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Higher-order corrections to precision observables in the Standard Model and the MSSM[1]

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Abstract

A summary of recent results obtained for higher-order corrections to precision observables in the Standard Model and the Minimal Supersymmetric Standard Model is given. In the Standard Model electroweak two-loop results valid for arbitrary values of the masses of the top quark, the Higgs boson and the gauge bosons are discussed. For the example of two specific diagrams the exact two-loop result is compared with the result of an expansion in the top-quark mass up to next-to-leading order. Furthermore the Higgs-mass dependence of the two-loop corrections to the relation between the gauge-boson masses is analyzed. In the MSSM the exact gluonic corrections to Δr are derived. They are compared with the leading contribution entering via the ρ parameter.

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1. Introduction

The experimental accuracy meanwhile reached for the electroweak precision observables allows to test the theory, namely the electroweak Standard Model (SM) and its extensions, most prominently the Minimal Supersymmetric Standard Model (MSSM), at the quantum level, where all parameters of the model enter the theoretical predictions.

In this way one is able to set constraints in the SM on the mass of the Higgs boson, which is the last missing ingredient of the minimal SM. From the most recent global SM fits to all available data one obtains an upper bound for the Higgs-boson mass of 420 GeV at 95% C.L. [1]. This bound is considerably affected by the error in the theoretical predictions due to missing higher-order corrections, which gives rise to an uncertainty of the upper bound of about 100 GeV. The main uncertainty in this context comes from the electroweak two-loop corrections, for which the results obtained so far have been restricted to expansions for asymptotically large values of the top-quark mass, $m_{\rm t}$, or the Higgs-boson mass, $M_{\rm H}$ [2, 3].

In order to improve this situation, an exact evaluation of electroweak two-loop contributions would be desirable, where no expansion in $m_{\rm t}$ or $M_{\rm H}$ is made. In this paper this is illustrated for the example of two specific diagrams, for which the exact result is compared with the result of an expansion in $m_{\rm t}$ up to next-to-leading order. Then exact results [4] recently obtained for the Higgs-mass dependence of Δr, i.e. the relation between the gauge-boson masses, are briefly reviewed.

The MSSM provides the most predictive framework beyond the SM. While the direct search for supersymmetric particles has not been successful yet, the precision tests of the theory provide the possibility for constraining the parameter space of the model and could eventually allow to distinguish between the SM and Supersymmetry via their respective virtual effects. In contrast to the SM case the predictions for Δr and the Z-boson observables in the MSSM are known at one-loop order only [5]. In order to treat the MSSM at the same level of accuracy as the SM, higher-order contributions should be incorporated. Recently the QCD corrections to the ρ parameter in the MSSM have been evaluated [6, 7]. In this paper the exact result for the gluonic contribution to Δr is presented. It is compared with the approximation based on the contribution to the ρ parameter.

2. Comparison of exact calculation and expansion in the top-quark mass

The calculation of top-quark or Higgs-boson contributions to Δr and other processes with light external fermions at low energies requires in par-

ticular the evaluation of two-loop self-energies on-shell, i.e. at non-zero external momentum, while vertex and box contributions can mostly be reduced to vacuum integrals. The problems encountered in such a calculation are due to the large number of contributing Feynman diagrams, their complicated tensor structure, the fact that scalar two-loop integrals are in general not expressible in terms of polylogarithmic functions [8], and due to the need for a two-loop renormalization, which has not yet been worked out in full detail.

The methods used for the calculations discussed in this paper have been outlined in Ref. [9]. The generation of the diagrams and counterterm contributions is done with the help of the computer-algebra program *FeynArts* [10]. As an example, in Fig. 1 those diagrams contributing to the two-loop W-boson self-energy are given that contain both the top quark and the Higgs boson. The corresponding counterterm graphs are also listed.

Making use of two-loop tensor-integral decompositions, the generated amplitudes are reduced to a minimal set of standard scalar integrals with the program *TwoCalc* [11]. The renormalization is performed within the complete on-shell scheme [12], i.e. physical parameters are used throughout. The two-loop scalar integrals are evaluated numerically with one-dimensional integral representations [13]. These allow a very fast calculation of the integrals with high precision without any approximation in the masses.

As an example, we consider the two diagrams marked as (a) and (b) in Fig. 1 and compare the exact result with the result of an expansion in the top-quark mass up to next to leading order, which takes into account terms of order $m_{\rm t}^4$ and $m_{\rm t}^2$ (see Ref. [3]). These diagrams represent the two main topologies for the self-energy of the W boson. The comparison of the two methods of evaluation for just two diagrams can of course only provide an estimate of the relative difference that one can expect in the results for physical observables. It has nevertheless the advantage that the direct comparison of unrenormalized two-loop diagrams is not affected by differences in the renormalization schemes and in the treatment of subleading higher-order corrections, which usually are present when predictions for physical observables are compared to each other.

The method for the asymptotic expansion in $m_{\rm t}$ has been described in Ref. [3].[2] It is performed in two regions, namely in the light Higgs region ("light Higgs expansion"), for which $M_{\rm W}, M_{\rm Z}, M_{\rm H} \ll m_{\rm t}$ is assumed, and in the heavy Higgs region ("heavy Higgs expansion"), which means $M_{\rm W}, M_{\rm Z} \ll M_{\rm H}, m_{\rm t}$.

We have checked that for asymptotically large values of the top-quark mass the difference between the exact result and the relevant expansion

[2] P. Gambino has kindly provided us with his results for the expansions of the two diagrams.

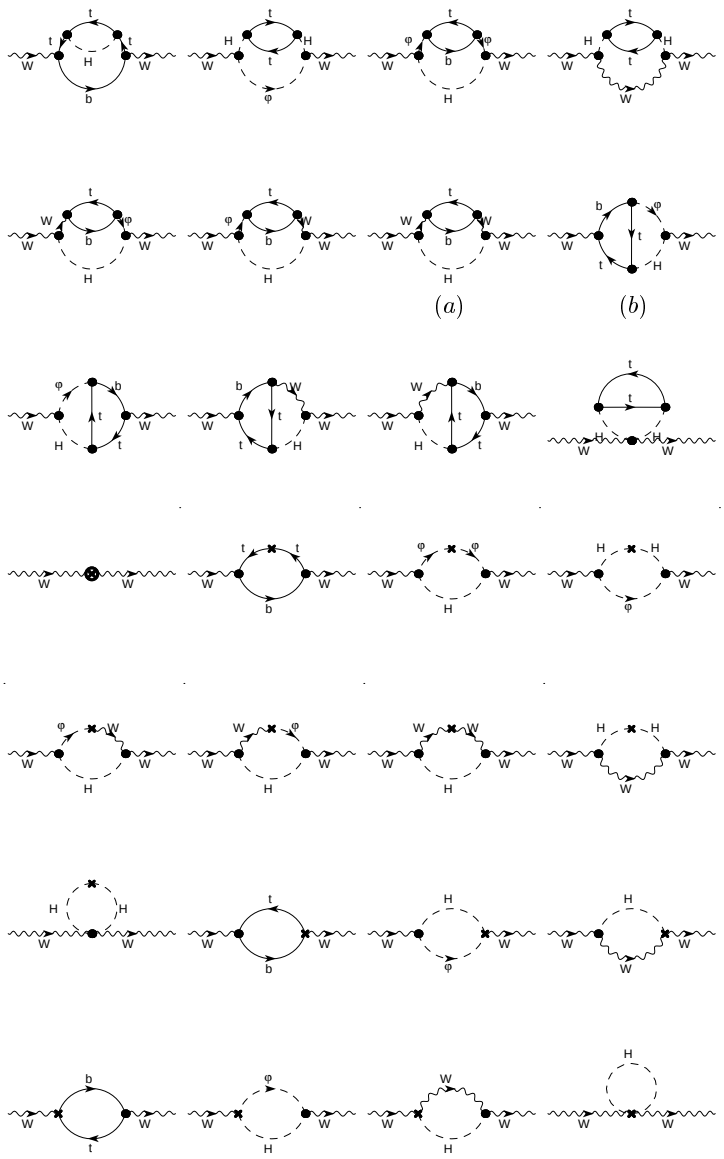

Fig. 1. The two-loop top-quark corrections (including counterterm graphs) to the W-boson self-energy that depend on the Higgs-boson mass. The diagrams (a) and (b) are marked for later reference.

grows slower than $m_{\rm t}^2$, as it is required as a matter of consistency.

In the physical parameter region, i.e. for $m_{\rm t} = 175$ GeV, the comparison between the exact result and the expansions is displayed in Fig. 2 and Fig. 3 for diagram (a) and (b), respectively. The plots show the finite parts of the diagrams (divided by $M_{\rm W}^2$ and in units of $\frac{\alpha^2}{4\pi^2}$) in dimensional regularization and are given as a function of the Higgs-boson mass and of the external momentum $\sqrt{s}$. In the latter plots $M_{\rm H} = 300$ GeV has been used and only the curve for the heavy Higgs expansion is shown since the light Higgs expansion is not valid for this value of $M_{\rm H}$.

For diagram (a) the expansion in the heavy Higgs region yields zero, and the deviations between the exact result and the expansions are relatively large for most values of $M_{\rm H}$ and $\sqrt{s}$. For diagram (b) we find relatively good agreement between the full result and the expansions. It should be noted, however, that the numerical contribution of diagram (b) is rather large so that in the final result considerable cancellations can be expected. The absolute deviation is for both diagrams of the order of $10 \times \frac{\alpha^2}{4\pi^2}$ to $50 \times \frac{\alpha^2}{4\pi^2}$.

3. Higgs-mass dependence of two-loop corrections to Δr

The relation between the vector-boson masses in terms of the Fermi constant G_μ reads [14]

$$M_{\rm W}^2 \left(1 - \frac{M_{\rm W}^2}{M_{\rm Z}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}\,(1 + \Delta r)\,, \tag{1}$$

where the radiative corrections are contained in the quantity Δr. In the context of this paper we treat Δr without resummations, i.e. as being fully expanded up to two-loop order,

$$\Delta r = \Delta r_{(1)} + \Delta r_{(2)} + \dots\ . \tag{2}$$

The theoretical predictions for Δr are obtained by calculating radiative corrections to muon decay.

We study the variation of the two-loop contributions to Δr with the Higgs-boson mass by considering the subtracted quantity

$$\Delta r_{(2),\rm subtr}(M_{\rm H}) = \Delta r_{(2)}(M_{\rm H}) - \Delta r_{(2)}(M_{\rm H} = 65\ {\rm GeV}), \tag{3}$$

where $\Delta r_{(2)}(M_{\rm H})$ denotes the two-loop contribution to Δr. Potentially large $M_{\rm H}$-dependent contributions are the corrections associated with the top quark, since the Yukawa coupling of the Higgs to the top quark is proportional to $m_{\rm t}$, and the contributions which are proportional to $\Delta\alpha$. We

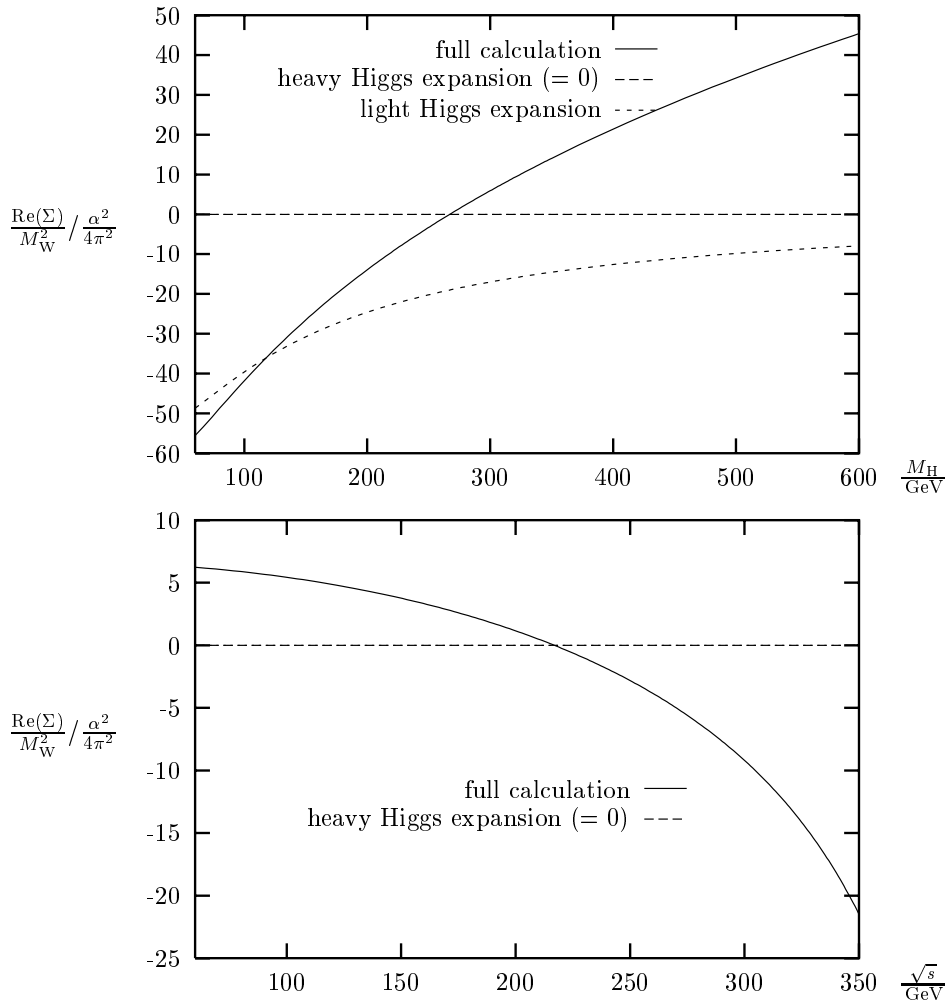

Fig. 2. Comparison between the exact result and the expansions up to next-to-leading order in m_t for diagram (a); $m_\mathrm{t} = 175$ GeV. The upper plot shows the full result as well as the results for the light Higgs and heavy Higgs expansion (the latter is zero) as a function of M_H for $s = M_\mathrm{W}^2$. The lower plot shows the full result and the heavy Higgs expansion as a function of $\sqrt{s}$ for $M_\mathrm{H} = 300$ GeV.

present here exact results for the fermionic contributions, while the purely bosonic corrections, which contain no specific source of enhancement, have been estimated to give rise to only a relatively small shift in the W-boson mass of the order of 2 MeV over the range $65\,\mathrm{GeV} \leq M_\mathrm{H} \leq 1\,\mathrm{TeV}$ [4].

We begin with the Higgs-mass dependence of the two-loop top-quark

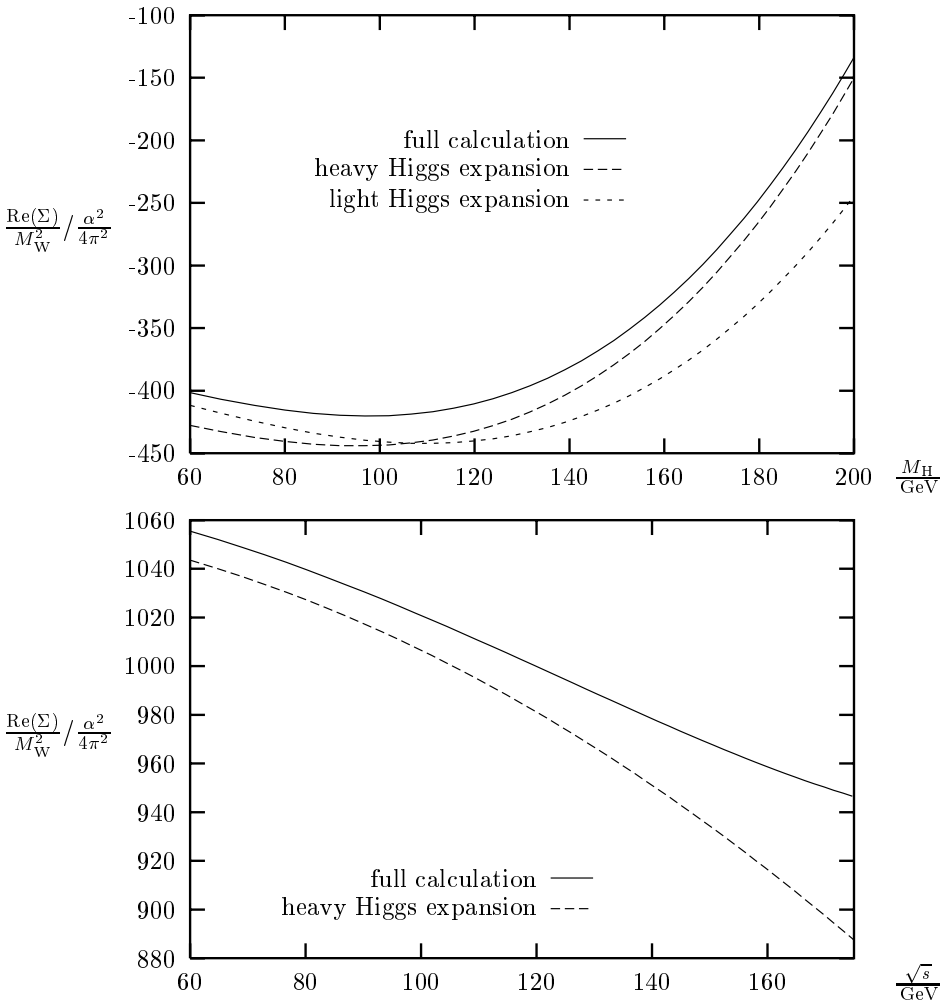

Fig. 3. Comparison between the exact result and the expansions up to next-to-leading order in $m_{\rm t}$ for diagram (b); $m_{\rm t} = 175$ GeV. The upper plot shows the full result and the results for the expansions as a function of $M_{\rm H}$ for $s = M_{\rm W}^2$, while the lower plot shows the full result and the heavy Higgs expansion as a function of $\sqrt{s}$ for $M_{\rm H} = 300$ GeV.

contributions and consider the quantity $\Delta r^{\rm top}_{(2),{\rm subtr}}(M_{\rm H})$ which denotes the contribution of the top/bottom doublet to $\Delta r_{(2),{\rm subtr}}(M_{\rm H})$. From the one-particle irreducible diagrams obviously those graphs contribute to $\Delta r^{\rm top}_{(2),{\rm subtr}}$ that contain both the top and/or bottom quark and the Higgs boson. For

the W self-energy the relevant graphs are the ones shown in Fig. 1. It is easy to see that only two-point functions enter in $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$, since all graphs where the Higgs boson couples to the muon or the electron may safely be neglected. Although no two-loop three-point function enters, there is nevertheless a contribution from the two-loop and one-loop vertex counterterms. If the field renormalization constants of the W boson are included (which cancel in the complete result), the vertex counterterms are separately finite. The technically most complicated contributions arise from the mass and mixing-angle renormalization. Since it is performed in the on-shell scheme, the evaluation of the W- and Z-boson self-energies are required at non-zero momentum transfer.

The contribution of the terms proportional to $\Delta\alpha$ has the simple form $\Delta r^{\Delta\alpha}_{(2),\rm subtr}(M_{\rm H}) = 2\Delta\alpha\,\Delta r_{(1),\rm subtr}(M_{\rm H})$ and can easily be obtained by a proper resummation of one-loop terms. The remaining fermionic contribution, $\Delta r^{\rm lf}_{(2),\rm subtr}$, is the one of the light fermions, i.e. of the leptons and of the quark doublets of the first and second generation, which is not contained in $\Delta\alpha$. Its structure is analogous to $\Delta r^{\rm top}_{(2),\rm subtr}$, but because of the negligible coupling of the light fermions to the Higgs boson much less diagrams contribute.

The total result for the one-loop and fermionic two-loop contributions to Δr, subtracted at $M_{\rm H} = 65\,\text{GeV}$, reads

$$\Delta r_{\rm subtr} \equiv \Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr} + \Delta r^{\Delta\alpha}_{(2),\rm subtr} + \Delta r^{\rm lf}_{(2),\rm subtr}. \tag{4}$$

It is shown in Fig. 4, where separately also the one-loop contribution $\Delta r_{(1),\rm subtr}$, as well as $\Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr}$, and $\Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr} + \Delta r^{\Delta\alpha}_{(2),\rm subtr}$ are shown for $m_{\rm t} = 175.6\,\text{GeV}$. The two-loop contributions $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ and $\Delta r^{\Delta\alpha}_{(2),\rm subtr}(M_{\rm H})$ turn out to be of similar size and to cancel each other to a large extent. In total, the inclusion of the higher-order contributions discussed here leads to a slight increase in the sensitivity to the Higgs-boson mass compared to the pure one-loop result.

We have compared the result for $\Delta r^{\rm top,\Delta\alpha}_{\rm subtr} \equiv \Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr} + \Delta r^{\Delta\alpha}_{(2),\rm subtr}$ with the result obtained via an expansion in $m_{\rm t}$ up to next-to-leading order, i.e. $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$ [3]. The results agree within about 30% of $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$, which amounts to a difference in $M_{\rm W}$ of up to about 4 MeV [4].

In Tab. 1 the shift in $M_{\rm W}$ corresponding to $\Delta r_{\rm subtr}(M_{\rm H})$, i.e. the change in the theoretical prediction for $M_{\rm W}$ when varying the Higgs-boson mass from 65 GeV to 1 TeV, is shown for three values of the top-quark mass, $m_{\rm t} = 170, 175, 180\,\text{GeV}$. The dependence on the precise value of $m_{\rm t}$ is

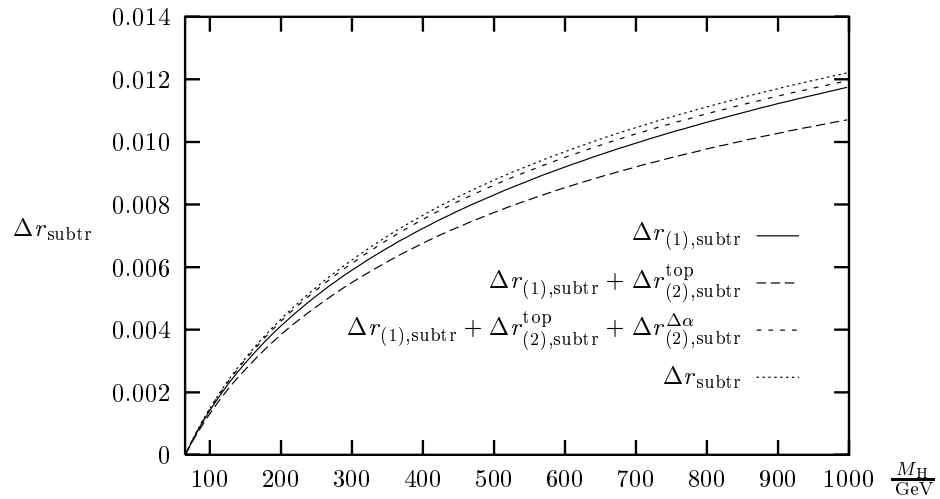

Fig. 4. One-loop and two-loop contributions to Δr subtracted at $M_{\rm H} = 65\,\text{GeV}$. $\Delta r_{\rm subtr}$ is the result for the full one-loop and fermionic two-loop contributions to Δr, as defined in the text.

$M_{\rm H}/\,\text{GeV}$	65	100	200	300	400	500	600	1000
$m_{\rm t} = 170\,\text{GeV}$	0	−22.6	−65.8	−94.5	−116	−133	−147	−185
$m_{\rm t} = 175\,\text{GeV}$	0	−22.8	−66.3	−95.2	−117	−134	−148	−187
$m_{\rm t} = 180\,\text{GeV}$	0	−23.0	−66.8	−96.0	−118	−135	−149	−188

Table 1. The shift in MeV in the theoretical prediction for $M_{\rm W}$ caused by varying the Higgs-boson mass in the interval $65\,\text{GeV} \leq M_{\rm H} \leq 1\,\text{TeV}$ for three values of $m_{\rm t}$.

rather mild, which is expected from the fact that $m_{\rm t}$ enters here only at the two-loop level and that $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ has a local maximum in the region of $m_{\rm t} = 175$ GeV (see Ref. [4]).

4. Gluonic corrections to Δr in the MSSM

In the MSSM, the leading contributions of scalar quarks to Δr and the leptonic Z-boson observables enter via the ρ parameter. The contribution of squark loops to the ρ parameter can be written in terms of the transverse

parts of the W- and Z-boson self-energies at zero momentum-transfer,

$$\Delta\rho = \frac{\Sigma^{\rm Z}(0)}{M_{\rm Z}^2} - \frac{\Sigma^{\rm W}(0)}{M_{\rm W}^2}. \tag{5}$$

The one-loop result for the stop/sbottom doublet in the MSSM reads [15]

$$\begin{aligned}\Delta\rho_0^{\rm SUSY} &= \frac{3G_\mu}{8\sqrt{2}\pi^2}\Big[-\sin^2\theta_{\tilde{\rm t}}\cos^2\theta_{\tilde{\rm t}}F_0\left(m^2_{\tilde{\rm t}_1}, m^2_{\tilde{\rm t}_2}\right) \\ &\quad + \cos^2\theta_{\tilde{\rm t}}F_0\left(m^2_{\tilde{\rm t}_1}, m^2_{\tilde{\rm b}_L}\right) + \sin^2\theta_{\tilde{\rm t}}F_0\left(m^2_{\tilde{\rm t}_2}, m^2_{\tilde{\rm b}_L}\right)\Big],\end{aligned} \tag{6}$$

where $\theta_{\tilde{\rm t}}$ is the mixing angle in the stop sector. Only the left-handed sbottom mass, $m_{\tilde{\rm b}_L}$, contributes since mixing in the sbottom sector has been neglected. The function $F_0(x,y)$ has the form

$$F_0(x,y) = x + y - \frac{2xy}{x-y}\log\frac{x}{y}. \tag{7}$$

It vanishes if the squarks are degenerate in mass, $F_0(m^2,m^2) = 0$. In the limit of a large mass splitting between the squarks it is proportional to the heavy squark mass squared, $F_0(m^2,0) = m^2$. This is in analogy to the case of the top/bottom doublet in the SM [16],

$$\Delta\rho_0^{\rm SM} = \frac{3G_\mu}{8\sqrt{2}\pi^2}F_0(m_{\rm t}^2, m_{\rm b}^2) \approx \frac{3G_\mu m_{\rm t}^2}{8\sqrt{2}\pi^2}. \tag{8}$$

Since the contribution of a squark doublet vanishes if all masses are degenerate, in most SUSY scenarios only the third generation contributes. While the scalar partners of the light quarks are almost mass degenerate in most SUSY scenarios, in the third generation the top-quark mass enters the mass matrix of the scalar partners of the top quark (see eq. (15) below). It can give rise to a large mixing in the stop sector and to a large splitting between the stop and sbottom masses.

Recently the QCD corrections to the ρ parameter in the MSSM have been evaluated [6, 7]. The two-loop Feynman diagrams of the squark loop contributions to $\Delta\rho$ at $\mathcal{O}(\alpha\alpha_s)$ can be divided into diagrams in which a gluon is exchanged, into diagrams with gluino exchange, and into pure scalar diagrams. After the inclusion of the corresponding counterterms the contribution of the pure scalar diagrams vanishes and the other two sets are separately ultraviolet finite and gauge-invariant (see Ref. [7]).

The result for the gluon-exchange contribution is given by a simple expression resembling the one-loop result of eq. (6),

$$\begin{aligned}\Delta\rho_{1,\rm gluon}^{\rm SUSY} &= \frac{G_\mu\alpha_s}{4\sqrt{2}\pi^3}\Big[-\sin^2\theta_{\tilde{\rm t}}\cos^2\theta_{\tilde{\rm t}}F_1\left(m^2_{\tilde{\rm t}_1}, m^2_{\tilde{\rm t}_2}\right) \\ &\quad + \cos^2\theta_{\tilde{\rm t}}F_1\left(m^2_{\tilde{\rm t}_1}, m^2_{\tilde{\rm b}_L}\right) + \sin^2\theta_{\tilde{\rm t}}F_1\left(m^2_{\tilde{\rm t}_2}, m^2_{\tilde{\rm b}_L}\right)\Big].\end{aligned} \tag{9}$$

The two-loop function $F_1(x,y)$ is given in terms of dilogarithms by

$$F_1(x,y) = x+y-2\frac{xy}{x-y}\log\frac{x}{y}\left[2+\frac{x}{y}\log\frac{x}{y}\right] + \frac{(x+y)x^2}{(x-y)^2}\log^2\frac{x}{y} - 2(x-y)\mathrm{Li}_2\left(1-\frac{x}{y}\right). \qquad (10)$$

It is symmetric in the interchange of x and y. It vanishes for degenerate masses, $F_1(m^2,m^2)=0$, while in the case of large mass splitting it increases with the heavy scalar quark mass squared: $F_1(m^2,0)=m^2(1+\pi^2/3)$.

It is remarkable that contrary to the Standard Model case [17],

$$\Delta\rho_1^{\mathrm{SM}} = -\Delta\rho_0^{\mathrm{SM}}\,\frac{2}{3}\frac{\alpha_s}{\pi}(1+\frac{\pi^2}{3}), \qquad (11)$$

where the QCD corrections are negative and screen the one-loop value, $\Delta\rho_{1,\mathrm{gluon}}^{\mathrm{SUSY}}$ enters with the same sign as the one-loop contribution. It therefore enhances the sensitivity in the search of the virtual effects of scalar quarks in high-precision electroweak measurements.

The analytical expression for the gluino-exchange contribution is much more complicated than eq. (9). In general the gluino-exchange diagrams give smaller contributions compared to gluon exchange. Only for gluino and squark masses close to the experimental lower bounds they compete with the gluon-exchange contributions. For higher values of $m_{\tilde{\mathrm{g}}}$, the contribution decreases rapidly since the gluino decouples in the large-mass limit.

The leading contribution to Δr in the MSSM can be approximated by the contribution to the ρ parameter according to

$$\Delta r \approx -\frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\Delta\rho, \qquad (12)$$

where $c_{\mathrm{W}}^2 = 1-s_{\mathrm{W}}^2 = M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2$.

We have derived the exact result for the gluon-exchange contribution to Δr, so that the accuracy of the approximation eq. (12) can be tested in this case. The gluon-exchange correction to the contribution of a squark doublet to Δr is given by

$$\Delta r_{\mathrm{gluon}}^{\mathrm{SUSY}} = \Pi^{\gamma}(0) - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\left(\frac{\delta M_{\mathrm{Z}}^2}{M_{\mathrm{Z}}^2}-\frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2}\right) + \frac{\Sigma^{\mathrm{W}}(0)-\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2}, \qquad (13)$$

where $\delta M_{\mathrm{W}}^2 = \mathrm{Re}\,\Sigma^{\mathrm{W}}(M_{\mathrm{W}}^2)$, $\delta M_{\mathrm{Z}}^2 = \mathrm{Re}\,\Sigma^{\mathrm{Z}}(M_{\mathrm{Z}}^2)$, and Π^{γ}, Σ^{W}, and Σ^{Z} denote the transverse parts of the two-loop gluon-exchange contributions to the photon vacuum polarization and the W-boson and Z-boson self-energies,

respectively, which all are understood to contain the subloop renormalization.

In contrast to the ρ parameter, the evaluation of eq. (13) requires the calculation of two-loop two-point functions at non-zero momentum transfer. Since the gluon is massless an analytical result in terms of polylogarithmic functions can be obtained. For deriving this result we have used the expressions for the relevant two-loop scalar integrals given in Ref. [18].

The gluon-exchange correction to the contribution of the stop/sbottom loops to Δr is shown in Fig. 5 together with the $\Delta\rho$ approximation according to eq. (12) as a function of the common scalar mass parameter

$$m_{\tilde{q}} = M_{\tilde{t}_L} = M_{\tilde{t}_R} = M_{\tilde{b}_L} = M_{\tilde{b}_R}. \tag{14}$$

The $M_{\tilde{q}_i}$ are the soft SUSY breaking parameters appearing in the stop and sbottom mass matrices,

$$\mathcal{M}^2_{\tilde{t}} = \begin{pmatrix} M^2_{\tilde{t}_L} + m^2_{\mathrm{t}} + d_{\tilde{t}_L} & m_{\mathrm{t}}\, M_t^{LR} \\ m_{\mathrm{t}}\, M_t^{LR} & M^2_{\tilde{t}_R} + m^2_{\mathrm{t}} + d_{\tilde{t}_R} \end{pmatrix} \tag{15}$$

$$\mathcal{M}^2_{\tilde{b}} = \begin{pmatrix} M^2_{\tilde{b}_L} + m^2_{\mathrm{b}} + d_{\tilde{b}_L} & m_{\mathrm{b}}\, M_b^{LR} \\ m_{\mathrm{b}}\, M_b^{LR} & M^2_{\tilde{b}_R} + m^2_{\mathrm{b}} + d_{\tilde{b}_R} \end{pmatrix}, \tag{16}$$

with $M_t^{LR} = A_t - \mu\, \cot\beta$, $M_b^{LR} = A_b - \mu \tan\beta$, and the $d_{\tilde{q}_i}$ are specified e.g. in Ref. [7]. In this scenario, the scalar top mixing angle is either very small, $\theta_{\tilde{t}} \sim 0$, or almost maximal, $\theta_{\tilde{t}} \sim -\pi/4$, in most of the MSSM parameter space. The plots are shown for the two cases $M_t^{LR} = 0$ (no mixing) and $M_t^{LR} = 200$ GeV (maximal mixing) for $\tan\beta = 1.6$.

The two-loop contribution $\Delta r^{\mathrm{SUSY}}_{\mathrm{gluon}}$ is of the order of 10–15% of the one-loop result. It yields a shift in the W-boson mass of up to 20 MeV for low values of $m_{\tilde{q}}$ in the no-mixing case. If the parameter M_t^{LR} is made large or the relation eq. (14) is relaxed, much bigger effects are possible [7]. As can be seen in Fig. 5, the $\Delta\rho$ contribution approximates the full result rather well. The two results agree within 10–15%.

From the good agreement between full result and $\Delta\rho$ approximation for the gluonic contributions one can expect for the gluino-exchange correction, whose contribution to $\Delta\rho$ is in general smaller than the gluonic part, that it should be sufficiently well approximated by the $\Delta\rho$ contribution.

5. Conclusions

Recent results obtained at two-loop order for the relation between the gauge-boson masses in the Standard Model and the MSSM have been reviewed. For the example of two typical SM diagrams contributing to the

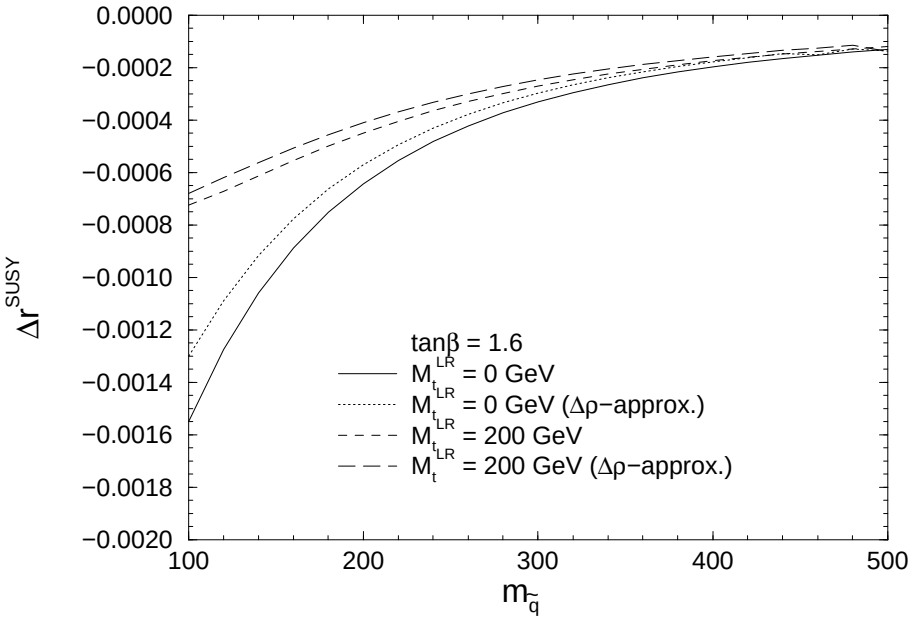

Fig. 5. Contribution of the gluon-exchange diagrams to $\Delta r^{\rm SUSY}$ as a function of the common scalar mass parameter $m_{\tilde{q}}$ for the scenario of no mixing ($M_t^{LR} = 0$) and of maximal mixing ($M_t^{LR} = 200$ GeV) in the stop sector. The exact result is compared with the approximation derived from the contribution of $\Delta\rho$.

W-boson self-energy exact two-loop results have been evaluated and compared with an asymptotic expansion in the top-quark mass up to next-to-leading order. While for asymptotically large values of the top-quark mass the consistency between the two results has been verified, for the physical value of the top-quark mass the relative difference between exact result and expansion can be sizable.

In the Standard Model, the Higgs-mass dependence of the relation between the gauge-boson masses has been studied. Exact two-loop results have been given for the fermionic contributions, while the extra shift coming from the purely bosonic two-loop corrections is expected to be negligible. As far as the Higgs-mass dependence of Δr is concerned, with the results presented here the theoretical uncertainty due to unknown higher-order corrections should now be under control.

In Supersymmetry, the exact result for the gluon-exchange correction to the contribution of squark loops to Δr has been presented. It gives rise to a shift in the W-boson mass of up to 20 MeV. The result has been compared with the recently obtained leading contribution entering via the

ρ parameter, and good agreement has been found.

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