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One–loop weak dipole moments of heavy fermions in the MSSM*

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The MSSM predictions at the one-loop level for the weak dipole moments of the τ lepton and the b quark are presented. The imaginary part of the AWMDM is of the order of the SM contribution whereas the real part may be a factor 5 (20) larger for the τ (b) in the high $\tan\beta$ scenario, still a factor five below the QCD contribution in the b case. More interestingly, a contribution up to twelve orders of magnitude larger than in the SM may be obtained, already at the one-loop level, for the WEDM in a MSSM with complex parameters.

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1. Introduction

The investigation of the electric and magnetic dipole moments provides very accurate tests of the quantum structure of the Standard Model (SM) and its possible extensions. As a generalization of the electromagnetic dipole moments of fermions (AMDM and EDM), one can define weak dipole moments (WDMs), corresponding to couplings with a Z boson instead of a photon. The most general Lorentz structure of the vertex function that couples a Z boson and two on-shell fermions (with outgoing momenta q and $\bar{q}$) can be written in term of form factors $F_i(s \equiv (q+\bar{q})^2)$ as

$$\begin{aligned}\Gamma_\mu^{Zff} = ie\Bigg\{&\gamma_\mu\left[\left(F_{\mathrm{V}} - \frac{v_f}{2s_W c_W}\right) - \left(F_{\mathrm{A}} - \frac{a_f}{2s_W c_W}\right)\gamma_5\right] \\ &+(q-\bar{q})_\mu[F_{\mathrm{M}} + F_{\mathrm{E}}\gamma_5] - (q+\bar{q})_\mu[F_{\mathrm{S}} + F_{\mathrm{P}}\gamma_5]\Bigg\}\,, \qquad (1)\end{aligned}$$

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where $v_f \equiv (I_3^f - 2s_W^2 Q_f)$, $a_f \equiv I_3^f$. The form factors $F_{\rm M}$ and $F_{\rm E}$ are related to the anomalous weak magnetic and electric dipole moments of the fermion f with mass m_f as follows:

$$\begin{aligned} \mathrm{AWMDM} \equiv a_f^W &= -2m_f \; F_{\rm M}(M_Z^2) \;, \\ \mathrm{WEDM} \equiv d_f^W &= ie \; F_{\rm E}(M_Z^2) \;, \end{aligned}$$

The $F_{\rm M}$ ($F_{\rm E}$) form factors are the coefficients of the *chirality-flipping* term of the CP-conserving (CP-violating) effective Lagrangian describing Z-fermion couplings.

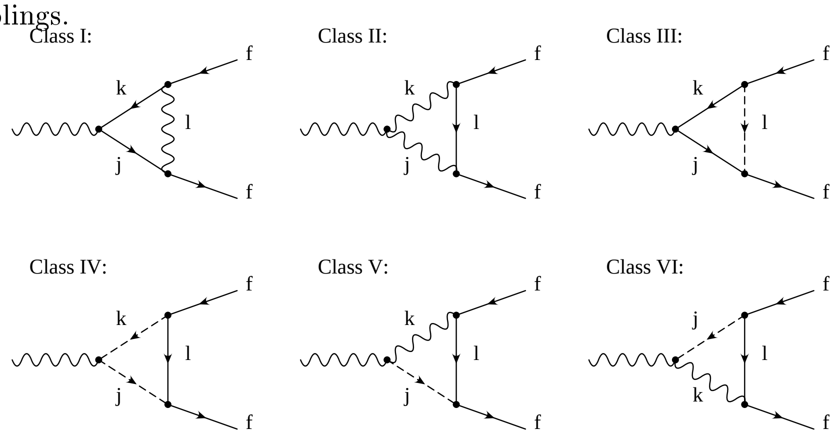

Fig. 1. The one-loop topologies for the Zff vertex.

In a renormalizable theory there is no contribution to the A(W)MDM or the (W)EDM at tree level, as they correspond to operators of dimension 5. At one loop, one can classify all the possible diagrams in six topologies or classes of triangle graphs [1] (Fig. 1). Analytical expressions for these moments have been obtained [2, 3] in the 't Hooft-Feynman gauge to one loop. The global result, adding all the diagrams, is gauge independent, as it is defined at $s = M_Z^2$. The contribution of every class of diagrams is expressed in terms of standard three-point one-loop integrals and generic couplings, which allow for the implementation of different theories. Every term in the expressions is proportional to some fermion mass, consistently with the chirality flipping character of the dipoles. This means that heavier fermions are expected to have larger WDMs. For an on-shell Z boson the b quark and τ lepton are the most promising candidates. Unlike the electromagnetic dipole moments, defined at $s = 0$, the WDMs can be complex due to the possibility of pair-produce particles below the M_Z threshold. Here the predictions for the MSSM are presented and compared to the SM ones.

2. Anomalous Weak Magnetic Dipole Moments

The contribution to the AWMDM, for the τ lepton and for heavy quarks, at the one-loop level has been calculated by Bernabéu *et al.* in the SM [4]. There the only free parameter is the SM Higgs boson mass M_{H^0} whose value does not significantly affect the result[1] for $a_\tau^W = (2.10 + 0.61\ i) \times 10^{-6}$ but it is more important for the real part of $a_b^W = [(1.1; 2.0; 2.4) - 0.2\ i] \times 10^{-6}$, with $M_{H^0} = M_Z$, $2M_Z$, $3M_Z$ respectively. Including the QCD contribution (a gluon exchange in class I diagrams) in the case of the b quark dramatically enlarges the result to $a_b^W = (-2.96 + 1.56\ i) \times 10^{-4}$.

In the MSSM the Higgs sector is different. It consists of a constrained 2HDM mainly controlled by the pseudoscalar Higgs boson mass M_A, the μ parameter and the ratio of VEVs $\tan\beta$. Besides, several soft-susy-breaking terms must be introduced. A simplified set is given by the assumption of universal scalar mass terms $m_{\tilde{q}}$ for squarks, and $m_{\tilde{l}}$ for sleptons, trilinear terms A_τ, A_b and A_t (for the third family) and gaugino mass terms related by the GUT constraint: $\alpha M_3 = \alpha_s s_W^2 M_2 = 3/5\ \alpha_s c_W^2 M_1$. In Ref. [2] a complete scan of the susy parameter space is performed. The results for the AWMDM are summarized below.

The Higgs sector can provide the only contribution to the imaginary part, of the order of the SM contribution, assuming the present experimental limits on the masses of the superpartners and MSSM Higgs bosons (except for still possible light neutralinos). The real part is typically negative and not very large: $-\mathrm{Re} \lesssim 0.3 \times 10^{-6}$ for the τ and $-\mathrm{Re} \lesssim 10^{-6}$ for the b.

The neutralino contribution to the real part is also small, and has opposite sign than μ in most of the parameter space: $|\mathrm{Re}(a_\tau^W)| \lesssim 0.02(0.4) \times 10^{-6}$ and $|\mathrm{Re}(a_b^W)| \lesssim 0.2(12) \times 10^{-6}$ for $\tan\beta = 1.6(50)$ respectively.

The chargino contribution is the dominant one being real and with the same sign as μ: $|\mathrm{Re}(a_\tau^W)| \lesssim 0.2(7) \times 10^{-6}$ and $|\mathrm{Re}(a_b^W)| \lesssim 1(30) \times 10^{-6}$.

The gluinos compete in importance with the charginos for the b and their contribution is always negative: $-\mathrm{Re} \lesssim 2(40) \times 10^{-6}$.

The sum of the MSSM contributions can amount to $|\mathrm{Re}(a_\tau^W)| \lesssim 0.5(7) \times 10^{-6}$ and $|\mathrm{Re}(a_b^W)| \lesssim 2(50) \times 10^{-6}$ for not so extreme and not excluded regions of the susy parameter space. Decoupling is observed for large values of the parameters.

[1] An opposite global sign is quoted here for comparison, due to our different conventions.

3. Weak Electric Dipole Moments

In the SM there is only one source of CP violation, the $\delta_{\rm CKM}$ phase of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix for quarks.[2] The only place where CP violation has currently been measured, the neutral K system, fixes the value of this phase but does not constitute itself a test for the origin of CP violation. Many extensions of the SM contain new CP violating phases, in particular, the supersymmetric models. The most significant effect of the CP violating phases in the phenomenology is their contribution to electric dipole moments. Their investigation may shed some light on the intriguing problem of CP violation.

The SM contribution to the (W)EDM comes at three loops and hence it is as small as $\sim eG_F m_f \alpha^2 \alpha_s J/(4\pi)^5$, with $J \equiv c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta$ (an invariant under reparametrizations of the CKM matrix). That is the (W)EDM$\sim$ 3×10^{-34} (10^{-33}) $e$cm for the τ (b).

In the MSSM new physical phases, provided by the soft-breaking terms, are brought into the game and their effects manifest themselves already at the one-loop level. For simplicity, we restrict to generation-diagonal trilinear soft-breaking terms to prevent from FCNC. In our analysis we do not make any additional assumption, except for the unification of the soft-breaking gaugino masses at the GUT scale. However we do not assume unification of the scalar mass parameters or trilinear mass parameters. In such a constrained framework the following susy parameters can be complex: the μ parameter, the gaugino masses, the bilinear mixing mass parameter m_{12}^2 and the trilinear soft-susy-breaking parameters. Not all of these phases are physical. Namely, the MSSM has two additional U(1) symmetries for vanishing μ and soft-breaking terms: the Peccei-Quinn and the R-symmetry. For non-vanishing μ and soft-breaking terms these symmetries can be used to absorb two of the phases by redefinition of the fields [5]. In addition, the GUT constraint leads to only one common phase for the gaugino mass terms. After these considerations, our choice of CP violating physical phases is:[3] $\varphi_\mu \equiv \arg(\mu)$, $\varphi_{\tilde f} \equiv \arg(m_{\rm LR}^f)$ $(f=\tau,\ t,\ b)$ with $m_{\rm LR}^t \equiv A_t - \mu^* \cot\beta$ and $m_{\rm LR}^{\tau,b} \equiv A_{\tau,b} - \mu^* \tan\beta$. The scan of the the parameter space leads to numerical results of the same order as the AWMDM expressed in *magnetons* μ_f for the values of the intervening CP violating phases that maximize the effect ($1\ \mu_f \equiv e/2m_f = 1.7(0.7)\times 10^{-15}$ $e$cm for τ (b), respectively). The expressions contain only imaginary parts of combinations of couplings [3].

[2] The QCD lagrangian includes an additional source of CP violation, the $\theta_{\rm QCD}$, but it will not be considered here.

[3] The (common) phase of the complex gaugino mass terms as well as the phase of m_{12}^2 are absorbed. With this choice, the susy preserving μ parameter remains complex.

The MSSM Higgs sector is CP conserving and hence it does not contribute to the (W)EDM.

The diagrams with neutralinos involve φ_μ and $\varphi_{\tilde{\tau}}$ ($\varphi_{\tilde{b}}$) for the τ (b) case and its contribution is maximal for these phases being $\pi/2$.

The chargino diagrams only involve φ_μ in the τ case (there is no scalar neutrino mixing) and also $\varphi_{\tilde{t}}$ for the b. The former contribution is enhanced for $\varphi_\mu = \pi/2$ and the two phases conspire in the latter to yield a maximum effect for $\varphi_\mu = \pi/2$ and $\varphi_{\tilde{t}} = \pi$.

The gluinos contribute maximally to the b WEDM for $\varphi_{\tilde{b}} = \pi/2$.

The maximal global pure supersymmetric contribution amounts to $|\mathrm{Re}(d_\tau^W)| \lesssim 0.3(12) \times 10^{-21}$ $e$cm, $|\mathrm{Re}(d_b^W)| \lesssim 1.4(35) \times 10^{-21}$ $e$cm.

4. Conclusions

The MSSM predictions for the WDMs of the τ lepton and the b quark are the following:

• The real part of the AWMDM can reach values 5 (20) times larger than the electroweak SM predictions for the τ lepton (b quark) in the high $\tan\beta$ scenario, still a factor five below the QCD contribution in the b case.

• In a generalized MSSM with complex parameters and generation-diagonal trilinear soft-susy-breaking terms a contribution to the WEDM is possible already at the one-loop level. It is governed by two physical phases (φ_μ, $\varphi_{\tilde{\tau}}$) in the τ case and three physical phases (φ_μ, $\varphi_{\tilde{b}}$ and $\varphi_{\tilde{t}}$) in the b case. The WEDM can be as much as twelve orders of magnitude larger than the SM prediction.

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