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Calculation of two-loop top-quark and Higgs-boson corrections in the electroweak Standard Model

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Abstract

A combination of algebraical and numerical techniques for calculating two-loop top-quark and Higgs-boson corrections to electroweak precision observables like Δr or the ρ-parameter is presented. The renormalization is performed within the on-shell scheme. The results of the calculations are valid for arbitrary values of m_{t}, M_{H} and of the gauge-boson masses. An example is treated where the full result is compared to the result obtained via an expansion up to next-to-leading order in m_{t}. As an application, results for the Higgs-mass dependent top-contributions to Δr are given.

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Calculation of Two-loop Top-quark and Higgs-boson Corrections in the Electroweak Standard Model

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A combination of algebraical and numerical techniques for calculating two-loop top-quark and Higgs-boson corrections to electroweak precision observables like Δr or the ρ-parameter is presented. The renormalization is performed within the on-shell scheme. The results of the calculations are valid for arbitrary values of $m_{\rm t}$, $M_{\rm H}$ and of the gauge-boson masses. An example is treated where the full result is compared to the result obtained via an expansion up to next-to-leading order in $m_{\rm t}$. As an application, results for the Higgs-mass dependent top-contributions to Δr are given.

1 Introduction

After the discovery of the top-quark the Higgs-boson remains the only missing ingredient of the (minimal) electroweak Standard Model (SM). At the moment, the mass of the Higgs-boson, $M_{\rm H}$, can only very mildly be constrained by confronting the SM with precision data. In order to improve on this situation, a further reduction of the experimental and theoretical errors is necessary. Concerning the reduction of the theoretical error due to missing higher order corrections, in particular two-loop top-quark corrections and higher order QCD corrections to the quantity Δr derived from muon decay and to the ρ-parameter are of interest. Recently the leading two-loop top-quark and Higgs-boson corrections to these quantities [1] have been supplemented by the calculation of the full Higgs-boson dependence of the leading $m_{\rm t}^4$ contribution [2] and furthermore by inclusion of the next-to-leading top-quark contributions [3]. Since both the Higgs-mass dependence of the leading $m_{\rm t}^4$ contribution and the inclusion of the next-to-leading term in the $m_{\rm t}$ expansion turned out to yield sizable contributions, a more complete calculation of the top-quark corrections would be desirable, where no expansion in $m_{\rm t}$ or $M_{\rm H}$ is made.

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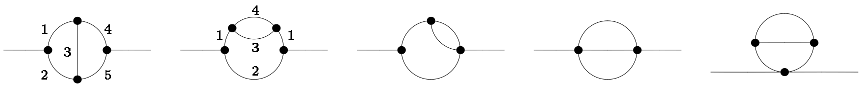

Fig. 1. One-particle irreducible topologies of two-loop self-energies which do not factorize into one-loop contributions.

In this article we describe techniques with which such a calculation can be carried out. The calculation of top-quark contributions to Δr and other processes with light external fermions at low energies requires in particular the evaluation of two-loop self-energies at non-zero external momentum, while vertex and box contributions can mostly be reduced to vacuum integrals. We perform the renormalization within the complete on-shell scheme, i.e. we use physical parameters throughout. All calculations are performed in an arbitrary R_ξ gauge with free gauge parameters ξ_i, $i = A, Z, W$.

2 Algebraic Evaluation

The problem of evaluating top-quark or Higgs-boson corrections to four-fermion processes with light external particles at low energies in the on-shell renormalization scheme requires the calculation of two-loop self-energies with massive particles at non-zero external momentum. The algebraic evaluation of these contributions can be carried out in a highly automatized way by means of the *Mathematica* packages *FeynArts* 2.0 [4] and *TwoCalc* [5].

The package *FeynArts* 2.0 is used for generating the relevant one-loop and two-loop diagrams and counterterm graphs. For this purpose, we have implemented the complete one-loop counterterm contributions of the SM (using the complete on-shell renormalization scheme) as well as the appropriate two-loop terms into the model file of the package.

The package *TwoCalc* reduces general two-loop self-energies to a minimal set of standard scalar two-loop integrals of the form

$$T_{i_1 i_2 \ldots i_l}(p^2; m_1^2, m_2^2, \ldots, m_l^2) = \int \frac{d^D q_1}{i\pi^2 (2\pi\mu)^{D-4}} \int \frac{d^D q_2}{i\pi^2 (2\pi\mu)^{D-4}} \frac{1}{\left[k_{i_1}^2 - m_1^2\right]\left[k_{i_2}^2 - m_2^2\right] \cdots \left[k_{i_l}^2 - m_l^2\right]}, \quad (1)$$

which correspond to the basic two-loop topologies depicted in Fig. 1. In (1) k_{i_l} denotes the momentum of the i-th propagator and m_l its mass. The k_{i_l} are related to the integration momenta q_1 and q_2 and the external momentum p via $k_1 = q_1$, $k_2 = q_1 + p$, $k_3 = q_2 - q_1$, $k_4 = q_2$, $k_5 = q_2 + p$. The topologies shown in Fig. 1 correspond to the scalar integrals T_{12345}, T_{11234}, T_{1234}, T_{234}, and T_{1134}, respectively. The analytical expression for T_{11234} can be obtained from T_{1234} by partial fractioning or taking the derivative with respect to m_1^2. Other integrals with higher powers of propagators are treated in the same way. For the general case one therefore needs to evaluate only four different types of two-loop scalar integrals.

The algebraic evaluation with *TwoCalc* is performed for arbitrary values of all particle masses, the invariant momentum p^2 and the space-time dimension D. A general R_ξ gauge is used which is specified by one gauge parameter ξ_i, $i = A, Z, W$, for each vector boson. The algorithm of *TwoCalc* has been described in detail in Refs. [5]. For the contraction of Lorentz indices, reduction of the Dirac algebra and evaluation of Dirac traces, which can be worked out like in the one-loop case, the routines of the program *FeynCalc* [6] are used. The further evaluation is based on a method for the tensor integral decomposition of two-loop self-energies [5] and furthermore makes use of certain symmetry properties of the two-loop integrals.

The counterterm contributions are also calculated with *TwoCalc*. The sum of unrenormalized two-loop diagrams and diagrams with counterterm insertions is expressed in terms of a minimal basis of standard scalar integrals. This is a very convenient feature, since it allows to directly check at the algebraic level whether the gauge-parameter dependence of the result drops out. As a further nontrivial check which can directly be read off from the algebraic result, all field renormalization constants of internal particles must drop out in the sum of all contributing diagrams. After inserting the divergent part of the two-loop integrals and decomposing the one-loop integrals into divergent and finite parts, also the UV-finiteness of the result can be checked algebraically, i.e. all terms proportional to $1/\delta^2$ and $1/\delta$, where $\delta = (4 - D)/2$, must cancel.

3 Two-loop On-shell Renormalization

For our calculations we use the complete on-shell renormalization scheme at the two-loop level, i.e. we use as parameters the masses of the physical particles and the electric charge e, and we introduce field renormalizations such that the residues of all propagators are equal to one (see e.g. Ref. [7]). The formulation in terms of physical parameters is particularly important in view of calculating corrections to the quantity Δr derived from muon decay, which relates the Fermi constant G_μ to the W-boson and Z-boson masses, M_W and M_Z, and to the electromagnetic fine structure constant $\alpha = e^2/(4\pi)$.

We perform the renormalization in such a way that the gauge-fixing term in the Lagrangian does not give rise to counterterm contributions, i.e. the renormalization of the parameters and fields in the gauge-fixing term is canceled by appropriate renormalizations of the gauge parameters. It should be noted that the renormalization prescription chosen for the gauge-fixing term determines the renormalization of the ghost sector, which at the two-loop level also enters the quantities in the physical sector of the theory. According to the described renormalization procedure we have derived the counterterms for the two-point and three-point vertices containing ghost fields. For the example of the complete two-loop top-quark contribution to the W-boson self-energy we have checked that insertion of these renormalization constants does in fact yield a finite result for the two-loop self-energy.

As a specific example we briefly consider the Higgs-dependent top-quark corrections to Δr. It is easy to see that Higgs-dependent top-quark corrections can, except for the renormalization, enter Δr only via two-point functions. Besides these self-energy corrections one also has to consider contributions arising from the renormalization of the relevant one-loop and two-loop three-point functions. The two-loop renormalization constants receiving Higgs-dependent top-quark corrections are the charge renormalization constant, $\delta Z_{e,(2)}$, and the counterterm of the electroweak mixing angle, $\delta s^2_{\mathrm{W},(2)}$, while the field renormalization constant $\delta Z_{W,(2)}$ (and also $\delta Z_{W,(1)}$) cancels in the sum of the contributing diagrams.

4 Decomposition and numerical Evaluation of scalar Integrals

It is known since several years that massive two-loop integrals are in general not expressible in terms of polylogarithmic functions [8]. Our approach for a numerical evaluation of scalar self-energy integrals are one-dimensional integral representations. They allow for a very fast calculation of these functions with high precision.

Such a numerical approach can of course only be applied to finite functions, while UV- and IR-divergencies of the two-loop integrals have to be kept under control. We decompose the integrals into divergent and finite parts and check algebraically that the divergencies cancel. For some integrals we make use of the fact that the two-loop vacuum integrals can be calculated analytically (see e.g. Ref. [9]). Then we numerically calculate suitable combinations of the non-vacuum and the vacuum integrals. For some diagrams it proved useful to separate the two-particle cut and the three-particle cut contributions [10], or to combine both methods. As an example the momentum derivative of T_{11234} can be calculated as

$$\frac{\partial}{\partial p^2} T_{11234}(p^2, m_1^2, m_2^2, m_3^2, m_4^2) = \frac{\partial}{\partial m_1^2}\left(B_0(m_1^2, m_3^2, m_4^2)\frac{\partial}{\partial p^2} B_0(p^2, m_1^2, m_2^2)\right) + DT_{11234C3}(p^2, m_1^2, m_2^2, m_3^2, m_4^2)\,, \qquad (2)$$

where $DT_{11234C3}$, the three-particle cut contribution, is finite, while the divergencies are contained in one-loop B_0-integrals, which can be calculated analytically.

For the numerical evaluation there remains an expression for the finite part which is in general very extensive. The main ingredients of the numerical evaluation are one-dimensional integral representations which we have implemented in C++. For the integration we apply an adaptive Gauss-Kronrod algorithm [11].

For those topologies which contain a one-loop self-energy insertion we make use of a dispersion representation of the subloop which has been discussed in detail in Refs. [10]. In these representations the two-particle cut contributions have been separated as products of B_0-functions. All momentum or mass derivatives of the functions T_{234} and T_{1234} can be calculated starting from these formulae. For the master topology, T_{12345}, we refer to the formulae presented in Ref. [12].

The basic functions in our integration kernels are B_0-functions and the discontinuities of B_0-functions. In some cases it is useful for the numerical stability to subtract the asymptotic behavior of the B_0-function for a large mass-variable. It is given by

$$B_0(p^2; s, m^2) = \frac{1}{\delta} + 1 - \log \frac{s}{4\pi\mu^2} + \frac{m^2}{s} \log \frac{m^2}{s} + \frac{p^2}{2\,s} + B_{0rest}(p^2; s, m^2)\,. \quad (3)$$

Calculating physical processes we encountered in some parameter regions huge cancellations among the contributions of the diagrams or the scalar integrals. Therefore we perform in these cases the calculations inside the integral representations with quadruple precision [13]. Typical computation times are 0.03 seconds for the evaluation of the functions T_{234} and T_{1234} and 0.3 seconds for the evaluation of T_{12345} to ten digits precision on a workstation DEC 3000 AXP. Using quadruple precision slows down the calculations by a factor of about 10, which results e.g. in 40 seconds for the calculation of the Higgs-dependent top-quark contributions to the W-boson self-energy for one set of parameters.

5 Results

5.1 Comparison with the expansion in $m_{\rm t}$

In this section we compare our results for the two diagrams shown in Fig. 2 with the results of an expansion up to next-to-leading order in $m_{\rm t}$ [1] (see Ref. [3]), which takes into account terms of order $m_{\rm t}^4$ and $m_{\rm t}^2$. This expansion is performed in two regions, namely in the light Higgs region ("light Higgs expansion") and in the heavy Higgs region ("heavy Higgs expansion") [3].

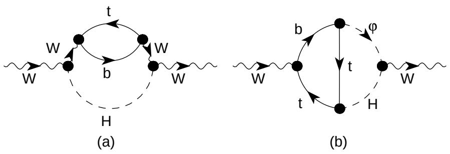

Fig. 2. The diagrams for which we compare our results with the expansion in $m_{\rm t}$.

We first consider asymptotically large values of $m_{\rm t}$ and check whether our full result and the expansion in $m_{\rm t}$ agree in this region. In order to compare with the heavy Higgs expansion, both $m_{\rm t}$ and $M_{\rm H}$ are made asymptotically large, while for comparison with the light Higgs expansion $M_{\rm H}$ is kept fixed. Fig. 3 shows for diagram (a) the difference between the finite parts of our result and the heavy Higgs expansion, divided by $m_{\rm t}$, as a function of $m_{\rm t}$. For the self-energies we have here and below used units of $M_{\rm W}^2 \frac{\alpha^2}{4\pi^2}$, which amounts with $\alpha(M_{\rm Z}^2) = \frac{1}{128.9}$ to $1.52 \times 10^{-6} M_{\rm W}^2$. The difference between the full result and the expansions in $m_{\rm t}$ is expected to be at most of the order $m_{\rm t}$, as it is the case in Fig. 3. We have checked that also for the light Higgs expansion of diagram (a) and for both expansions of diagram (b) one finds agreement between the full result and the expansions in the asymptotic region.

[1] P. Gambino has kindly provided us with his results for the expansions of these diagrams.

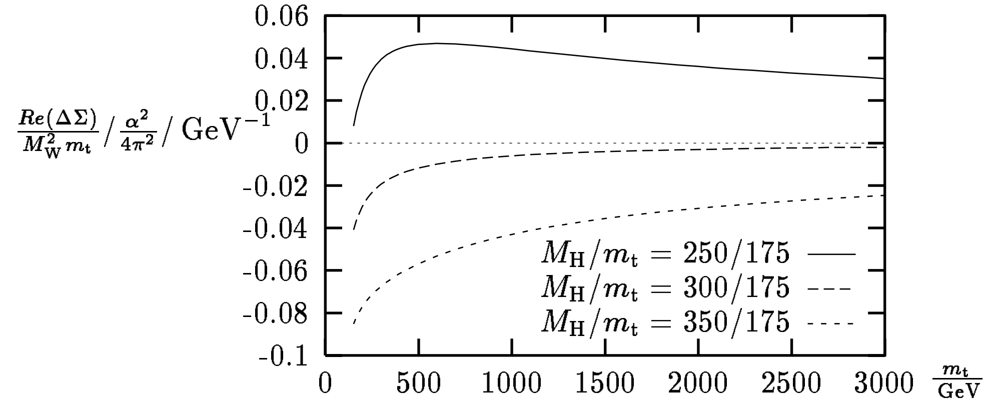

Fig. 3. Diagram (a): Difference between full calculation and heavy Higgs expansion, divided by m_t ($p^2 = M_W^2$).

Next we consider the physical parameter region, i.e. we put $m_t = 175\,\mathrm{GeV}$, and compare the full result with the expansions for different values of M_H. This is shown in Fig. 4 for diagram (a) and in Fig. 5 for diagram (b). We find relatively good agreement for diagram (b), while for diagram (a) for most values of M_H the relative deviations are larger. It should be noted, however, that the numerical contribution of diagram (b) is very large so that in the final result large cancellations can be expected.

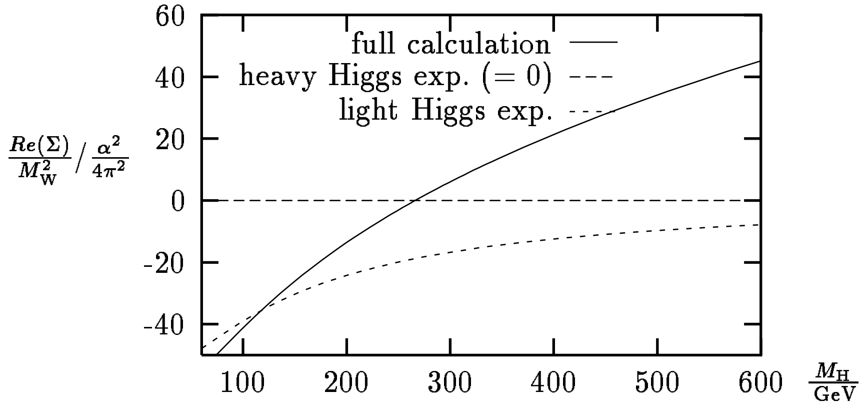

Fig. 4. Diagram (a): Full calculation and expansions ($m_t = 175\,\mathrm{GeV}$, $p^2 = M_W^2$).

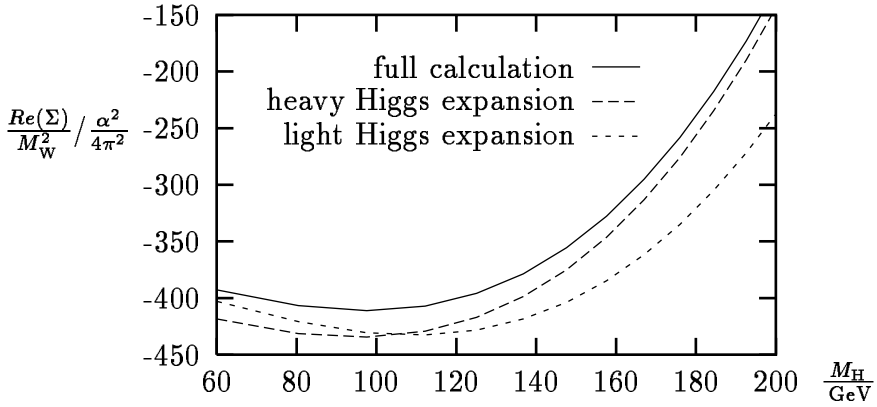

Fig. 5. Diagram (b): Full calculation and expansions ($m_t = 175\,\mathrm{GeV}$, $p^2 = M_W^2$).

5.2 *Sensitivity of the two-loop top-quark corrections to Δr on the Higgs-boson mass*

In this section we study the sensitivity of the two-loop top-quark corrections to Δr to the Higgs-boson mass by considering the quantity

$$\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}}) = \Delta r^{\mathrm{top}}_{(2)}(M_{\mathrm{H}}) - \Delta r^{\mathrm{top}}_{(2)}(M_{\mathrm{H}} = 60\,\mathrm{GeV}), \tag{4}$$

where $\Delta r^{\mathrm{top}}_{(2)}(M_{\mathrm{H}})$ denotes the complete two-loop top-quark contribution to Δr. In Fig. 6 the variation of $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ with the Higgs-boson mass is shown in the interval $60\,\mathrm{GeV} \leq \mathrm{M_H} \leq 1\,\mathrm{TeV}$ for various values of m_{t}. The change in $\Delta r^{\mathrm{top}}_{(2)}(M_{\mathrm{H}})$ induced by varying M_{H} in this interval is found to be about 0.001. It is interesting to note that the absolute value of the correction is maximal just in the region of $m_{\mathrm{t}} = 175\,\mathrm{GeV}$, i.e. for the physical value of the top-quark mass.

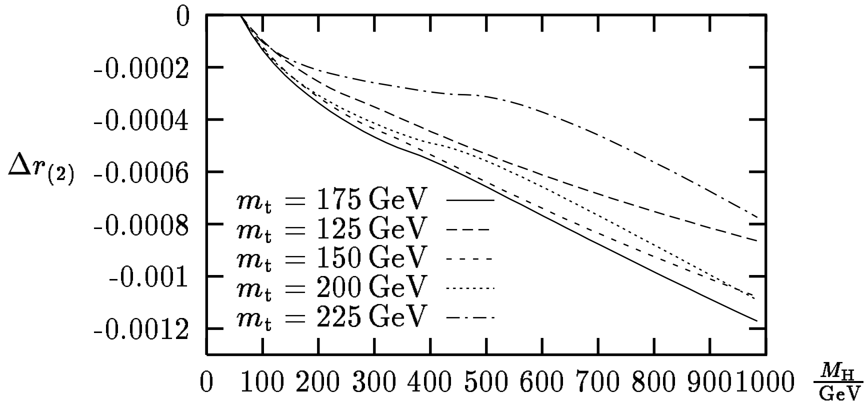

Fig. 6. Two-loop top-quark contributions to Δr subtracted at $M_{\mathrm{H}} = 60\,\mathrm{GeV}$.

In Fig. 7 the corresponding one-loop correction,

$$\Delta r_{(1),\mathrm{subtr}}(M_{\mathrm{H}}) = \Delta r_{(1)}(M_{\mathrm{H}}) - \Delta r_{(1)}(M_{\mathrm{H}} = 60\,\mathrm{GeV}) \tag{5}$$

is shown together with the combined contribution $\Delta r_{(1),\mathrm{subtr}}(M_{\mathrm{H}}) + \Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$. As one can see in the plot, the one-loop and two-loop corrections enter with different sign, i.e. the sensitivity of Δr to the Higgs-boson mass is lowered by the inclusion of the two-loop top-quark corrections. The size of the two-loop correction $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ is about 10 percent of the one-loop contribution.

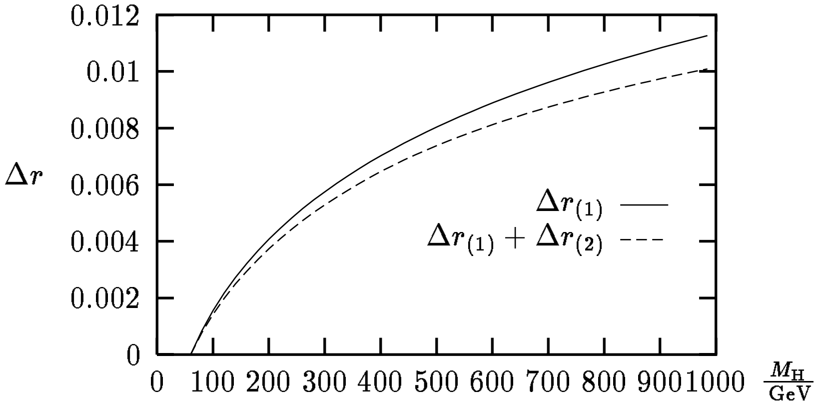

Fig. 7. One-loop and two-loop top-quark contrib. to Δr subtracted at $M_{\mathrm{H}} = 60\,\mathrm{GeV}$.

6 Conclusions

In this article we have outlined techniques for the calculation of electroweak two-loop corrections associated with the top quark or the Higgs boson to precision observables like Δr. No expansion in the particle masses is performed, i.e. the calculations are valid for arbitrary values of m_{t}, M_{H} and the gauge-boson masses. The calculations are performed with the help of computer-algebra packages, which provide a high degree of automatization, and very efficient C-routines for the numerics, which are based on one-dimensional integral representations of the two-loop scalar integrals. For an example we have compared our results to those obtained via an expansion in the top-quark mass up to next-to-leading order. We have furthermore calculated the sensitivity of the two-loop top-quark corrections to Δr with respect to the Higgs-boson mass and compared these contributions to the one-loop result.

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