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ELECTROWEAK DEFECTS *

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A brief, non-technical review is given of certain defect-like configurations in the electroweak standard model, which may have played an important role in the physics of the early universe.

The history of the early universe is, for the most part, rather uneventful: the universe expands and cools down. There are, however, brief moments when something interesting does happen. These are, in reverse order, the epochs of

- recombination at temperatures $T = \mathrm{O(eV)}$,
- nucleosynthesis at $T = \mathrm{O(MeV)}$,
- quark confinement at $T = \mathrm{O(GeV)}$,
- electroweak phase transition at $T = \mathrm{O(TeV)}$,

where the temperatures indicate the physics involved, namely of the atomic, nuclear, chromodynamic and weak interactions (units are such that $\hbar = c = k = 1$). What happened in the universe at even higher temperatures is a complete mystery, because we do not know how the elementary particles behave at collision energies much above, say, 1 TeV. Still, it is conceivable that already the electroweak phase transition epoch may have some bearing on the dark

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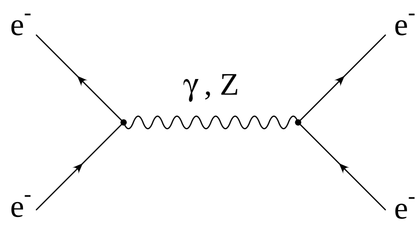

Figure 1: Feynman diagram.

matter problem (the nature and origin of what constitutes the bulk of our present universe). For this reason, we like to present at this workshop some recent results on a somewhat unusual aspect of electroweak physics.

Elementary particle physics, over the years, has established a so-called standard model. It consists of two parts: the electroweak interactions and the strong (chromodynamic) interactions. A priori, these two interactions seem to be unrelated. The really mysterious part is the electroweak sector, with the violation of parity (P) and charge conjugation (C) discovered in the 1950's. The non-conservation of these two discrete symmetries is nowadays incorporated into the electroweak standard model, right from the start. In fact, the non-conservation already shows up in the fields present and their basic interaction structure (e.g. the absence of an interacting right-handed neutrino field).

This brings us to the main point of our talk, namely that the electroweak standard model has been established, so far, in perturbation theory only. The Feynman diagram of Fig. 1, for example, contributes to the scattering of two electrons (e^-) by the exchange of a virtual photon (γ) or neutral vector boson (Z^0). Essentially, the electrons are free moving plane waves (wave packets, rather), which now and then emit or absorb a photon, with probability amplitude proportional to e (the electric charge of the electron). Standard perturbation theory is the collection of all relevant Feynman diagrams, which can be ordered as an infinite series in powers of e^2 for the probabilities (Fig. 1, for example, contributes in order e^4 to the scattering cross-section). However, there may be more to the electroweak theory than Feynman diagrams. In fact, we now believe that there are certain non-perturbative processes, proportional to $\exp[-1/e^2]$, that may lead to violation of time reversal symmetry (T) and fermion number conservation ($B+L$, with B the baryon number and L the lepton number). Most importantly, these processes occur for interacting fields only. For these processes there is no longer a meaningful distinction between a free and an interacting part of the Hamiltonian, all terms being equally important. The field configurations involved are thus very different from plane waves (free particles) and generically go under the name of "defects". Let us first review two such defects in simple models and then see wether or not defect-like configurations also appear in the electroweak standard model.

Consider the Abelian Higgs model in 3+1 dimensions. The cartesian coordinates are written as (x^1, x^2, x^3, x^0) and, for later use, we define the cylindrical coordinates ρ, ϕ, z and t by $(\rho\cos\phi, \rho\sin\phi, z, t) = (x^1, x^2, x^3, x^0)$. Static (time-independent) fields have an energy density

$$\epsilon = \left| \partial_k \, \Phi + i \, \frac{e}{2} \, A_k \, \Phi \right|^2 + \frac{1}{4} \, (\partial_k A_l - \partial_l A_k)^2 + \lambda \left(\Phi^\star \Phi - \frac{v^2}{2} \right)^2 , \qquad (1)$$

with indices k, l running over 1, 2, 3, and ∂_k standing for the partial derivative $\partial/\partial x^k$. Here A_k is the real gauge field and $\Phi = \phi_1 + i\phi_2$ the complex scalar Higgs field, with vacuum expectation value v, quartic coupling constant λ and electric charge $\frac{1}{2}\, e$. The theory has an $U(1)$ gauge invariance

$$\begin{aligned} \Phi(x) &\to e^{i\omega(x)} \, \Phi(x) \, , \\ A_k(x) &\to A_k(x) - \frac{2}{e} \, \partial_k \, \omega(x) \, , \end{aligned} \qquad (2)$$

with $\omega(x)$ an arbitrary real function, so that $e^{i\omega(x)} \in U(1)$. The gauge symmetry group is called Abelian, because a successive transformation with first ω_1 and then ω_2 gives the same result as with the order reversed (i. e. the group addition is commutative). The so-called Higgs symmetry breaking mechanism transforms the gauge field A into a massive vector field, with a vector boson mass given by $M_A = \frac{1}{2}\, e\, v$.

As a further restriction we impose z-independence, so that we have an essentially 2-dimensional theory with fields $\Phi(x^1, x^2)$ and $A_k(x^1, x^2)$ for $k = 1$, 2 . Field configurations with finite string tension (total energy per unit length in the z-direction) obey the following condition:

$$\lim_{\rho \to \infty} |\Phi| \equiv |\Phi^\infty| = \frac{v}{\sqrt{2}} \ , \tag{3}$$

because otherwise the scalar potential term in (1) would give an infinite contribution upon integration. The field values $|\Phi| = v/\sqrt{2}$ define the vacuum manifold $M_{\rm vac}$, which has the topology of a circle S_1. Continuous, finite string tension field configurations are thus characterized by their behaviour at spatial infinity S_1^∞ (a limiting circle in the plane $z = 0$, say). In short, there is the map

$$\Phi^\infty \quad : \quad S_1^\infty \to M_{\rm vac} = S_1 \ . \tag{4}$$

This map $\Phi^\infty(\phi) \in M_{\rm vac}$ classifies the different field configurations according to the number (n) of times the complex phase of the field Φ winds around, for azimuthal angle ϕ running from 0 to 2π, see Fig. 2.

Intuitively, it is clear that any configuration in the $n = 0$ sector can relax to the classical vacuum solution ($A_k = 0, \Phi = v/\sqrt{2}$, modulo gauge transformations), whereas a configuration in the $n = \pm 1$ sector cannot. One therefore expects the existence of another classical solution to which $n = 1$ configurations may relax. This is the well-known vortex solution [1]

$$\begin{aligned} \Phi &= \bar{f}(\rho)\, e^{i\phi}\, \frac{v}{\sqrt{2}} \ , \\ \vec{A} &= -\frac{2}{e}\, \frac{\bar{g}(\rho)}{\rho}\, \hat{e}_\phi \ , \end{aligned} \tag{5}$$

with $\hat{e}_\phi$ the azimuthal unit vector and $\bar{f}(\rho)$, $\bar{g}(\rho)$ radial functions that solve the reduced field equations under boundary conditions

$$\bar{f}(0) = \bar{g}(0) = 0 \ , \ \ \bar{f}(\infty) = \bar{g}(\infty) = 1 \ . \tag{6}$$

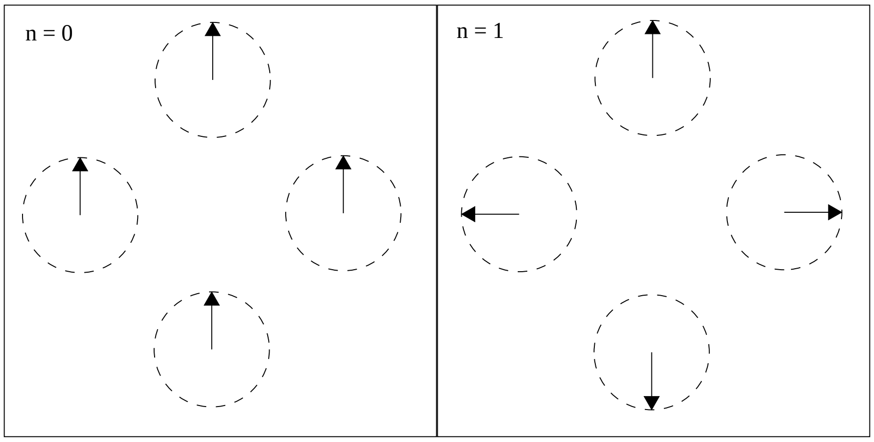

Figure 2: Winding number sectors.

The anti-vortex solution, with winding number $n = -1$, has the signs of ϕ and $\hat{e}_\phi$ in (5) reversed. Only the behaviour of the scalar field was sketched in Fig. 2, but the gauge field certainly plays an important role. The asymptotic gauge field (5) cancels the gradient of Φ and makes the first term on the right-hand side of (1) vanish as $\rho \to \infty$, thereby keeping the string tension finite. Moreover, the gauge field carries a quantized magnetic flux in the z-direction. The solution (5, 6) is a flux tube in the z-direction, with the Higgs symmetry breaking absent at the core ($|\Phi| << v/\sqrt{2}$ for $\rho \sim 0$), so that locally the vector field is massless again.

The existence of the vortex solution is governed by the topological classification $n = \pm 1$, as sketched in Fig. 2 above. For this reason, the vortex solution is called a topological defect. Also, if the highly symmetric vortex configuration is the lowest energy classical solution in the $n = 1$ sector, then it must necessarily be *stable*, since there is no other solution to decay to (admittedly, the argument is somewhat circular, but local stability of the vortex solution can be proven rigorously). To return to a point made in the beginning, all terms of the energy density (1) are of equal importance in the vortex solution and there is no separation possible into a free and O(e) interacting part. Indeed, the interaction term $|e\, A_k\, \Phi|^2$ is O(1), due to the large, O($1/e$), amplitude of the vortex gauge field (5). All in all, the vortex solution is very different from the classical vacuum, or small fluctuations around it.

Another interesting example of a topological defect occurs in $SO(3)$ Yang-Mills-Higgs theory. We consider fully 3-dimensional static fields and introduce the standard spherical coordinates r, θ and ϕ by $(r \sin\theta \cos\phi,\ r \sin\theta \sin\phi,\ r\cos\theta) = (x^1, x^2, x^3)$. This theory has a real Higgs triplet

$$\Psi = \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} , \tag{7}$$

on which a non-Abelian (non-commutative) gauge symmetry operates, $\Psi(x) \to \Omega(x)\Psi(x)$ with $\Omega(x) \in SO(3)$. Finite total energy requires $|\Psi| \to v/\sqrt{2}$ for $r \to \infty$ and the vacuum manifold has the topology of a sphere, $M_{\rm vac} = S_2$. The relevant map is from the sphere at spatial infinity into the vacuum manifold

$$\Psi^\infty \quad : \quad S_2^\infty \to M_{\rm vac} = S_2 \ . \tag{8}$$

Analogous to Fig. 2, there is now a 3-dimensional picture with winding number $n = 1$ (imagine the field vectors pointing outwards, just as for a hedgehog). Such a configuration with $n = 1$ cannot relax to the vacuum solution, but can to another classical solution, the magnetic-monopole solution [2]. This concludes our brief review of so-called topological defects. These stable defects are primarily relevant to the *static* properties of the theory, notably the spectrum of the full Hamiltonian.

We now come to the crucial question: can these stable, topological defects be carried over to the electroweak standard model (EWSM)? The short answer is: no. The reason is that the electroweak theory has a complex Higgs doublet

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_3 + i\phi_4 \end{pmatrix} , \tag{9}$$

so that $M_{\rm vac} = S_3$. This implies that the mappings from either the circle S_1^∞ or the sphere S_2^∞ at infinity into $M_{\rm vac} = S_3$ are always topologically trivial (in Coleman's words: you cannot lasso a basketball). Hence, there are no static topological defects in the $3+1$ dimensional EWSM. Still (and this is the long answer to the question above), there *are* other types of defects in the EWSM, but they turn out to be *unstable*. It is believed that these unstable defects may be relevant to the *dynamic* properties of the theory. Here, we will mention two such electroweak defects, the sphaleron and the Z-string.

The existence of the sphaleron solution in the EWSM again follows from a topological argument. However, rather than considering the behaviour of the field configurations directly, such as was done in Fig. 2, one considers instead the topology of configuration space. Configuration space here is the abstract, mathematical space of static, finite energy configurations (with the gauge freedom eliminated). Each point of configuration space corresponds to a snapshot of the fields. There is, for example, one point in configuration space which identifies the vacuum configuration (Higgs field constant and gauge fields vanishing). The energy functional $E_{\rm B}$ defines a surface over configuration space, with the stationary points corresponding to the solutions of the classical field equations. This configuration space is truly enormous: infinite dimensional and non-compact. Moreover, the topology of configuration space turns out to be non-trivial. Configuration space is not simply an infinite dimensional euclidian space $\mathbb{R}^\infty$. In fact, there are *holes* and the point at the "top" of one such hole corresponds to a new classical solution, the so-called sphaleron [a] [3]. The sphaleron S has typical field energy

$$E_{\rm S} = {\rm O}(M_W/\alpha_w) \sim 10\,{\rm TeV}\ , \tag{10}$$

with $M_W \equiv \frac{1}{2}\, g\, v = 80.4\,{\rm GeV}$ the mass of the $W^\pm$ vector bosons, $v = 247\,{\rm GeV}$ the Higgs vacuum expectation value and $\alpha_w \equiv g^2/4\pi = 1/29.6$ the strength of the $SU(2)$ gauge interactions. The shape of the sphaleron S is slightly elongated and the electromagnetic field has a large magnetic dipole moment,

[a] From the Greek adjective 'sphaleros', meaning ready to fall.

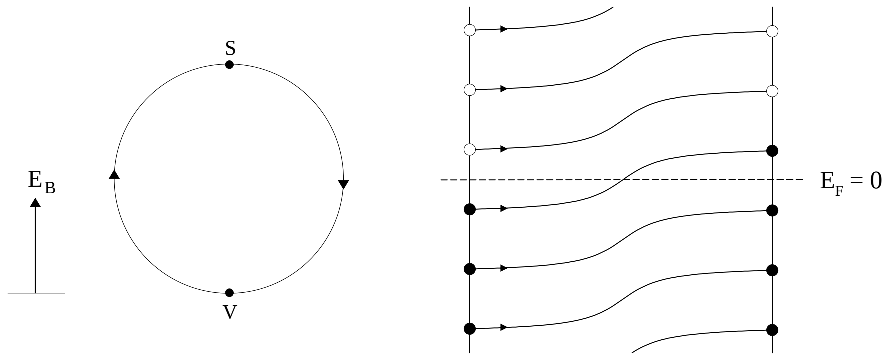

Figure 3: Non-contractible loop and spectral flow.

which may be interpreted [4] as coming from a tight monopole - antimonopole pair. In this way, we find magnetic monopoles already in the EWSM!

For completeness, we should mention that the hole in configuration space is captured by the non-trivial map

$$S_1 \times S_2^\infty \to M_{\mathrm{vac}} = S_3 \; , \tag{11}$$

where S_1 parametrizes a loop of configurations, each of which is characterized by its behaviour at spatial infinity S_2^∞. The sphaleron solution is just one "point" on this non-contractible loop (Fig. 3, left-hand side) and by itself has trivial topology ($S_2^\infty \to S_3$), in contrast to, for example, the magnetic monopole solution based on the non-trivial map (8).

Without saying explicitely, we have considered up till now only the bosonic fields (scalar and gauge fields), with total energy E_{B}. We now look at the response of (quantized) fermionic fields to external (classical) bosonic fields. Specifically, we take a loop of bosonic field configurations that encircles the configuration space hole mentioned above, starting and ending at the vacuum V, while passing througth the sphaleron S. This is a non-contractible loop (NCL) of configurations, precisely because it encircles a hole. If we now consider the Dirac eigenvalues ($E_{\rm F}$) for the different bosonic configurations of the NCL, we observe the surprising phenomenon of *spectral flow*. The NCL starts and ends at the same configuration (V) and the Dirac eigenvalue spectrum there is the same, but along the NCL there is a non-trivial flow of eigenvalues (Fig. 3). Numerical calculations appear to confirm this behaviour [5]. In the electroweak context this spectral flow[b] leads to the violation of fermion number conservation [6]

$$\Delta(B+L) = \pm 2\, N_{\rm fam} \;, \tag{12}$$

where $N_{\rm fam}(=3)$ is the number of families of quarks and leptons. The selection rule (12) is simply the total contribution of all left-handed fermion doublets (quarks and leptons) responding to the $SU(2)$ gauge fields of the NCL.

Originally, $B+L$ violation was considered [6] to proceed via quantum tunneling (instantons), but the picture of Fig. 3 suggests the possibility of $B+L$ violation at large temperatures or collision energies, where one passes *over* the energy barrier $E_{\rm S}$, not through. As expected the tunneling rate is extremely small, with a WKB factor $\exp[-16\pi^2/g^2\hbar] \sim \exp[-372]$, whereas the thermal rate, with a Boltzmann factor $\exp[-E_{\rm S}/kT]$, increases with temperature. These thermal reactions turn out to have been important in the very early universe [3,7]. However, the precise scenario for the creation of the present baryon number of the universe is not clear yet. This remains one of the outstanding problems of cosmology and particle physics.

Another classical solution in the EWSM is the Z-string [8], which is simply the embedded vortex solution (5),

$$\begin{aligned}
\Phi &= \bar{f}(\rho) \begin{pmatrix} 0 \\ e^{i\phi} \end{pmatrix} \frac{v}{\sqrt{2}} \;, \\
\vec{Z} &= -\,\frac{2\cos\theta_w}{g}\,\frac{\bar{g}(\rho)}{\rho}\,\hat{e}_\phi \;, \\
\vec{W}^{\pm} &= 0 \;,
\end{aligned}$$

[b] Note that the Dirac sea (the set of filled negative energy states of the Dirac Hamiltonian) is truly bottomless, which allows for an overall shift of the occupied states, as in Fig. 3.

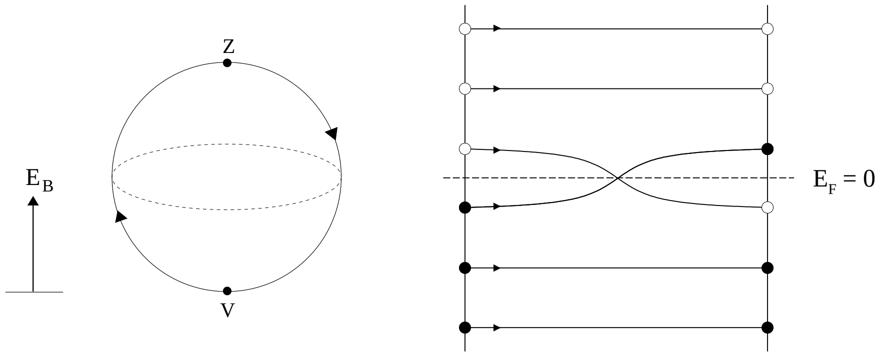

Figure 4: Non-contractible sphere and level crossing.

$$\vec{A} = 0 \ , \tag{13}$$

with A the massless photon field and Z^0, $W^{\pm}$ the massive vector boson fields (mass ratio $M_W/M_Z = \cos\theta_w$, in terms of the weak mixing angle $\theta_w = 0.491$). The extra degrees of freedom, compared to the Abelian Higgs model, turn the Z-string into an unstable solution. In fact, it can be shown [9] that there is a hole in the space of z-independent configurations, around which a non-contractible *sphere* (NCS) may be constructed, with the Z-string at the top (Fig. 4, left hand side, where $E_{\rm B}$ indicates the 2-dimensional "energy", i. e. the string tension in the 3-dimensional context). The instability of the Z-string is manifest, with at least two unstable directions. The topologically non-trivial map is now

$$S_2 \times S_1^{\infty} \rightarrow M_{\rm vac} = S_3 \ , \tag{14}$$

where S_2 parametrizes a sphere of configurations, each of which is characterized by its behaviour at spatial infinity S_1^{∞}. Again, there is spectral flow of the Dirac eigenvalues over the NCS [10]. The behaviour along an arbitrary meridian of the NCS is sketched in Fig. 4. In this case there is no net change of quantum number, since particles and anti-particles are created simultaneously.

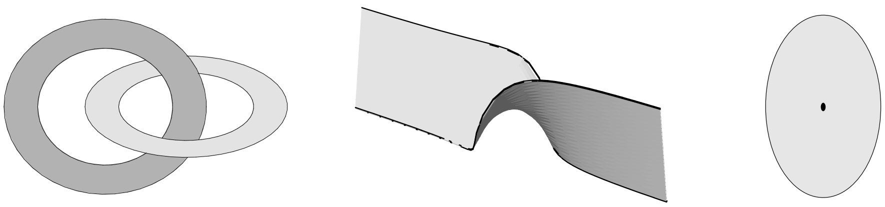

Figure 5: Linked Z-strings, twisted Z-string segment and sphaleron S.

To conclude, we mention an interesting observation, which relates these two electroweak solutions. The observation [11] is that *linked* Z-string loops and *twisted* Z-string segments[c] (Fig. 5) have non-vanishing Chern-Simons number, just as the sphaleron S does, and may therefore lead to violation of $B+L$. This ties in with our earlier remark that the sphaleron solution S may be viewed as a bound monopole and antimonopole, which are then connected by a very short segment of twisted Z-string.

All these unstable defects of the electroweak standard model (Fig. 5) are expected to play an important role in the electroweak phase transition of the early universe ($T \sim 100-1000\,\mathrm{GeV}$). Extended models of the electroweak (and strong) interactions may even have (meta-)stable defects, which could have been created [12] at an earlier phase transition ($T > 1\,\mathrm{TeV}$). The cosmological creation of defects can be simulated perhaps in appropriate low-temperature condensed matter experiments [13]. Hopefully the coming decennia will provide us with clues, both from astronomical observations and from physics experiments on Earth, to help explore the cosmological epoch of the electroweak phase transition and before.

[c] Note that these particular configurations are, strictly speaking, not exact static solutions of the field equations; they evolve with time.

References

1. H. Nielsen and P. Olesen, *Nucl. Phys.* B **61**, 45 (1973)
2. G. 't Hooft, *Nucl. Phys.* B **79**, 276 (1974);
   A. Polyakov, *Sov. Phys. JETP Lett.* **20**, 194 (1974)
3. F. Klinkhamer and N. Manton, *Phys. Rev.* D **30**, 2212 (1984)
4. M. Hindmarsh and M. James, *Phys. Rev.* D **49**, 6109 (1994)
5. J. Kunz and Y. Brihaye, *Phys. Lett.* B **304**, 141 (1993)
6. G. 't Hooft, *Phys. Rev. Lett.* **37**, 8 (1976)
7. V. Kuzmin, V. Rubakov and M. Shaposhnikov, *Phys. Lett.* B **155**, 36 (1985); P. Arnold and L. McLerran, *Phys. Rev.* D **36**, 581 (1987);
   D. Diakonov, M. Polyakov, P. Sieber, J. Schaldach and K. Goeke, *Phys. Rev.* D **53**, 3366 (1996)
8. Y. Nambu, *Nucl. Phys.* B **130**, 505 (1977);
   T. Vachaspati, *Nucl. Phys.* B **397**, 648 (1993)
9. F. Klinkhamer and P. Olesen, *Nucl. Phys.* B **422**, 227 (1994)
10. C. Rupp, *Spektraler Fluss und der Z-Vortex*, Diplomarbeit Universität Karlsruhe, April 1996
11. T. Vachaspati and G. Field, *Phys. Rev. Lett.* **73**, 373 (1994)
12. T. Kibble, *J. Phys. A : Math. & Gen.* **9**, 1387 (1976)
13. W. Zurek, *Phys. Rep.* in press, preprint cond-mat/9607135