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Two-loop top-Higgs contributions to muon decay[1]

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Abstract

The sensitivity of the two-loop contributions to muon decay to the Higgs-boson mass is analyzed. Exact results are given for the Higgs-dependent two-loop corrections associated with the fermions, i.e. no expansion in the top-quark and the Higgs-boson mass is made. The remaining theoretical uncertainties in the Higgs-mass dependence of Δr are discussed.

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1 Introduction

The experimental accuracy meanwhile reached for the electroweak precision observables allows to test the electroweak Standard Model (SM) at its quantum level, where all parameters of the model enter the theoretical predictions. In this way one is able to obtain constraints on the mass of the Higgs boson, which is the last missing ingredient of the minimal SM. From the most recent global SM fits to all available data one obtains an upper bound for the Higgs-boson mass of 420 GeV at 95% C.L. [1]. This bound is considerably affected by the error in the theoretical predictions due to missing higher-order corrections, which gives rise to an uncertainty of the upper bound of about 100 GeV. The main uncertainty in this context comes from the electroweak two-loop corrections, for which the results obtained so far have been restricted to expansions for asymptotically large values of the top-quark mass, m_{t}, or the Higgs-boson mass, M_{H} [2, 3].

In order to improve this situation, an exact evaluation of electroweak two-loop contributions would be desirable, where no expansion in m_{t} or M_{H} is made. In this talk exact results [4] recently obtained for the Higgs-mass dependence of Δr, i.e. of the prediction for muon decay, are summarized.

2 Muon decay

Very precise data is available for the decay constant of the muon. The global fit of the Particle Data Group to all available measurements is $\tau_\mu = 1/\Gamma_\mu = (2.19703 \pm .00004)10^{-6}\,\mathrm{s}$ [5]. The main decay channel is $\mu^- \to \mathrm{e}^-\bar{\nu}_{\mathrm{e}}\nu_\mu$. Additional photons are produced in 0.014 parts of the decays, and an additional $\mathrm{e}^+\mathrm{e}^-$-pair in 3.4×10^{-5} parts of the decays. Other decay channels are negligible.

The muon decay constant is related to the Fermi constant, the basic parameter of the Fermi model, as

$$\Gamma_\mu = \frac{1}{\tau_\mu} = \frac{G_\mu^2 m_\mu^5}{192\pi^3} F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_{\mathrm{W}}^2}\right)\left(1 + \frac{\alpha(m_\mu)}{2\pi}\left(\frac{25}{4} - \pi^2\right)\right), \tag{1}$$

$$\text{with } F(x) = 1 - 8x + 8x^3 - x^4 - 12x^2\log(x), \tag{2}$$

$$\text{and } \alpha(m_\mu) = \alpha_0\left(1 + \frac{2\alpha}{3\pi}\log\frac{m_\mu}{m_{\mathrm{e}}}\right). \tag{3}$$

The relation between the vector-boson masses in terms of the Fermi constant reads [6]

$$M_{\mathrm{W}}^2\left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}(1 + \Delta r), \tag{4}$$

where the radiative corrections are contained in the quantity Δr. In the context of this paper we treat Δr without resummations, i.e. as being fully expanded up to two-loop order,

$$\Delta r = \Delta r_{(1)} + \Delta r_{(2)} + \dots . \tag{5}$$

According to (1) and (4) the theoretical predictions for Δr are obtained by calculating radiative corrections to muon decay.

3 One-loop contributions

The one-loop contributions to Δr are well known [6] and can be split into three parts,

$$\Delta r_{(1)} = \Delta\alpha - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\Delta\rho_{(1),\mathrm{top}} + \Delta r_{(1),\mathrm{rem}}. \tag{6}$$

They are dominated by the first two terms, which yield a contribution of about 6% and 3.3%, respectively, while the remainder part $\Delta r_{(1),\mathrm{rem}}$ amounts to about 1%. Here $\Delta\alpha$ denotes the QED induced shift in the fine structure constant arising from the contributions of the light fermions to the vacuum polarization of the photon,

$$\Delta\alpha \;=\; \Pi^{AA}_{\rm lf}(0) - {\rm Re}\left(\Pi^{AA}_{\rm lf}(M_{\rm Z}^2)\right), \tag{7}$$

where the one-loop contribution of the leptons can be approximated by

$$\Delta\alpha_{\rm lept} = \frac{\alpha}{3\pi}\sum_l\left(\log\frac{M_{\rm Z}^2}{m_l^2}-\frac{5}{3}\right), \tag{8}$$

and the hadronic contribution is obtained from experimental data via a dispersion relation, $\Delta\alpha_{\rm had} = 0.0280 \pm 0.0007$ [7]. The second term, $\Delta\rho_{(1),{\rm top}}$, is the contribution of the top/bottom doublet to the ρ parameter [8],

$$\Delta\rho_{(1),{\rm top}} \;=\; \frac{3\alpha}{16\pi s_{\rm W}^2 c_{\rm W}^2}\frac{m_{\rm t}^2}{M_{\rm Z}^2}, \tag{9}$$

with $s_{\rm W}^2 \equiv 1 - c_{\rm W}^2 = 1 - M_{\rm W}^2/M_{\rm Z}^2$.

4 Two-loop contributions

From a technical point of view the calculation of top-quark and Higgs-boson contributions to Δr and other processes with light external fermions at low energies requires in particular the evaluation of two-loop self-energies on-shell, i.e. at non-zero external momentum, while vertex and box contributions can mostly be reduced to vacuum integrals.

The problems encountered in the calculation of two-loop contributions to Δr are due to the large number of contributing Feynman diagrams, their complicated tensor structure, the fact that scalar two-loop integrals are in general not expressible in terms of polylogarithmic functions [9], and due to the need for a two-loop renormalization, which has not yet been worked out in full detail.

4.1 Two-loop renormalization

Since we are interested in the relation between the Fermi constant and the physical (i.e. pole) masses $M_{\rm W}$ and $M_{\rm Z}$, the two-loop calculations are performed in a full on-shell renormalization scheme [10, 11]. For the field renormalization of the photon and the Z boson a matrix renormalization is used, defined by the conditions of on-shell decoupling and a normalized residue of the propagators. The charge renormalization is performed with the use of Ward identities. The formulae for the two-loop counterterms that are relevant for the calculation of Δr read

$$\delta M^2_{{\rm W},(2)} \;=\; {\rm Re}\left(\Sigma^{WW}_{{\rm T},(2)}(M_{\rm W}^2)\right) - \delta Z^{W}_{(1)}\delta M^2_{{\rm W},(1)}\,, \tag{10}$$

$$\delta M^2_{{\rm Z},(2)} \;=\; {\rm Re}\left(\Sigma^{ZZ}_{{\rm T},(2)}(M_{\rm Z}^2)\right) - \delta Z^{ZZ}_{(1)}\delta M^2_{{\rm Z},(1)} + \frac{M_{\rm Z}^2}{4}(\delta Z^{AZ}_{(1)})^2\,, \tag{11}$$

$$\begin{aligned}\delta Z_{e,(2)} \;=&\; -\frac{1}{2}\,\delta Z^{AA}_{(2)} - \frac{s_{\rm W}}{2\,c_{\rm W}}\,\delta Z^{ZA}_{(2)} \\ &+ (\delta Z_{e,(1)})^2 + \frac{1}{8}(\delta Z^{AA}_{(1)})^2 - \frac{1}{2\,c_{\rm W}^3}\,\delta Z^{ZA}_{(1)}\delta s_{{\rm W},(1)}\,,\end{aligned} \tag{12}$$

$$\delta Z^{AA}_{(2)} \;=\; -\left.\frac{\partial\Sigma^{AA}_{{\rm T},(2)}(k^2)}{\partial k^2}\right|_{k^2=0} - \frac{1}{4}(\delta Z^{ZA}_{(1)})^2\,, \tag{13}$$

$$\delta Z^{ZA}_{(2)} \;=\; 2\,\frac{\Sigma^{AZ}_{{\rm T},(2)}(0)}{M_{\rm Z}^2} - \frac{\delta M^2_{{\rm Z},(1)}}{M_{\rm Z}^2}\delta Z^{ZA}_{(1)} - \frac{1}{2}\delta Z^{ZZ}_{(1)}\delta Z^{ZA}_{(1)}\,, \tag{14}$$

$$\delta Z^{l,{\rm L}}_{(2)} \;=\; -\,{\rm Re}\left(\Sigma^{l,{\rm L}}_{(2)}(m_l^2) + m_l^2\frac{\partial}{\partial p^2}(\Sigma^{l,{\rm L}}_{(2)}(p^2) + \Sigma^{l,{\rm R}}_{(2)}(p^2) + 2\,\Sigma^{l,{\rm S}}_{(2)}(p^2))|_{p^2=m_l^2}\right), \tag{15}$$

$$\delta Z^{l,{\rm R}}_{(2)} \;=\; -\,{\rm Re}\left(\Sigma^{l,{\rm R}}_{(2)}(m_l^2) + m_l^2\frac{\partial}{\partial p^2}(\Sigma^{l,{\rm L}}_{(2)}(p^2) + \Sigma^{l,{\rm R}}_{(2)}(p^2) + 2\,\Sigma^{l,{\rm S}}_{(2)}(p^2))|_{p^2=m_l^2}\right), \tag{16}$$

$$\delta Z^{\nu,{\rm L}}_{(2)} \;=\; -\,{\rm Re}\left(\Sigma^{\nu,{\rm L}}_{(2)}(0)\right), \tag{17}$$

where all two-loop self-energies are understood to contain the subloop renormalization, and l and ν stand for leptons and neutrinos, respectively. The one-loop counterterms appearing in (10)–(17) have the form specified in Ref. [11].

4.2 The calculations

The methods that we use for carrying out the calculations have been outlined in Ref. [12]. The generation of the diagrams and counterterm contributions is done with the help of the computer-algebra program *FeynArts* [13]. Making use of two-loop tensor-integral decompositions, the generated amplitudes are reduced to a minimal set of standard scalar integrals with the program *TwoCalc* [14]. The two-loop scalar integrals are evaluated numerically with one-dimensional integral representations [15]. These allow a very fast calculation of the integrals with high precision without any approximation in the masses. As a check of our calculations, all results presented here have been derived using a general R_ξ gauge, which is specified by one gauge parameter $\xi_i,\ i = \gamma, Z, W$, for each vector boson, and we have explicitly verified that the gauge parameters actually drop out.

In order to study the Higgs-mass dependence of the two-loop contributions to Δr we consider the subtracted quantity

$$\Delta r_{(2),\mathrm{subtr}}(M_\mathrm{H}) = \Delta r_{(2)}(M_\mathrm{H}) - \Delta r_{(2)}(M_\mathrm{H} = 65\,\mathrm{GeV}), \tag{18}$$

where $\Delta r_{(2)}(M_\mathrm{H})$ denotes the two-loop contribution to Δr.

Potentially large M_H-dependent contributions are the corrections associated with the top quark, since the Yukawa coupling of the Higgs boson to the top quark is proportional to m_t, and the contributions which are proportional to $\Delta\alpha$. We first consider the Higgs-mass dependence of the two-loop top-quark contributions and calculate the quantity $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H})$, which denotes the contribution of the top/bottom doublet to $\Delta r_{(2),\mathrm{subtr}}(M_\mathrm{H})$.

From the one-particle irreducible diagrams obviously those graphs contribute to $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}$ that contain both the top and/or bottom quark and the Higgs boson. It is easy to see that only two-point functions enter in this case, since all graphs where the Higgs boson couples to the muon or the electron may safely be neglected. Although no two-loop three-point function enters, there is nevertheless a contribution from the two-loop and one-loop vertex counterterms. If the field renormalization constants of the W boson are included (which cancel in the complete result), the vertex counterterms are separately finite.

The technically most complicated contributions arise from the mass and mixing-angle renormalization. Since it is performed in the on-shell scheme, the evaluation of the W- and Z-boson self-energies is required at non-zero momentum transfer.

Expressed in terms of the one-loop and two-loop contributions to the transverse part of the W-boson self-energy $\Sigma^\mathrm{W}(p^2)$ and the counterterm δZ^{vert} to the $W^-\bar{e}\nu_e$ vertex, the quantity $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H})$ reads

$$\begin{aligned}
\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H}) = \Bigg[& \frac{\Sigma^\mathrm{W}_{(2)}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(2)}(M_\mathrm{W}^2)}{M_\mathrm{W}^2} + 2\delta Z^{\mathrm{vert}}_{(2)} \\
& + 2\frac{\left(\Sigma^\mathrm{W}_{(1),\mathrm{t}}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(1),\mathrm{t}}(M_\mathrm{W}^2)\right)\left(\Sigma^\mathrm{W}_{(1),\mathrm{H}}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(1),\mathrm{H}}(M_\mathrm{W}^2)\right)}{M_\mathrm{W}^4} \\
& + 2\frac{\left(\Sigma^\mathrm{W}_{(1),\mathrm{t}}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(1),\mathrm{t}}(M_\mathrm{W}^2)\right)\delta Z^{\mathrm{vert}}_{(1),\mathrm{H}}}{M_\mathrm{W}^2} \\
& + 2\frac{\left(\Sigma^\mathrm{W}_{(1),\mathrm{H}}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(1),\mathrm{H}}(M_\mathrm{W}^2)\right)\delta Z^{\mathrm{vert}}_{(1),\mathrm{t}}}{M_\mathrm{W}^2} + 2\delta Z^{\mathrm{vert}}_{(1),\mathrm{t}}\delta Z^{\mathrm{vert}}_{(1),\mathrm{H}}\Bigg]_{\mathrm{subtr}},
\end{aligned} \tag{19}$$

where it is understood that the two-loop contributions to the self-energies contain the subloop renormalization. The two-loop terms denote those graphs that contain both the top quark and the Higgs boson, while for the one-loop terms the top-quark and the Higgs-boson contributions are indicated by a subscript. The two-loop vertex counterterm is expressible in terms of the charge counterterm δZ_e and the mixing-angle counterterm $\delta s_{\mathrm{W}}/s_{\mathrm{W}}$,

$$\delta Z^{\mathrm{vert}}_{(2)} = \delta Z_{e,(2)} - \frac{\delta s_{\mathrm{W},(2)}}{s_{\mathrm{W}}} + 2\frac{\delta s_{\mathrm{W},(1),\mathrm{t}}}{s_{\mathrm{W}}}\frac{\delta s_{\mathrm{W},(1),\mathrm{H}}}{s_{\mathrm{W}}} - \delta Z_{e,(1),\mathrm{t}}\frac{\delta s_{\mathrm{W},(1),\mathrm{H}}}{s_{\mathrm{W}}}, \tag{20}$$

and similarly the one-loop vertex counterterm is given by

$$\delta Z^{\mathrm{vert}}_{(1)} = \delta Z_{e,(1)} - \frac{\delta s_{\mathrm{W},(1)}}{s_{\mathrm{W}}}. \tag{21}$$

For the Higgs-dependent fermionic contributions the charge counterterm given in (12) simplifies to

$$\delta Z_{e,(2)} = -\frac{1}{2}\delta Z_{AA,(2)} = \frac{1}{2}\Pi^{AA}_{(2)}(0), \tag{22}$$

which is familiar from QED. The validity of (22) can also be understood by observing that the contributions considered here are precisely the same as the ones obtained within the framework of the background-field method [16].

The mixing angle counterterm $\delta s_{\mathrm{W},(2)}/s_{\mathrm{W}}$ is expressible in terms of the on-shell two-loop W-boson and Z-boson self-energies and additional one-loop contributions,

$$\begin{aligned}\frac{\delta s_{\mathrm{W},(2)}}{s_{\mathrm{W}}} = &-\frac{1}{2}\frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\left(\frac{\mathrm{Re}\,\Sigma^{\mathrm{W}}_{(2)}(M_{\mathrm{W}}^2)}{M_{\mathrm{W}}^2} - \frac{\mathrm{Re}\,\Sigma^{\mathrm{ZZ}}_{(2)}(M_{\mathrm{Z}}^2)}{M_{\mathrm{Z}}^2}\right)\\ &- \frac{\delta s_{\mathrm{W},(1),\mathrm{t}}}{s_{\mathrm{W}}}\frac{\mathrm{Re}\,\Sigma^{\mathrm{ZZ}}_{(1),\mathrm{H}}(M_{\mathrm{Z}}^2)}{M_{\mathrm{Z}}^2} - \frac{\delta s_{\mathrm{W},(1),\mathrm{H}}}{s_{\mathrm{W}}}\frac{\mathrm{Re}\,\Sigma^{\mathrm{ZZ}}_{(1),\mathrm{t}}(M_{\mathrm{Z}}^2)}{M_{\mathrm{Z}}^2} - \frac{\delta s_{\mathrm{W},(1),\mathrm{t}}}{s_{\mathrm{W}}}\frac{\delta s_{\mathrm{W},(1),\mathrm{H}}}{s_{\mathrm{W}}},\end{aligned} \tag{23}$$

where

$$\frac{\delta s_{\mathrm{W},(1)}}{s_{\mathrm{W}}} = -\frac{1}{2}\frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\left(\frac{\mathrm{Re}\,\Sigma^{\mathrm{W}}_{(1)}(M_{\mathrm{W}}^2)}{M_{\mathrm{W}}^2} - \frac{\mathrm{Re}\,\Sigma^{\mathrm{ZZ}}_{(1)}(M_{\mathrm{Z}}^2)}{M_{\mathrm{Z}}^2}\right). \tag{24}$$

In (19)–(21) the field renormalization constants of the W boson have been omitted. In our calculation of $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ we have explicitly kept the field renormalization constants of all internal fields and have checked that they actually cancel in the final result.

4.3 Results

The result for $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ is shown in Fig. 1 for various values of m_{t}. The Higgs-boson mass is varied in the interval $65\,\mathrm{GeV} \leq M_{\mathrm{H}} \leq 1\,\mathrm{TeV}$. The change in $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ over this interval is about 0.001, which corresponds to a shift in M_{W} of about 20 MeV. It is interesting to note that the absolute value of the correction is maximal just in the region of $m_{\mathrm{t}} = 175\,\mathrm{GeV}$, i.e. for the physical value of the top-quark mass. For $m_{\mathrm{t}} \sim 175\,\mathrm{GeV}$ the correction $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ amounts to about 10% of the one-loop contribution, $\Delta r_{(1),\mathrm{subtr}}(M_{\mathrm{H}})$, which is defined in analogy to (18).

The other M_{H}-dependent two-loop correction that is expected to be sizable is the contribution of the terms proportional to $\Delta\alpha$. It reads

$$\begin{aligned}\Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}}(M_{\mathrm{H}}) &= 2\Delta\alpha\left[\frac{\Sigma^{\mathrm{W}}_{(1),\mathrm{H}}(0) - \mathrm{Re}\,\Sigma^{\mathrm{W}}_{(1),\mathrm{H}}(M_{\mathrm{W}}^2)}{M_{\mathrm{W}}^2} - 2\frac{\delta s_{\mathrm{W},(1),\mathrm{H}}}{s_{\mathrm{W}}}\right]_{\mathrm{subtr}}\\ &= 2\Delta\alpha\,\Delta r_{(1),\mathrm{subtr}}(M_{\mathrm{H}}),\end{aligned} \tag{25}$$

and can easily be obtained by a proper resummation of one-loop terms [17].

The remaining fermionic contribution, $\Delta r^{\mathrm{lf}}_{(2),\mathrm{subtr}}$, is the one of the light fermions, i.e. of the leptons and of the quark doublets of the first and second generation, which is not contained in $\Delta\alpha$. Its structure is analogous to (19), but due to the negligible coupling of the light fermions to the Higgs boson much less diagrams contribute. The scalar two-loop integrals needed for the light-fermion contribution can be solved analytically in terms of polylogarithmic functions. They can be found in Ref. [18].

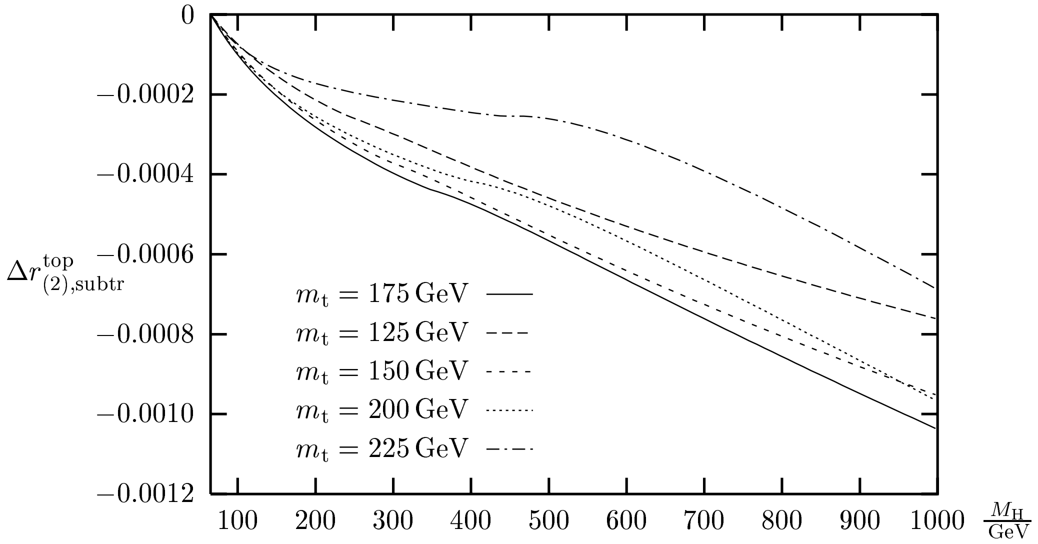

Figure 1: Two-loop top-quark contribution to Δr subtracted at $M_{\rm H} = 65\,\rm GeV$.

The total result for the one-loop and fermionic two-loop contributions to Δr, subtracted at $M_{\rm H} = 65\,\rm GeV$, reads

$$\Delta r_{\rm subtr} \equiv \Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr} + \Delta r^{\Delta\alpha}_{(2),\rm subtr} + \Delta r^{\rm lf}_{(2),\rm subtr}. \tag{26}$$

It is shown in Fig. 2, where separately also the one-loop contribution $\Delta r_{(1),\rm subtr}$, as well as $\Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr}$, and $\Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr} + \Delta r^{\Delta\alpha}_{(2),\rm subtr}$ are shown for $m_{\rm t} = 175.6\,\rm GeV$. It can be seen that the higher-order contributions $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ and $\Delta r^{\Delta\alpha}_{(2),\rm subtr}(M_{\rm H})$ are of about the same size and to a large extent cancel each other. The light-fermion contributions which are not contained in $\Delta\alpha$ add a relatively small correction. Over the full range of the Higgs-boson mass it amounts to about 4 MeV. In total, the inclusion of the higher-order contributions discussed here leads to a slight increase in the sensitivity to the Higgs-boson mass compared to the pure one-loop result.

We have compared the result for $\Delta r^{\rm top}_{(2),\rm subtr}$ with the result obtained via an expansion in $m_{\rm t}$ up to next-to-leading order, i.e. $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_Z^2)$ [3]. The results agree within about 30% of $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$, which amounts to a difference in $M_{\rm W}$ of up to about 4 MeV [4].

In Tab. 1 the shift in $M_{\rm W}$ corresponding to $\Delta r_{\rm subtr}(M_{\rm H})$, i.e. the change in the theoretical prediction for $M_{\rm W}$ when varying the Higgs-boson mass from 65 GeV to 1 TeV, is shown for three values of the top-quark mass, $m_{\rm t} = 170, 175, 180\,\rm GeV$. The dependence on the precise value of $m_{\rm t}$ is rather mild, which is expected from the fact that $m_{\rm t}$ enters here only at the two-loop level and that $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ has a local maximum in the region of $m_{\rm t} = 175$ GeV (see Fig. 1).

Regarding the remaining Higgs-mass dependence of Δr at the two-loop level, there are only purely bosonic corrections left, which contain no specific source of enhancement. They can be expected to yield a contribution to $\Delta r_{(2),\rm subtr}(M_{\rm H})$ of about the same size as $\left.\left(\Delta r^{\rm bos}_{(1)}(M_{\rm H})\right)^2\right|_{\rm subtr}$, where $\Delta r^{\rm bos}_{(1)}$ denotes the bosonic contribution to Δr at the one-loop level. The contribution of $\left.\left(\Delta r^{\rm bos}_{(1)}(M_{\rm H})\right)^2\right|_{\rm subtr}$ amounts to only about 10% of $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ corresponding to a shift of about 2 MeV in the W-boson mass. This estimate agrees well with the values obtained for the Higgs-mass dependence from the formula in Ref. [19] for the leading term proportional to $M_{\rm H}^2$ in an asymptotic expansion for large Higgs-boson mass. The Higgs-mass dependence of the term proportional to $M_{\rm H}^2$ amounts to less than 15% of $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ for reasonable values of $M_{\rm H}$.

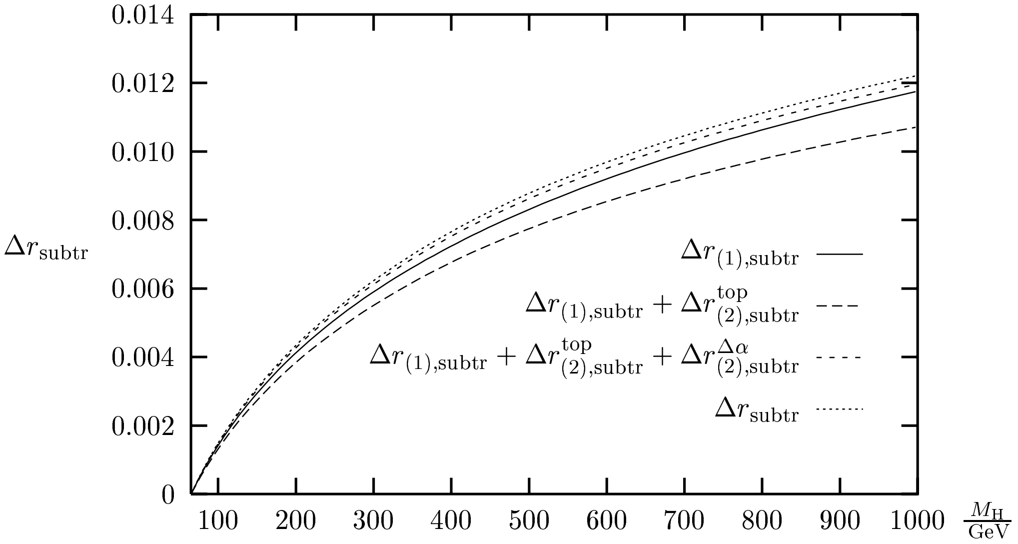

Figure 2: One-loop and two-loop contributions to Δr subtracted at $M_{\mathrm{H}} = 65\,\mathrm{GeV}$. $\Delta r_{\mathrm{subtr}}$ is the result for the full one-loop and fermionic two-loop contributions to Δr, as defined in the text.

$M_{\mathrm{H}}/\,\mathrm{GeV}$	65	100	200	300	400	500	600	1000
$\Delta M_{\mathrm{W}}(M_{\mathrm{H}})/\,\mathrm{MeV},\ m_{\mathrm{t}} = 170\,\mathrm{GeV}$	0	−22.6	−65.8	−94.5	−116	−133	−147	−185
$\Delta M_{\mathrm{W}}(M_{\mathrm{H}})/\,\mathrm{MeV},\ m_{\mathrm{t}} = 175\,\mathrm{GeV}$	0	−22.8	−66.3	−95.2	−117	−134	−148	−187
$\Delta M_{\mathrm{W}}(M_{\mathrm{H}})/\,\mathrm{MeV},\ m_{\mathrm{t}} = 180\,\mathrm{GeV}$	0	−23.0	−66.8	−96.0	−118	−135	−149	−188

Table 1: The shift in the theoretical prediction for M_{W} caused by varying the Higgs-boson mass in the interval $65\,\mathrm{GeV} \leq M_{\mathrm{H}} \leq 1\,\mathrm{TeV}$ for three values of m_{t}.

5 Conclusion

In summary, we have analyzed the Higgs-mass dependence of the relation between the gauge-boson masses at the two-loop level by considering the subtracted quantity $\Delta r_{\mathrm{subtr}}(M_{\mathrm{H}}) = \Delta r(M_{\mathrm{H}}) - \Delta r(M_{\mathrm{H}} = 65\,\mathrm{GeV})$. Exact results have been presented for the fermionic contributions, i.e. no expansion in the top-quark and the Higgs-boson mass has been made. The extra shift coming from the purely bosonic two-loop corrections has been estimated to be relatively small. Considering the envisaged experimental error of M_{W} from the measurements at LEP2 and the Tevatron of $\sim 20\,\mathrm{MeV}$, we conclude that the theoretical uncertainties due to unknown higher-order corrections in the Higgs-mass dependence of Δr are now under control.

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