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Sensitivity of Two-Loop Corrections to Muon Decay to the Higgs-Boson Mass[1]

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The Higgs-mass dependence of the two-loop contributions to muon decay is analyzed at the two-loop level. Exact results are given for the Higgs-dependent two-loop corrections associated with the fermions, i.e. no expansion in the top-quark and the Higgs-boson mass is made. The remaining theoretical uncertainties in the Higgs-mass dependence of Δr are discussed.

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1 Introduction

The experimental accuracy meanwhile reached for the electroweak precision observables allows to test the electroweak Standard Model (SM) at its quantum level, where all parameters of the model enter the theoretical predictions. In this way one is able to derive constraints on the mass of the Higgs boson, which is the last missing ingredient of the minimal SM. From the most recent global SM fits to all available data one obtains an upper bound for the Higgs-boson mass of 420 GeV at 95% C.L. [1]. This bound is considerably affected by the error in the theoretical predictions due to missing higher-order corrections, which gives rise to an uncertainty of the upper bound of about 100 GeV. The main uncertainty in this context comes from the electroweak two-loop corrections, for which the results obtained so far have been restricted to expansions for asymptotically large values of the top-quark mass, m_{t}, or the Higgs-boson mass, M_{H} [2, 3].

In order to improve this situation, an exact evaluation of electroweak two-loop contributions would be desirable, where no expansion in m_{t} or M_{H} is made. In this paper the Higgs-mass dependence of the two-loop contributions to Δr in the SM is studied [4]. Exact results for the corrections associated with the fermions are presented.

2 Higgs-mass dependence of Δr

The relation between the vector-boson masses in terms of the Fermi constant G_μ reads [5]

$$M_{\mathrm{W}}^2 \left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu} \left(1 + \Delta r\right) , \tag{1}$$

where the radiative corrections are contained in the quantity Δr. In the context of this paper we treat Δr without resummations, i.e. as being fully expanded up to two-loop order, $\Delta r = \Delta r_{(1)} + \Delta r_{(2)} + \ldots$. The theoretical predictions for Δr are obtained by calculating radiative corrections to muon decay. We study the variation of the two-loop contributions to Δr with the Higgs-boson mass by considering the subtracted quantity

$$\Delta r_{(2),\mathrm{subtr}}(M_{\mathrm{H}}) = \Delta r_{(2)}(M_{\mathrm{H}}) - \Delta r_{(2)}(M_{\mathrm{H}} = 65\,\mathrm{GeV}), \tag{2}$$

where $\Delta r_{(2)}(M_{\mathrm{H}})$ denotes the two-loop contribution to Δr. Potentially large M_{H}-dependent contributions are the corrections associated with the top quark, since the Yukawa coupling of the Higgs boson to the top quark is proportional to m_{t}, and the contributions which are proportional to $\Delta\alpha$.

The methods used for the calculations discussed in this paper have been outlined in Ref. [6]. The generation of the diagrams and counterterm contributions is done with the help of the computer-algebra program *FeynArts* [7]. Making use of two-loop tensor-integral decompositions, the generated amplitudes are reduced to a minimal set of standard scalar integrals with the program *TwoCalc* [8]. The renormalization is performed within the complete on-shell scheme [9], i.e. physical parameters are used throughout. The two-loop scalar integrals are evaluated numerically with one-dimensional integral representations [10]. These allow a very fast calculation of the integrals with high precision without any approximation in the masses.

We first consider the contribution of the top/bottom doublet, which is denoted as $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$. From the one-particle irreducible diagrams obviously those graphs contribute to $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}$ that contain both the top and/or bottom quark and the Higgs boson. The technically most complicated contributions arise from the mass and mixing-angle renormalization. Since it is performed in the on-shell scheme, the evaluation of the W- and Z-boson self-energies is required at non-zero momentum transfer.

The contribution of the terms proportional to $\Delta\alpha$ has the simple form $\Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}}(M_{\mathrm{H}}) = 2\Delta\alpha\,\Delta r_{(1),\mathrm{subtr}}(M_{\mathrm{H}})$ and can easily be obtained by a proper resummation of one-loop terms [11]. The remaining fermionic contribution, $\Delta r^{\mathrm{lf}}_{(2),\mathrm{subtr}}$, is the one of the light fermions, i.e. of the leptons and of the quark doublets of the first and second generation, which is not contained in $\Delta\alpha$. Its structure is analogous to $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}$, but because of the negligible coupling of the light fermions to the Higgs boson much less diagrams contribute.

The total result for the one-loop and fermionic two-loop contributions to Δr, subtracted at $M_{\mathrm{H}} = 65\,\mathrm{GeV}$, reads

$$\Delta r_{\mathrm{subtr}} \equiv \Delta r_{(1),\mathrm{subtr}} + \Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}} + \Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}} + \Delta r^{\mathrm{lf}}_{(2),\mathrm{subtr}}. \tag{3}$$

It is shown in Fig. 1, where separately also the one-loop contribution $\Delta r_{(1),\mathrm{subtr}}$, as well as $\Delta r_{(1),\mathrm{subtr}} + \Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}$, and $\Delta r_{(1),\mathrm{subtr}} + \Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}} + \Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}}$ are shown for $m_{\mathrm{t}} = 175.6\,\mathrm{GeV}$. The two-loop contributions $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ and $\Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ turn out to be of similar size and to cancel each other to a large extent. In total, the inclusion of the higher-order contributions discussed here leads to a slight increase in the sensitivity to the Higgs-boson mass compared to the pure one-loop result.

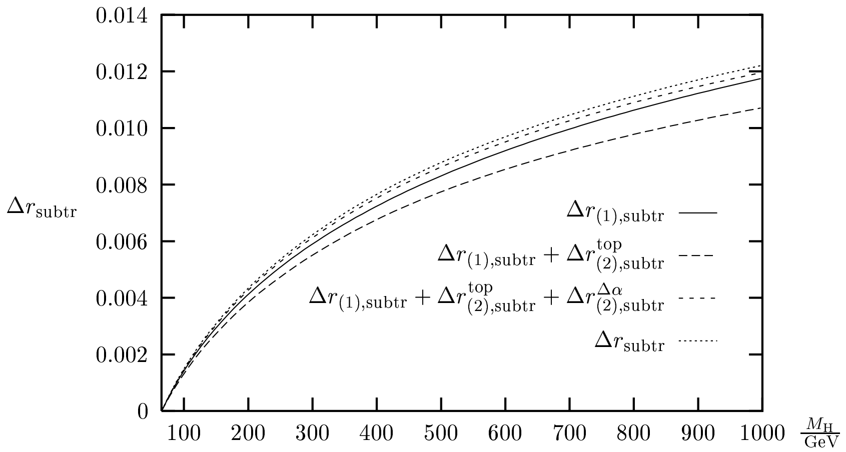

Fig. 1. One-loop and two-loop contributions to Δr subtracted at $M_{\mathrm{H}} = 65\,\mathrm{GeV}$. $\Delta r_{\mathrm{subtr}}$ is the result for the full one-loop and fermionic two-loop contributions to Δr, as defined in the text.

We have compared the result for $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}$ with the result obtained via an expansion in m_{t} up to next-to-leading order, i.e. $\mathcal{O}(G_\mu^2 m_{\mathrm{t}}^2 M_{\mathrm{Z}}^2)$ [3]. The results agree within about 30% of $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$, which amounts to a difference in M_{W} of up to about 4 MeV [4].

Regarding the remaining Higgs-mass dependence of Δr at the two-loop level, there are only purely bosonic corrections left, which contain no specific source of enhancement. They can be expected to yield a contribution to $\Delta r_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ of about the same size as $\left.\left(\Delta r^{\mathrm{bos}}_{(1)}(M_{\mathrm{H}})\right)^2\right|_{\mathrm{subtr}}$, where $\Delta r^{\mathrm{bos}}_{(1)}$ denotes the bosonic contribution to Δr at the one-loop level. The contribution of $\left.\left(\Delta r^{\mathrm{bos}}_{(1)}(M_{\mathrm{H}})\right)^2\right|_{\mathrm{subtr}}$ amounts to only about 10% of $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ corresponding to a shift of about 2 MeV in the W-boson mass. This estimate agrees well with the values obtained for the Higgs-mass dependence from the formula in Ref. [12] for the leading term proportional to M_{H}^2 in an asymptotic expansion for large Higgs-boson mass. The Higgs-mass dependence of the term proportional to M_{H}^2 amounts to less than 15% of $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ for reasonable values of M_{H}.

3 Conclusions

We have analyzed the Higgs-mass dependence of the relation between the gauge-boson masses at the two-loop level by considering the subtracted quantity $\Delta r_{\mathrm{subtr}}(M_{\mathrm{H}}) = \Delta r(M_{\mathrm{H}}) - \Delta r(M_{\mathrm{H}} = 65\,\mathrm{GeV})$. Exact results have been presented for the fermionic contributions, i.e. no expansion in the top-quark and the Higgs-boson mass has been made. The extra shift coming from the purely bosonic two-loop corrections has been estimated to be relatively small. Considering the envisaged experimental error of M_{W} from the measurements at LEP2 and the Tevatron of $\sim 20\,\mathrm{MeV}$, we conclude that the theoretical uncertainties due to unknown higher-order corrections in the Higgs-mass dependence of Δr are now under control.

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