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Higgs-mass dependence of two-loop corrections to Δr

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Abstract

The Higgs-mass dependence of the Standard Model contributions to the correlation between the gauge-boson masses is studied at the two-loop level. Exact results are given for the Higgs-dependent two-loop corrections associated with the fermions, i.e. no expansion in the top-quark and the Higgs-boson mass is made. The results for the top quark are compared with results of an expansion up to next-to-leading order in the top-quark mass. Agreement is found within 30% of the two-loop result. The remaining theoretical uncertainties in the Higgs-mass dependence of Δr are discussed.

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The remarkable accuracy of the electroweak precision data allows to thoroughly test the predictions of the electroweak Standard Model (SM) at its quantum level, where all the parameters of the model enter the theoretical predictions. In this way it has been possible to predict the value of the top-quark mass, $m_{\rm t}$, within the SM prior to its actual experimental discovery [1], and the predicted value turned out to be in impressive agreement with the experimental result.

After the discovery of the top quark the Higgs boson remains the only missing ingredient of the minimal SM. At the moment, the mass of the Higgs boson, $M_{\rm H}$, can still only rather mildly be constrained by confronting the SM with precision data [2]. An important goal for the future is therefore a further reduction of the experimental and theoretical errors, not only in order to obtain stronger bounds for the Higgs-boson mass but also to achieve improved sensitivity to effects of physics beyond the SM.

Concerning the reduction of the theoretical error due to missing higher-order corrections, in particular a precise prediction for the basic relation between the masses $M_{\rm W}$, $M_{\rm Z}$ of the vector bosons, the Fermi constant G_μ, and the fine structure constant α is of interest. This relation is commonly expressed in terms of the quantity Δr [3] derived from muon decay. With the prospect of the improving accuracy of the measurement of the W-boson mass at LEP2 and the Tevatron, the importance of Δr for testing the electroweak theory becomes even more pronounced.

At the one-loop level the largest contributions to Δr in the SM are the QED induced shift in the fine structure constant, $\Delta\alpha$, and the contribution of the top/bottom weak isospin doublet, which gives rise to a term that grows as $m_{\rm t}^2$. The SM one-loop result for Δr [3] has been supplemented by resummations of certain one-loop contributions [4, 5]. While QCD corrections at $\mathcal{O}(\alpha\alpha_{\rm s})$ [6, 7] and $\mathcal{O}(\alpha\alpha_{\rm s}^2)$ [8] are available and the remaining theoretical uncertainty of the QCD corrections has been estimated to be small [9, 10], the electroweak results at the two-loop level have so far been restricted to expansions in either $m_{\rm t}$ or $M_{\rm H}$. The leading top-quark and Higgs-boson contributions have been evaluated in Refs. [11, 12]. The full Higgs-boson dependence of the leading $G_\mu^2 m_{\rm t}^4$ contribution was calculated in Ref. [13], and recently also the next-to-leading top-quark contributions of $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$ were derived [14].

In the global SM fits to all available data [2], where the $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$ correction obtained in Ref. [14] is not yet included, the error due to missing higher-order corrections has a strong effect on the resulting value of $M_{\rm H}$, shifting the upper bound for $M_{\rm H}$ at 95% C.L. by $\sim +100\,\mathrm{GeV}$. For the different precision observables this error is comparable to the error caused by the parametric uncertainty related to the experimental error of the hadronic contribution to $\alpha(M_{\rm Z}^2)$ [15]. In Refs. [10, 16] it is argued that inclusion of the $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$ contribution will lead to a significant reduction of the error from the missing higher-order corrections.

Both the Higgs-mass dependence of the leading $m_{\rm t}^4$ contribution and the inclusion of the next-to-leading term in the $m_{\rm t}$ expansion turned out to yield important corrections. In order to further settle the issue of theoretical uncertainty due to missing

higher-order corrections therefore a more complete calculation would be desirable, where no expansion in $m_{\rm t}$ or $M_{\rm H}$ is made.

In this paper the Higgs-mass dependence of the two-loop contributions to Δr in the SM is studied. The corrections associated with the fermions are evaluated exactly, i.e. without an expansion in the masses. The results are compared with the expansion up to next-to-leading order in the top-quark mass.

The relation between the vector-boson masses in terms of the Fermi constant reads [3]

$$M_{\rm W}^2 \left(1 - \frac{M_{\rm W}^2}{M_{\rm Z}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu}\,(1 + \Delta r)\,, \tag{1}$$

where the radiative corrections are contained in the quantity Δr. In the context of this paper we treat Δr without resummations, i.e. as being fully expanded up to two-loop order,

$$\Delta r = \Delta r_{(1)} + \Delta r_{(2)} + \mathcal{O}(\alpha^3). \tag{2}$$

The theoretical predictions for Δr are obtained by calculating radiative corrections to muon decay.

From a technical point of view the calculation of top-quark and Higgs-boson contributions to Δr and other processes with light external fermions at low energies requires in particular the evaluation of two-loop self-energies on-shell, i.e. at non-zero external momentum, while vertex and box contributions can mostly be reduced to vacuum integrals. The problems encountered in such a calculation are due to the large number of contributing Feynman diagrams, their complicated tensor structure, the fact that scalar two-loop integrals are in general not expressible in terms of polylogarithmic functions [17], and due to the need for a two-loop renormalization, which has not yet been worked out in full detail.

The methods that we use for carrying out such a calculation have been outlined in Ref. [18]. The generation of the diagrams and counterterm contributions is done with the help of the computer-algebra program *FeynArts* [19]. Making use of two-loop tensor-integral decompositions, the generated amplitudes are reduced to a minimal set of standard scalar integrals with the program *TwoCalc* [20]. The renormalization is performed within the complete on-shell scheme [21], i.e. physical parameters are used throughout. The two-loop scalar integrals are evaluated numerically with one-dimensional integral representations [22]. These allow a very fast calculation of the integrals with high precision without any approximation in the masses. As a check of our calculations, all results presented in this paper have been derived using a general R_ξ gauge, which is specified by one gauge parameter ξ_i, $i = \gamma, Z, W$, for each vector boson, and we have explicitly verified that the gauge parameters actually drop out. As input parameters for our numerical analysis we use unless otherwise stated the following values: $\alpha(M_{\rm Z}^2)^{-1} \equiv (1 - \Delta\alpha)/\alpha(0) = 128.896$, $G_\mu = 1.16639\,10^{-5}\,\mathrm{GeV}^{-2}$, $M_{\rm Z} = 91.1863\,\mathrm{GeV}$, $m_{\rm t} = 175.6\,\mathrm{GeV}$, $m_{\rm b} = 4.7\,\mathrm{GeV}$.

In order to study the Higgs-mass dependence of the two-loop contributions to Δr we consider the subtracted quantity

$$\Delta r_{(2),\mathrm{subtr}}(M_\mathrm{H}) = \Delta r_{(2)}(M_\mathrm{H}) - \Delta r_{(2)}(M_\mathrm{H} = 65\,\mathrm{GeV}), \tag{3}$$

where $\Delta r_{(2)}(M_\mathrm{H})$ denotes the two-loop contribution to Δr.

Potentially large M_H-dependent contributions are the corrections associated with the top quark, since the Yukawa coupling of the Higgs to the top quark is proportional to m_t, and the contributions which are proportional to $\Delta\alpha$. We first consider the Higgs-mass dependence of the two-loop top-quark contributions and calculate the quantity $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H})$ which denotes the contribution of the top/bottom doublet to $\Delta r_{(2),\mathrm{subtr}}(M_\mathrm{H})$.

From the one-particle irreducible diagrams obviously those graphs contribute to $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H})$ that contain both the top and/or bottom quark and the Higgs boson. It is easy to see that only two-point functions enter in this case, since all graphs where the Higgs boson couples to the muon or the electron may safely be neglected. Although no two-loop three-point function enters, there is nevertheless a contribution from the two-loop and one-loop vertex counterterms. If the field renormalization constants of the W boson are included (which cancel in the complete result), the vertex counterterms are separately finite.

The technically most complicated contributions arise from the mass and mixing-angle renormalization. Since it is performed in the on-shell scheme, the evaluation of the W- and Z-boson self-energies are required at non-zero momentum transfer.[2]

Expressed in terms of the one-loop and two-loop contributions to the transverse part of the W-boson self-energy $\Sigma^\mathrm{W}(p^2)$ and the counterterm δZ^{vert} to the $W^-\bar{e}\nu_e$ vertex, the quantity $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H})$ reads

$$\begin{aligned}
\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H}) = \Bigg[& \frac{\Sigma^\mathrm{W}_{(2)}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(2)}(M_\mathrm{W}^2)}{M_\mathrm{W}^2} + 2\delta Z^{\mathrm{vert}}_{(2)} \\
& + 2\frac{\left(\Sigma^\mathrm{W}_{(1),\mathrm{t}}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(1),\mathrm{t}}(M_\mathrm{W}^2)\right)\left(\Sigma^\mathrm{W}_{(1),\mathrm{H}}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(1),\mathrm{H}}(M_\mathrm{W}^2)\right)}{M_\mathrm{W}^4} \\
& + 2\frac{\left(\Sigma^\mathrm{W}_{(1),\mathrm{t}}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(1),\mathrm{t}}(M_\mathrm{W}^2)\right)\delta Z^{\mathrm{vert}}_{(1),\mathrm{H}}}{M_\mathrm{W}^2} \\
& + 2\frac{\left(\Sigma^\mathrm{W}_{(1),\mathrm{H}}(0) - \mathrm{Re}\,\Sigma^\mathrm{W}_{(1),\mathrm{H}}(M_\mathrm{W}^2)\right)\delta Z^{\mathrm{vert}}_{(1),\mathrm{t}}}{M_\mathrm{W}^2} + 2\delta Z^{\mathrm{vert}}_{(1),\mathrm{t}}\delta Z^{\mathrm{vert}}_{(1),\mathrm{H}}\Bigg]_{\mathrm{subtr}},
\end{aligned} \tag{4}$$

where it is understood that the two-loop contributions to the self-energies contain the subloop renormalization. The two-loop terms denote those graphs that contain

[2]It should be noted at this point that in the context of this paper the question is immaterial whether the mass definition of unstable particles at the two-loop level should be based on the real part of the complex pole of the S matrix or on the real pole. For the contributions investigated here both definitions are equivalent.

both the top quark and the Higgs boson, while for the one-loop terms the top-quark and the Higgs-boson contributions are indicated by a subscript. The two-loop vertex counterterm is expressible in terms of the charge counterterm δZ_e and the mixing-angle counterterm $\delta s_{\rm W}/s_{\rm W}$,

$$\delta Z^{\rm vert}_{(2)} = \delta Z_{e,(2)} - \frac{\delta s_{\rm W,(2)}}{s_{\rm W}} + 2\frac{\delta s_{\rm W,(1),t}}{s_{\rm W}}\frac{\delta s_{\rm W,(1),H}}{s_{\rm W}} - \delta Z_{e,(1),{\rm t}}\frac{\delta s_{\rm W,(1),H}}{s_{\rm W}}, \tag{5}$$

and similarly the one-loop vertex counterterm is given by

$$\delta Z^{\rm vert}_{(1)} = \delta Z_{e,(1)} - \frac{\delta s_{\rm W,(1)}}{s_{\rm W}}, \tag{6}$$

with $s_{\rm W}^2 \equiv 1 - c_{\rm W}^2 = 1 - M_{\rm W}^2/M_{\rm Z}^2$. For the Higgs-dependent fermionic contributions the charge counterterm is related to the photon vacuum polarization according to

$$\delta Z_{e,(2)} = -\frac{1}{2}\delta Z_{AA,(2)} = \frac{1}{2}\Pi^{AA}_{(2)}(0), \tag{7}$$

which is familiar from QED. The validity of (7) can also be understood by observing that the contributions considered here are precisely the same as the ones obtained within the framework of the background-field method [23].

The mixing angle counterterm $\delta s_{\rm W,(2)}/s_{\rm W}$ is expressible in terms of the on-shell two-loop W-boson and Z-boson self-energies and additional one-loop contributions,

$$\begin{aligned}\frac{\delta s_{\rm W,(2)}}{s_{\rm W}} = {} & -\frac{1}{2}\frac{c_{\rm W}^2}{s_{\rm W}^2}\left(\frac{{\rm Re}\,\Sigma^{\rm W}_{(2)}(M_{\rm W}^2)}{M_{\rm W}^2} - \frac{{\rm Re}\,\Sigma^{\rm ZZ}_{(2)}(M_{\rm Z}^2)}{M_{\rm Z}^2}\right) \\ & - \frac{\delta s_{\rm W,(1),t}}{s_{\rm W}}\frac{{\rm Re}\,\Sigma^{\rm ZZ}_{(1),{\rm H}}(M_{\rm Z}^2)}{M_{\rm Z}^2} - \frac{\delta s_{\rm W,(1),H}}{s_{\rm W}}\frac{{\rm Re}\,\Sigma^{\rm ZZ}_{(1),{\rm t}}(M_{\rm Z}^2)}{M_{\rm Z}^2} - \frac{\delta s_{\rm W,(1),t}}{s_{\rm W}}\frac{\delta s_{\rm W,(1),H}}{s_{\rm W}},\end{aligned} \tag{8}$$

where

$$\frac{\delta s_{\rm W,(1)}}{s_{\rm W}} = -\frac{1}{2}\frac{c_{\rm W}^2}{s_{\rm W}^2}\left(\frac{{\rm Re}\,\Sigma^{\rm W}_{(1)}(M_{\rm W}^2)}{M_{\rm W}^2} - \frac{{\rm Re}\,\Sigma^{\rm ZZ}_{(1)}(M_{\rm Z}^2)}{M_{\rm Z}^2}\right). \tag{9}$$

In (4)–(6) the field renormalization constants of the W boson have been omitted. In our calculation of $\Delta r^{\rm top}_{(2),{\rm subtr}}(M_{\rm H})$ we have explicitly kept the field renormalization constants of all internal fields and have checked that they actually cancel in the final result.

The result for $\Delta r^{\rm top}_{(2),{\rm subtr}}(M_{\rm H})$ is shown in Fig. 1 for various values of $m_{\rm t}$. The Higgs-boson mass is varied in the interval $65\,{\rm GeV} \leq M_{\rm H} \leq 1\,{\rm TeV}$. The change in $\Delta r^{\rm top}_{(2),{\rm subtr}}(M_{\rm H})$ over this interval is about 0.001, which corresponds to a shift in $M_{\rm W}$ of about 20 MeV. It is interesting to note that the absolute value of the correction is maximal just in the region of $m_{\rm t} = 175\,{\rm GeV}$, i.e. for the physical value of the top-quark mass. For $m_{\rm t} \sim 175\,{\rm GeV}$ the correction $\Delta r^{\rm top}_{(2),{\rm subtr}}(M_{\rm H})$ amounts to about 10% of the one-loop contribution, $\Delta r_{(1),{\rm subtr}}(M_{\rm H})$, which is defined in analogy to (3).

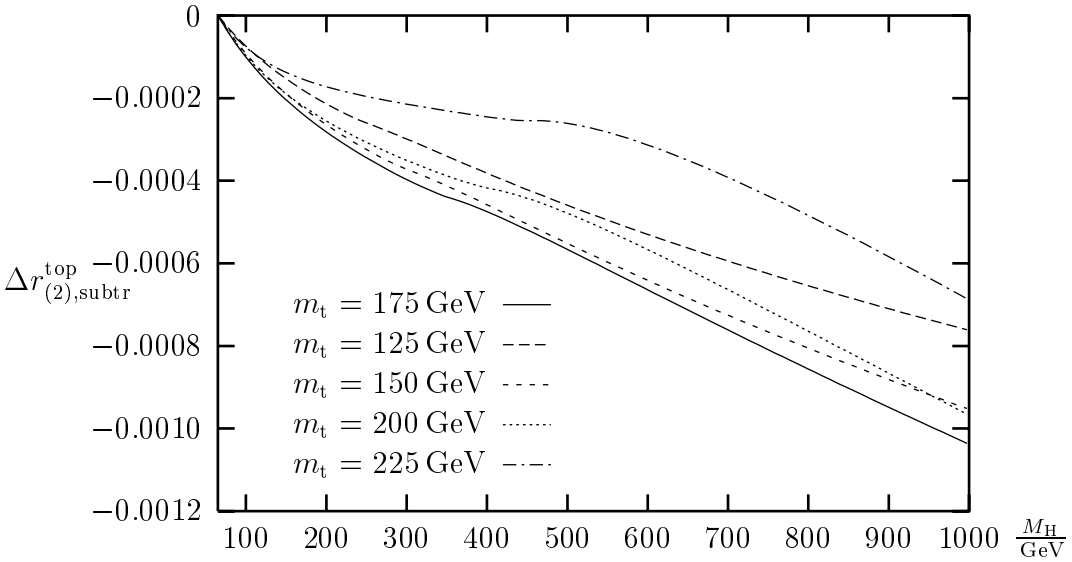

Figure 1: Two-loop top-quark contribution to Δr subtracted at $M_{\rm H} = 65\,\rm GeV$.

The other $M_{\rm H}$-dependent two-loop correction that is expected to be sizable is the contribution of the terms proportional to $\Delta\alpha$. It reads

$$\begin{aligned}\Delta r^{\Delta\alpha}_{(2),\rm subtr}(M_{\rm H}) &= 2\Delta\alpha \left[\frac{\Sigma^{\rm W}_{(1),\rm H}(0) - \mathrm{Re}\,\Sigma^{\rm W}_{(1),\rm H}(M^2_{\rm W})}{M^2_{\rm W}} - 2\frac{\delta s_{\rm W,(1),H}}{s_{\rm W}}\right]_{\rm subtr} \\ &= 2\Delta\alpha\,\Delta r_{(1),\rm subtr}(M_{\rm H}),\end{aligned} \tag{10}$$

and can easily be obtained by a proper resummation of one-loop terms [5].

The remaining fermionic contribution, $\Delta r^{\rm lf}_{(2),\rm subtr}$, is the one of the light fermions, i.e. of the leptons and of the quark doublets of the first and second generation, which is not contained in $\Delta\alpha$. Its structure is analogous to (4), but due to the negligible coupling of the light fermions to the Higgs boson much less diagrams contribute. The scalar two-loop integrals needed for the light-fermion contribution can be solved analytically in terms of polylogarithmic functions. They can be found in Ref. [24].

The total result for the one-loop and fermionic two-loop contributions to Δr, subtracted at $M_{\rm H} = 65\,\rm GeV$, reads

$$\Delta r_{\rm subtr} \equiv \Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr} + \Delta r^{\Delta\alpha}_{(2),\rm subtr} + \Delta r^{\rm lf}_{(2),\rm subtr}. \tag{11}$$

It is shown in Fig. 2, where separately also the one-loop contribution $\Delta r_{(1),\rm subtr}$, as well as $\Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr}$, and $\Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr} + \Delta r^{\Delta\alpha}_{(2),\rm subtr}$ are shown for $m_{\rm t} = 175.6\,\rm GeV$. In Tab. 1 numerical values for the different contributions are given for several values of $M_{\rm H}$. It can be seen that the higher-order contributions $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ and $\Delta r^{\Delta\alpha}_{(2),\rm subtr}(M_{\rm H})$ are of about the same size and to a large extent

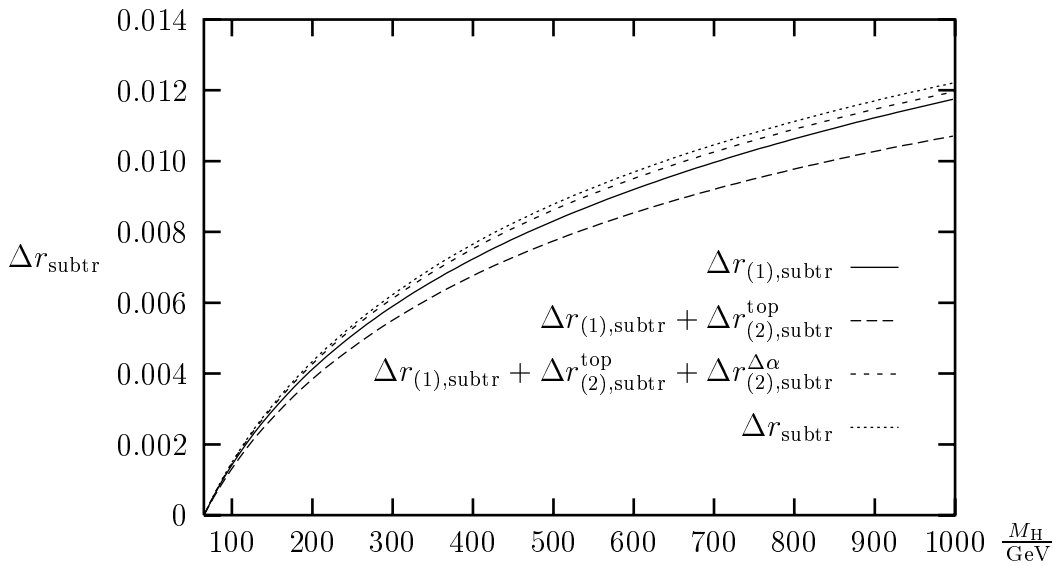

Figure 2: One-loop and two-loop contributions to Δr subtracted at $M_{\rm H} = 65\,\text{GeV}$. $\Delta r_{\rm subtr}$ is the result for the full one-loop and fermionic two-loop contributions to Δr, as defined in the text.

cancel each other. The light-fermion contributions which are not contained in $\Delta\alpha$ add a relatively small correction. Over the full range of the Higgs-boson mass it amounts to about 4 MeV. In total, the inclusion of the higher-order contributions discussed here leads to a slight increase in the sensitivity to the Higgs-boson mass compared to the pure one-loop result.

Regarding the remaining Higgs-mass dependence of Δr at the two-loop level, there are only purely bosonic corrections left, which contain no specific source of enhancement. They can be expected to yield a contribution to $\Delta r_{(2),\rm subtr}(M_{\rm H})$ of about the same size as $\left(\Delta r^{\rm bos}_{(1)}(M_{\rm H})\right)^2\Big|_{\rm subtr}$, where $\Delta r^{\rm bos}_{(1)}$ denotes the bosonic contribution to Δr at the one-loop level. The contribution of $\left(\Delta r^{\rm bos}_{(1)}(M_{\rm H})\right)^2\Big|_{\rm subtr}$ amounts to only about 10% of $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ corresponding to a shift of about 2 MeV in the W-boson mass. This estimate agrees well with the values obtained for the Higgs-mass dependence from the formula in the second paper of Ref. [7] for the leading term proportional to $M_{\rm H}^2$ in an asymptotic expansion for large Higgs-boson mass. The Higgs-mass dependence of the term proportional to $M_{\rm H}^2$ amounts to less than 15% of $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ for reasonable values of $M_{\rm H}$.

The result for $\Delta r^{\rm top,\Delta\alpha}_{\rm subtr} \equiv \Delta r_{(1),\rm subtr} + \Delta r^{\rm top}_{(2),\rm subtr} + \Delta r^{\Delta\alpha}_{(2),\rm subtr}$ can be compared to the result obtained via an expansion in $m_{\rm t}$ up to next-to-leading order, i.e. $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$ [14, 16]. The results for $M_{\rm W}$ as a function of $M_{\rm H}$ according to this

$M_{\rm H}$/ GeV	$\Delta r_{(1),\rm subtr}/10^{-3}$	$\Delta r^{\rm top}_{(2),\rm subtr}/10^{-3}$	$\Delta r^{\Delta\alpha}_{(2),\rm subtr}/10^{-3}$	$\Delta r_{\rm subtr}/10^{-3}$
65	0	0	0	0
100	1.42	−0.101	0.148	1.49
200	4.12	−0.282	0.431	4.34
300	5.90	−0.397	0.620	6.23
400	7.24	−0.474	0.762	7.66
500	8.31	−0.565	0.876	8.78
600	9.20	−0.663	0.971	9.69
1000	11.8	−1.04	1.25	12.2

Table 1: The dependence of one-loop and two-loop contributions to Δr on the Higgs-boson mass for $m_{\rm t} = 175.6\,\text{GeV}$ (see text).

$M_{\rm H}$/ GeV	65	100	300	600	1000
$M_{\rm W}$/ GeV	80.4819	80.4584	80.3837	80.3294	80.2901

Table 2: The results for $M_{\rm W}$ as a function of $M_{\rm H}$ (without QCD corrections; $m_{\rm t} = 175.6\,\text{GeV}$) obtained via an expansion up to next-to-leading order in $m_{\rm t}$ [25].

expansion (without QCD corrections; $m_{\rm t} = 175.6\,\text{GeV}$) are given in Tab. 2 [25]. Extracting from Tab. 2 the corresponding values of Δr and subtracting at $M_{\rm H} = 65\,\text{GeV}$ yields the values $\Delta r^{\rm top,\Delta\alpha,expa}_{\rm subtr}(M_{\rm H})$ as results of the expansion in $m_{\rm t}$. These are compared to the exact result $\Delta r^{\rm top,\Delta\alpha}_{\rm subtr}(M_{\rm H})$ in Tab. 3. In the last column of Tab. 3 the approximate shift in $M_{\rm W}$ is given which corresponds to the difference between exact result and expansion. The results agree within about 30% of $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$, which amounts to a difference in $M_{\rm W}$ of up to about 4 MeV.

In Tab. 4 the shift in $M_{\rm W}$ corresponding to $\Delta r_{\rm subtr}(M_{\rm H})$, i.e. the change in the theoretical prediction for $M_{\rm W}$ when varying the Higgs-boson mass from 65 GeV to 1 TeV, is shown for three values of the top-quark mass, $m_{\rm t} = 170, 175, 180\,\text{GeV}$. The dependence on the precise value of $m_{\rm t}$ is rather mild, which is expected from the fact that $m_{\rm t}$ enters here only at the two-loop level and that $\Delta r^{\rm top}_{(2),\rm subtr}(M_{\rm H})$ has a local maximum in the region of $m_{\rm t} = 175\,\text{GeV}$ (see Fig. 1). From the shift in $M_{\rm W}$ given in Tab. 4 the theoretical prediction for the absolute value of $M_{\rm W}$ can be obtained using as input one value of $M_{\rm W}$ for a given $M_{\rm H}$. From Ref. [26] we infer for the subtraction point $M_{\rm H} = 65\,\text{GeV}$ the corresponding values $M_{\rm W} = 80.374, 80.404, 80.435\,\text{GeV}$ for $m_{\rm t} = 170, 175, 180\,\text{GeV}$, respectively. The accuracy of this subtraction point, being taken from the expansion up to $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$, is of course lower than the accuracy of the shift in $M_{\rm W}$ as given in Tab. 4.

$M_{\rm H}/\,{\rm GeV}$	$\Delta r^{\rm top,\Delta\alpha}_{\rm subtr}/10^{-3}$	$\Delta r^{\rm top,\Delta\alpha,expa}_{\rm subtr}/10^{-3}$	$\delta M_{\rm W}/\,{\rm MeV}$
65	0	0	0
100	1.48	1.52	0.6
300	6.16	6.32	2.5
600	9.56	9.79	3.6
1000	12.0	12.3	4.1

Table 3: Comparison between the exact result, $\Delta r^{\rm top,\Delta\alpha}_{\rm subtr}(M_{\rm H})$, and the result of an expansion up to next-to-leading order in $m_{\rm t}$, $\Delta r^{\rm top,\Delta\alpha,expa}_{\rm subtr}(M_{\rm H})$. In the last column the approximate shift in $M_{\rm W}$ is displayed which corresponds to the difference between the two results.

$M_{\rm H}/\,{\rm GeV}$	65	100	200	300	400	500	600	1000
$\Delta M_{\rm W}(M_{\rm H})/\,{\rm MeV},\ m_{\rm t} = 170\,{\rm GeV}$	0	-22.6	-65.8	-94.5	-116	-133	-147	-185
$\Delta M_{\rm W}(M_{\rm H})/\,{\rm MeV},\ m_{\rm t} = 175\,{\rm GeV}$	0	-22.8	-66.3	-95.2	-117	-134	-148	-187
$\Delta M_{\rm W}(M_{\rm H})/\,{\rm MeV},\ m_{\rm t} = 180\,{\rm GeV}$	0	-23.0	-66.8	-96.0	-118	-135	-149	-188

Table 4: The shift in the theoretical prediction for $M_{\rm W}$ caused by varying the Higgs-boson mass in the interval $65\,{\rm GeV} \leq M_{\rm H} \leq 1\,{\rm TeV}$ for three values of $m_{\rm t}$.

In summary, we have discussed the Higgs-mass dependence of the two-loop corrections to Δr by considering the subtracted quantity $\Delta r_{\rm subtr}(M_{\rm H}) = \Delta r(M_{\rm H}) - \Delta r(M_{\rm H} = 65\,{\rm GeV})$. Exact results have been presented for the Higgs-dependent fermionic contributions, i.e. no expansion in the top-quark and the Higgs-boson mass has been made. The contribution associated with the top quark has been compared with the result of an expansion up to next-to-leading order in $m_{\rm t}$. Agreement within about 30% of the two-loop top-quark correction has been found, which corresponds to a difference in $M_{\rm W}$ of about 4 MeV in the range $65\,{\rm GeV} \leq M_{\rm H} \leq 1\,{\rm TeV}$ of the Higgs-boson mass. The Higgs-dependence of the light-fermion contributions leads to a shift of $M_{\rm W}$ of up to 4 MeV. The only missing part in the Higgs-dependence of Δr at the two-loop level are the purely bosonic contributions, which have been estimated to yield a relatively small correction of up to about 2 MeV in the W-boson mass. Considering the envisaged experimental error of $M_{\rm W}$ from the measurements at LEP2 and the Tevatron of ~ 20 MeV, we conclude that the theoretical uncertainties due to unknown higher-order corrections in the Higgs-mass dependence of Δr are now under control.

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