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QCD Corrections to Electroweak Precision Observables in SUSY Theories[1]

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Abstract

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1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) provides the most predictive framework beyond the Standard Model (SM). While the direct search for supersymmetric particles has not been successful yet, the precision tests of the theory provide the possibility for constraining the parameter space of the model and could eventually allow to distinguish between the SM and Supersymmetry via their respective virtual effects. While the SM predictions for Δr and the Z-boson observables include leading terms at two-loop and three-loop order, the corresponding predictions within the MSSM have been restricted so far to one-loop order [1]. In order to treat the MSSM at the same level of accuracy as the SM, higher-order contributions should be incorporated. In this paper results for the QCD corrections to the ρ parameter in the MSSM are presented [2, 3]. In addition, the result for the gluonic contribution to Δr is derived and compared with the approximation based on the contribution entering via the ρ parameter.

2 QCD corrections to the ρ parameter in the MSSM

In the MSSM, the leading contributions of scalar quarks to Δr and the leptonic Z-boson observables enter via the ρ parameter. The contribution of squark loops to the ρ parameter can be written in terms of the transverse parts of the W- and Z-boson self-energies at zero momentum-transfer,

$$\Delta\rho = \frac{\Sigma^{\mathrm{Z}}(0)}{M_{\mathrm{Z}}^2} - \frac{\Sigma^{\mathrm{W}}(0)}{M_{\mathrm{W}}^2}. \tag{1}$$

The one-loop result for the stop/sbottom doublet in the MSSM reads [4]

$$\Delta\rho_0^{\mathrm{SUSY}} = \frac{3G_\mu}{8\sqrt{2}\pi^2}\left[-scF_0(m_{\tilde{\mathrm{t}}_1}^2, m_{\tilde{\mathrm{t}}_2}^2) + cF_0(m_{\tilde{\mathrm{t}}_1}^2, m_{\tilde{\mathrm{b}}_L}^2) + sF_0(m_{\tilde{\mathrm{t}}_2}^2, m_{\tilde{\mathrm{b}}_L}^2)\right],$$

where $s = \sin^2\theta_{\tilde{t}}$, $c = \cos^2\theta_{\tilde{t}}$, $\theta_{\tilde{t}}$ is the stop mixing angle, and mixing in the sbottom sector has been neglected. The function $F_0(x,y)$ has the form $F_0(x,y) = x+y-\frac{2xy}{x-y}\log\frac{x}{y}$. It vanishes if the squarks are degenerate in mass. In the limit of a large mass splitting between the squarks it is proportional to the heavy squark mass squared. This is in analogy to the case of the top/bottom doublet in the SM [5], $\Delta\rho_0^{\rm SM} = \frac{3G_\mu}{8\sqrt{2}\pi^2}F_0(m_{\rm t}^2, m_{\rm b}^2) \approx \frac{3G_\mu m_{\rm t}^2}{8\sqrt{2}\pi^2}$.

Since the contribution of a squark doublet vanishes if all masses are degenerate, in most SUSY scenarios only the third generation contributes. In the third generation the top-quark mass enters the mass matrix of the scalar partners of the top quark and can give rise to a large mixing in the stop sector and to a large splitting between the stop and sbottom masses.

The two-loop Feynman diagrams of the squark loop contributions to $\Delta\rho$ at $\mathcal{O}(\alpha\alpha_s)$ can be divided into diagrams in which a gluon is exchanged, into diagrams with gluino exchange, and into pure scalar diagrams. After the inclusion of the corresponding counterterms the contribution of the pure scalar diagrams vanishes and the other two sets are separately ultraviolet finite and gauge-invariant (see Ref. [3]).

The result for the gluon-exchange contribution is given by a simple expression resembling the one-loop result

$$\Delta\rho_{1,\rm gluon}^{\rm SUSY} = \frac{G_\mu \alpha_s}{4\sqrt{2}\pi^3}\left[-scF_1(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2) + cF_1(m_{\tilde{t}_1}^2, m_{\tilde{b}_L}^2) + sF_1(m_{\tilde{t}_2}^2, m_{\tilde{b}_L}^2)\right].$$

The two-loop function $F_1(x,y)$ is given in terms of dilogarithms by

$$\begin{aligned} F_1(x,y) &= x + y - 2\frac{xy}{x-y}\log\frac{x}{y}\left[2 + \frac{x}{y}\log\frac{x}{y}\right] \\ &\quad + \frac{(x+y)x^2}{(x-y)^2}\log^2\frac{x}{y} - 2(x-y)\mathrm{Li}_2\left(1-\frac{x}{y}\right). \end{aligned} \tag{2}$$

It is symmetric in the interchange of x and y and vanishes for degenerate masses, $F_1(m^2, m^2) = 0$, while in the case of large mass splitting it increases with the heavy scalar quark mass squared: $F_1(m^2, 0) = m^2(1 + \pi^2/3)$.

The gluon-exchange contribution is of the order of 10–15% of the one-loop result [2, 3]. It is remarkable that contrary to the Standard Model case [6], $\Delta\rho_1^{\rm SM} = -\Delta\rho_0^{\rm SM}\frac{2}{3}\frac{\alpha_s}{\pi}(1+\frac{\pi^2}{3})$, where the QCD corrections are negative and screen the one-loop result, $\Delta\rho_{1,\rm gluon}^{\rm SUSY}$ enters with the same sign as the one-loop contribution. It therefore enhances the sensitivity in the search for the virtual effects of scalar quarks in high-precision electroweak measurements.

The analytical expression for the gluino-exchange contribution is much more complicated than for gluon-exchange. In general the gluino-exchange diagrams give smaller contributions compared to gluon exchange. Only for gluino and squark masses close to the experimental lower bounds they compete with the gluon-exchange contributions. In this case, the gluon and gluino contributions add up to about 30% of the one-loop value for maximal mixing [2]. For higher values of $m_{\tilde{g}}$, the contribution decreases rapidly since the gluino decouples in the large-mass limit.

3 Gluonic corrections to Δr

The leading contribution to Δr in the MSSM can be approximated by the contribution to the ρ parameter according to $\Delta r \approx -c_{\mathrm{W}}^2/s_{\mathrm{W}}^2 \Delta\rho$, where $c_{\mathrm{W}}^2 = 1 - s_{\mathrm{W}}^2 = M_{\mathrm{W}}^2/M_{\mathrm{Z}}^2$. In order to test the accuracy of this approximation, we have derived the exact result for the gluon-exchange correction to the contribution of a squark doublet to Δr. It is given by

$$\Delta r_{\mathrm{gluon}}^{\mathrm{SUSY}} = \Pi^{\gamma}(0) - \frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\left(\frac{\delta M_{\mathrm{Z}}^2}{M_{\mathrm{Z}}^2} - \frac{\delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2}\right) + \frac{\Sigma^{\mathrm{W}}(0) - \delta M_{\mathrm{W}}^2}{M_{\mathrm{W}}^2}, \qquad (3)$$

where $\delta M_{\mathrm{W}}^2 = \mathrm{Re}\, \Sigma^{\mathrm{W}}(M_{\mathrm{W}}^2)$, $\delta M_{\mathrm{Z}}^2 = \mathrm{Re}\, \Sigma^{\mathrm{Z}}(M_{\mathrm{Z}}^2)$, and Π^{γ}, Σ^{W}, and Σ^{Z} denote the transverse parts of the two-loop gluon-exchange contributions to the photon vacuum polarization and the W-boson and Z-boson self-energies, respectively, which all are understood to contain the subloop renormalization.

The gluon-exchange correction to the contribution of the stop/sbottom loops to Δr is shown in Fig. 1 together with the $\Delta\rho$ approximation, $\Delta r \approx -c_{\mathrm{W}}^2/s_{\mathrm{W}}^2 \Delta\rho$, as a function of the common scalar mass parameter $m_{\tilde{q}} = M_{\tilde{\mathrm{t}}_L} = M_{\tilde{\mathrm{t}}_R} = M_{\tilde{\mathrm{b}}_L} = M_{\tilde{\mathrm{b}}_R}$, where the $M_{\tilde{q}_i}$ are the soft SUSY breaking parameters appearing in the stop and sbottom mass matrices as specified in Ref. [3]. In this scenario, the scalar top mixing angle is either very small, $\theta_{\tilde{\mathrm{t}}} \sim 0$, or almost maximal, $\theta_{\tilde{\mathrm{t}}} \sim -\pi/4$, in most of the MSSM parameter space. The plots are shown for the two cases $M_{\mathrm{t}}^{LR} = 0$ (no mixing) and $M_{\mathrm{t}}^{LR} = 200$ GeV (maximal mixing) for $\tan\beta = 1.6$.

The two-loop contribution $\Delta r_{\mathrm{gluon}}^{\mathrm{SUSY}}$ is of the order of 10–15% of the one-loop result. It yields a shift in the W-boson mass of up to 20 MeV for low values of $m_{\tilde{q}}$ in the no-mixing case. If the parameter M_{t}^{LR} is made large or the assumption of a common scalar mass parameter is relaxed, much bigger effects are possible [3]. As can be seen in Fig. 1, the $\Delta\rho$ contribution approximates the full result rather well. The two results agree within 10–15%.

4 Conclusions

The two-loop $\mathcal{O}(\alpha_s)$ corrections to the ρ parameter has been derived in the MSSM. The gluonic corrections are of $\mathcal{O}(10\%)$: they are positive and increase the sensitivity in the search for scalar quarks through their virtual effects in high-precision electroweak observables. The gluino contributions are in general smaller except for relatively light gluinos and scalar quarks; the contribution vanishes for large gluino masses. The exact result for the gluon-exchange correction to the contribution of squark loops to Δr has also

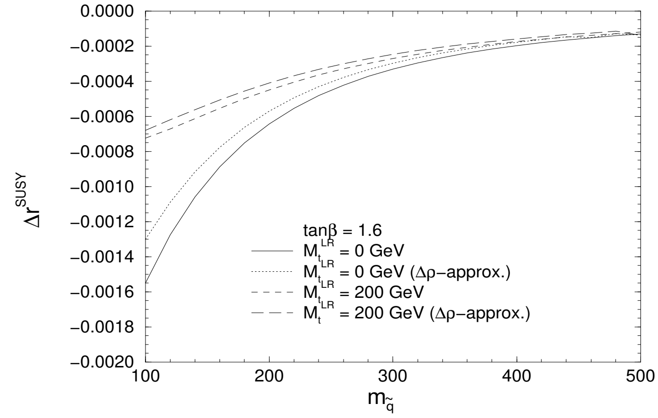

Fig. 1. Contribution of the gluon-exchange diagrams to Δr^{SUSY}. The exact result is compared with the $\Delta\rho$ approximation.

been presented. It gives rise to a shift in the W-boson mass of up to 20 MeV. The result has been compared with the leading contribution entering via the ρ parameter, and good agreement has been found.

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