KA–TP–20–96
September 1996

QCD Corrections to Scalar Quark Decays

A. DJOUADI*, W. HOLLIK, and C. JÜNGER

Institut für Theoretische Physik, Universität Karlsruhe,
D–76128 Karlsruhe, Germany.

Abstract

In supersymmetric theories, the main decay modes of scalar quarks are decays into quarks plus charginos or neutralinos, if the gluinos are heavy enough. We calculate the $\mathcal{O}(\alpha_s)$ QCD corrections to these decay modes in the minimal supersymmetric extension of the Standard Model. In the case of scalar top and bottom quarks, where mixing effects can be important, these corrections can reach values of the order of a few ten percent. They can be either positive or negative and increase logarithmically with the gluino mass. For the scalar partners of light quarks, the corrections do not exceed in general the level of ten percent for gluino masses less than 1 TeV.

*Supported by Deutsche Forschungsgemeinschaft DFG (Bonn).

1. Introduction

Supersymmetric theories (SUSY) [1, 2] are the best motivated extensions of the Standard Model (SM) of the electroweak and strong interactions. They provide an elegant way to stabilize the huge hierarchy between the Grand Unification or Planck scale and the Fermi scale, and its minimal version, the Minimal Supersymmetric Standard Model (MSSM) allows for a consistent unification of the gauge coupling constants and a natural solution of the Dark Matter problem [3].

Supersymmetry predicts the existence of a left– and right–handed scalar partner to each Standard Model (SM) quark. The current eigenstates, $\tilde{q}_L$ and $\tilde{q}_R$, mix to give the mass eigenstates $\tilde{q}_1$ and $\tilde{q}_2$; the mixing angle is proportional to the quark mass and is therefore important only in the case of the third generation squarks [4]. In particular, due to the large value of the top mass m_t, the mixing between the left– and right–handed scalar partners of the top quark, $\tilde{t}_L$ and $\tilde{t}_R$, is very large and after diagonalization of the mass matrix, the lightest scalar top quark mass eigenstate $\tilde{t}_1$ can be much lighter than the top quark and all the scalar partners of the light quarks [4].

If the gluinos [the spin 1/2 superpartners of the gluons] are heavy enough, scalar quarks will mainly decay into quarks and charginos and/or neutralinos [mixtures of the SUSY partners of the electroweak gauge bosons and Higgs bosons]. These are in general tree–level two–body decays, except in the case of the lightest top squark which could decay into a charm quark and a neutralino through loop diagrams if the decay into a chargino and a bottom quark is not overwhelming [5]. These decays have been extensively discussed in the Born approximation [6]. In this paper we will extend these analyses by including the $\mathcal{O}(\alpha_s)$ corrections, which due to the relatively large value of the strong coupling constant, might be large and might affect significantly the decay rates and the branching ratios[1].

The particular case of the QCD corrections to scalar quark decays into massless quarks and photinos has been discussed in Refs. [9, 7]. In the general case that we will address here, there are three [related] features which complicate the analysis, the common denominator of all these features being the finite value of quark masses: (i) In the case of the decays of top and bottom squarks, one needs to take into account the finite value of the top quark mass in the phase space as well as in the loop diagrams. (ii) Scalar quark mixing will introduce a new parameter which will induce additional contributions; since the mixing angle appears in the Born approximation, it needs to be renormalized. (iii) The finite quark mass [which enters the coupling between scalar quarks, quarks and the neutralino/chargino states] needs also to be renormalized.

The QCD corrections to the reaction $\tilde{q} \to q\chi$ analyzed in the present paper are very similar to the case of the reverse process, $t \to \tilde{t}\chi^0$ and $t \to \tilde{b}\chi^+$ recently discussed in Ref. [10] (see also Ref. [11]). During the preparation of this paper, we received a report by Kraml et al. [12], where a similar analysis has been conducted. Our analytical results agree

[1]If the gluinos are lighter than squarks, then squarks will mainly decay into quarks plus gluinos; the QCD corrections to these processes have been recently discussed in Refs.[7, 8].

with those given in this paper[2]. We extend their numerical analysis, which focused on the decay of the lightest top squark into the lightest charginos and neutralinos, by discussing the decays into the heavier charginos and neutralinos and by studying the case of bottom squarks and the SUSY partners of light squarks.

2. Born Approximation

In the Minimal Supersymmetric Standard Model [1, 2], there are two charginos $\chi_i^+ [i=1,2]$ and four neutralinos χ_i^0 $[i=1$–$4]$. Their masses and their couplings to squarks and quarks are given in terms of the Higgs–higgsino mass parameter μ, the ratio of the vacuum expectation values $\tan\beta$ of the two Higgs doublet MSSM fields needed to break the electroweak symmetry, and the wino mass parameter M_2. The bino and gluino masses are related to the parameter M_2 $[M_1 \sim M_2/2$ and $m_{\tilde{g}} \sim 3.5 M_2]$ when the gaugino masses and the three coupling constants of SU(3)×SU(2)×U(1) are unified at the Grand Unification scale.

The squark masses are given in terms of the parameters μ and $\tan\beta$, as well as the left– and right–handed scalar masses $M_{\tilde{q}_L}$ and $M_{\tilde{q}_R}$ [which in general are taken to be equal] and the soft–SUSY breaking trilinear coupling A_q. The top and bottom squark mass eigenstates, and their mixing angles, are determined by diagonalizing the following mass matrices

$$\mathcal{M}^2_{\tilde{t}} = \left(\begin{array}{cc} M^2_{\tilde{t}_L} + m_t^2 + \cos 2\beta (\frac{1}{2} - \frac{2}{3} s_W^2)\, M_Z^2 & m_t\, M_t^{LR} \\ m_t\, M_t^{LR} & M^2_{\tilde{t}_R} + m_t^2 + \frac{2}{3} \cos 2\beta \; s_W^2\, M_Z^2 \end{array} \right) \tag{1}$$

$$\mathcal{M}^2_{\tilde{b}} = \left(\begin{array}{cc} M^2_{\tilde{t}_L} + m_b^2 + \cos 2\beta (-\frac{1}{2} + \frac{1}{3} s_W^2)\, M_Z^2 & m_b\, M_b^{LR} \\ m_b\, M_b^{LR} & M^2_{\tilde{b}_R} + m_b^2 - \frac{1}{3} \cos 2\beta \; s_W^2\, M_Z^2 \end{array} \right) \tag{2}$$

where $M^{LR}_{t,b}$ in the off–diagonal terms read: $M_t^{LR} = A_t - \mu \cot\beta$ and $M_b^{LR} = A_b - \mu \tan\beta$.

In the Born approximation, the partial widths for the decays $\tilde{t}_i \to t\chi_j^0$, $\tilde{t}_i \to b\chi_j^+$ can be written as $[q \equiv t$ or b, and we drop the indices of the neutralino/chargino states]

$$\Gamma_0(\tilde{t}_i \to q\chi) = \frac{\alpha}{4\, m^3_{\tilde{t}_i}} \left[({c_L^i}^2 + {c_R^i}^2)\, (m^2_{\tilde{t}_i} - m_q^2 - m_\chi^2) - 4\, c_L^i\, c_R^i\, m_q\, m_\chi\, \epsilon_\chi \right] \lambda^{1/2}(m^2_{\tilde{t}_i}, m_q^2, m_\chi^2) \tag{3}$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2\,(xy + xz + yz)$ is the usual two–body phase space function and ϵ_χ is the sign of the eigenvalue of the neutralino χ. The couplings $c^i_{L,R}$ for the neutral current process, $\tilde{t}_i \to t\chi^0$, are given by

$$\begin{aligned} \left\{ \begin{array}{c} c_R^1 \\ c_R^2 \end{array} \right\} &= b\, m_t \left\{ \begin{array}{c} s_{\theta_t} \\ c_{\theta_t} \end{array} \right\} + f_L \left\{ \begin{array}{c} c_{\theta_t} \\ -s_{\theta_t} \end{array} \right\} \\ \left\{ \begin{array}{c} c_L^1 \\ c_L^2 \end{array} \right\} &= b\, m_t \left\{ \begin{array}{c} c_{\theta_t} \\ -s_{\theta_t} \end{array} \right\} + f_R \left\{ \begin{array}{c} s_{\theta_t} \\ c_{\theta_t} \end{array} \right\} \end{aligned} \tag{4}$$

[2]We thank the Vienna group and in particular S. Kraml for their cooperation in resolving some discrepancies with some of the formulae and plots given in the early version of the paper Ref. [12]. We also thank T. Plehn for checking independently the results.

$$
\begin{aligned}
b &= \frac{1}{\sqrt{2}\, M_W \sin\beta\, s_W}\, N_{j4} \\
f_L &= \sqrt{2}\left[\frac{2}{3}\, N'_{j1} + \left(\frac{1}{2} - \frac{2}{3}\, s_W^2\right)\frac{1}{c_W s_W}\, N'_{j2}\right] \\
f_R &= -\sqrt{2}\left[\frac{2}{3}\, N'_{j1} - \frac{2}{3}\frac{s_W}{c_W}\, N'_{j2}\right] \;,
\end{aligned} \tag{5}
$$

and for the charged current process, $\tilde{t}_i \to b\chi^+$,

$$
\begin{aligned}
\left\{ \begin{array}{c} c_L^1 \\ c_L^2 \end{array} \right\} &= \frac{m_b\, U_{j2}}{\sqrt{2}\, s_W\, M_W\, \cos\beta} \left\{ \begin{array}{c} -c_{\theta_t} \\ s_{\theta_t} \end{array} \right\} \\
\left\{ \begin{array}{c} c_R^1 \\ c_R^2 \end{array} \right\} &= \frac{V_{j1}}{s_W} \left\{ \begin{array}{c} c_{\theta_t} \\ -s_{\theta_t} \end{array} \right\} - \frac{m_t\, V_{j2}}{\sqrt{2}\, s_W\, M_W\, \sin\beta} \left\{ \begin{array}{c} s_{\theta_t} \\ c_{\theta_t} \end{array} \right\} .
\end{aligned} \tag{6}
$$

In these equations, θ_t is the $\tilde{t}$ mixing angle [which as discussed previously can be expressed in terms of the Higgs–higgsino SUSY mass parameter μ, $\tan\beta$ and the soft–SUSY breaking trilinear coupling A_t] with $s_\theta = \sin\theta$, $c_\theta = \cos\theta$ etc.; $s_W^2 = 1 - c_W^2 \equiv \sin^2\theta_W$ and $N, U/V$ are the diagonalizing matrices for the neutralino and chargino states [13] with

$$
N'_{j1} = c_W N_{j1} + s_W N_{j2} \quad , \quad N'_{j2} = -s_W N_{j1} + c_W N_{j2} \;. \tag{7}
$$

A similar expression eq. (3) can be obtained for the neutral and charged decays of bottom squarks, $\tilde{b}_i \to b\chi_j^0$ and $\tilde{b} \to t\chi_j^-$

$$
\Gamma_0(\tilde{b}_i \to q\chi) = \frac{\alpha}{4\, m_{\tilde{b}_i}^3}\left[({c_L^i}^2 + {c_R^i}^2)\,(m_{\tilde{b}_i}^2 - m_q^2 - m_\chi^2) - 4\, c_L^i\, c_R^i\, m_q\, m_\chi\, \epsilon_\chi\right] \lambda^{1/2}(m_{\tilde{b}_i}^2, m_q^2, m_\chi^2) \tag{8}
$$

with the couplings $c_{L,R}^i$ in the neutral decay $\tilde{b} \to b\chi^0$ given by [θ_b is the $\tilde{b}$ mixing angle]

$$
\begin{aligned}
\left\{ \begin{array}{c} c_R^1 \\ c_R^2 \end{array} \right\} &= b\, m_b \left\{ \begin{array}{c} s_{\theta_b} \\ c_{\theta_b} \end{array} \right\} + f_L \left\{ \begin{array}{c} c_{\theta_b} \\ -s_{\theta_b} \end{array} \right\} \\
\left\{ \begin{array}{c} c_L^1 \\ c_L^2 \end{array} \right\} &= b\, m_b \left\{ \begin{array}{c} c_{\theta_b} \\ -s_{\theta_b} \end{array} \right\} + f_R \left\{ \begin{array}{c} s_{\theta_b} \\ c_{\theta_b} \end{array} \right\}
\end{aligned} \tag{9}
$$

$$
\begin{aligned}
b &= \frac{1}{\sqrt{2}\, M_W \cos\beta\, s_W}\, N_{j3} \\
f_L &= \sqrt{2}\left[-\frac{1}{3}\, N'_{j1} + \left(-\frac{1}{2} + \frac{1}{3}\, s_W^2\right)\frac{1}{c_W s_W}\, N'_{j2}\right] \\
f_R &= -\sqrt{2}\left[-\frac{1}{3}\, N'_{j1} + \frac{1}{3}\frac{s_W}{c_W}\, N'_{j2}\right] \;,
\end{aligned} \tag{10}
$$

and for the charged current process, $\tilde{b}_i \to t\chi^-$,

$$
\begin{aligned}
\left\{ \begin{array}{c} c_L^1 \\ c_L^2 \end{array} \right\} &= \frac{m_t\, V_{j2}}{\sqrt{2}\, s_W\, M_W\, \sin\beta} \left\{ \begin{array}{c} -c_{\theta_b} \\ s_{\theta_b} \end{array} \right\} \\
\left\{ \begin{array}{c} c_R^1 \\ c_R^2 \end{array} \right\} &= \frac{U_{j1}}{s_W} \left\{ \begin{array}{c} c_{\theta_b} \\ -s_{\theta_b} \end{array} \right\} - \frac{m_b\, U_{j2}}{\sqrt{2}\, s_W\, M_W\, \cos\beta} \left\{ \begin{array}{c} s_{\theta_b} \\ c_{\theta_b} \end{array} \right\} .
\end{aligned} \tag{11}
$$

In the case where the mass of the final quark and the squark mixing angle are neglected [as it is the case for the first and second generation squarks], the decay widths simplify to

$$\Gamma_0(\tilde{q}_i \to q\chi) = \frac{\alpha}{4}\, m_{\tilde{q}_i} \left(1 - \frac{m_\chi^2}{m_{\tilde{q}_i}^2}\right)^2 f_i^2 \tag{12}$$

where the f_i's [with now $i = L, R$ since there is no squark mixing] in the case of the neutral decays, $\tilde{q} \to q\chi^0$, are given in terms of the quark isospin I_{3L}^q and charge e_q, by

$$\begin{aligned} f_L &= \sqrt{2}\left[e_q\, N'_{j1} + \left(I_{3L}^q - e_q s_W^2\right) \frac{1}{c_W s_W}\, N'_{j2}\right] \\ f_R &= -\sqrt{2}\left[e_q\, N'_{j1} - e_q\, \frac{s_W}{c_W}\, N'_{j2}\right] \ , \end{aligned} \tag{13}$$

while for the charged decays, $\tilde{q} \to q'\chi^+$ one has for up–type (down–type) squarks:

$$f_L = V_{j1}/s_W \ (U_{j1}/s_W) \quad , \quad f_R = 0 \ . \tag{14}$$

3. QCD corrections to Top Squark Decays

The QCD corrections to the top squark decay width, eq. (3), consist of virtual corrections Figs.1a–d, and real corrections with an additional gluon emitted off the initial $\tilde{t}$ or final t [for the neutral decay] or b [for the charged decay] quark states, Fig. 1e. The $\mathcal{O}(\alpha_s)$ virtual contributions can be split into gluon and gluino exchange in the q–$\tilde{t}$–χ [$q = t, b$] vertex as well as mixing diagrams and the $\tilde{t}$ and t/b wave function renormalization constants. The renormalization of the q–$\tilde{t}$–χ coupling is achieved by renormalizing the top/bottom quark masses and the $\tilde{t}$ mixing angle. We will use the dimensional reduction scheme[3] to regularize the ultraviolet divergencies, and a fictitious gluon mass λ is introduced to regularize the infrared divergencies.

3.1 Virtual Corrections

The QCD virtual corrections to the $\tilde{t}_i$–χ–q interaction vertex can be cast into the form

$$\delta\Gamma^i = ie\, \frac{\alpha_s}{3\pi} \sum_{j=g,\tilde{g},\mathrm{mix},\mathrm{ct}} \left[G^i_{j,L} P_L + G^i_{j,R} P_R\right] \tag{15}$$

where G^i_g, $G^i_{\tilde{g}}$, G^i_{mix} and G^i_{ct} denote the gluon and gluino exchanges in the vertex, and the mixing and counterterm contributions, respectively.

The contribution of the gluonic exchange [Fig. 1a] can be written as

$$G^i_{g,L,R} = c^i_{L,R}\, F^i_1 + c^i_{R,L}\, F^i_2 \tag{16}$$

[3]The quark mass and wave-function counterterms will be different in the dimensional regularization [14] and dimensional reduction schemes [15]. Since dimensional reduction is the scheme which preserves supersymmetry, we will present our results in this scheme.

with the form factors $F^i_{1,2}$ given by

$$
\begin{aligned}
F^i_1 &= B_0 + 2\,m_q^2\,C_0 - 2\,m_{\tilde{t}_i}^2\,(C_{11}-C_{12}) + 2\,m_\chi^2\,C_{11} \\
F^i_2 &= -2\,m_q\,m_\chi\,(C_0 + C_{11})
\end{aligned}
\tag{17}
$$

with $q \equiv t$ for the neutral and $q \equiv b$ for the charged decays; the two and three–point Passarino–Veltman functions, $B_0 \equiv B_0(m_{\tilde{t}_i}^2, \lambda, m_{\tilde{t}_i})$ and $C_{..} \equiv C_{..}(m_q^2, m_{\tilde{t}_i}^2, m_\chi^2,\ m_q^2, \lambda^2, m_{\tilde{t}_i}^2)$ can be found in Ref. [16].

The gluino exchange contributions [Fig. 1b], are given by

$$
\begin{aligned}
G^i_{\tilde{g},L,R} = -2 \sum_{k=1,2} & d^k_{L,R} \Big[(v^k_{\tilde{q}} v^i_{\tilde{t}} + a^k_{\tilde{q}} a^i_{\tilde{t}}) F^{ik}_4 \mp (a^k_{\tilde{q}} v^i_{\tilde{t}} + v^k_{\tilde{q}} a^i_{\tilde{t}}) F^{ik}_5 \\
& + (v^k_{\tilde{q}} v^i_{\tilde{t}} - a^k_{\tilde{q}} a^i_{\tilde{t}}) F^{ik}_6 \mp (a^k_{\tilde{q}} v^i_{\tilde{t}} - v^k_{\tilde{q}} a^i_{\tilde{t}}) F^{ik}_7 \Big] \\
& + d^k_{R,L} \Big[(v^k_{\tilde{q}} v^i_{\tilde{t}} + a^k_{\tilde{q}} a^i_{\tilde{t}}) F^{ik}_1 \mp (a^k_{\tilde{q}} v^i_{\tilde{t}} + v^k_{\tilde{q}} a^i_{\tilde{t}}) F^{ik}_1 \\
& + (v^k_{\tilde{q}} v^i_{\tilde{t}} - a^k_{\tilde{q}} a^i_{\tilde{t}}) F^{ik}_2 \mp (a^k_{\tilde{q}} v^i_{\tilde{t}} - v^k_{\tilde{q}} a^i_{\tilde{t}}) F^{ik}_3 \Big]
\end{aligned}
\tag{18}
$$

with again $q = t$ for the neutral decay and $q = b$ for the charged one; the form factors $F^{ik}_{1,..,7}$ read

$$
\begin{aligned}
F^{ik}_1 &= m_{\tilde{g}}\,m_\chi\,[C_0 + C_{12}] \\
F^{ik}_{2,3} &= m_\chi\,[\pm m_q\,(C_0 + C_{11}) + m_t\,C_{12}] \\
F^{ik}_{4,5} &= m_{\tilde{g}}\,[m_t\,C_0 \pm m_q\,(C_{11} - C_{12})] \\
F^{ik}_{6,7} &= m_{\tilde{q}_k}^2\,C_0 \pm m_t\,m_q\,[C_0 + C_{11} - C_{12}] + m_q^2\,[C_{11} - C_{12}] + m_\chi^2\,C_{12} + B_0
\end{aligned}
\tag{19}
$$

with the two– and three–point functions $B_0 \equiv B_0(m_{\tilde{t}_i}^2, m_{\tilde{g}}, m_t)$ and $C_{..} \equiv C_{..}(m_q^2, m_{\tilde{t}_i}^2, m_\chi^2, m_{\tilde{q}}^2,$ $m_{\tilde{g}}^2, m_t^2)$. The couplings $d^k_{R,L}$ are given by

$$
d^k_{L,R} \ = c^k_{R,L} \tag{20}
$$

for neutralinos, while for the charginos one has

$$
\begin{aligned}
\left\{ \begin{array}{c} d^1_L \\ d^2_L \end{array} \right\} &= \frac{U_{j1}}{s_W} \left\{ \begin{array}{c} c_{\theta_b} \\ -s_{\theta_b} \end{array} \right\} - \frac{m_b\,U_{j2}}{\sqrt{2}\,s_W\,M_W\,\cos\beta} \left\{ \begin{array}{c} s_{\theta_b} \\ c_{\theta_b} \end{array} \right\} \\
\left\{ \begin{array}{c} d^1_R \\ d^2_R \end{array} \right\} &= \frac{m_t\,V_{j2}}{\sqrt{2}\,s_W\,M_W\,\sin\beta} \left\{ \begin{array}{c} -c_{\theta_b} \\ s_{\theta_b} \end{array} \right\} .
\end{aligned}
\tag{21}
$$

The $v^i_{\tilde{q}}$ and $a^i_{\tilde{q}}$ couplings read

$$
\begin{aligned}
v^1_{\tilde{q}} &= \tfrac{1}{2}\,(c_{\theta_q} - s_{\theta_q})\ , \qquad & v^2_{\tilde{q}} &= -\tfrac{1}{2}\,(c_{\theta_q} + s_{\theta_q})\ , \\
a^1_{\tilde{q}} &= \tfrac{1}{2}\,(c_{\theta_q} + s_{\theta_q})\ , \qquad & a^2_{\tilde{q}} &= \tfrac{1}{2}\,(c_{\theta_q} - s_{\theta_q})\ .
\end{aligned}
\tag{22}
$$

Finally, the mixing contributions due to the diagrams Fig. 1c, yield the expressions

$$\begin{aligned} G^i_{\rm mix,L,R} &= \frac{(-1)^i\,(\delta_{1i}\,c^2_{L,R}+\delta_{2i}\,c^1_{L,R})}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\Big[4m_t\,m_{\tilde g}\,c_{2\theta_t}\,B_0(m^2_{\tilde t_i},m_t,m_{\tilde g}) \\ &\qquad + c_{2\theta_t}s_{2\theta_t}(A_0(m^2_{\tilde t_2})-A_0(m^2_{\tilde t_1}))\Big]\,. \end{aligned} \tag{23}$$

Therein, A_0 is the Passarino–Veltman one–point function. Note that all these contributions are the same in both the dimensional reduction and dimensional regularization schemes.

3.2 Counterterms

The counterterm contributions in eq. (15) are due to the $\tilde t$ and t/b wave function renormalizations [Fig. 1d] as well as the renormalization of the quark mass m_t or m_b and the mixing angle θ_t, which appear in the Born couplings.

For the neutral decay process, $\tilde t_i \to t\chi^0_j$, the counterterm contribution is given by

$$\begin{aligned} G^{1,2}_{\rm ct,L} &= \frac{1}{2}\,c^{1,2}_L\,(\delta Z^t_R+\delta Z_{\tilde t_{1,2}})+b\,\{c_{\theta_t},-s_{\theta_t}\}\,\delta m_t-b\,m_t\,\{s_{\theta_t},c_{\theta_t}\}\,\delta\theta_t+f_R\,\{c_{\theta_t},-s_{\theta_t}\}\,\delta\theta_t \\ G^{1,2}_{\rm ct,R} &= \frac{1}{2}\,c^{1,2}_R\,(\delta Z^t_L+\delta Z_{\tilde t_{1,2}})+b\,\{s_{\theta_t},c_{\theta_t}\}\,\delta m_t+b\,m_t\,\{c_{\theta_t},-s_{\theta_t}\}\,\delta\theta_t-f_L\,\{s_{\theta_t},c_{\theta_t}\}\,\delta\theta_t\;, \end{aligned} \tag{24}$$

whereas for the charged current process, $\tilde t_i \to b\chi^+_j$, one obtains,

$$\begin{aligned} G^{1,2}_{\rm ct,L} &= \frac{1}{2}\,c^{1,2}_L\left[\delta Z^b_R+\delta Z_{\tilde t_{1,2}}+2\,\frac{\delta m_b}{m_b}\right]+\frac{m_b U_{j2}}{\sqrt2\,s_W\,M_W\,\cos\beta}\,\{s_{\theta_t},c_{\theta_t}\}\,\delta\theta_t \\ G^{1,2}_{\rm ct,R} &= \frac{1}{2}\,c^{1,2}_R\left[\delta Z^b_L+\delta Z_{\tilde t_{1,2}}\right]-\frac{\delta m_t\,V_{j2}}{\sqrt2\,s_W\,M_W\,\sin\beta}\,\{s_{\theta_t},c_{\theta_t}\} \\ &\quad -\frac{V_{j1}}{s_W}\,\{s_{\theta_t},c_{\theta_t}\}\,\delta\theta_t-\frac{m_t V_{j2}}{\sqrt2\,s_W\,M_W\,\sin\beta}\,\{c_{\theta_t},-s_{\theta_t}\}\,\delta\theta_t\;. \end{aligned} \tag{25}$$

In the on–shell scheme, the quark and squark masses are defined as the poles of the propagators and the wave–function renormalization constants follow from the residues at the poles; the corresponding counterterms are given by (see also Refs. [10, 12])

$$\begin{aligned} \frac{\delta m_q}{m_q} &= \frac{1}{2}\Big[\Sigma^q_R(m^2_q)+\Sigma^q_L(m^2_q)\Big]+\Sigma^q_S(m^2_q) \\ \delta Z^q_L &= -\Sigma^q_L(m^2_q)-m^2_q\Big[\Sigma^{q\,\prime}_L(m^2_q)+\Sigma^{q\,\prime}_R(m^2_q)+2\,\Sigma^{q\,\prime}_S(m^2_q)\Big] \\ \delta Z^q_R &= -\Sigma^q_R(m^2_q)-m^2_q\Big[\Sigma^{q\,\prime}_L(m^2_q)+\Sigma^{q\,\prime}_R(m^2_q)+2\,\Sigma^{q\,\prime}_S(m^2_q)\Big] \\ \delta Z_{\tilde t_i} &= -\left(\Sigma^{ii}_{\tilde t}\right)'(m^2_{\tilde t_i}) \end{aligned} \tag{26}$$

In the dimensional reduction scheme, the self–energies Σ and their derivatives Σ', up to a factor $\alpha_s/3\pi$ which has been factorized out, are given by [10, 12]

$$
\begin{aligned}
\Sigma^q_L(k^2) &= -\Big[2\,B_1(k^2,m_q,\lambda)+(1+c_{2\theta_q})B_1(k^2,m_{\tilde g},m_{\tilde q_1})+(1-c_{2\theta_q})B_1(k^2,m_{\tilde g},m_{\tilde q_2})\Big] \\
\Sigma^q_R(k^2) &= -\Big[2\,B_1(k^2,m_q,\lambda)+(1-c_{2\theta_q})B_1(k^2,m_{\tilde g},m_{\tilde q_1})+(1+c_{2\theta_q})B_1(k^2,m_{\tilde g},m_{\tilde q_2})\Big] \\
\Sigma^q_S(k^2) &= -\Big[4\,B_0(k^2,m_q,\lambda)+\frac{m_{\tilde g}}{m_q}\,s_{2\theta_q}\left(B_0(k^2,m_{\tilde g},m_{\tilde q_1})-B_0(k^2,m_{\tilde g},m_{\tilde q_2})\right)\Big] \\
(\Sigma^{ii}_{\tilde t})'(k^2) &= -2\Big[-2\,B_1(k^2,m_{\tilde t_i},\lambda)-2\,k^2\,B_1'(k^2,m_{\tilde t_i},\lambda)+(m_t^2+m_{\tilde g}^2-k^2)\,B_0'(k^2,m_t,m_{\tilde g}) \\
&\qquad -B_0(k^2,m_t,m_{\tilde g})+(-1)^i\,2\,s_{2\theta}\,m_t\,m_{\tilde g}B_0'(k^2,m_t,m_{\tilde g})\Big]\;. \qquad (27)
\end{aligned}
$$

Using dimensional regularization, the quark self–energies differ from the previous expressions by a constant; in terms of the their values in the dimensional reduction scheme, they are given by

$$
\Sigma^q_{L,R}\Big|_{\rm dim.\ reg.}=\Sigma^q_{L,R}-2\quad,\quad \Sigma^q_S\big|_{\rm dim.\ reg.}=\Sigma^q_S+2\;. \qquad (28)
$$

Finally, we need a prescription to renormalize the $\tilde t$ mixing angle θ_t. Following Ref. [17], we choose this condition in such a way that it cancels exactly the mixing contributions eq. (23) for the decay $\tilde t_2\to t\chi^0$

$$
\delta\theta_t = \frac{1}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\left[4\,m_t\,m_{\tilde g}\,c_{2\theta_t}\,B_0(m^2_{\tilde t_2},m_t,m_{\tilde g})+c_{2\theta_t}s_{2\theta_t}(A_0(m^2_{\tilde t_2})-A_0(m^2_{\tilde t_1}))\right]\;. \qquad (29)
$$

Alternatively, since the lightest top squark $\tilde t_1$ can be lighter than the top quark and then is more likely to be discovered first in the top decays $t\to\tilde t_1\chi_0$, one can choose the renormalization condition such that the mixing contributions are cancelled in the latter process; this leads to a counterterm similar to eq. (29) but with $B_0(m^2_{\tilde t_2},m_t,m_{\tilde g})$ replaced by $B_0(m^2_{\tilde t_1},m_t,m_{\tilde g})$. The difference between the two renormalization conditions,

$$
\Delta\delta\theta_t=\frac{4m_t\,m_{\tilde g}\,c_{2\theta_t}}{m^2_{\tilde t_1}-m^2_{\tilde t_2}}\left[B_0(m^2_{\tilde t_1},m_t,m_{\tilde g})-B_0(m^2_{\tilde t_2},m_t,m_{\tilde g})\right] \qquad (30)
$$

is, however, very small numerically. Indeed, if $m_{\tilde t_1}$ is a few GeV away from $m_{\tilde t_2}$, one has $\theta_t\simeq-\pi/4$ and therefore $c_{2\theta_t}\sim 0$, leading to a difference which is less than one permille for the scenario of Figs. 2a/b. For degenerate top squarks, one has $\Delta\delta\theta=4m_tm_{\tilde g}c_{2\theta_t}B_0'(m^2_{\tilde t_2},m_t,m_{\tilde g})$ which is also very small numerically [less than $\sim 1\%$ for the scenarios of Fig. 2.]

The complete virtual corrections to the $\tilde t_i\to q\chi$ decay width is then given by

$$
\begin{aligned}
\Gamma^V(\tilde t_i\to q\chi) &= \frac{\alpha}{6\,m^3_{\tilde t_i}}\frac{\alpha_s}{\pi}\,{\rm Re}\,\Big\{(c^i_L\,G^i_L+c^i_R\,G^i_R)\,(m^2_{\tilde t_i}-m^2_q-m^2_\chi) \\
&\qquad -2\,(c^i_L\,G^i_R+c^i_R\,G^i_L)\,m_q\,m_\chi\epsilon_\chi\Big\}\,\lambda^{1/2}(m^2_{\tilde t_i},m^2_q,m^2_\chi)\;. \qquad (31)
\end{aligned}
$$

The sum of all virtual contributions including the counterterms are ultraviolet finite as it should be, but they are still infrared divergent; the infrared divergencies will be cancelled after adding the real corrections.

3.3 Real Corrections

The contributions to the squark decay widths from the real corrections, with an additional gluon emitted from the initial $\tilde{t}$ or final t/b states, can be cast into the form

$$
\begin{aligned}
\Gamma^i_{\rm real} &= \frac{2\,\alpha}{3\,m_{\tilde{t}_i}}\frac{\alpha_s}{\pi}\Big\{8\;c^i_L\,c^i_R\;m_q\,m_\chi\epsilon_\chi\,[\;(m^2_{\tilde{t}_i}+m^2_q-m^2_\chi)\,I_{01}+m^2_{\tilde{t}_i}\,I_{00}+m^2_q\,I_{11}+I_0+I_1] \\
&\quad +(c^{i\,2}_L+c^{i\,2}_R)\,[\;2\,(m^2_q+m^2_\chi-m^2_{\tilde{t}_i})\,(m^2_{\tilde{t}_i}\,I_{00}+m^2_q\,I_{11}+I_0+I_1) \\
&\qquad +2\,(m^4_q-\;(m^2_\chi-m^2_{\tilde{t}_i})^2)\,I_{01}-I-I^0_1]\Big\} \qquad (32)
\end{aligned}
$$

where the phase space integrals $I(m_{\tilde{t}_i},m_q,m_\chi)\equiv I$ are given by [18]

$$
\begin{aligned}
I_{00} &= \frac{1}{4\,m^4_{\tilde{t}_i}}\Big[\kappa\,\ln\Big(\frac{\kappa^2}{\lambda\,m_{\tilde{t}_i}\,m_q\,m_\chi}\Big)-\kappa-(m^2_q-m^2_\chi)\ln\Big(\frac{\beta_1}{\beta_2}\Big)-m^2_{\tilde{t}_i}\,\ln(\beta_0)\Big] \\
I_{11} &= \frac{1}{4\,m^2_q\,m^2_{\tilde{t}_i}}\Big[\kappa\,\ln\Big(\frac{\kappa^2}{\lambda\,m_{\tilde{t}_i}\,m_q\,m_\chi}\Big)-\kappa-(m^2_{\tilde{t}_i}-m^2_\chi)\ln\Big(\frac{\beta_0}{\beta_2}\Big)-m^2_q\,\ln(\beta_1)\Big] \\
I_{01} &= \frac{1}{4\,m^2_{\tilde{t}_i}}\Big[-2\,\ln\Big(\frac{\lambda\,m_{\tilde{t}_i}\,m_q\,m_\chi}{\kappa^2}\Big)\,\ln(\beta_2)+2\,\ln^2(\beta_2)-\ln^2(\beta_0)-\ln^2(\beta_1) \\
&\quad +2\,{\rm Li}_2\,(1-\beta^2_2)-{\rm Li}_2\,(1-\beta^2_0)-{\rm Li}_2\,(1-\beta^2_1)\Big] \\
I &= \frac{1}{4\,m^2_{\tilde{t}_i}}\Big[\frac{\kappa}{2}(m^2_{\tilde{t}_i}+m^2_q+m^2_\chi)+2\,m^2_{\tilde{t}_i}\,m^2_q\,\ln(\beta_2)+2\,m^2_{\tilde{t}_i}\,m^2_\chi\,\ln(\beta_1)+2\,m^2_q\,m^2_\chi\,\ln(\beta_0)\Big] \\
I_0 &= \frac{1}{4\,m^2_{\tilde{t}_i}}\Big[-2\,m^2_q\,\ln(\beta_2)-2\,m^2_\chi\,\ln(\beta_1)-\kappa\Big] \\
I_1 &= \frac{1}{4\,m^2_{\tilde{t}_i}}\Big[-2\,m^2_{\tilde{t}_i}\,\ln(\beta_2)-2\,m^2_\chi\,\ln(\beta_0)-\kappa\Big] \\
I^0_1 &= \frac{1}{4\,m^2_{\tilde{t}_i}}\Big[m^4_{\tilde{t}_i}\,\ln(\beta_2)-m^2_\chi\,(2\,m^2_q-2\,m^2_{\tilde{t}_i}+m^2_\chi)\,\ln(\beta_0)-\frac{\kappa}{4}\,(m^2_q-3\,m^2_{\tilde{t}_i}+5\,m^2_\chi)\Big]\;. \qquad (33)
\end{aligned}
$$

with $\kappa=\lambda^{1/2}(m^2_{\tilde{t}_i},m_q,m_\chi)$ and

$$
\beta_0=\frac{m^2_{\tilde{t}_i}-m^2_q-m^2_\chi+\kappa}{2\,m_q\,m_\chi},\quad \beta_1=\frac{m^2_{\tilde{t}_i}-m^2_q+m^2_\chi-\kappa}{2\,m_{\tilde{t}_i}\,m_\chi},\quad \beta_2=\frac{m^2_{\tilde{t}_i}+m^2_q-m^2_\chi-\kappa}{2\,m_{\tilde{t}_i}\,m_q}\;. \qquad (34)
$$

Our analytical results agree with the results obtained recently in Ref. [12].

4. QCD corrections to other squark decays

4.1 Bottom Squark Decays

In the case of the bottom squark decays, $\tilde{b}_i \to b\chi^0$ and $\tilde{b}_i \to t\chi^-$, the analytical expressions of the QCD corrections are just the same as in the previous section once the proper changes of the squark $[m_{\tilde{t}_i} \to m_{\tilde{b}_i}]$, the quark $[q \equiv b$ and $q \equiv t$ for the neutral and charged decays] masses and the mixing angles $[\theta_t \to \theta_b]$ are performed. The couplings for $\tilde{b}$ decays are as given in section 2: for the $d^k_{L,R}$ couplings, one has in the case of the neutral decay $\tilde{b}_i \to b\chi^0$

$$d^k_{L,R} \quad = c^k_{R,L} \;, \tag{35}$$

with $c^k_{L,R}$ of eq. (11), while in the charged decay $\tilde{b}_i \to t\chi^-$, they read

$$\begin{aligned} \left\{ \begin{array}{c} d^1_L \\ d^2_L \end{array} \right\} &= \frac{V_{j1}}{s_W} \left\{ \begin{array}{c} c_{\theta_t} \\ -s_{\theta_t} \end{array} \right\} - \frac{m_t\, V_{j2}}{\sqrt{2}\, s_W\, M_W\, \sin\beta} \left\{ \begin{array}{c} s_{\theta_t} \\ c_{\theta_t} \end{array} \right\} \\ \left\{ \begin{array}{c} d^1_R \\ d^2_R \end{array} \right\} &= \frac{m_b\, U_{j2}}{\sqrt{2}\, s_W\, M_W\, \cos\beta} \left\{ \begin{array}{c} -c_{\theta_t} \\ s_{\theta_t} \end{array} \right\} . \end{aligned} \tag{36}$$

The counterterm contributions are the same as in eq. (24) with the change $(t,\tilde{t}) \to (b,\tilde{b})$ in the neutral decay; in the charged decay mode they are different due to different couplings (see also Refs. [10, 12]):

$$\begin{aligned} G^{1,2}_{\rm ct,L} &= \frac{1}{2}\, c^{1,2}_L \left[\delta Z^t_R + \delta Z_{\tilde{b}_{1,2}} + 2\,\frac{\delta m_t}{m_t} \right] + \frac{m_t V_{j2}}{\sqrt{2}\, s_W\, M_W\, \sin\beta} \left\{ s_{\theta_b}, c_{\theta_b} \right\} \delta\theta_b \\ G^{1,2}_{\rm ct,R} &= \frac{1}{2}\, c^{1,2}_R \left[\delta Z^t_L + \delta Z_{\tilde{b}_{1,2}} \right] - \frac{\delta m_b\, U_{j2}}{\sqrt{2}\, s_W\, M_W\, \cos\beta} \left\{ s_{\theta_b}, c_{\theta_b} \right\} \\ &\quad - \frac{U_{j1}}{s_W} \left\{ s_{\theta_b}, c_{\theta_b} \right\} \delta\theta_b - \frac{m_b U_{j2}}{\sqrt{2}\, s_W\, M_W\, \cos\beta} \left\{ c_{\theta_b}, -s_{\theta_b} \right\} \delta\theta_b \;. \end{aligned} \tag{37}$$

where again the $c^k_{L,R}$ are given by eq. (11). Except for very large values of $\tan\beta$, the $\tilde{b}$ mixing angle [as well as the bottom quark mass] can be set to zero and the analytical expressions simplify considerably[4]. The case of the neutral decay $\tilde{b} \to b\chi^0$ is even simpler since one can also neglect the mass of the final b quark. In fact, the latter situation corresponds to the case of decays of first and second generation squarks into light quarks and charginos/neutralinos, which will be discussed now.

4.2 Light Quark Partners Decays

Neglecting the squark mixing angle as well as the mass of the final quarks, the virtual corrections of the processes $\tilde{q}_i \to q\chi$ [where the subscript i stands now for the chirality of

[4]In the absence of mixing, the left– and right–handed bottom squarks are, to a very good approximation, degenerate if $M_{\tilde{q}_L} = M_{\tilde{q}_R}$. In the rest of the discussion, $\tilde{b}_L$ and $\tilde{b}_R$ [and a *fortiori* the partners of the light quarks $\tilde{q}_L$ and $\tilde{q}_R$] will be considered as degenerate.

the squark, since in the absence of squark mixing one has $\tilde{q}_{L,R} = \tilde{q}_{1,2}$] are given by the sum of the gluon and gluino exchange vertices and the wave–function counterterm, plus the real correction. The total width can then be written as

$$\Gamma^i = \Gamma_0^i \left[1 + \frac{4}{3} \frac{\alpha_s}{\pi} \left(F_{\rm g} + F_{\tilde{\rm g}} + F_{\rm ct} + F_{\rm r} \right) \right] \tag{38}$$

where the decay width in the Born approximation Γ_0^i has been given in eq. (12). In terms of the ratio $\kappa = m_\chi^2/m_{\tilde{q}}^2$, the gluon exchange corrections are given by [$\Delta = 1/(4-n)$ with n the space-time dimension, and μ is the renormalization scale]

$$F_{\rm g} \;=\; \frac{\Delta}{2} + 1 - \frac{1}{2} \ln \frac{m_{\tilde{q}}^2}{\mu^2} - \frac{1}{4} \ln^2 \frac{\lambda^2/m_{\tilde{q}}^2}{(1-\kappa)^2} - \ln \frac{\lambda^2/m_{\tilde{q}}^2}{1-\kappa} - {\rm Li}_2(\kappa) \; . \tag{39}$$

The gluino exchange contribution, with $\gamma = m_{\tilde{g}}^2/m_{\tilde{q}}^2$, is given by

$$F_{\tilde{\rm g}} = \sqrt{\kappa\gamma} \left[\frac{1}{\kappa} \ln(1-\kappa) + \frac{1}{1-\kappa} \left[\gamma \ln \gamma - (\gamma-1) \ln(\gamma-1) \right] + \frac{\kappa+\gamma-2}{(1-\kappa)^2}\, I \right] \tag{40}$$

with

$$I \;\equiv\; \frac{1}{m_{\tilde{q}_i}^2 \, (1-\kappa)} \, C_0(0, m_{\tilde{q}}^2, m_\chi^2, m_{\tilde{q}}^2, m_{\tilde{g}}^2, 0) \; . \tag{41}$$

In terms of dilogarithms, the function I is given for $\kappa\gamma < 1$ by

$$I = {\rm Li}_2\left(\frac{\gamma-1}{\gamma\kappa-1}\right) - {\rm Li}_2\left(\kappa\frac{\gamma-1}{\gamma\kappa-1}\right) - {\rm Li}_2\left(\frac{\gamma+\kappa-2}{\gamma\kappa-1}\right) + {\rm Li}_2\left(\kappa\frac{\gamma+\kappa-2}{\gamma\kappa-1}\right) \tag{42}$$

and for $\kappa\gamma > 1$ one has

$$\begin{aligned} I \;=\; & -{\rm Li}_2\left(\frac{\gamma\kappa-1}{\gamma-1}\right) + {\rm Li}_2\left(\frac{\gamma\kappa-1}{\gamma+\kappa-2}\right) + {\rm Li}_2\left(\frac{\gamma\kappa-1}{\kappa(\gamma-1)}\right) - {\rm Li}_2\left(\frac{\gamma\kappa-1}{\kappa(\gamma+\kappa-2)}\right) \\ & - \ln(\kappa) \ln \frac{\gamma+\kappa-2}{\gamma-1} \; . \end{aligned} \tag{43}$$

The counterterm contribution, consisting of the sum of the squark and quark wave–function renormalization constants, reads

$$\begin{aligned} F_{\rm ct} \;=\; & -\frac{\Delta}{2} + \frac{\gamma}{4\,(1-\gamma)} - \frac{\gamma}{2} - \frac{15}{8} + \frac{1}{2} \ln \frac{m_{\tilde{q}}^2}{\mu^2} - \frac{1}{4} \ln \frac{\lambda^2}{m_{\tilde{q}}^2} \\ & - \frac{1}{2}(\gamma^2-1) \ln \frac{\gamma-1}{\gamma} + \frac{1}{4} \left[\frac{2\,\gamma-1}{(1-\gamma)^2} + 3 \right] \ln \gamma \; . \end{aligned} \tag{44}$$

Finally, the real corrections with massless quarks in the final state contribute

$$\begin{aligned} F_{\rm r} \;=\; & \frac{1}{4} \ln^2 \frac{\lambda^2/m_{\tilde{q}}^2}{(1-\kappa)^2} + \frac{5}{4} \ln \frac{\lambda^2/m_{\tilde{q}}^2}{(1-\kappa)^2} - \frac{\kappa\,(4-3\kappa)}{4\,(1-\kappa)^2} \ln \kappa \\ & - {\rm Li}_2(\kappa) - \ln \kappa \ln(1-\kappa) - \frac{3\kappa-5}{8\,(\kappa-1)} - \frac{\pi^2}{3} + 4 \; . \end{aligned} \tag{45}$$

We see explicitly that the ultraviolet divergences $\Delta/2$ and the scale μ cancel when F_g^i and $F_{\rm ct}^i$ are added, and that the infrared divergences $\ln^2(\lambda^2/m_{\tilde q}^2)$ and $\ln(\lambda^2/m_{\tilde q}^2)$ disappear when F_g, $F_{\rm ct}$ and $F_{\rm r}$ are summed. The gluino exchange contribution eq. (40) does not contain any ultraviolet or infrared divergences. The total correction in eq. (38) then reads

$$\begin{aligned}
F_{\rm tot} &= F_{\rm g} + F_{\tilde g} + F_{\rm ct} + F_{\rm r} \\
&= -\frac{1}{8}\left(\frac{4\,\gamma^2 - 27\,\gamma + 25}{\gamma - 1} + \frac{3\,\kappa - 5}{\kappa - 1}\right) - \frac{\pi^2}{3} - 2\,{\rm Li}_2(\kappa) - \frac{1}{2}\,(\gamma^2 - 1)\,\ln\frac{\gamma - 1}{\gamma} \\
&\quad + \frac{3\,\gamma^2 - 4\,\gamma + 2}{4\,(1-\gamma)^2}\,\ln\gamma - \frac{3}{2}\,\ln(1-\kappa) + \frac{3\,\kappa^2 - 4\,\kappa}{4\,(\kappa - 1)^2}\,\ln\kappa - \ln\kappa\,\ln(1-\kappa) \\
&\quad + \sqrt{\kappa\gamma}\left[\frac{1}{\kappa}\ln(1-\kappa) + \frac{1}{1-\kappa}\left[\gamma\ln\gamma - (\gamma - 1)\ln(\gamma - 1)\right] + \frac{\kappa + \gamma - 2}{(1-\kappa)^2}\,I\right]. \qquad (46)
\end{aligned}$$

In the limit where the mass of the final neutralino or chargino is much smaller than the mass of the initial squark, the analytical expression of the QCD correction further simplifies:

$$F_{\rm tot} = \frac{3\gamma^2 - 4\gamma + 2}{4(\gamma - 1)^2}\ln\gamma - \frac{1}{2}(\gamma^2 - 1)\ln\frac{\gamma - 1}{\gamma} - \frac{2\gamma^2 - 11\gamma + 10}{4(\gamma - 1)} - \frac{\pi^2}{3}\,. \qquad (47)$$

Note the explicit logarithmic dependence on the gluino mass in the correction. This logarithmic behaviour, leading to a non-decoupling of the gluinos for very large masses,

$$F_{\rm tot} = \frac{3}{4}\ln\frac{m_{\tilde g}^2}{m_{\tilde q}^2} + \frac{5}{2} - \frac{\pi^2}{3} \quad \text{for} \quad m_{\tilde g} \gg m_{\tilde q} \qquad (48)$$

is due to the wave function renormalization and is a consequence of the breakdown of SUSY as discussed in Ref. [9]. Had we chosen the $\overline{\rm MS}$ scheme when renormalizing the squark/quark wave functions [i.e. subtracting only the poles and the related constants in the expression eq. (23)] we would have been left with contributions which increase linearly with the gluino mass.

Our analytical results in the case of massless final quarks agree with the corresponding results obtained in Refs.[9, 7], where the QCD corrections to the decay of a squark into a massless quark and a photino have been derived, after correcting the sign of $F_{\tilde g}$ in Ref. [9]; see also the discussion given in Ref. [7].

5. Numerical Analysis and Discussion

In the numerical analysis of the QCD corrections to squark decays, we will choose $m_t = 180$ GeV (consistent with [19]) and $m_b = 5$ GeV for the top and bottom quark masses and a constant value for the strong coupling constant $\alpha_s = 0.12$ [the value of α_s in the running from a scale of 0.1 to 1 TeV does not change significantly]; the other fixed input parameters are $\alpha = 1/128$, $M_Z = 91.187$ GeV and $s_W^2 = 0.23$ [20]. For the SUSY parameters, we will take into account the experimental bounds from the Tevatron and LEP1.5 data [21], and in some cases use the values favored by fits of the electroweak precision data from LEP1 [22].

Fig. 2 shows the partial widths for the decays of the lightest top squark into the two charginos $\chi^+_{1,2}$ and a bottom quark [2a] and into the lightest neutralino χ^0_1 and the sum of all neutralinos [the opening of the neutralino thresholds can be seen in the curves] and a top quark [2b]. In these figures, $\tan\beta$ is fixed to $\tan\beta = 1.6$, a value favored by b–τ Yukawa coupling unification [23]. The solid, dashed and dot–dashed curves correspond to the (M_2, μ) values [in GeV]: $(70, -500), (70, -70)$ and $(300, -70)$ in Fig. 2a [which give approximately the same value for the lightest chargino mass, $m_{\chi_1^+} \simeq 70$ GeV] and $(100, -500), (100, -100)$ and $(250, -50)$ in Fig. 2b [giving an LSP mass of $m_{\chi_1^0} \sim 50$ GeV]. These values correspond to the scenarios $M_2 \ll |\mu|$, $M_2 \simeq \mu$ and $M_2 \gg |\mu|$, and have been chosen to allow for a comparison with the numerical analysis given in [12]. The parameters in the $\tilde{t}$ mass matrix are fixed by requiring $m_{\tilde{t}_2} = 600$ GeV and varying $M_{\tilde{t}_L}$. The mixing angle is then completely fixed assuming $M_{\tilde{t}_R} = M_{\tilde{t}_L}$ ($\theta_{\tilde{t}} \approx -\pi/4$ except for $m_{\tilde{t}_1}$ very close to $m_{\tilde{t}_2}$); in the bottom squark sector we have $m_{\tilde{b}_1} = 220$ GeV, $m_{\tilde{b}_2} \sim 230$ GeV and $\theta_{\tilde{b}} \simeq 0$.

Fig. 3 shows the magnitude of the QCD corrections relative to the Born width to the decays of the lightest top squark into charginos+bottom [3a/b] and neutralinos+stop [3c/d] for the scenarios described in Fig. 2a [for Figs.3a/b] and Fig. 2b [for Figs.3c/d]. For both the neutral and charged decays, the QCD corrections can be rather large and vary in a wide margin: from $\sim \pm 10\%$ for light top squarks up to $\sim -40\%$ for $m_{\tilde{t}_1} \sim m_{\tilde{t}_2}$ and some (M_2, μ) values.

The small spikes near $m_{\tilde{t}_1} \sim 425$ (530) GeV for $\chi^+ b$ ($\chi^0 t$) decays are due to thresholds in the top squark wave function renormalization constants from the channel $\tilde{t}_1 \to \tilde{g} t$. For the depicted $m_{\tilde{t}_1}$ range, this happens only for the value $M_2 = 70$ (100) GeV which leads to $m_{\tilde{g}} \simeq 3.5 M_2 \sim 245(350)$ GeV. Note, however, that when this occurs, the channel $\tilde{t}_1 \to \tilde{g} t$ becomes by far the main decay mode, and the chargino/neutralino modes are very rare.

In Fig. 4 the variation of the QCD corrections for the decay $\tilde{t}_1 \to b\chi_1^+$ [4a] and $\tilde{t}_1 \to t\chi_1^0$ [4b] is displayed as a function of the gluino mass, for two values of $\mu = -50$ and -500 GeV and $\tan\beta = 1.6$ and 20. The top squark masses are fixed to $m_{\tilde{t}_1} = 300$ and $m_{\tilde{t}_2} = 600$ GeV ($\theta_{\tilde{t}} = -\pi/4$) and the $\tilde{b}$ masses are as in Fig. 2. M_2 and hence the chargino and neutralino masses are fixed by $m_{\tilde{g}}$. The figure exhibits a slight dependence of the QCD correction on the gluino mass. For the chosen set of squark mass parameters, the variation of the QCD correction with μ is rather pronounced, while the variation with $\tan\beta$ is milder.

Fig. 5 shows the partial decay widths for the decays of the lightest bottom squark [which in our convention is denoted by $\tilde{b}_1$ and is almost left–handed] into the lightest chargino χ_1^- and a top quark [5a] and into the lightest neutralino χ_1^0 and a bottom quark [5b]. As in Fig. 2, $\tan\beta$ is fixed to $\tan\beta = 1.6$ and $m_{\tilde{t}_1} = 600$ GeV; the mass difference between the two squarks is $\simeq 10$ GeV and we have for the mixing angle $\theta_{\tilde{b}} \simeq 0$. The solid, dashed and dot–dashed curves correspond to the (M_2, μ) values [in GeV]: $(60, -500), (70, -60)$ and $(300, -60)$ in Fig. 5a and $(100, -500), (100, -100)$ and $(250, -50)$ in Fig. 5b. The decay $\tilde{b}_1 \to t\chi_1^-$ is by far dominant when the channel $\tilde{b}_1 \to \tilde{g} b$ is closed, since its decay width is almost two orders of magnitude larger than the $\tilde{b}_1 \to$ LSP+bottom decay width.

Fig. 6 presents the magnitude of the relative QCD corrections to the decays $\tilde{b}_1 \to t\chi_1^-$ [6a] and $\tilde{b}_1 \to b\chi_1^0$ [6b] as a function of the bottom squark mass, for the same scenarios as in Fig. 5. Again, depending on the values of μ, M_2 and $m_{\tilde{b}_1}$, the QCD corrections vary from ($\pm$) a few percent up to -50%.

Finally, Fig. 7 displays the QCD corrections to the decays of the SUSY partners of massless quarks into neutralinos, $\tilde{q} \to q\chi_0$, as a function of the ratio $\kappa = m_\chi^2/m_{\tilde{q}}^2$ for several values of the ratio $\gamma = m_{\tilde{g}}^2/m_{\tilde{q}}^2, \gamma = 1.2, 1.5$ and 2 [7a] and as a function of γ for several values of $\kappa, \kappa = 0.2, 0.5$ and 0.8 [7b]. The quark mass and the squark mixing angle are set to zero and all squarks are taken to be degenerate. The corrections then depend only on the two parameters, κ and γ since the dependence on the other SUSY parameters factorizes in the Born term. The QCD corrections vary from small [most of the time negative] values for small κ values and small gluino masses, up to $\sim 20\%$ near threshold.

For the decays $\tilde{q}_L \to q'\chi_j^\pm$ [the right–handed squark does not decay into charginos], the matrix elements in the chargino mass matrix do not factorize in the Born expressions and the QCD corrections further depend on the ratios U_{j1}/V_{j1} through the contribution $F_{\tilde{g}}$. This dependence is, however, rather mild since first the ratio U_{j1}/V_{j1} is of order unity in most of the relevant SUSY parameter space [in particular for $|\mu| > M_2$] and second the contribution $F_{\tilde{g}}$ is small compared to the other contributions for gluino masses below 1 TeV. The QCD corrections for the decays $\tilde{q}_L \to q'\chi^\pm$ are thus approximately the same as in the case of the decays into neutralinos.

In conclusion: we have calculated the $\mathcal{O}(\alpha_s)$ QCD corrections to decay modes of scalar squarks into quarks plus charginos or neutralinos in the Minimal Supersymmetric Standard Model. We have paid a particular attention to the case of $\tilde{t}$ [and also $\tilde{b}$] squarks, where mixing effects are important. In the case of top squark decays, the QCD corrections can reach values of the order of a few ten percent depending on the various SUSY parameters. They can be either positive or negative and increase logarithmically with the gluino mass. For the scalar partners of light quarks, the corrections do not exceed the level of ten to twenty percent for gluino masses less than 1 TeV.

Acknowledgements:

We thank Tilman Plehn and Peter Zerwas for discussions and for the comparison between their results Ref. [8] and ours, and the Vienna group, in particular Sabine Kraml, for discussions about Ref. [12].

References

[1] For reviews on Supersymmetry see:
H. P. Nilles, Phys. Rep. 117 (1985) 1;
P. Nath, R. Arnowitt and A. Chamseddine, *Applied N=1 Supergravity*, ICTP series in Theoretical Physics, World Scientific, Singapore, 1984.

[2] H. Haber and G. Kane, Phys. Rep. 117 (1985) 75.

[3] For a recent review on the motivations of SUSY, see: H. Haber, *The Supersymmetric top ten lists*, talk given at the *International Workshop on Recent Advances in the Superworld*, Woodlands Superworld (1993), p.27-51 (hep-ph/9308209).

[4] J. Ellis and D. Rudaz, Phys. Lett. B128 (1983) 248.

[5] K. Hikasa and M. Kobayashi, Phys. Rev. D36 (1987) 724;
K. Hikasa and M. Drees, Phys. Lett. B252 (1990) 127.

[6] H. Baer, V. Barger, D. Karatas and X. Tata, Phys. Rev. D36 (1987) 96;
R. M. Barnett, J. F. Gunion and H.E. Haber, Phys. Rev. D37 (1988) 1892;
H. Baer, X. Tata and J. Woodside, Phys. Rev. D42 (1990) 1568;
A. Bartl, W. Majerotto, B. Mösslacher and N. Oshimo, Z. Phys. C52 (1991) 477;
A. Bartl, W. Majerotto and W. Porod, Z. Phys. C64 (1994) 499.

[7] W. Beenakker, R. Höpker and P.M. Zerwas, Phys. Lett. B378 (1996) 159.

[8] W. Beenakker, R. Höpker, T. Plehn and P.M. Zerwas, to be published.

[9] K. Hikasa and Y. Nakamura, Z. Phys. C70 (1996) 139.

[10] A. Djouadi, W. Hollik and C. Jünger, Report KA–TP–14–96, hep-ph/9605340; to appear in Phys. Rev. D.

[11] C.S. Li, R.J. Oakes and J.M. Yang, Report NUHEP-TH-96-3, hep-ph/9606385

[12] S. Kraml, H. Eberl, A. Bartl, W. Majerotto and W. Porod, Report UWThPh–1996–35, hep–ph/9505412.

[13] J. Gunion and H. Haber, Nucl. Phys. B272 (1986) 1.

[14] G. 't Hooft and M. Veltman, Nucl. Phys. B44 (1972) 189;
P. Breitenlohner and D. Maison, Commun. Math. Phys. 52 (1977) 11.

[15] W. Siegel, Phys. Lett. B84 (1979) 193;
D. M. Capper, D.R.T. Jones, P. van Nieuwenhuizen, Nucl. Phys. B167 (1980) 479;
S. Martin and M. Vaughn, Phys. Lett. B318 (1993) 331.

[16] G. Passarino and M. Veltman, Nucl. Phys. B160 (1979) 151;
G. 't Hooft and M. Veltman, Nucl. Phys. B153 (1979) 365;
G.J. Oldenborgh, Comput. Phys. Comm. 66 (1991) 1.

[17] H. Eberl, A. Bartl and W. Majerotto, Report UWThPh–1996–6, hep–ph/9603206.

[18] A. Denner, Fortschr. Phys. 41 (1993) 4.

[19] F. Abe et al., CDF Coll., Phys. Rev. Lett. 74 (1995) 2626;
S. Abachi et al., D0 Coll., Phys. Rev. Lett. 74 (1995) 2632.

[20] The LEP Electroweak Working Group, Report CERN-PPE95/172.

[21] S. Abachi et al. (D0 Coll.), Phys. Rev. Lett. 76 (1996) 2222;
D. Buskulic et al. (ALEPH Coll.), Phys. Lett. B373 (1996) 246;
P. Abreu et al. (DELPHI Coll.), Report CERN-PPE-96-075;
M. Acciarri et al. (L3 Coll.), Report CERN-PPE-96-029;
G. Alexander et al. (OPAL Coll.), Reports CERN-PPE-96-019 and CERN-PPE-96-020.

[22] For a recent account, see for instance, W. de Boer, A. Dabelstein, W. Hollik, W. Mösle and U. Schwickerath, hep-ph/9609202.

[23] V. Barger, M.S. Berger and P. Ohmann, Phys. Rev. D47 (1993) 1093;
M. Carena, S. Pokorski and C.E.M. Wagner, Nucl. Phys. B406 (1993) 59;
W. de Boer et al., Report IEKP–KA/96–04, hep-ph/9603350.

Figure Captions

Fig. 1: Feynman diagrams relevant for the $\mathcal{O}(\alpha_s)$ QCD corrections to the decay of a squark into a quark and a neutralino or chargino.

Fig. 2: (a) Partial widths [in GeV] for the decays of the lightest top squark $\tilde{t}_1$ into the two charginos χ_1^+ and χ_2^+ and a bottom quark; $\tan\beta = 1.6$, $m_{\tilde{t}_2} = 600$ GeV. The solid, dashed and dot–dashed curves correspond to the (M_2, μ) values [in GeV]: $(70, -500)$, $(70, -70)$ and $(300, -70)$. (b) Partial widths [in GeV] for the decays of $\tilde{t}_1$ into a top quark and the lightest neutralino χ_1^0 as well as the sum of all neutralinos [the thresholds can be read off the curves]; $\tan\beta = 1.6$, $m_{\tilde{t}_2} = 600$ GeV. The solid, dashed and dot–dashed curves correspond to the (M_2, μ) values [in GeV]: $(100, -500), (100, -100)$ and $(250, -50)$.

Fig. 3: Relative size [in %] of the $\mathcal{O}(\alpha_s)$ QCD corrections to the decay rates (a) $\tilde{t}_1 \to b\chi_1^+$, (b) $\tilde{t}_1 \to b\chi_2^+$, (c) $\tilde{t}_1 \to t\chi_1^0$ and (d) $\tilde{t}_1 \to \sum_i t\chi_i^0$, as a function of the top squark mass. The set of (M_2, μ) parameters is as in Fig. 2a for Figs. 3a/b and as in Fig. 2b for Figs. 3c/d.

Fig. 4: Relative size [in %] of the $\mathcal{O}(\alpha_s)$ QCD corrections to the decay rates (a) $\tilde{t}_1 \to b\chi_1^+$ and (b) $\tilde{t}_1 \to t\chi_1^0$ as a function of the gluino mass. M_2 is fixed in terms of $m_{\tilde{g}}$ with the GUT relation. The solid, dashed, dotted and dot–dashed curves correspond to the $(\tan\beta, \mu)$ values [μ is in GeV]: (1.6, -50), (20, -50), (1.6, -500) and (20, -500).

Fig. 5: (a) Partial widths [in GeV] for the decays of $\tilde{b}_1$ into the lightest chargino χ_1^+ and a top quark; $\tan\beta = 1.6$, $m_{\tilde{t}_2} = 600$ GeV. The solid, dashed and dot–dashed curves correspond to the (M_2, μ) values [in GeV]: (60, -500), (70, -60) and (300,-60). (b) Partial widths [in GeV] for the decays of $\tilde{b}_1$ to lightest neutralino χ_1^0 and a bottom quark; again $\tan\beta = 1.6$, $m_{\tilde{t}_2} = 600$ GeV. The solid, dashed and dot–dashed curves correspond to the (M_2, μ) values [in GeV]: (100, -500), (100, -100) and (250,-50).

Fig. 6: Relative size [in %] of the $\mathcal{O}(\alpha_s)$ QCD corrections to the decay rates (a) $\tilde{b}_1 \to t\chi_1^-$ and (b) $\tilde{b}_1 \to b\chi_1^0$ and as a function of the bottom squark mass. The set of (M_2, μ) parameters is as in Fig. 5a/b.

Fig. 7: The size of the QCD corrections for the decays $\tilde{q}_L \to q\chi^0$ as a function of the ratios $m_\chi^2/m_{\tilde{q}}^2$ (a) and $m_{\tilde{g}}^2/m_{\tilde{q}}^2$ (b). The solid, dashed and dot–dashed lines correspond respectively to $\gamma = 1.2, 1.5$ and 2 in (a) and $\kappa = 0.2, 0.5$ and 0.8 in (b).