

- The light fermionic content of the subtracted photon vacuum polarization corresponds to a QED induced shift in the electromagnetic fine structure constant. The recent update of the evaluation of the light quark content [6,7] yield the result

$$(\Delta\alpha)_{had} = 0.0280 \pm 0.0007\,. \tag{1}$$

  Other determinations [8] agree within one standard deviation. Together with the leptonic content, $\Delta\alpha$ can be resummed resulting in an effective fine structure constant at the Z mass scale:

$$\alpha(M_Z^2) \;=\; \frac{\alpha}{1-\Delta\alpha} \;=\; \frac{1}{128.89 \pm 0.09}\,. \tag{2}$$

- The electroweak mixing angle is related to the vector boson masses by

$$\sin^2\theta = 1 - \frac{M_W^2}{M_Z^2} + \frac{M_W^2}{M_Z^2}\Delta\rho + \cdots \equiv s_W^2 + c_W^2\,\Delta\rho + \cdots \tag{3}$$

  where the main contribution to the higher order quantity $\Delta\rho$ is from the (t,b) doublet [9], in 1-loop and neglecting m_b given by:

$$\Delta\rho^{(1)} = 3x_t, \quad x_t = \frac{G_\mu m_t^2}{8\pi^2\sqrt{2}} \tag{4}$$

  Higher order irreducible contributions have become available, modifying $\Delta\rho$ according to

$$\Delta\rho = 3x_t \cdot [1 + x_t\,\rho^{(2)} + \delta\rho_{\,QCD}] \tag{5}$$

  The electroweak 2-loop part [10,11] is described by the function $\rho^{(2)}(M_H/m_t)$ derived in [11] for general Higgs masses. $\delta\rho_{\,QCD}$ is the QCD correction to the leading $G_\mu m_t^2$ term [12,13]

$$\delta\rho_{\,QCD} = -2.86a_s - 14.6a_s^2, \quad a_s = \frac{\alpha_s(m_t)}{\pi}\,. \tag{6}$$

  The Higgs contribution to ρ is only logarithmic for large Higgs masses.

2.2 *The vector boson masses*

The correlation between the masses M_W, M_Z of the vector bosons in terms of the Fermi constant G_μ, in 1-loop order is given by [14]:

$$\frac{G_\mu}{\sqrt{2}} = \frac{\pi\alpha}{2s_W^2 M_W^2}[1 + \Delta r(\alpha, M_W, M_Z, M_H, m_t)] \,. \tag{7}$$

The decomposition

$$\Delta r = \Delta\alpha - \frac{c_W^2}{s_W^2}\,\Delta\rho^{(1)} + (\Delta r)_{remainder}\,. \tag{8}$$

separates the leading fermionic contributions $\Delta\alpha$ and $\Delta\rho$. All other terms are collected in the $(\Delta r)_{remainder}$, the typical size of which is of the order ~ 0.01.

The presence of large terms in Δr requires the consideration of higher than 1-loop effects. The modification of Eq. (7) according to

$$\begin{aligned} 1 + \Delta r \quad &\rightarrow \quad \frac{1}{(1-\Delta\alpha)\cdot(1+\frac{c_W^2}{s_W^2}\Delta\rho) \;-\; (\Delta r)_{remainder}} \\ &\equiv \quad \frac{1}{1-\Delta r} \end{aligned} \tag{9}$$

accommodates the following higher order terms (Δr in the denominator is an effective correction including higher orders):

- The leading log resummation [15] of $\Delta\alpha$: $1 + \Delta\alpha \,\rightarrow\, (1-\Delta\alpha)^{-1}$
- The resummation of the leading m_t^2 contribution [16] in terms of $\Delta\rho$ in Eq. (5). Beyond the $G_\mu m_t^2 \alpha_s$ approximation through the ρ-parameter, the complete $O(\alpha\alpha_s)$ corrections to the self energies are available from perturbative calculations [17] and by means of dispersion relations [18].
- With the quantity $(\Delta r)_{remainder}$ in the denominator non-leading higher order terms containing mass singularities of the type $\alpha^2 \log(M_Z/m_f)$ from light fermions are also incorporated [19].

2.3 *Z boson observables*

With M_Z as a precise input parameter, the predictions for the partial widths as well as for the asymmetries can conveniently be calculated in terms of effective neutral current coupling constants for the various fermions. The effective couplings follow from the set of 1-loop diagrams without virtual photons, the non-QED or weak corrections. These weak corrections can be written in terms of fermion-dependent overall normalizations ρ_f and effective mixing angles s_f^2 in the NC vertices:

$$\begin{aligned} J_\nu^{NC} &= \left(\sqrt{2}G_\mu M_Z^2\right)^{1/2} (g_V^f\,\gamma_\nu - g_A^f\,\gamma_\nu\gamma_5) \qquad (10) \\ &= \left(\sqrt{2}G_\mu M_Z^2\rho_f\right)^{1/2} \left((I_3^f - 2Q_f s_f^2)\gamma_\nu - I_3^f\gamma_\nu\gamma_5\right). \end{aligned}$$

ρ_f and s_f^2 contain universal parts (i.e. independent of the fermion species) and non-universal parts which explicitly depend on the type of the external fermions. The universal parts arise from the self-energies and contain the Higgs mass dependence. The Higgs contributions to the non-universal vertex corrections are suppressed by the small Yukawa couplings.

Asymmetries and mixing angles: The effective mixing angles are of particular interest since they determine the on-resonance asymmetries via the combinations

$$A_f = \frac{2g_V^f g_A^f}{(g_V^f)^2 + (g_A^f)^2}\,. \qquad (11)$$

Measurements of the asymmetries hence are measurements of the ratios

$$g_V^f/g_A^f = 1 - 2Q_f s_f^2 \qquad (12)$$

or the effective mixing angles, respectively.

Z width and partial widths: The total Z width Γ_Z can be calculated essentially as the sum over the fermionic partial decay widths

$$\Gamma_Z = \sum_f \Gamma_f + \cdots, \quad \Gamma_f = \Gamma(Z \to f\bar{f}) \qquad (13)$$

The dots indicate other decay channels which, however, are not significant. The fermionic partial widths, when expressed in terms of the effective coupling constants read up to 2nd order in the fermion masses:

$$\Gamma_f \;=\; \Gamma_0\left((g_V^f)^2 + (g_A^f)^2(1 - \frac{6m_f^2}{M_Z^2})\right)\cdot(1 + Q_f^2\,\frac{3\alpha}{4\pi}) + \Delta\Gamma_{QCD}^f$$

with

$$\Gamma_0 \,=\, N_C^f\,\frac{\sqrt{2}G_\mu M_Z^3}{12\pi}, \quad N_C^f = 1 \text{ (leptons)}, \;\; = 3 \text{ (quarks)}.$$

and the QCD corrections $\Delta\Gamma_{QCD}^f$ for quark final states [20].

2.4 *Accuracy of the standard model predictions*

For a discussion of the theoretical reliability of the standard model predictions one has to consider the various sources contributing to their uncertainties:

The experimental error of the hadronic contribution to $\alpha(M_Z^2)$, Eq. (2), leads to $\delta M_W = 13$ MeV in the W mass prediction, and $\delta \sin^2\theta = 0.00023$ common to all of the mixing angles, which matches with the experimental precision.

The uncertainties from the QCD contributions, besides the 3 MeV in the hadronic Z width, can essentially be traced back to those in the top quark loops for the ρ-parameter. They can be combined into the following errors [21]:

$$\delta(\Delta\rho) \simeq 1.5 \cdot 10^{-4}, \; \delta s_\ell^2 \simeq 0.0001$$

for $m_t = 174$ GeV.

The size of unknown higher order contributions can be estimated by different treatments of non-leading terms of higher order in the implementation of radiative corrections in electroweak observables ('options') and by investigations of the scheme dependence. Explicit comparisons between the results of 5 different computer codes based on on-shell and $\overline{MS}$ calculations for the Z resonance observables are documented in the "Electroweak Working Group Report" [22] in ref. [5]. Table 1 shows the uncertainty in a selected set of precision observables. Quite recently (not included in table 1) the non-leading 2-loop corrections $\sim G_\mu^2 m_t^2 M_Z^2$ have been calculated [23] for Δr and s_ℓ^2. They reduce the uncertainty in M_W and s_ℓ^2 considerably, by about a factor 0.2.

3 Precision data and virtual Higgs bosons

In table 2 the standard model predictions for Z pole observables and the W mass are put together for a light and a heavy Higgs particle with $m_t = 175$ GeV. The last column is the variation of the prediction according to $\Delta m_t = \pm 6$ GeV. The input value $\alpha_s = 0.123$ is the one from QCD observables at the Z peak [24]. Not included are the uncertainties from $\delta\alpha_s = 0.006$, which amount to 3 MeV for the hadronic Z width. The experimental results on the Z observables are from combined LEP and SLD data. ρ_ℓ and s_ℓ^2 are the leptonic neutral current couplings in eq. (10), derived from partial widths and asymmetries under the assumption of lepton universality. The table illustrates the sensitivity of the various quantities to the Higgs mass. The effective mixing angle turns out to be the most sensitive observable, where both the experimental error and the uncertainty from m_t are small compared to the variation with M_H. Since a light Higgs boson goes along with a low value of s_ℓ^2, the strongest upper

Table 1: Largest half-differences among central values (Δ_c) and among maximal and minimal predictions (Δ_g) for $m_t = 175\,\text{GeV}$, $60\,\text{GeV} < M_H < 1\,\text{TeV}$, $\alpha_s(M_Z^2) = 0.125$ (from ref.[22])

Observable O	$\Delta_c O$	$\Delta_g O$
M_W (GeV)	4.5×10^{-3}	1.6×10^{-2}
Γ_e (MeV)	1.3×10^{-2}	3.1×10^{-2}
Γ_Z (MeV)	0.2	1.4
s_e^2	5.5×10^{-5}	1.4×10^{-4}
s_b^2	5.0×10^{-5}	1.5×10^{-4}
R_{had}	4.0×10^{-3}	9.0×10^{-3}
R_b	6.5×10^{-5}	1.7×10^{-4}
R_c	2.0×10^{-5}	4.5×10^{-5}
σ_0^{had} (nb)	7.0×10^{-3}	8.5×10^{-3}
A_{FB}^l	9.3×10^{-5}	2.2×10^{-4}
A_{FB}^b	3.0×10^{-4}	7.4×10^{-4}
A_{FB}^c	2.3×10^{-4}	5.7×10^{-4}
A_{LR}	4.2×10^{-4}	8.7×10^{-4}

bound on M_H is from A_{LR} at the SLC [25], whereas LEP data alone allow to accommodate also a relatively heavy Higgs (see figure 1). Further constraints on M_H are to be expected in the future from more precise M_W measurements at LEP 2.

Besides the direct measurement of the W mass, the quantity s_W^2 resp. the ratio M_W/M_Z is indirectly measured in deep-inelastic neutrino scattering, in particular in the NC/CC neutrino cross section ratio for isoscalar targets. The world average [1] from CCFR, CDHS and CHARM, including the new CCFR result [26]

$$s_W^2 = 1 - M_W^2/M_Z^2 = 0.2244 \pm 0.0044$$

is fully consistent with the direct vector boson mass measurements.

Standard model fits and Higgs mass range: Assuming the validity of the standard model a global fit to all electroweak results from LEP, SLD, M_W, νN and m_t, allows to derive information on the allowed range for the Higgs mass. Although the Higgs mass dependence of the electroweak parameters is only logarithmic, the already quite accurate value for m_t leads to some sensitivity to M_H. The Higgs mass dependence of the χ^2 of an overall fit is shown in figure 2 [27]. As one can see, the impact of R_b, which is on the way to the standard

Table 2: Precision observables: experimental results [1] and standard model predictions.

observable	exp. (1996)	$M_H = 65$ GeV	$M_H = 1$ TeV	Δm_t
M_Z (GeV)	91.1863 ± 0.0020	input	input	
Γ_Z (GeV)	2.4946 ± 0.0027	2.5015	2.4923	± 0.0015
σ_0^{had} (nb)	41.508 ± 0.056	41.441	41.448	± 0.003
Γ_{had}/Γ_e	20.778 ± 0.029	20.798	20.770	± 0.002
Γ_b/Γ_{had}	0.2178 ± 0.0011	0.2156	0.2157	± 0.0002
Γ_c/Γ_{had}	0.1715 ± 0.0056	0.1724	0.1723	± 0.0001
ρ_ℓ	1.0043 ± 0.0014	1.0056	1.0036	± 0.0006
s_ℓ^2	0.23165 ± 0.00024	0.23115	0.23265	± 0.0002
M_W (GeV)	80.356 ± 0.125	80.414	80.216	± 0.038

model value, is only marginal whereas A_{LR} is decisive for a restrictive upper bound for M_H:
including A_{LR}:

$$M_H = 146^{+112}_{-68}\text{GeV}, \quad M_H < 364\text{GeV}(95\% C.L.) \tag{14}$$

without A_{LR}:

$$M_H = 250^{+187}_{-112}\text{GeV}, \quad M_H < 622\text{GeV}(95\% C.L.) \tag{15}$$

Similar results have been obtained by Passarino [28]. The fit results by the LEP-EWWG [1,29] are slightly higher:
all data:

$$M_H = 149^{+148}_{-82}\text{GeV}, \quad M_H < 450\text{GeV}(95\% C.L.) \tag{16}$$

Thee numbers do not yet include the theoretical uncertainties of the standard model predictions. The LEP-EWWG [1,29] has performed a study of the influence of the various 'options' discussed in section 2.4 on the bounds for the Higgs mass with the result that the 95% C.L. upper bound is shifted by +100 GeV to higher values. It has to be kept in mind, however, that this error estimate is based on the uncertainties as given in table 1. Since the recent improvement in the theoretical prediction [23] is going to reduce the theoretical uncertainty especially in the effective mixing angle one may expect also a significant smaller theoretical error on the Higgs mass bounds once the 2-loop terms $\sim G_\mu^2 m_t^2 M_Z^2$ are implemented in the codes used for the fits. At the present stage the codes are without the new terms.

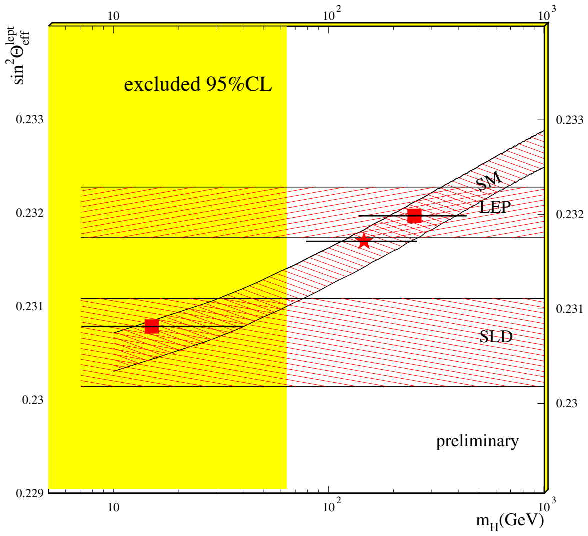

Figure 1: Dependence of the leptonic mixing angle on the Higgs mass. The theoretical predictions correspond to $m_t = 175 \pm 6$ GeV. The SLD [25] (0.23061 ± 0.00047) and LEP [1] (0.23200 ± 0.00027) measurements are separately shown. The star is the result of a combined fit to LEP and SLD data, the squares are for separate fits (from ref.[27], updated version)

4 Conclusions

The quantum structure of the electroweak standard model allows in principle to probe the mass of the as yet experimentally unknown Higgs boson through its contribution to the radiative corrections for electroweak precision observables. Although the dependence on M_H is only logarithmic, the experimental precision in the Z boson parameters and the top quark mass have meanwhile reached a level where a sensitvity to the Higgs mass becomes visible, with preference to a light Higgs. The present upper bound on M_H is dominated by the result on A_{LR}. The instability of the Higgs mass range obtained from global fits with or without A_{LR} recommends to consider the present mass bound with some caution. The only safe conclusion is that we are well below the critical range where the standard Higgs becomes non-perturbative. For the future, the reduction of the theoretical uncertainties and more precise experimental values for M_W and m_t will be the important ingredients in improving the indirect Higgs search.

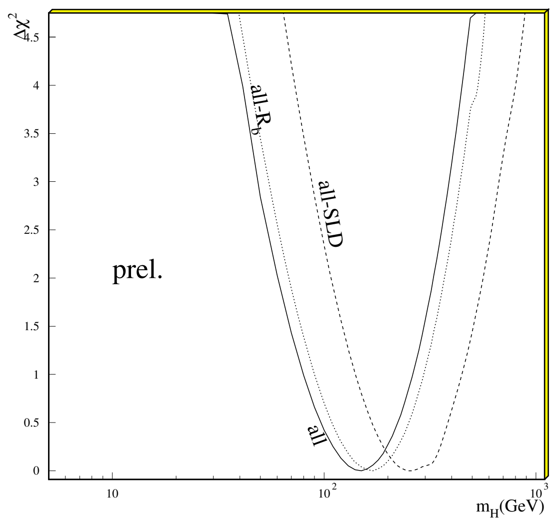

Figure 2: Dependence of $\Delta\chi^2 = \chi^2 - \chi^2_{min}$ on the Higgs mass (from ref.[27], updated version)

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