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The background-field formulation of the electroweak Standard Model[†‡]

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Abstract

The application of the background-field method to the electroweak Standard Model and its virtues are reviewed. Special emphasis is directed to the Ward identities that follow from the gauge invariance of the background-field effective action. They are compatible with on-shell renormalization and imply a decent behavior of the background-field vertex functions. Via the usual construction of connected Green functions they transfer to Ward identities for connected Green functions which, in distinction to the conventional formalism, remain exactly valid in finite orders of perturbation theory even if a Dyson summation of self-energies (within a systematic use of one-particle irreducible building blocks) is performed. Finally, we comment on the interplay between gauge invariance and gauge-parameter (in-)dependence of vertex and Green functions and the uniqueness of resummation procedures.

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1 Introduction

Gauge invariance is the guiding principle for the construction of the Standard Model of elementary particle physics. However, in order to quantize gauge theories, gauge invariance is broken by adding a gauge-fixing term to the Lagrangian. The ambiguity in fixing the gauge introduces a gauge dependence of vertex and Green functions which becomes manifest through the appearance of gauge parameters. The gauge symmetry of the Lagrangian is restricted to BRS invariance which leads to complicated, non-linear Slavnov–Taylor identities for Green functions.

Physical observables, such as S-matrix elements, are gauge-independent in each complete order of perturbation theory. However, the use of incomplete orders of the perturbative expansion is sometimes unavoidable. For example, the introduction of finite-width effects for unstable particles or of running couplings can only be achieved by a summation of self-energy corrections. This so-called Dyson summation in general yields gauge-dependent answers in orders of perturbation theory that are not completely taken into account. Moreover Dyson summation in general violates Slavnov–Taylor identities, and thus gauge cancellations and the consistency of the predictions may be destroyed.

The situation is considerably improved in the framework of the background-field method [1–3] (BFM) where quantization is performed without losing gauge invariance of the effective action. This manifests itself in simple (QED-like) Ward identities which imply a decent behavior of the vertex functions. Moreover, the Ward identities for connected Green functions are not violated by a consistent Dyson summation. In addition, the BFM provides a number of technical advantages. The purpose of this article is to review the basic features of the formulation of the SM in the framework of the BFM [4–7].

Section 2 contains a discussion of the background-field effective action and vertex functions. The construction of the gauge-invariant effective action is sketched in Sect. 2.1. The corresponding Ward identities for the vertex functions are considered in Sect. 2.2. These Ward identities remain valid after the usual on-shell renormalization if the field renormalization is chosen appropriately, as described in Sect. 2.3. As an illustration of the improved properties of BFM vertex functions, in Sect. 2.4 we define running couplings which possess a gauge-independent high-energy behavior and are governed by the renormalization group.

Section 3 deals with S-matrix elements and connected Green functions. In Sect. 3.1 we sketch the construction of the generating functional for connected Green functions from which the S-matrix is obtained as usual by the reduction formula. This construction naturally introduces the full propagators (including the Dyson-summed self-energy corrections) without violating the Ward identities for connected Green functions in finite orders of perturbation theory. This fact and some consequences are explained in Sect. 3.2.

Section 4 provides a discussion of the connection between the gauge-parameter (in-)dependence of vertex functions and the existence of Ward identities for these vertex functions which are related to the invariance of the classical Lagrangian.

2 Effective action and vertex functions

2.1 The gauge-invariant effective action for the Standard Model

The BFM is a technique for quantizing gauge theories without losing explicit gauge invariance of the effective action [1,2]. This is done by decomposing the usual fields $\hat{\varphi}$ in the classical Lagrangian $\mathcal{L}_{\mathrm{C}}$ into background fields $\hat{\varphi}$ and quantum fields φ,

$$\mathcal{L}_{\mathrm{C}}(\hat{\varphi}) \to \mathcal{L}_{\mathrm{C}}(\hat{\varphi}+\varphi). \tag{1}$$

While the background fields are treated as external sources, only the quantum fields are variables of integration in the functional integral. A gauge-fixing term is added which breaks only the invariance with respect to quantum-field gauge transformations but retains the invariance of the functional integral with respect to background-field gauge transformations. From the functional integral an effective action $\Gamma[\hat{\varphi}]$ for the background fields is derived which is invariant under gauge transformations of the background fields and thus gauge-invariant.

A detailed treatment of the SM, which has been presented in Ref. [5], is beyond the scope of this short review. Therefore, we restrict our discussion to the basic differences to the conventional approach. While the gauge fields are treated as specified in (1), the complex scalar $\mathrm{SU(2)_W}$ doublet field of the minimal Higgs sector is written as the sum of a background Higgs field $\hat{\Phi}$, having the usual non-vanishing vacuum expectation value v, and a quantum Higgs field Φ, whose vacuum expectation value is zero:

$$\hat{\Phi}(x) = \begin{pmatrix} \hat{\phi}^+(x) \\ \frac{1}{\sqrt{2}}(v+\hat{H}(x)+i\hat{\chi}(x)) \end{pmatrix}, \qquad \Phi(x) = \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(H(x)+i\chi(x)) \end{pmatrix}. \tag{2}$$

Here $\hat{H}$ and H denote the physical background and quantum Higgs field, respectively, and $\hat{\phi}^+, \hat{\chi}, \phi^+, \chi$ represent the unphysical Goldstone-boson fields.

The generalization of the 't Hooft gauge fixing to the BFM [8] reads

$$\begin{aligned}\mathcal{L}_{\mathrm{GF}} = & -\frac{1}{2\xi_Q^W}\Big[(\delta^{ac}\partial_\mu + g_2\varepsilon^{abc}\hat{W}_\mu^b)W^{c,\mu} - ig_2\xi_Q^W\frac{1}{2}(\hat{\Phi}_i^\dagger\sigma_{ij}^a\Phi_j - \Phi_i^\dagger\sigma_{ij}^a\hat{\Phi}_j)\Big]^2 \\ & -\frac{1}{2\xi_Q^B}\Big[\partial_\mu B^\mu + ig_1\xi_Q^B\frac{1}{2}(\hat{\Phi}_i^\dagger\Phi_i - \Phi_i^\dagger\hat{\Phi}_i)\Big]^2,\end{aligned} \tag{3}$$

where W_μ^a, a=1,2,3, represents the triplet of gauge fields associated with the weak isospin group $\mathrm{SU(2)_W}$, and B_μ the gauge field associated with the group $\mathrm{U(1)_Y}$ of weak hypercharge Y_{W}. The Pauli matrices are denoted by σ^a, $a=1,2,3$, and ξ_Q^W and ξ_Q^B are parameters associated with the gauge fixing of the quantum fields, one for $\mathrm{SU(2)_W}$ and one for $\mathrm{U(1)_Y}$. In order to avoid tree-level mixing between the quantum photon and Z-boson fields, we set $\xi_Q = \xi_Q^W = \xi_Q^B$ in the following. Background-field gauge invariance requires that the background gauge fields appear only within a covariant derivative in the gauge-fixing term and that the terms in brackets transform according to the adjoint representation of the gauge group. The gauge-fixing term of (3) translates to the conventional one upon replacing the background Higgs field by its vacuum expectation value and omitting the background $\mathrm{SU(2)_W}$ triplet field $\hat{W}_\mu^a$.

The vertex functions can be calculated from Feynman rules that distinguish between quantum and background fields. Whereas the quantum fields appear only inside loops, the background fields are associated with external lines. Apart from doubling of the gauge and Higgs fields, the BFM Feynman rules differ from the conventional ones only owing to the gauge-fixing and ghost terms. Because these terms are quadratic in the quantum fields, they affect only vertices that involve exactly two quantum fields and additional background fields. Since the gauge-fixing term is non-linear in the fields, the gauge parameter enters also the gauge-boson vertices. The fermion fields can be treated as usual, i.e. they have the conventional Feynman rules, and no distinction needs to be made between external and internal fields. A complete set of BFM Feynman rules for the electroweak SM has been given in Ref. [5].

The BFM was also applied to the non-linear scalar realization of the SM [9], which is physically equivalent to the linear scalar representation (2) but, e.g., more convenient for studying effects of a heavy Higgs-boson mass. In the following all formulae are given for the more familiar linear scalar realization.

2.2 Ward identities

The invariance of the background-field effective action under background-field gauge transformations with associated group parameters $\hat{\theta}^a$,

$$\frac{\delta\Gamma}{\delta\hat{\theta}^a} = 0, \qquad a = A, Z, \pm, \tag{4}$$

implies *linear* identities for the vertex functions that are precisely the Ward identities related to the classical Lagrangian. This is in contrast to the conventional formalism where, owing to the gauge-fixing procedure, explicit gauge invariance is lost, and Ward identities are obtained only from invariance under BRS transformations. These Slavnov–Taylor identities have a more complicated non-linear structure and in general involve ghost contributions[1].

The BFM Ward identities are valid in all orders of perturbation theory and hold for arbitrary values of the quantum gauge parameter ξ_Q. We give some examples for illustration. Concerning the notation and conventions for the vertex functions we follow Ref. [5] throughout. Some of the Ward identities for two-point functions involving neutral gauge bosons read:

$$\begin{aligned} k^\mu\Gamma^{\hat{A}\hat{A}}_{\mu\nu}(k) = 0, \qquad k^\mu\Gamma^{\hat{A}\hat{Z}}_{\mu\nu}(k) = 0, \qquad k^\mu\Gamma^{\hat{Z}\hat{Z}}_{\mu\nu}(k) - iM_Z\Gamma^{\hat{\chi}\hat{Z}}_{\nu}(k) &= 0, \\ k^\mu\Gamma^{\hat{Z}\hat{\chi}}_{\mu}(k) - iM_Z\Gamma^{\hat{\chi}\hat{\chi}}(k) + \frac{ie}{2s_W c_W}\Gamma^{\hat{H}}(0) &= 0. \end{aligned} \tag{5}$$

For the photon–fermion and the photon-W-boson vertices QED-like Ward identities are derived, e.g.

$$k^\mu\Gamma^{\hat{A}\bar{f}f}_{\mu}(k,\bar{p},p) = -eQ_f\left[\Gamma^{\bar{f}f}(\bar{p}) - \Gamma^{\bar{f}f}(-p)\right],$$

[1]Note that also in the BFM a BRS invariance involving the quantum fields is still valid and gives rise to Slavnov–Taylor identities for Green functions with external quantum fields, which appear as substructures in the BFM vertex functions.

$$
\begin{aligned}
k^\mu \Gamma^{\hat{A}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) &= e\left[\Gamma^{\hat{W}^+\hat{W}^-}_{\rho\sigma}(k_+) - \Gamma^{\hat{W}^+\hat{W}^-}_{\rho\sigma}(-k_-)\right], \\
k_+^\rho \Gamma^{\hat{A}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) &- M_{\mathrm{W}}\Gamma^{\hat{A}\hat{\phi}^+\hat{W}^-}_{\mu\sigma}(k,k_+,k_-) = \\
& e\left[\Gamma^{\hat{W}^+\hat{W}^-}_{\mu\sigma}(-k_-) - \Gamma^{\hat{A}\hat{A}}_{\mu\sigma}(k) + \frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\Gamma^{\hat{A}\hat{Z}}_{\mu\sigma}(k)\right].
\end{aligned} \tag{6}
$$

Further Ward identities are listed in Refs. [5,6].

2.3 Gauge-invariant on-shell renormalization

In the on-shell renormalization scheme the parameters M_{W}, M_{Z}, M_{H}, m_f are identified with the physical masses (propagator poles), and e with the electric unit charge fixed in the Thomson limit. Of course this choice of physical parameters is still possible within the BFM. However, the BFM gauge invariance has important consequences for the structure of the field renormalization constants necessary to render Green functions and S-matrix elements finite. The arguments which we give in the following are made explicit for the one-loop level.[2] It is easy, however, to extend them by induction to arbitrary orders in perturbation theory. Because the renormalization of the fermionic sector is similar to the one in the conventional formalism, we suppress it here.

We introduce the following renormalization constants for the parameters:

$$
\begin{aligned}
M^2_{\mathrm{W},0} &= M^2_{\mathrm{W}} + \delta M^2_{\mathrm{W}}, \qquad M^2_{\mathrm{Z},0} = M^2_{\mathrm{Z}} + \delta M^2_{\mathrm{Z}}, \qquad M^2_{\mathrm{H},0} = M^2_{\mathrm{H}} + \delta M^2_{\mathrm{H}}, \\
e_0 &= Z_e e = (1+\delta Z_e)e, \qquad t_0 = t + \delta t,
\end{aligned} \tag{7}
$$

where the subscript "0" denotes bare quantities. The tadpole counterterm δt renormalizes the term $t\hat{H}(x)$ in the Lagrangian linear in the Higgs field $\hat{H}$. It corrects for the shift in the minimum of the Higgs potential owing to radiative corrections. Choosing v as the correct vacuum expectation value of the Higgs field $\hat{\Phi}$ is equivalent to the vanishing of t.

Following the QCD treatment of Ref. [2], we introduce field renormalization only for the background fields,

$$
\begin{aligned}
\hat{W}^\pm_0 &= Z^{1/2}_{\hat{W}}\hat{W}^\pm = (1+\frac{1}{2}\delta Z_{\hat{W}})\hat{W}^\pm, \\
\begin{pmatrix} \hat{Z}_0 \\ \hat{A}_0 \end{pmatrix} &= \begin{pmatrix} Z^{1/2}_{\hat{Z}\hat{Z}} & Z^{1/2}_{\hat{Z}\hat{A}} \\ Z^{1/2}_{\hat{A}\hat{Z}} & Z^{1/2}_{\hat{A}\hat{A}} \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{A} \end{pmatrix} = \begin{pmatrix} 1+\frac{1}{2}\delta Z_{\hat{Z}\hat{Z}} & \frac{1}{2}\delta Z_{\hat{Z}\hat{A}} \\ \frac{1}{2}\delta Z_{\hat{A}\hat{Z}} & 1+\frac{1}{2}\delta Z_{\hat{A}\hat{A}} \end{pmatrix} \begin{pmatrix} \hat{Z} \\ \hat{A} \end{pmatrix}, \\
\hat{S}_0 &= Z^{1/2}_{\hat{S}}\hat{S} = (1+\frac{1}{2}\delta Z_{\hat{S}})\hat{S}, \qquad \hat{S} = \hat{H}, \hat{\chi}, \hat{\phi}.
\end{aligned} \tag{8}
$$

In order to preserve the background-field gauge invariance, the renormalized effective action has to be invariant under background-field gauge transformations. This restricts the possible counterterms and relates the renormalization constants introduced above. These relations can be derived from the requirement that the renormalized vertex functions fulfill Ward identities of the same form as the unrenormalized ones. As a consequence, also the

[2] We implicitly assume the existence of an invariant regularization scheme.

counterterms have to fulfill these Ward identities. An analysis of the Ward identities yields [5]:

$$\begin{aligned}
\delta Z_{\hat{A}\hat{A}} &= -2\delta Z_e, \qquad \delta Z_{\hat{Z}\hat{A}} = 0, \qquad \delta Z_{\hat{A}\hat{Z}} = 2\frac{c_{\mathrm{W}}}{s_{\mathrm{W}}}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}, \\
\delta Z_{\hat{Z}\hat{Z}} &= -2\delta Z_e - \frac{c_{\mathrm{W}}^2 - s_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}, \qquad \delta Z_{\hat{W}} = -2\delta Z_e - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2}, \\
\delta Z_{\hat{H}} &= \delta Z_{\hat{\chi}} = \delta Z_{\hat{\phi}} = -2\delta Z_e - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2} + \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2},
\end{aligned} \tag{9}$$

where

$$c_{\mathrm{W}}^2 = \frac{M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2} = 1 - s_{\mathrm{W}}^2, \qquad \frac{\delta c_{\mathrm{W}}^2}{c_{\mathrm{W}}^2} = \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2} - \frac{\delta M_{\mathrm{Z}}^2}{M_{\mathrm{Z}}^2}. \tag{10}$$

The relations (9) express the field renormalization constants of all gauge bosons and scalars completely in terms of the renormalization constants of the electric charge and the particle masses. With this set of renormalization constants all background-field vertex functions become finite[3]. This is evident since the divergences of the vertex functions are subject to the same restrictions as the counterterms. In Ref. [5] it has been verified explicitly at one-loop order that a renormalization based on the on-shell definition of all parameters can consistently be used in the BFM. It renders all vertex functions finite while respecting the full gauge symmetry of the BFM.

As the field renormalization constants are fixed by (9), the propagators in general acquire residues being different from unity but finite, and different fields can mix on shell. This is similar to the minimal on-shell scheme of the conventional formalism [10]. Therefore, when calculating S-matrix elements, one has to introduce (UV-finite) wave-function renormalization constants, which have been explicitly given in Ref. [7] for the gauge fields. However, just as in QED, the on-shell definition of the electric charge together with gauge invariance automatically fixes the residue of the photon propagator to unity.

As a consequence of the relations between the renormalization constants, the counterterm vertices of the background fields have a much simpler structure than the ones in the conventional formalism (see e.g. Ref. [11]). In fact, all vertices originating from a separately gauge-invariant term in the Lagrangian acquire the same renormalization constants. The explicit form of the counterterm vertices at one-loop order has been given in Ref. [5].

As the renormalized parameters are identified with the physical electron charge and the physical particle masses, they are manifestly gauge-independent. Moreover, the original bare parameters in the Lagrangian are obviously gauge-independent, as they represent free parameters of the theory. The same is true for the bare charge and the bare weak mixing angle as these are directly related to the free bare parameters. Consequently, the counterterms δZ_e and δc_{W}^2 for the gauge couplings are gauge-independent. The relations (9) therefore imply that the field renormalizations of all gauge-boson fields are

[3]Beyond one-loop order one needs in addition a renormalization of the quantum gauge parameters [2]. At one-loop level these counterterms do not enter the background-field vertex functions, because ξ_Q does not appear in pure background-field vertices. Clearly, the renormalization of gauge parameters is irrelevant for gauge-independent quantities such as S-matrix elements at any order.

gauge-independent. This is in contrast to the conventional formalism where the field renormalizations in the on-shell scheme are gauge-dependent.[4]

2.4 Properties of background-field vertex functions

The QED-like Ward identities valid for the BFM vertex functions (for all values of ξ_Q) give rise to improved theoretical properties of form factors defined within the BFM compared to their conventional counterparts [4,5].

As an example, we consider the asymptotic behavior of gauge-boson self-energies. Just as in QED, one can define running couplings in the BFM for the SM via naïve Dyson summation of self-energies as follows:

$$\begin{aligned} e^2(q^2) &= \frac{e_0^2}{1+\mathrm{Re}\,\Pi_0^{\hat{A}\hat{A}}(q^2)} = \frac{e^2}{1+\mathrm{Re}\,\Pi^{\hat{A}\hat{A}}(q^2)}, \\ g_2^2(q^2) &= \frac{g_{2,0}^2}{1+\mathrm{Re}\,\Pi_0^{\hat{W}\hat{W}}(q^2)} = \frac{g_2^2}{1+\mathrm{Re}\,\Pi^{\hat{W}\hat{W}}(q^2)}, \end{aligned} \tag{11}$$

where $g_{2,0} = e_0/s_{\mathrm{W},0}$ and $g_2 = e/s_{\mathrm{W}}$. The quantities $\Pi^{\hat{V}\hat{V}'}$ are related to the transverse parts of the gauge-boson self-energies $\Sigma_{\mathrm{T}}^{\hat{V}\hat{V}'}$ as follows:

$$\Pi^{\hat{V}\hat{V}'}(q^2) = \frac{\Sigma_{\mathrm{T}}^{\hat{V}\hat{V}'}(q^2) - \Sigma_{\mathrm{T}}^{\hat{V}\hat{V}'}(0)}{q^2}. \tag{12}$$

The relations (9) give rise to the following properties of these running couplings: As indicated in (11), the renormalization constants cancel. Consequently, the running couplings are finite without renormalization and thus independent of the renormalization scheme (as long as it respects BFM gauge invariance). Their asymptotic behavior is gauge-independent and governed by the renormalization group. In particular, the coefficients of the leading logarithms in the self-energies are equal to the ones appearing in the β-functions associated with the running couplings. All these properties are completely analogous to those of the running coupling in QED; they follow in the same way from the relations (9) as in QED from $Z_e = Z_{\hat{A}\hat{A}}^{-1/2}$.

3 S-matrix and connected Green functions

3.1 Construction

S-matrix elements and connected Green functions are constructed by forming trees with vertex functions from the effective action $\Gamma[\hat{\varphi}]$ joined by background-field propagators. These propagators are defined by adding a gauge-fixing term to $\Gamma[\hat{\varphi}]$, resulting in

$$\Gamma^{\mathrm{full}} = \Gamma + i\int \mathrm{d}^4x\, \mathcal{L}_{\mathrm{GF}}^{\mathrm{BF}}. \tag{13}$$

[4]In contrast to δZ_e and δc_{W}^2 the mass counterterms are not gauge-independent. The bare masses depend on the bare vacuum expectation value v_0 of the Higgs field, which is not a free parameter of the theory. See Ref. [5] for a discussion.

The gauge-fixing term $\mathcal{L}_{\mathrm{GF}}^{\mathrm{BF}}$ is not related to the term (3) that fixes the gauge of the quantum fields, and the associated gauge parameters ξ_B^i enter only tree-level quantities but not the higher-order contributions to the vertex functions.

The generating functional of connected Green functions, Z_{c}, is obtained from Γ^{full} (as usual) by a Legendre transformation [7],

$$Z_{\mathrm{c}}[J_{\hat{F}}, J_f, J_{\bar{f}}] = \Gamma^{\mathrm{full}}[\hat{F}, f, \bar{f}] + i\int \mathrm{d}^4x \Big[\sum_{\hat{F}} J_{\hat{F}^\dagger}\hat{F} + \sum_f (\bar{f}J_f + J_{\bar{f}}f)\Big] \tag{14}$$

with $\hat{F} = \hat{A}, \hat{Z}, \hat{W}^+, \hat{W}^-, \hat{H}, \hat{\chi}, \hat{\phi}^+, \hat{\phi}^-$, where $\hat{F}^\dagger$ denotes the complex conjugate of $\hat{F}$, and

$$iJ_{\hat{F}^\dagger} = -\frac{\delta\Gamma^{\mathrm{full}}}{\delta\hat{F}}, \qquad iJ_{\bar{f}} = \frac{\delta\Gamma^{\mathrm{full}}}{\delta f}, \qquad iJ_f = -\frac{\delta\Gamma^{\mathrm{full}}}{\delta\bar{f}}. \tag{15}$$

As a consequence, the 1-particle reducible Green functions and S-matrix elements are composed as in the conventional formalism from a tree structure of vertex functions. While the vertices in these trees are directly given by the background-field vertex functions, the propagators are determined as the inverse of the two-point vertex functions resulting from Γ^{full}. Note that these propagators contain by construction all self-energy insertions. The Dyson summation of self-energy corrections has already been taken care of by this formalism. Of course, one can expand the propagators and recover the ordinary perturbative expansion.

The S matrix follows from Z_{c} by the usual reduction formula. The equivalence of the S-matrix in the BFM to the conventional one has been proven in Refs. [3,12].

Despite the distinction between background and quantum fields, calculations in the BFM become in general simpler than in the conventional formalism. This is in particular the case in the 't Hooft–Feynman gauge ($\xi_Q = 1$) for the quantum fields where many vertices simplify. Moreover, the gauge fixing of the background fields is totally unrelated to the gauge fixing of the quantum fields [12]. This freedom can be used to choose a particularly suitable background gauge, e.g. the unitary gauge. In this way the number of Feynman diagrams can considerably be reduced.

The Ward identities (4) for the effective action Γ, which generates the vertex functions, translate into Ward identities for the functional Z_{c}, which generates the connected Green functions. These Ward identities were explicitly derived in Ref. [7] in a 't Hooft gauge for the background fields. The (renormalized) two-point functions involving neutral gauge bosons obey for instance:

$$k^\mu G^{\hat{A}\hat{A}}_{\mu\nu}(k) = \frac{-i\hat{\xi}_A k_\nu}{k^2}, \qquad k^\mu G^{\hat{A}\hat{Z}}_{\mu\nu}(k) = 0, \qquad k^\mu G^{\hat{Z}\hat{Z}}_{\mu\nu}(k) + i\hat{\xi}_Z M_Z G^{\hat{\chi}\hat{Z}}_{\nu}(k) = \frac{-\hat{\xi}_Z i k_\nu}{k^2 - \hat{\xi}_Z M_Z^2},$$
$$k^\mu G^{\hat{Z}\hat{\chi}}_{\mu}(k) + i\hat{\xi}_Z M_Z G^{\hat{\chi}\hat{\chi}}(k) = \frac{-\hat{\xi}_Z M_Z}{k^2 - \hat{\xi}_Z M_Z^2}. \tag{16}$$

For the photon–fermion and the photon-W-boson vertices we find

$$\frac{i}{\hat{\xi}_A} k^2 k^\mu G^{\hat{A}\bar{f}f}_{\mu}(k,\bar{p},p) = -eQ_f\left[G^{\bar{f}f}(\bar{p}) - G^{\bar{f}f}(-p)\right],$$

$$\frac{i}{\hat{\xi}_A}k^2 k^\mu G^{\hat{A}\hat{W}^+\hat{W}^-}_{\mu\rho\sigma}(k,k_+,k_-) = e\left[G^{\hat{W}^+\hat{W}^-}_{\rho\sigma}(k_+) - G^{\hat{W}^+\hat{W}^-}_{\rho\sigma}(-k_-)\right]$$
$$+ e\frac{1}{k_+^2-\hat{\xi}_W M_\mathrm{W}^2}k_{+,\rho}\left[k_+^\mu G^{\hat{W}^+\hat{W}^-}_{\mu\sigma}(-k_-) + \hat{\xi}_W M_\mathrm{W} G^{\hat{\phi}^+\hat{W}^-}_{\sigma}(-k_-)\right]$$
$$- e\frac{1}{k_-^2-\hat{\xi}_W M_\mathrm{W}^2}k_{-,\sigma}\left[k_-^\mu G^{\hat{W}^+\hat{W}^-}_{\rho\mu}(k_+) - \hat{\xi}_W M_\mathrm{W} G^{\hat{W}^+\hat{\phi}^-}_{\rho}(k_+)\right], \qquad (17)$$

where we have used a Ward identity for the W-boson two-point function to simplify the last equation. The terms in the last line result from the gauge-fixing of the background fields.

After amputating the Green functions and putting fields on shell many terms drop out in the Ward identities. Denoting amputated Green functions by $G_{\hat{\phi}_i\hat{\phi}_j\ldots}$, we find for example the following identities

$$k^\nu G_{\hat{A}\ldots,\nu} = 0, \qquad k^\nu G_{\hat{Z}\ldots,\nu} = iM_\mathrm{Z} G_{\hat{\chi}\ldots}, \qquad k^\nu G_{\hat{W}^\pm\ldots,\nu} = \pm M_\mathrm{W} G_{\hat{\phi}^\pm\ldots}, \qquad (18)$$

where the ellipses stand for any on-shell fields. The first of these identities expresses electromagnetic current conservation, the others imply the well-known Goldstone-boson equivalence theorem, as discussed in Ref. [7] in detail.

3.2 Dyson summation without violating Ward identities

A particularly important property of the BFM is the fact that the BFM Ward identities for connected Green functions are not violated even in finite orders of perturbation theory by Dyson summation of self-energies, as was proven in Ref. [7]. This is in contrast to the Slavnov–Taylor identities, which in general only hold for connected Green functions in a given order of perturbation theory if all contributions, including the propagators, are expanded up to this order.

The crucial difference with respect to the Slavnov–Taylor identities lies in the fact that the BFM Ward identities for vertex functions $\Gamma^{\hat{\varphi}_i\hat{\varphi}_j\cdots}$ are *linear* in all vertex functions. Consequently they are exactly valid loop order by loop order and the background-field effective action truncated at n-loop order, $\Gamma|_{n-\mathrm{loop}}$, is exactly gauge-invariant. Thus, the connected Green functions defined from $\Gamma|_{n-\mathrm{loop}}$ via a Legendre transformation fulfill exactly the same Ward identities as those defined from the full effective action containing all orders. This implies that the Ward identities valid for the full connected Green functions in the BFM also hold exactly for any fixed loop order in perturbation theory if the inverse propagators, which are just the two-point vertex functions, are calculated in the same loop order as all other vertex functions. This means that in the BFM Dyson summation does not destroy the Ward identities for connected Green functions and the related gauge cancellations at high energies. As an immediate consequence, the Goldstone-boson equivalence theorem is valid in the BFM after Dyson summation.

Dyson summation is of particular importance for the treatment of finite-width effects of unstable particles. The finite width of a particle P is introduced in field theory by Dyson summing the self-energy $\Sigma^{PP}(k^2)$,

$$-\left[\Gamma^{PP}(k^2)\right]^{-1} = \frac{i}{k^2-M^2} + \frac{i}{k^2-M^2}i\Sigma^{PP}(k^2)\frac{i}{k^2-M^2} + \cdots$$

$$= \frac{i}{k^2 - M^2 + \Sigma^{PP}(k^2)}, \tag{19}$$

and relating the finite width to the imaginary part of the self-energy. However, since the summation mixes different orders in perturbation theory, the result in general will not be gauge-invariant in finite orders of perturbation theory.

This problem has been investigated recently in connection with the process $e^+e^- \to W^+W^- \to 4f$, where a finite width has to be introduced for the W boson. In lowest order the W boson decays only into fermions, i.e. only fermion loops contribute to the relevant imaginary part of the one-loop W-boson self-energy. The same is true for the Z boson. In Ref. [13] it was argued that finite-width effects of W and Z bosons can be introduced in tree-level amplitudes without destroying the Ward identities (and thus also the gauge cancellations) by Dyson-summing the fermion-loop contributions to the self-energies *and* including also all the other fermion-loop contributions in one-loop order[5].

Within the framework of the BFM it is easy to understand why this prescription indeed preserves the Ward identities. The fermion-loop contributions at one-loop order in the conventional formalism coincide with those in the BFM, and as explained above in the BFM Dyson summation does not violate the Ward identities for connected Green functions.

Obviously, the same procedure could be used within the BFM for the general case, where also bosonic loop corrections contribute to the imaginary part of the self-energy. In contrast to the conventional formalism Dyson summing the complete fermionic and bosonic corrections in the BFM still preserves the Ward identities for connected Green functions. However, both in the BFM and in the conventional formalism one is in general faced with a gauge-parameter dependence at the incompletely calculated loop level. As discussed in the following section, so far—to the best of our knowledge—no prescription is available that yields a unique unambiguous result in the general case.

4 Gauge invariance versus gauge-parameter independence

Equipped with the gauge-invariant BFM effective action it is interesting to investigate the connection between gauge invariance and gauge-parameter (in-)dependence of vertex functions. Motivated by the gauge independence of complete S-matrix elements, several authors have performed rearrangements of gauge-dependent parts between different vertex functions resulting in definitions of separately gauge-parameter-independent building blocks [14,15]. In particular, the so-called pinch technique (PT) [15] provides a quite general algorithm for a rearrangement at the one-loop level which leads to gauge-parameter-free "vertex functions" with improved theoretical properties. However, having no solid field-theoretical basis, the PT suffers from a number of conceptual and technical problems, like the unclear field-theoretical meaning of building blocks constructed by rearranging parts between different vertex functions. Moreover, the question of universality

[5]In the process $e^+e^- \to W^+W^- \to 4f$ this amounts to inclusion of the fermion-loop corrections to the triple-gauge-boson vertex.

and process-independence of the so-defined quantities could only be verified by additional assumptions or a (necessarily incomplete) case-by-case study. Finally, the generalization of these methods to higher orders is not straightforward.

It was shown in Ref. [4] that the PT "vertex functions" coincide with the special case $\xi_Q = 1$ of the corresponding BFM vertex functions[6] and that the improved theoretical properties of the constructed building blocks are a consequence of simple classical Ward identities. In the BFM, these Ward identities are a direct consequence of the gauge invariance and hold in all orders of perturbation theory. It is instructive to investigate the origin of these Ward identities within the PT. The crucial observation is that in this formalism the S-matrix elements are composed of gauge-parameter-free "vertex functions" connected by gauge-parameter-dependent tree-level propagators. As the complete S-matrix element is independent of the gauge parameters, certain non-trivial symmetry relations between the new "vertex functions" must exist that enforce the cancellation of the remaining gauge-parameter dependence. This fact together with some additional assumptions on the independence of propagator-, vertex-, and box-like structures, as explained in some detail in Ref. [6], leads to PT "vertex functions" that fulfill the classical Ward identities. Note that the Ward identities do not uniquely fix these "vertex functions", since one can always shift appropriate parts between the "vertex functions" that by themselves fulfill the Ward identities.

Thus, the validity of these non-trivial symmetry relations is not based on the actual gauge-parameter independence of the new "vertex functions", but—more generally—on the independence of the gauge parameters in the tree-level propagators from the gauge fixing within loop diagrams. The prescription given within the PT is just a special case of decoupling the gauge fixing in the loops from the tree lines, like it is the case in the BFM. From these considerations it should be clear that, as far as gauge invariance and gauge or prescription independence is concerned, application of methods like the PT *within* the BFM is not meaningful, since the gauge fixings in the loops and tree lines are already decoupled, and the elimination of ξ_Q can *not* be distinguished from trivially putting ξ_Q to any specific value. In this context it is interesting to note that a generalized PT algorithm was proposed in Ref. [17] which reproduces the BFM vertex functions of QCD for arbitrary quantum gauge parameter ξ_Q at one loop. This shows explicitly how the gauge dependence of the BFM vertex functions corresponds to an arbitrariness in fixing the PT algorithm.

As explained above, once resummations are involved, physical predictions in fixed orders of perturbation theory depend on the gauge and any other prescription used to define the vertex functions. This raises the question whether one of these prescriptions is distinguished on physical grounds. Because the BFM Ward identities, and thus the decent theoretical properties of the BFM (or PT) vertex functions, hold equally well for any choice of ξ_Q, these Ward identities are not sufficient to provide a distinction. The authors of Ref. [18] argue that the PT (or equivalently the BFM with $\xi_Q = 1$) is distinguished. Their only argument for rejecting the BFM for $\xi_Q \neq 1$ is the appearance of unphysical thresholds in the corresponding vertex functions. For $\xi_Q = 1$ the unphysical thresholds happen to appear at the same locations as the physical thresholds and cannot be distinguished in Green functions. The starting point of the PT are S-matrix elements which evidently do

[6]In QCD this fact was also pointed out in Ref. [16].

not involve unphysical thresholds. However, the PT "vertex functions" result from a split of the S-matrix elements into propagator-, vertex- and box-like contributions. It is by far not obvious that this split does not introduce unphysical thresholds in the individual contributions, which appear at the same locations as the physical thresholds.

As long as no physically distinguished definition of vertex functions can be found, the existence of various prescriptions signals the inherent ambiguity in defining form factors, resummations etc. on the basis of off-shell vertex functions. In view of applications for resummations of bosonic loop contributions, for instance, this means that the ambiguity found there is not removed by a prescription like the PT but is only traded on cost of the specific definition used to eliminate the gauge parameters. Finally we note that in a different context the authors of Ref. [19] also arrived at the conclusion that off-shell quantities are ambiguous even if gauge invariance is imposed.

5 Conclusion

We have reviewed some basic features of the application of the background-field method (BFM) to the electroweak Standard Model (SM).

The gauge invariance of the BFM effective action implies simple (QED-like) Ward identities for the vertex functions, which as a consequence possess desirable theoretical properties like an improved high-energy, UV and IR behavior. The BFM gauge invariance not only admits the usual on-shell renormalization but even simplifies its technical realization. Moreover, the formalism provides additional advantages such as simplifications in the Feynman rules and the possibility to use different gauges for tree and loop lines in Feynman diagrams, thus allowing to reduce the number of graphs.

In contrast to the Slavnov–Taylor identities, the BFM Ward identities are not violated by Dyson summation if the connected Green functions are constructed from the complete set of vertex functions of a fixed loop order. Consequently, gauge cancellations, and in particular the Goldstone-boson equivalence theorem, are not disturbed if Dyson summation is applied. This fact is important for the incorporation of finite-width effects of unstable particles within perturbation theory which requires a summation of self-energy corrections. Despite of this important improvement in comparison to the conventional formalism, also in the BFM a gauge-parameter dependence remains at the incompletely calculated loop level. At present it is not known how or whether at all this problem can be avoided.

The decoupling of the different gauge fixings of tree and loop lines does not uniquely determine the BFM vertex functions, as already signaled by their dependence on the quantum gauge parameter ξ_Q. This kind of ambiguity is also inherent in all those methods that eliminate the gauge-parameter dependence from vertex functions by redistributing the gauge-dependent parts. This is in particular the case for the pinch-technique algorithm which reproduces the choice $\xi_Q = 1$ of the BFM. Indeed the improved behavior of the BFM or pinch-technique "vertex functions" can be traced back to the Ward identities, which hold in the BFM for arbitrary ξ_Q. These Ward identities follow in the BFM directly from gauge invariance, but can only be derived in the pinch technique on the basis of additional assumptions.

In conclusion, the BFM provides an alternative framework for quantizing gauge theories which compared to the conventional method has several advantages both on conceptual and technical grounds.

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