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How reliably can the Higgs-Boson Mass be predicted from Electroweak Precision Data?[†]

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Abstract

From the LEP precision data, upon excluding the observables R_b, R_c in a combined fit of the top-quark mass, m_t, and the Higgs-boson mass, M_H, within the Standard Model, we find the weak 1σ bound of $40\,\mathrm{GeV} \lesssim \mathrm{M_H} \lesssim 1\,\mathrm{TeV}$. Stronger upper bounds on M_H, sometimes presented in the literature, rely heavily on the inclusion of R_b in the data sample. Upon including R_b, the quality of the fit drastically decreases, and m_t comes out significantly below the experimental value of $m_\mathrm{t} = 180 \pm 12\,\mathrm{GeV}$, thus implying that the stronger bounds are not reliable. Moreover, the stronger bounds on M_H are lost if the deviation between theory and experiment in R_b is ascribed to contributions of new physics. Replacing $\bar{s}^2_\mathrm{w}(\mathrm{LEP})$ by $\bar{s}^2_\mathrm{w}(\mathrm{SLD})$ in the data sample leads to the tight bound of $M_\mathrm{H} \lesssim 35\,\mathrm{GeV}$ at the 1σ level, which is in conflict, however, with $M_\mathrm{H} \gtrsim 65.2\,\mathrm{GeV}$ from direct searches.

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The discovery [1] of the top quark and the direct determination of its mass of $m_{\mathrm{t}} = 180 \pm 12\,\mathrm{GeV}$ open the possibility of improving the constraints on the mass of the Higgs boson, M_{H}, from the body of the precision electroweak data at the Z-boson resonance [2, 3] and the experimental value of the W-boson mass, M_{W} [4]. In this note we present our results for m_{t} and M_{H}, obtained by performing fits to the precision data and M_{W} within the Standard Model (SM). The dependence of the fits on the experimental data on R_{b}, R_{c}, and $\bar{s}^2_{\mathrm{W}}$ is investigated, and the effects of varying the SM input parameters $\alpha(M_{\mathrm{Z}}^2)$ and $\alpha_{\mathrm{s}}(M_{\mathrm{Z}}^2)$ in the allowed range are discussed. We also examine how the results of these fits are influenced, if one allows for non-standard $\mathrm{Z} \to \mathrm{b\bar{b}},\mathrm{c\bar{c}}$ vertices. Even though several papers on this subject have appeared recently [5, 6, 7], additional investigations combined with comments on the interpretation of the results seem useful.

It is the main purpose of the present work to analyze the data in several distinct steps in a most transparent and objective manner. In a first step we concentrate on the leptonic observables Γ_{l} and $\bar{s}^2_{\mathrm{W}}(\mathrm{LEP})$, and the W-boson mass (the set of data to be referred to as "leptonic sector"), and include Γ_{T} and Γ_{h} from the set of hadronic observables (referring to this set of data as "all data $\setminus R_{\mathrm{b}}, R_{\mathrm{c}}$"), thus ignoring in the fits at this stage the partial Z-boson decays $\mathrm{Z} \to \mathrm{b\bar{b}},\mathrm{c\bar{c}}$. In a second step we include the $\mathrm{Z} \to \mathrm{b\bar{b}}$ decay mode and determine m_{t} and M_{H} in a two-parameter fit again. In a third step we finally discuss how the results of the fits change when the decay $\mathrm{Z} \to \mathrm{c\bar{c}}$ is included, and we also investigate the effect of replacing $\bar{s}^2_{\mathrm{W}}(\mathrm{LEP})$ by $\bar{s}^2_{\mathrm{W}}(\mathrm{SLD})$ in the set of data. The procedure adopted here obviously allows us to identify the dependence of the results for m_{t} and M_{H} on the set of experimental data used in the fits. The various steps are motivated by the discrepancies [2] between SM prediction and experiment observed in the $\mathrm{Z} \to \mathrm{b\bar{b}},\mathrm{c\bar{c}}$ decays and the difference between the LEP and SLD results for $\bar{s}^2_{\mathrm{W}}$. In a final step we discuss fits in which non-standard contributions are allowed.

The set of experimental data and the input parameters are listed in Tab. 1. The theoretical SM results at the one-loop level, taking into account leading two-loop contributions, are taken from Refs. [8, 9][1]. Therefore, we provide an analysis which is completely independent of results presented by other authors [5, 6, 7].

We turn to the discussion of the results for the parameters m_{t} and M_{H} obtained within the SM. Figure 1 shows the results in the $(m_{\mathrm{t}}, M_{\mathrm{H}})$ plane. The corresponding numerical values and the 1σ bounds obtained for m_{t} and M_{H}, as well as the shifts of m_{t} and M_{H} resulting from replacing $\alpha(M_{\mathrm{Z}}^2)^{-1}$ and $\alpha_{\mathrm{s}}(M_{\mathrm{Z}}^2)$ by their upper and lower limits (see Tab. 1) in the SM prediction are collected in Tab. 2.[2] In Tab. 2 we also give the results for m_{t} and M_{H} obtained upon including R_{c} in the data sample. These results have been omitted in Fig. 1 for compactness.

We first of all concentrate on the results of *the first step of our analysis*, namely the fits in Fig. 1 and Tab. 2 based on the data sets $\bar{s}^2_{\mathrm{W}}(\mathrm{LEP})$, Γ_{l} and M_{W} ("leptonic

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[1]We have supplemented the analytical results given in Refs. [8, 9] by the $\mathcal{O}(G_\mu m_{\mathrm{t}}^2 \alpha_{\mathrm{s}}^2)$ corrections [10] to the ρ-parameter.

[2]In the subsequent tables the entry $\gtrsim 1000$ indicates a result for the Higgs-boson mass in the region above 1 TeV, where a perturbative evaluation is no longer trustworthy. The upper limits for the top-quark mass appearing in conjunction with a Higgs-boson mass of $M_{\mathrm{H}} \gtrsim 1000\,\mathrm{GeV}$ also have to be treated with care.

leptonic sector	hadronic sector	
$\Gamma_{\mathrm{l}} = 83.93 \pm 0.14\,\mathrm{MeV}$	$R = 20.788 \pm 0.032$	$\Gamma_{\mathrm{T}} = 2496.3 \pm 3.2\,\mathrm{MeV}$
$\bar{s}^2_{\mathrm{W}}(\mathrm{LEP}) = 0.23186 \pm 0.00034$	$\sigma_{\mathrm{h}} = 41.488 \pm 0.078$	$\Gamma_{\mathrm{h}} = 1744.8 \pm 3.0\,\mathrm{MeV}$
$\bar{s}^2_{\mathrm{W}}(\mathrm{SLD}) = 0.23049 \pm 0.00050$	$R_{\mathrm{b}} = 0.2219 \pm 0.0017$	$\Gamma_{\mathrm{b}} = 387.2 \pm 3.0\,\mathrm{MeV}$
$\bar{s}^2_{\mathrm{W}}(\mathrm{LEP}+\mathrm{SLD}) = 0.23143 \pm 0.00028$	$R_{\mathrm{c}} = 0.1543 \pm 0.0074$	$\Gamma_{\mathrm{c}} = 269 \pm 13\,\mathrm{MeV}$
$M_{\mathrm{W}} = 80.26 \pm 0.16\,\mathrm{GeV}$		

input parameters
$M_{\mathrm{Z}} = 91.1884 \pm 0.0022\,\mathrm{GeV}$
$G_\mu = 1.16639(2) \cdot 10^{-5}\,\mathrm{GeV}^{-2}$
$\alpha(M^2_{\mathrm{Z}})^{-1} = 128.89 \pm 0.09$
$\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}}) = 0.123 \pm 0.006$
$m_{\mathrm{b}} = 4.5\,\mathrm{GeV}$

correlation matrices

	σ_{h}	R	Γ_{T}
σ_{h}	1.00	0.15	−0.12
R	0.15	1.00	−0.01
Γ_{T}	−0.12	−0.01	1.00

	R_{b}	R_{c}
R_{b}	1.00	−0.34
R_{c}	−0.34	1.00

Table 1: The precision data used in the fits, consisting of the LEP data [2], the SLD value [3] for $\bar{s}^2_{\mathrm{W}}$, and the world average [4] for M_{W}. The partial widths Γ_{l}, Γ_{h}, Γ_{b}, and Γ_{c} are obtained from the observables $R = \Gamma_{\mathrm{h}}/\Gamma_{\mathrm{l}}$, $\sigma_{\mathrm{h}} = (12\pi\Gamma_{\mathrm{l}}\Gamma_{\mathrm{h}})/(M^2_{\mathrm{Z}}\Gamma^2_{\mathrm{T}})$, $R_{\mathrm{b}} = \Gamma_{\mathrm{b}}/\Gamma_{\mathrm{h}}$, $R_{\mathrm{c}} = \Gamma_{\mathrm{c}}/\Gamma_{\mathrm{h}}$, and Γ_{T} using the given correlation matrices. The data in the upper left-hand column (using $\bar{s}^2_{\mathrm{W}}(\mathrm{LEP})$ if not otherwise specified) will be referred to as "leptonic sector" in the fits. Inclusion of the data in the upper right-hand column will be referred to as fitting "all data". The theoretical predictions are based on the input parameters [2, 11] given in the lower left-hand column of the table.

sector") and $\bar{s}^2_{\mathrm{W}}(\mathrm{LEP})$, Γ_{l}, M_{W}, Γ_{T}, Γ_{h} ("all data $\setminus$ $R_{\mathrm{b}}, R_{\mathrm{c}}$"). Both fits yield an excellent $\chi^2_{\mathrm{min}}/\mathrm{d.o.f.} < 1$, independently of whether the central values for $\alpha(M^2_{\mathrm{Z}})^{-1}$ and $\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}})$ of Tab. 1 (for which $\chi^2_{\mathrm{min}}/\mathrm{d.o.f.}$ is indicated in Fig. 1 and Tab. 2) or the upper and lower limits for $\alpha(M^2_{\mathrm{Z}})^{-1}, \alpha_{\mathrm{s}}(M^2_{\mathrm{Z}})$ are used in the fits. The figure shows that the results of the fits are strongly affected by variations of $\alpha(M^2_{\mathrm{Z}})^{-1}$. For instance, lowering $\alpha(M^2_{\mathrm{Z}})^{-1} = 128.89$ by one standard deviation to $\alpha(M^2_{\mathrm{Z}})^{-1} = 128.80$ also lowers the central value of M_{H} by approximately one standard deviation. Varying $\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}})$ from $\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}}) = 0.123$ to $\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}}) = 0.117$ shifts the upper 1σ limit of M_{H} from $\sim 1\,\mathrm{TeV}$ to $265\,\mathrm{GeV}$ in the fit in which Γ_{h} and Γ_{T} are included (second column of Fig. 1). We also note the somewhat low values of the top-quark mass of $m_{\mathrm{t}} = 157\,\mathrm{GeV}$ and $m_{\mathrm{t}} = 162\,\mathrm{GeV}$ obtained for the lower values of $\alpha(M^2_{\mathrm{Z}})^{-1}$ and $\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}})$, respectively, (compare Tab. 2, second line) which are below the 1σ lower limit of $m_{\mathrm{t}} = 168\,\mathrm{GeV}$ from the direct measurement of m_{t}. The fit results in the leptonic sector are stable under variation in the strong coupling constant, $\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}})$, since $\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}})$ only enters at the two-loop level. Altogether, we thus conclude that varying $\alpha(M^2_{\mathrm{Z}})^{-1}$ and $\alpha_{\mathrm{s}}(M^2_{\mathrm{Z}})$ within the 1σ bounds given in Tab. 1 leads to a considerable

	m_t / GeV	M_H / GeV	$\alpha_s(M_Z^2)$	χ^2_{min}/d.o.f.
leptonic sector	$174^{+36+29+1}_{-19-14-1}$	$247^{+\gtrsim 1000+\gtrsim 1000+1}_{-188-175-1}$		0.2/3
all data \ R_b, R_c	$168^{+29+6+36}_{-20-11-6}$	$167^{+737+113+\gtrsim 1000}_{-126-109-79}$		0.6/5
	167^{+44+26}_{-20-10}	$162^{+\gtrsim 1000+821}_{-123-105}$	$0.123^{+0.006+0.004}_{-0.004-0.002}$	0.6/5
all data \ R_b	$165^{+28+6+36}_{-19-10-5}$	$145^{+643+106+\gtrsim 1000}_{-107-91-62}$		6.4/6
	164^{+39+18}_{-18-8}	$133^{+\gtrsim 1000+365}_{-96-79}$	$0.123^{+0.005+0.002}_{-0.004-0.002}$	6.4/6
all data \ R_c	$148^{+14+2-3}_{-12-1+3}$	$54^{+88+26+2}_{-29-19-0}$		12/6
	148^{+14+2}_{-12-1}	53^{+86+26}_{-29-18}	$0.122^{+0.004+0.001}_{-0.004-0.000}$	12/6
all data	$149^{+15+2-2}_{-12-2+3}$	$57^{+100+28+6}_{-31-21-2}$		14/7
	149^{+14+2}_{-12-1}	56^{+97+29}_{-31-20}	$0.122^{+0.004+0.001}_{-0.004-0.000}$	14/7

Table 2: The fit parameters (m_t, M_H) and (m_t, M_H, α_s) obtained by fitting to different sets of experimental data within the SM. No entry for α_s means that $\alpha_s(M_Z^2) = 0.123$ is fixed in the corresponding two-parameter (m_t, M_H) fit. An entry for α_s, when given, is the result of the corresponding three-parameter (m_t, M_H, α_s) fit. The errors of m_t, M_H, and α_s quoted first correspond to the 1σ limits. The "error entries" in second place give the shifts in the corresponding central fit values resulting from the replacements of $\alpha(M_Z^2)^{-1} = 128.89$ by $\alpha(M_Z^2)^{-1} = 128.98$ (upper value) and by $\alpha(M_Z^2)^{-1} = 128.80$ (lower value) in the SM prediction. Likewise, the entries in third place show the shifts of the central values resulting from the replacements of $\alpha_s(M_Z^2) = 0.123$ by $\alpha_s(M_Z^2) = 0.129$ (upper value) and $\alpha_s(M_Z^2) = 0.117$ (lower value) in the SM prediction.

effect concerning the boundaries in the (m_t, M_H) plane. Concerning $\alpha_s(M_Z^2)$, note that the input of the central value of $\alpha_s(M_Z^2) = 0.123 \pm 0.006$ (from the event shape measurement at LEP [2]) used in the two-parameter (m_t, M_H) fit is supported by the fact that precisely this central value is reobtained as a result of the three-parameter (m_t, M_H, α_s) fit (compare Tab. 2). The corresponding values of m_t and M_H practically coincide with the ones of the two-parameter (m_t, M_H) fit.

Finally, concerning the range of M_H allowed by the results from the leptonic sector (with $\bar{s}_W^2$(LEP)) and Γ_T, Γ_h, i.e. by fitting the SM only to those data that agree with the theoretical predictions, according to the foregoing discussion of Fig. 1 and Tab. 2, it seems hardly possible to deduce stronger limits than

$$40\,\mathrm{GeV} \lesssim \mathrm{M_H} \lesssim 1\,\mathrm{TeV} \tag{1}$$

at the 1σ level, even upon taking into account the constraint of $m_t = 180 \pm 12\,\mathrm{GeV}$ from the direct observation of the top quark. The lowest values in (1) are of course excluded by the lower bound $M_H > 65.2\,\mathrm{GeV}$ [12] obtained from the direct searches.

We turn to the *second step of our analysis* and include R_b in the fit, which is thus based on the leptonic sector in conjunction with Γ_T, Γ_h and R_b. According to Fig. 1 and

m_t / GeV fixed	144	168	180	192
	M_H / GeV (χ^2_{min}/d.o.f.)			
leptonic sector	47^{+30}_{-18} (4.2/3)	160^{+104}_{-69} (0.2/3)	362^{+206}_{-136} (0.2/3)	792^{+444}_{-280} (0.4/3)
all data \ R_b, R_c	44^{+34}_{-20} (2.1/5)	172^{+110}_{-73} (0.6/5)	349^{+196}_{-131} (0.8/5)	682^{+368}_{-239} (1.3/5)
all data \ R_b	44^{+34}_{-20} (7.7/6)	174^{+111}_{-73} (6.4/6)	353^{+199}_{-133} (6.7/6)	689^{+375}_{-242} (7.3/6)
all data \ R_c	45^{+35}_{-21} (12/6)	176^{+112}_{-74} (14/6)	355^{+199}_{-133} (16/6)	685^{+368}_{-239} (19/6)
all data	45^{+35}_{-21} (14/7)	176^{+112}_{-74} (15/7)	355^{+199}_{-133} (17/7)	686^{+369}_{-240} (20/7)

Table 3: Fits of M_H to various sets of experimental data in the SM for fixed values of m_t. In all fits $\alpha(M_Z^2) = 1/128.89$ and $\alpha_s(M_Z^2) = 0.123$ are kept fixed and the LEP value of $\bar{s}_W^2$ is used in the input data.

Tab. 2, taking into account the data for the $Z \to b\bar{b}$ partial width leads to an increase of χ^2_{min}/d.o.f. by an order of magnitude to χ^2_{min}/d.o.f. = 12/6. Comparing the third column in Fig. 1 with the first and second columns, one observes a considerable shrinkage of the 1σ regions in the (m_t, M_H) plane and a drastic shift towards lower values of m_t and M_H. The sensitivity against variations of $\alpha(M_Z^2)^{-1}$ and $\alpha_s(M_Z^2)$ is considerably weaker in this sample of data. The central fit-value for the top-quark mass of $m_t = 148^{+14}_{-12}$ GeV is significantly below the central value of the direct measurement of $m_t = 180 \pm 12$ GeV, and the central value obtained for the Higgs-boson mass, $M_H = 54^{+88}_{-29}$ GeV, lies in the vicinity of the experimental lower bound $M_H > 65.2$ GeV.

The large increase of χ^2_{min}/d.o.f. to χ^2_{min}/d.o.f. = 12/6 when including R_b in the fit (to be compared with χ^2_{min}/d.o.f. = 0.6/5 obtained when ignoring R_b) signals the large discrepancy between theory and experiment in this fit. In particular, when evaluating R_b for the best-fit values of $(m_t, M_H) = (148^{+14}_{-12}\,\text{GeV}, 54^{+88}_{-29}\,\text{GeV})$, the resulting ($M_H$-insensitive) theoretical prediction, $R_b^{SM} = 0.2164^{-0.0005}_{+0.0004}$ (with the errors indicating the changes by varying m_t within the 1σ limits), still lies more than 3σ below the experimental value of $R_b = 0.2219 \pm 0.0017$.

In connection with the low central fit value of $m_t = 148$ GeV, it is illuminating to consider the results of single-parameter M_H fits, where m_t is kept fixed at certain (assumed) values. In Tab. 3, again for the previously selected sets of data, results of single-parameter M_H fits are shown. The known strong (m_t, M_H) correlation in SM fits leads to a remarkable stability of the resulting fit values for M_H. Once m_t is fixed, there is almost no dependence of the fit value for M_H on which set of input data is actually used in the fit. In particular, whenever a low value of m_t is chosen, one obtains a low value for M_H, independently of whether R_b is included in the fit or not.[3] Since the (M_H-insensitive) SM prediction for R_b increases with decreasing m_t, in the combined fit of (m_t, M_H) the inclusion of R_b lowers

[3]Conversely, if M_H is fixed, the values of m_t obtained in the fit are fairly stable, independently of wether R_b or R_c are included in the fit or not. This is consistent with the results of Ref. [2], where $M_H = 300$ GeV is kept fixed when fitting m_t.

the fit value of m_t, and via the (m_t, M_H) correlation also the value of M_H. As discussed above, the result of $m_t = 148\,\mathrm{GeV}$ is nothing but a kind of compromise, as it still leads to a 3σ discrepancy between theory and experiment in R_b. Moreover, this result for m_t is disfavored by the Tevatron result of $m_t = 180 \pm 12\,\mathrm{GeV}$.

In summary, the large value of $\chi^2_{\mathrm{min}}/\mathrm{d.o.f.}$ and the low fit value for m_t that is at variance with the Tevatron result, lead to the conclusion that the low value of $M_H = 54^{+88}_{-29}\,\mathrm{GeV}$ obtained when including the data for R_b does not seem reliable. It is an artefact of the procedure of describing the "non-standard" value of R_b by the unmodified SM in conjunction with the (m_t, M_H) correlation. This conclusion is strengthened by the fact that a simple phenomenological modification of the $Z \to b\bar{b}$ vertex, to be discussed below, leads to values of m_t compatible with the Tevatron result and removes the stringent upper bounds on M_H.

We turn to the third step of our analysis and consider the *impact of the observable* R_c. As can be seen in Tab. 2, the results for m_t and M_H are hardly affected by including R_c. This is a consequence of the fact that the contribution of R_c to χ^2 depends only very weakly on m_t and M_H, because the experimental error for R_c is much larger than the change in the SM prediction for R_c induced by varying m_t and M_H. Similarly to the case of R_b, including R_c in the set of data (and omitting R_b) leads to an enhanced value of χ^2_{min}, and to a tendency towards lower values of M_H.

So far the analysis has been based on the LEP experimental value of $\bar{s}^2_W(\mathrm{LEP})$. The last row of Fig. 1 shows the *effect of replacing* $\bar{s}^2_W(\mathrm{LEP})$ *by* $\bar{s}^2_W(\mathrm{SLD})$ and by the combined value of $\bar{s}^2_W(\mathrm{LEP+SLD})$. The change in the allowed (m_t, M_H) plane occurring as a consequence of these replacements is very strong. For the leptonic sector and all data $\setminus$ R_b, R_c the fit value of $m_t \simeq 170\,\mathrm{GeV}$ is consistent with the value from the direct measurements, $m_t = 180 \pm 12\,\mathrm{GeV}$, while the values for M_H resulting from using $\bar{s}^2_W(\mathrm{SLD})$ now have decreased to $M_H = 17^{+25}_{-9}\,\mathrm{GeV}$ and $M_H = 13^{+20}_{-7}\,\mathrm{GeV}$, respectively. The fit to all data $\setminus$ R_c yields $m_t = 161\,\mathrm{GeV}$ and a similarly low value for M_H, namely $M_H = 16^{+19}_{-8}\,\mathrm{GeV}$. Comparing these results to the lower bound from the direct Higgs-boson search, $M_H > 65.2\,\mathrm{GeV}$ [12], one arrives at a serious conflict between the unmodified SM and experiment. The discrepancy is weakened if the combined value of $\bar{s}^2_W(\mathrm{LEP+SLD})$ from Tab. 2 is used. In this case one obtains $m_t = 152^{+11}_{-9}\,\mathrm{GeV}$ and $M_H = 31^{+40}_{-15}\,\mathrm{GeV}$ for all data $\setminus$ R_c. We note in passing that our results of the fit to all data ($m_t = 153^{+11}_{-9}\,\mathrm{GeV}$, $M_H = 32^{+44}_{-16}\,\mathrm{GeV}$) are in good agreement with the results of $m_t = 156^{+14}_{-15}$ and $M_H = 36^{+56}_{-22}\,\mathrm{GeV}$ given in Ref. [6], where the data from the 1995 Winter Conferences [13] were analyzed.[4] A resolution of the LEP–SLD discrepancy on $\bar{s}^2_W$ is obviously one of the most important tasks with respect to the issue of M_H bounds via radiative corrections.

In order to accommodate the experimental result for R_b, we now allow for a *modification of the* $Z \to b\bar{b}$ *vertex* by a parameter Δy_b, as introduced in Ref. [9].[5] The possible origin

[4] In Ref. [6] also the available low-energy data were included in the analysis, which shows that the effect of these data on the results of the SM fits is rather small. The somewhat higher values of $M_H = 76^{+152}_{-50}\,\mathrm{GeV}$ and $M_H = 79^{+160}_{-54}\,\mathrm{GeV}$ quoted for the Higgs-boson mass in Ref. [6] and Ref. [7], respectively, are mainly due to the fact that in those fits the experimental value of the top-quark mass was used as an additional input for the fits, leading to higher values of m_t and consequently also of M_H.

[5] The parameter Δy_b is related to the parameter ε_b introduced in Ref. [14] via $\Delta y_b = -2\varepsilon_b - 0.2 \times 10^{-3}$ (see Ref. [9]).

$\alpha_s(M_Z^2)$	m_t / GeV	M_H / GeV	$\Delta y_b/10^{-3}$	χ^2_{min}/d.o.f.
0.123 fixed	173^{+28+6}_{-21-4}	$386^{+\gtrsim 1000+267}_{-270-130}$	$3.9^{+4.6-1.1}_{-4.6+1.2}$	12/6
0.110 fixed	173^{+32+11}_{-23-7}	$349^{+\gtrsim 1000+437}_{-261-155}$	$-8.7^{+4.6-1.1}_{-4.6+1.3}$	3.4/6
0.100 fixed	172^{+36+18}_{-24-10}	$281^{+\gtrsim 1000+749}_{-218-157}$	$-18.5^{+4.7-1.2}_{-4.6+1.6}$	0.9/6
0.097 ± 0.008 fitted	170^{+38+20}_{-24-12}	$254^{+\gtrsim 1000+833}_{-200-162}$	$-21.0^{+8.9-0.1}_{-9.0-0.5}$	0.8/6

Table 4: The results of the three-parameter $(m_t, M_H, \Delta y_b)$ fits to all data $\setminus$ R_c keeping $\alpha_s(M_Z^2)$ fixed as indicated (first three rows) and the four-parameter $(m_t, M_H, \Delta y_b, \alpha_s)$ fit (last row). The errors quoted first correspond to the 1σ limits, while the second entries indicate the shift of the central fit values when $\alpha(M_Z^2)^{-1}$ is set to $\alpha(M_Z^2)^{-1} = 128.98$ (upper) and $\alpha(M_Z^2)^{-1} = 128.80$ (lower).

of this modification of the SM predictions is left open for the time being, but in particular it includes the impact of new particles in conjunction with loop corrections at the $Z \to b\bar{b}$ vertex. We allow for values of $\alpha_s(M_Z^2)$ different [15] from the LEP value of $\alpha_s(M_Z^2) = 0.123$, in order to compensate for the enhanced value of the total hadronic Z-boson width, Γ_h, resulting from the enlarged theoretical value of Γ_b which is adjusted to be in agreement with experiment. Deviations of Δy_b from its (m_t-dependent) SM value Δy_b^{SM} [9] lead to an extra contribution X_b [2, 13] in the prediction for Γ_b,

$$\begin{aligned} X_b = \Gamma_b - \Gamma_b^{SM} &= \frac{\alpha(M_Z^2) M_Z}{24 s_0^2 c_0^2} (2 s_0^2 - 3)\, R_{QED}\, R_{QCD} \left(\Delta y_b - \Delta y_b^{SM}\right) \\ &= -0.421\,\mathrm{GeV} \times R_{QCD} \left(\Delta y_b - \Delta y_b^{SM}\right), \end{aligned} \tag{2}$$

where $s_0^2 c_0^2 = s_0^2(1 - s_0^2) = \pi\alpha(M_Z^2)/\sqrt{2} G_\mu M_Z^2$, $R_{QED} = 1 + \alpha/12\pi$, and $R_{QCD} = 1 + \alpha_s(M_Z^2)/\pi + 1.41(\alpha_s(M_Z^2)/\pi)^2 - 12.8(\alpha_s(M_Z^2)/\pi)^3$ according to Ref. [16].

In Tab. 4 we present our results for a three-parameter $(m_t, M_H, \Delta y_b)$ fit with fixed values of $\alpha_s(M_Z^2)$ taking the data set "all data $\setminus$ R_c" (where the LEP value of $\bar{s}_W^2$ is used), as well as the results of a four-parameter $(m_t, M_H, \Delta y_b, \alpha_s)$ fit. The conclusion from Tab. 4 is simple: once one allows for a modification of R_b by the parameter Δy_b, the bounds on M_H obtained by fitting within the unmodified SM are lost. The quality of the fit is improved considerably, if one allows for a value of $\alpha_s(M_Z^2)$ substantially below the LEP result from the event shape measurement [2] of $\alpha_s(M_Z^2) = 0.123 \pm 0.006$. The values of m_t in Tab. 4 roughly coincide with the ones obtained in the SM fits to the leptonic sector given in Tab. 2. For low $\alpha_s(M_Z^2)$ also the M_H bounds are similar to the results of the SM fit obtained for the leptonic sector (Tab. 2). Fitting also $\alpha_s(M_Z^2)$ leads to the extremely low best-fit value of $\alpha_s(M_Z^2) = 0.097 \pm 0.008$, which is even lower than the extrapolated value of $\alpha_s(M_Z^2)$ from low-energy deep inelastic scattering data, $\alpha_s(M_Z^2) = 0.112 \pm 0.004$ [16], and the value obtained from lattice QCD calculations, $\alpha_s(M_Z^2) = 0.115 \pm 0.003$ [16].

$\alpha_s(M_Z^2)$	0.123	0.099	0.123	0.099
$\chi^2_{\rm min}$/d.o.f.	0		9.3/6	0.5/6
$\Delta x^{\rm exp}/10^{-3}$	10.1 ± 4.2		9.9 ± 4.2	10.1 ± 4.2
$\Delta y^{\rm exp}/10^{-3}$	5.4 ± 4.3		7.0 ± 4.3	5.7 ± 4.3
$\varepsilon^{\rm exp}/10^{-3}$	-5.3 ± 1.6		-4.2 ± 1.5	-5.0 ± 1.5
$\Delta y_{\rm b}^{\rm exp}/10^{-3}$	-14.6 ± 6.9	-20.9 ± 7.0	0.7 ± 4.7	-20.5 ± 4.7
$\Delta y_{\rm h}^{\rm exp}/10^{-3}$	4.9 ± 2.3	-1.7 ± 2.3	-1.4 (from theory)	
$\Delta y_{\nu}^{\rm exp}/10^{-3}$	0.6 ± 5.2		-3.0 (from theory)	

Table 5: Experimental results for the effective parameters for $\alpha_s(M_Z^2) = 0.123$ and the low value $\alpha_s(M_Z^2) = 0.099$. The entries on the left-hand side are obtained by determining the six effective parameters from the six observables $\Gamma_{\rm l}$, $\bar{s}^2_{\rm W}({\rm LEP})$, $M_{\rm W}$, $\Gamma_{\rm T}$, $\Gamma_{\rm h}$, and $\Gamma_{\rm b}$. On the right-hand side $\Delta y_{\rm h}$ and Δy_ν have been taken from theory, and the remaining parameters have been fitted twice, namely for fixed $\alpha_s(M_Z^2) = 0.123$ and with $\alpha_s(M_Z^2)$ as additional fit parameter, resulting in $\alpha_s(M_Z^2) = 0.099$.

As in the previous case of the pure SM fits, the results do not change qualitatively when $R_{\rm c}$ is included in the data set. In the ($m_{\rm t}$,$M_{\rm H}$,$\Delta y_{\rm b}$,α_s) fit we obtain

$$\begin{aligned} m_{\rm t} &= 169^{+40+21}_{-23-11}, \qquad M_{\rm H} = 219^{+\gtrsim 1000+771}_{-171-138}, \qquad \alpha_s(M_Z^2) = 0.102 \pm 0.008^{+0.001}_{-0.003}, \\ \Delta y_{\rm b} &= (-15.4 \pm 8.6^{+0.0}_{-0.6}) \times 10^{-3}, \qquad (\chi^2_{\rm min}/{\rm d.o.f.} = 5.7/7). \end{aligned} \tag{3}$$

This is in good agreement with the results presented in Ref. [2] for a fit of $(m_{\rm t}, \alpha_s, X_{\rm b})$ for $M_{\rm H}$ fixed at $M_{\rm H} = 300\,{\rm GeV}$. The increased value of $\chi^2_{\rm min}$/d.o.f. in (3) relative to the corresponding value in Tab. 4 is of course a consequence of the 2.5σ discrepancy [2] in $R_{\rm c}$. However, it does not seem to be meaningful to introduce an additional non-standard parameter $\Delta y_{\rm c}$ in order to accommodate the $R_{\rm c}$ discrepancy. On the one hand, a modification of the $Z \to c\bar{c}$ vertex is much less motivated than in the case of the $Z \to b\bar{b}$ vertex (see e.g. the discussion in Ref. [9]); on the other hand, a fit in which a non-standard $\Delta y_{\rm c}$ is allowed yields the absurd value $\alpha_s(M_Z^2) = 0.19 \pm 0.04$, which was also obtained e.g. in Refs. [2, 7, 16].

As a final point, we compare the $(m_{\rm t}, M_{\rm H}, \Delta y_{\rm b}, \alpha_s)$-fit discussed above with a *model-independent analysis* based on phenomenological effective parameters. The six observables $\Gamma_{\rm l}$, $\bar{s}^2_{\rm W}({\rm LEP})$, $M_{\rm W}$ and $\Gamma_{\rm T}$, $\Gamma_{\rm h}$, $\Gamma_{\rm b}$ can be represented as linear combinations of six phenomenological parameters Δx, Δy, ε and $\Delta y_{\rm h} = (\Delta y_{\rm u} + \Delta y_{\rm d})/2 + (s_0^2/6c_0^2)(\Delta y_{\rm d} - \Delta y_{\rm u})$, $\Delta y_{\rm b}$, Δy_ν that describe possible sources of SU(2) violation within an effective Lagrangian for electroweak interactions at the Z-boson resonance [9]. We assume that the QCD corrections, such as $R_{\rm QCD}$, which enter $\Gamma_{\rm T}$, $\Gamma_{\rm h}$, and $\Gamma_{\rm b}$, have standard form. These corrections are extracted from the experimental data before the determination of the effective parameters (see Ref. [9]), which therefore quantify all electroweak corrections in a model-independent way.

The results of extracting the experimental values of the six parameters $\Delta x^{\rm exp}$ etc. from the six observables by inverting the system of linear equations is shown on the left-hand side of Tab. 5. The $\alpha_{\rm s}$-dependence only affects $\Delta y_{\rm b}^{\rm exp}$ and $\Delta y_{\rm h}^{\rm exp}$, since the leptonic sector by itself determines $\Delta x^{\rm exp}$, $\Delta y^{\rm exp}$ and $\varepsilon^{\rm exp}$. The values of $\Delta x^{\rm exp}$, $\Delta y^{\rm exp}$ and $\varepsilon^{\rm exp}$ are in excellent agreement with their SM predictions, as discussed in detail for the data of Refs. [2, 3] in Ref. [17]. For $\alpha_{\rm s}(M_{\rm Z}^2) = 0.123$, in addition to the non-standard value of $\Delta y_{\rm b}^{\rm exp}$, also the parameter $\Delta y_{\rm h}^{\rm exp}$ disagrees with the theoretical prediction [9] of $\Delta y_{\rm h}^{\rm SM} = -3.0 \times 10^{-3}$. Agreement between SM and experiment in $\Delta y_{\rm h}$ is achieved, however, for low values of $\alpha_{\rm s}(M_{\rm Z}^2)$, such as $\alpha_{\rm s}(M_{\rm Z}^2) = 0.099$.

Noting that the process-specific parameters $\Delta y_{\rm h}$ and Δy_ν only depend [9] on the empirically well-established[6] couplings between vector-bosons and light fermions (i.e. all fermions except for top and bottom quarks), we now impose the SM values for $\Delta y_{\rm h}$ and Δy_ν and determine the remaining parameters in a fit. According to the right-hand side of Tab. 5, for fixed $\alpha_{\rm s}(M_{\rm Z}^2) = 0.123$, we find a rather poor quality of the fit ($\chi^2_{\rm min}/{\rm d.o.f.} = 9.3/6$). Allowing for $\alpha_{\rm s}(M_{\rm Z}^2)$ as additional fit parameter, we obtain an excellent quality of the fit ($\chi^2_{\rm min}/{\rm d.o.f.} = 0.5/6$), and for $\alpha_{\rm s}(M_{\rm Z}^2)$ the low value of $\alpha_{\rm s}(M_{\rm Z}^2) = 0.099$ deliberately chosen before.

It has thus been shown that the low value for $\alpha_{\rm s}(M_{\rm Z}^2)$, as a consequence of the experimental value of $R_{\rm b}$, emerges independently of much of the details of electroweak radiative corrections. The two very weak assumptions made here, namely standard form of the QCD corrections and of Δy_ν and $\Delta y_{\rm h}$, already imply the very low value of $\alpha_{\rm s}(M_{\rm Z}^2) = 0.099 \pm 0.008$ in the fit which includes $R_{\rm b}$. This value is very close to the one obtained above in the $(m_{\rm t}, M_{\rm H}, \Delta y_{\rm b}, \alpha_{\rm s})$ fit, where non-standard contributions have only been allowed in the $\rm Z \to b\bar{b}$ vertex. Moreover, the values of $\Delta y_{\rm b} = 0.7 \pm 4.7$ for $\alpha_{\rm s}(M_{\rm Z}^2) = 0.123$ and of $\Delta y_{\rm b} = -20.5 \pm 4.7$ for $\alpha_{\rm s}(M_{\rm Z}^2) = 0.099$ obtained in the present analysis are in good agreement with the values given in Tab. 4 for the $(m_{\rm t}, M_{\rm H}, \Delta y_{\rm b})$ fit. In both treatments a decent value of $\chi^2_{\rm min}/{\rm d.o.f.}$ therefore requires a value of $\alpha_{\rm s}(M_{\rm Z}^2)$ that, in the best-fit case, is four standard deviations below $\alpha_{\rm s}(M_{\rm Z}^2) = 0.123 \pm 0.006$.

In summary, from our analysis of the precision data at the Z-boson resonance and $M_{\rm W}$, we find that a Higgs-boson mass lying in the perturbative regime of the Standard Model, i.e. below 1 TeV, is indeed favored at the 1σ level. Having investigated in much detail the impact of the data for the $\rm Z \to b\bar{b}, c\bar{c}$ decay modes and the experimental value of $\bar{s}_{\rm W}^2({\rm SLD})$, as well as the influence of the uncertainties connected with the input parameters $\alpha(M_{\rm Z}^2)^{-1}$ and $\alpha_{\rm s}(M_{\rm Z}^2)$, we conclude that a stronger upper bound on $M_{\rm H}$ can hardly be justified from the data at present. The stringent bounds on $M_{\rm H}$ that are obtained when the unmodified Standard Model is fitted to the complete data sample are immediately lost when $R_{\rm b}$ and $\bar{s}_{\rm W}^2({\rm SLD})$ are excluded from the analysis or, as demonstrated for the case of $R_{\rm b}$, if non-standard contributions are allowed in the theoretical model. The well-known fact that allowing for a non-standard contribution to $R_{\rm b}$ in the fits gives rise to an extremely low value of $\alpha_{\rm s}(M_{\rm Z}^2)$ has been shown to emerge already under the weak theoretical assumptions

[6]Here we ignore the $R_{\rm c}$ problem, previously commented upon.

of standard QCD corrections and standard form of the couplings of the gauge-bosons to the leptons and to the quarks of the first two generations.

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Figure 1: The results of the two-parameter $(m_{\mathrm{t}}, M_{\mathrm{H}})$ fits within the SM are displayed in the $(m_{\mathrm{t}}, M_{\mathrm{H}})$ plane. The three different columns refer to the three different sets of experimental data used in the corresponding fits,
(i) "leptonic sector": $\Gamma_{\mathrm{l}}, \bar{s}^2_{\mathrm{W}}(\mathrm{LEP}), M_{\mathrm{W}}$,
(ii) "all data $\backslash R_{\mathrm{c}}, R_{\mathrm{b}}$": $\Gamma_{\mathrm{T}}, \Gamma_{\mathrm{h}}$ are added to set (i),
(iii) "all data $\backslash R_{\mathrm{c}}$": $\Gamma_{\mathrm{T}}, \Gamma_{\mathrm{h}}, \Gamma_{\mathrm{b}}$ are added to the set (i).
The second and the third rows show the shift resulting from changing $\alpha(M_{\mathrm{Z}}^2)^{-1}$ and $\alpha_{\mathrm{s}}(M_{\mathrm{Z}}^2)$, respectively, by one standard deviation in the SM prediction. The fourth row shows the effect of replacing $\bar{s}^2_{\mathrm{W}}(\mathrm{LEP})$ by $\bar{s}^2_{\mathrm{W}}(\mathrm{SLD})$ and $\bar{s}^2_{\mathrm{W}}(\mathrm{LEP}+\mathrm{SLD})$ in the fits. Note that the 1σ boundaries given in the first row are repeated identically in each row, in order to facilitate comparison with other boundaries. In all plots the empirical value of the top-quark mass of $m_{\mathrm{t}} = 180 \pm 12\,\mathrm{GeV}$ is also indicated.

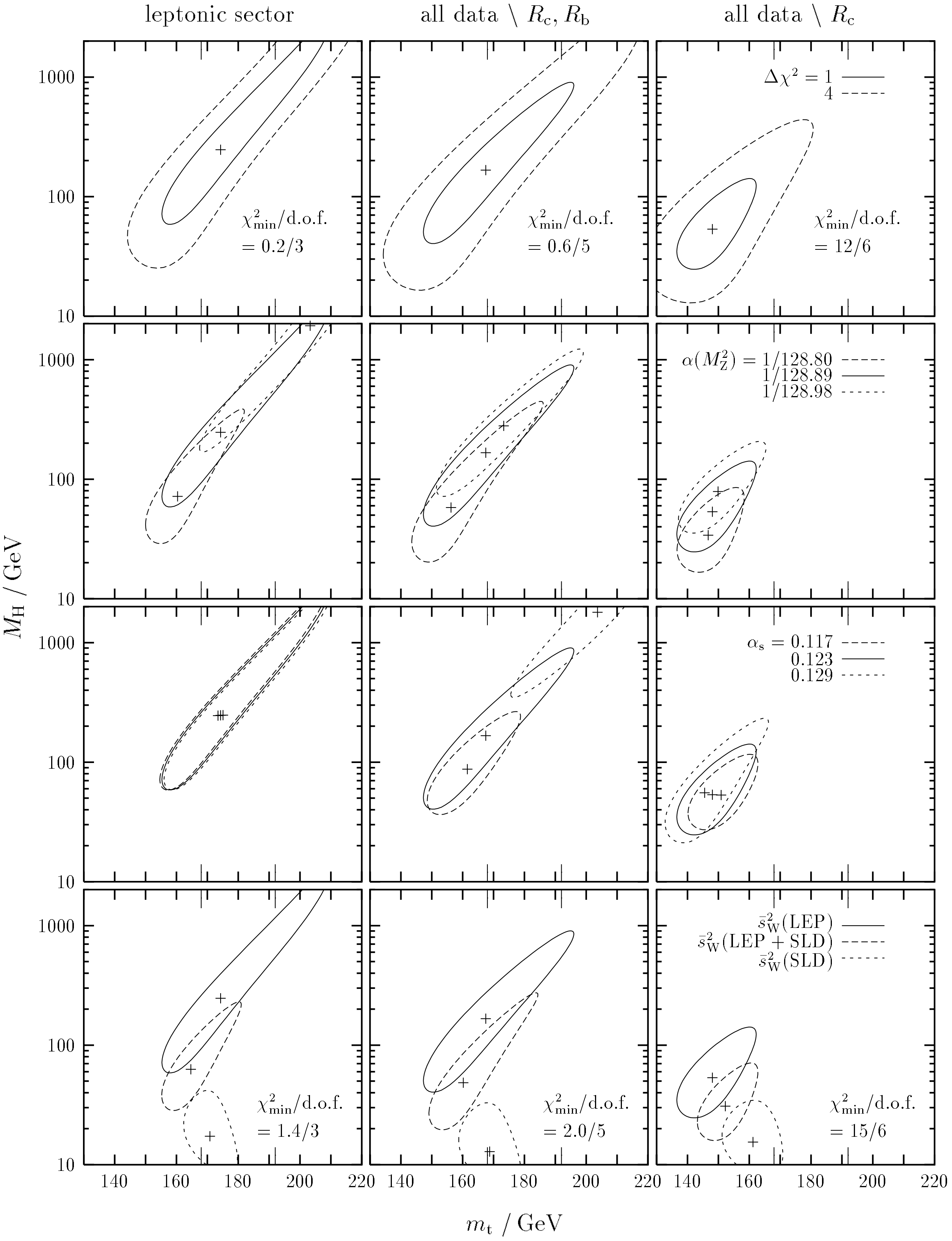
leptonic sector
all data \ R_c, R_b
all data \ R_c
$\Delta\chi^2 = 1$
4
χ^2_{min}/d.o.f. = 0.2/3
χ^2_{min}/d.o.f. = 0.6/5
χ^2_{min}/d.o.f. = 12/6
$\alpha(M_Z^2) = 1/128.80$
1/128.89
1/128.98
$\alpha_s = 0.117$
0.123
0.129
$\bar{s}^2_W$(LEP)
$\bar{s}^2_W$(LEP + SLD)
$\bar{s}^2_W$(SLD)
χ^2_{min}/d.o.f. = 1.4/3
χ^2_{min}/d.o.f. = 2.0/5
χ^2_{min}/d.o.f. = 15/6
M_H / GeV
m_t / GeV
10
100
1000
140
160
180
200
220

Figure 1: