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Precise Predictions for the W-Boson Mass[1]

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Abstract

Recent results for higher-order corrections to the relation between the vector-boson masses in the Standard Model and Supersymmetry are summarized. In the Standard Model, the Higgs-mass dependence of the two-loop contributions to Δr is studied. Exact results are given for the Higgs-dependent two-loop corrections associated with the fermions, i.e. no expansion in the top-quark and the Higgs-boson mass is made. The results for the top quark are compared with results of an expansion up to next-to-leading order in the top-quark mass. Agreement is found within 30% of the two-loop result. In Supersymmetry, the two-loop QCD corrections to the stop- and sbottom-loop contributions to the ρ parameter are presented. The two-loop corrections modify the one–loop contribution by up to 30%; the gluino decouples for large masses. Contrary to the SM case where the QCD corrections are negative and screen the one-loop value, the corresponding corrections in the supersymmetric case are in general positive, increasing the sensitivity in the search for scalar quarks through their virtual effects in high-precision electroweak observables.

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1 Introduction

With the prospect of the improving accuracy of the measurement of the W-boson mass at LEP2 and the Tevatron, the importance of the basic relation between the masses M_{W}, M_{Z} of the vector bosons, the Fermi constant G_μ and the fine structure constant α for testing the Standard Model (SM) and extensions of it, most prominently the Minimal Supersymmetric Standard Model (MSSM), becomes even more pronounced. This relation is commonly expressed in terms of the quantity Δr [1] derived from muon decay. After the discovery of the top quark [2], whose mass had already successfully been predicted by confronting the electroweak theory with the precision data, an important goal for the future is to further constrain the mass of the Higgs boson, M_{H}, for which at the moment only rather mild bounds exist (see e.g. Ref. [3]). In order to improve on this situation, and also to achieve a higher sensitivity to effects of physics beyond the SM, a further reduction of the experimental and theoretical errors is necessary.

Concerning the reduction of the theoretical error due to missing higher-order corrections, in particular a precise prediction for Δr is of interest. At the one-loop level the largest contributions to Δr in the SM are the QED induced shift in the fine structure constant, $\Delta\alpha$, and the contribution of the top/bottom weak isospin doublet, which gives rise to a term that grows as m_{t}^2. This contribution enters Δr via the ρ parameter [4], which measures the relative strength of the neutral to charged current processes at zero momentum-transfer. The SM one-loop result for Δr [1] has been supplemented by resummations of certain one-loop contributions [5, 6]. While QCD corrections at $\mathcal{O}(\alpha\alpha_{\mathrm{s}})$ [7, 8] and $\mathcal{O}(\alpha\alpha_{\mathrm{s}}^2)$ [9] are available, the electroweak results at the two-loop level have so far been restricted to expansions in either m_{t} or M_{H}. The leading top-quark and Higgs-boson contributions were evaluated in Refs. [10, 11]. The full Higgs-boson dependence of the leading $G_\mu^2 m_{\mathrm{t}}^4$ contribution was calculated in Ref. [12], and recently also the next-to-leading top-quark contributions of $\mathcal{O}(G_\mu^2 m_{\mathrm{t}}^2 M_{\mathrm{Z}}^2)$ were derived [13].

In the global SM fits to all available data (see e.g. Ref. [3]), where the $\mathcal{O}(G_\mu^2 m_{\mathrm{t}}^2 M_{\mathrm{Z}}^2)$ correction obtained in Ref. [13] is not yet included, the error due to missing higher-order corrections has a strong effect on the resulting value of M_{H}, shifting the upper bound for M_{H} at 95% C.L. by $\sim +100\,\mathrm{GeV}$. In Refs. [14] it is argued that inclusion of the $\mathcal{O}(G_\mu^2 m_{\mathrm{t}}^2 M_{\mathrm{Z}}^2)$ will lead to a significant reduction of this error.

Since both the Higgs-mass dependence of the leading m_{t}^4 contribution and the inclusion of the next-to-leading term in the m_{t} expansion turned out to yield important corrections, in order to further settle the issue of theoretical uncertainty due to missing higher-order corrections a more complete calculation would be desirable, where no expansion in m_{t} or M_{H} is made.

In the MSSM, the one-loop result for Δr is known [15]. The most important supersymmetric (SUSY) contribution is that of the stop and sbottom loops to the ρ parameter [16]. If there is a large splitting between the masses of these particles, in analogy to the SM case the contribution will grow with the squared mass of the heaviest scalar quark and can be sizable. In order to treat the SUSY loop contributions to the electroweak observables at the same level of accuracy as the standard contribution, higher-order corrections should be incorporated. In particular the QCD corrections, which because of the large value of the strong coupling constant can be rather important, are of interest.

In this article recent results obtained in the SM and the MSSM at the two-loop level are summarized. In the SM, the M_{H}-dependence of the two-loop contributions to Δr is studied and the corrections associated with the fermions are evaluated exactly [17, 18], i.e. without an expansion in the masses. In the MSSM, results for the two-loop QCD corrections to the ρ parameter are presented [19].

2 Higgs-mass dependence of two-loop corrections to Δr

The correlation between the vector-boson masses in terms of the Fermi constant reads [1]

$$M_{\mathrm{W}}^2 \left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu} \left(1 + \Delta r\right), \tag{1}$$

where the radiative corrections are contained in the quantity Δr. In the context of this paper we treat Δr without resummations, i.e. as being fully expanded up to two-loop order,

$$\Delta r = \Delta r_{(1)} + \Delta r_{(2)} + \mathcal{O}(\alpha^3). \tag{2}$$

The theoretical predictions for Δr are obtained by calculating radiative corrections to muon decay.

From a technical point of view the calculation of top-quark and Higgs-boson contributions to Δr and other processes with light external fermions at low energies requires in particular the evaluation of two-loop self-energies on-shell, i.e. at non-zero external momentum, while vertex and box contributions can mostly be reduced to vacuum integrals. The problems encountered in such a calculation are due to the large number of contributing Feynman diagrams, their complicated tensor structure, the fact that scalar two-loop integrals are in general not expressible in terms of polylogarithmic functions [20], and due to the need for a two-loop renormalization, which has not yet been worked out in full detail.

The methods that we use for carrying out such a calculation have been outlined in Ref. [17]. The generation of the diagrams and counterterm contributions is done with the help of the computer-algebra program *FeynArts* [21]. Making use of two-loop tensor-integral decompositions, the generated amplitudes are reduced to a minimal set of standard scalar integrals with the program *TwoCalc* [22]. The renormalization is performed within the complete on-shell scheme (see e.g. Ref. [23]), i.e. physical parameters are used throughout. The two-loop scalar integrals are evaluated numerically with one-dimensional integral representations [24]. These allow a very fast calculation of the integrals with high precision without any approximation in the masses.

As an application, we study here the Higgs-mass dependence of different two-loop contributions to Δr. To this end we consider the subtracted quantity

$$\Delta r_{(2),\mathrm{subtr}}(M_{\mathrm{H}}) = \Delta r_{(2)}(M_{\mathrm{H}}) - \Delta r_{(2)}(M_{\mathrm{H}} = 65\,\mathrm{GeV}), \tag{3}$$

where $\Delta r_{(2)}(M_{\mathrm{H}})$ denotes the two-loop contribution to Δr.

2.1 Higgs-mass dependence of two-loop top-quark contributions

Potentially large M_{H}-dependent contributions are the corrections associated with the top quark, since the Yukawa coupling of the Higgs to the top quark is proportional to m_{t}, and the contributions which are proportional to $\Delta\alpha$. We first consider the Higgs-mass dependence of the two-loop top-quark contributions and calculate the quantity $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ which denotes the contribution of the top/bottom doublet to $\Delta r_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$.

From the one-particle irreducible diagrams obviously those graphs contribute to $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ that contain both the top quark and the Higgs boson. It is easy to see that only two-point functions enter in this case, since all graphs where the Higgs boson couples to the muon or the electron may safely be neglected. Although no two-loop three-point function enters, there is nevertheless a contribution from the two-loop and one-loop vertex counterterms. If the field renormalization constants of the W boson are included (which cancel in the complete result), the vertex counterterms are separately finite.

Expressed in terms of the one-loop and two-loop contributions to the transverse part of the W-boson self-energy $\Sigma^{\mathrm{W}}(p^2)$ and the counterterm δZ^{vert} to the $W^-\bar{e}\nu_e$ vertex the quantity $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ reads

$$\begin{aligned}
\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}}) = \Bigg[& \frac{\Sigma^{\mathrm{W}}_{(2)}(0) - \mathrm{Re}\,\Sigma^{\mathrm{W}}_{(2)}(M_{\mathrm{W}}^2)}{M_{\mathrm{W}}^2} + 2\delta Z^{\mathrm{vert}}_{(2)} \\
& + 2\frac{\left(\Sigma^{\mathrm{W}}_{(1),\mathrm{t}}(0) - \mathrm{Re}\,\Sigma^{\mathrm{W}}_{(1),\mathrm{t}}(M_{\mathrm{W}}^2)\right)\left(\Sigma^{\mathrm{W}}_{(1),\mathrm{H}}(0) - \mathrm{Re}\,\Sigma^{\mathrm{W}}_{(1),\mathrm{H}}(M_{\mathrm{W}}^2)\right)}{M_{\mathrm{W}}^4} \\
& + 2\frac{\left(\Sigma^{\mathrm{W}}_{(1),\mathrm{t}}(0) - \mathrm{Re}\,\Sigma^{\mathrm{W}}_{(1),\mathrm{t}}(M_{\mathrm{W}}^2)\right)\delta Z^{\mathrm{vert}}_{(1),\mathrm{H}}}{M_{\mathrm{W}}^2} \\
& + 2\frac{\left(\Sigma^{\mathrm{W}}_{(1),\mathrm{H}}(0) - \mathrm{Re}\,\Sigma^{\mathrm{W}}_{(1),\mathrm{H}}(M_{\mathrm{W}}^2)\right)\delta Z^{\mathrm{vert}}_{(1),\mathrm{t}}}{M_{\mathrm{W}}^2} + 2\delta Z^{\mathrm{vert}}_{(1),\mathrm{t}}\delta Z^{\mathrm{vert}}_{(1),\mathrm{H}}\Bigg]_{\mathrm{subtr}},
\end{aligned} \tag{4}$$

where it is understood that the two-loop contributions to the self-energies contain the subloop renormalization. The two-loop terms denote those graphs that contain both the top quark and the Higgs boson, while for the one-loop terms the top-quark and the Higgs-boson contributions are indicated by a subscript. The two-loop vertex counterterm is expressible in terms of the charge counterterm δZ_e and the mixing-angle counterterm $\delta s_{\mathrm{W}}/s_{\mathrm{W}}$,

$$\delta Z^{\mathrm{vert}}_{(2)} = \delta Z_{e,(2)} - \frac{\delta s_{\mathrm{W},(2)}}{s_{\mathrm{W}}} + 2\frac{\delta s_{\mathrm{W},(1),\mathrm{t}}}{s_{\mathrm{W}}}\frac{\delta s_{\mathrm{W},(1),\mathrm{H}}}{s_{\mathrm{W}}} - \delta Z_{e,(1),\mathrm{t}}\frac{\delta s_{\mathrm{W},(1),\mathrm{H}}}{s_{\mathrm{W}}}, \tag{5}$$

and analogously for the one-loop vertex counterterm. For the considered contributions the charge counterterm is related to the photon vacuum polarization according to [25]

$$\delta Z_{e,(2)} = -\frac{1}{2}\delta Z_{AA,(2)} = \frac{1}{2}\Pi^{AA}_{(2)}(0), \tag{6}$$

and similarly to the one-loop case the mixing angle counterterm $\delta s_{\mathrm{W},(2)}/s_{\mathrm{W}}$ is expressible in terms of the on-shell two-loop W-boson and Z-boson self-energies and additional one-loop contributions [18]. In (4) and (5) the field renormalization constants of the W boson have been omitted. In our calculation of $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ we have explicitly kept the field renormalization constants of all internal fields and have checked that they actually cancel in the final result.

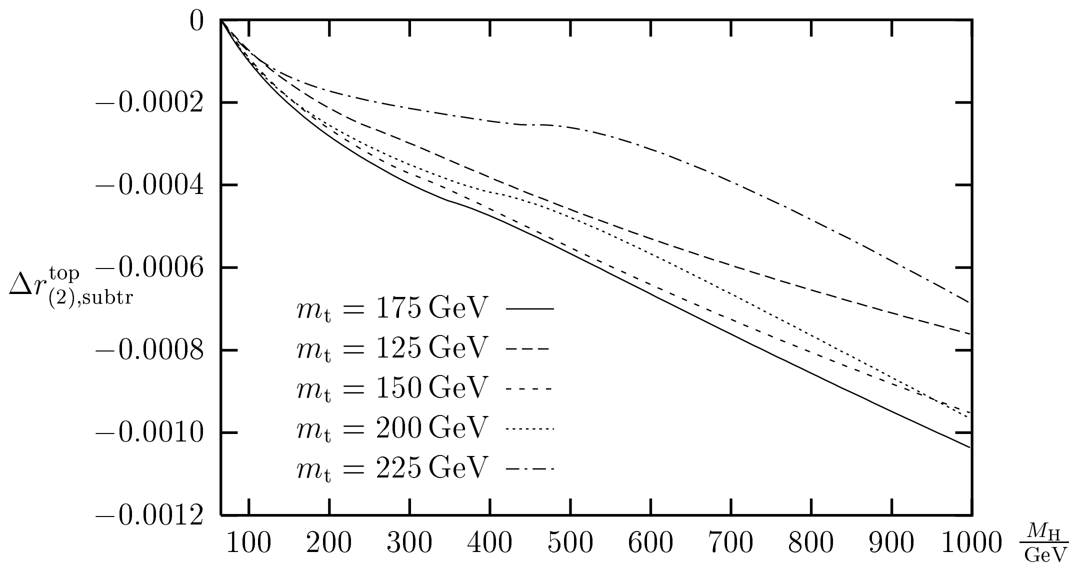

Figure 1: Two-loop top-quark contribution to Δr subtracted at $M_{\mathrm{H}} = 65\,\mathrm{GeV}$.

The result for $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ is shown in Fig. 1 for various values of m_{t}. The Higgs-boson mass is varied in the interval $65\,\mathrm{GeV} \leq M_{\mathrm{H}} \leq 1\,\mathrm{TeV}$. The change in $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ over this interval is about 0.001, which corresponds to a shift in M_{W} of about 20 MeV. It is interesting to note that the absolute value of the correction is maximal just in the region of $m_{\mathrm{t}} = 175\,\mathrm{GeV}$, i.e. for the physical value of the top-quark mass. For $m_{\mathrm{t}} \sim 175\,\mathrm{GeV}$ the correction $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_{\mathrm{H}})$ amounts to about 10% of the one-loop contribution, $\Delta r_{(1),\mathrm{subtr}}(M_{\mathrm{H}})$, which is defined in analogy to (3).

2.2 Higgs-mass dependence of the other fermionic contributions

The other M_{H}-dependent two-loop correction that is expected to be sizable is the contribution of the terms proportional to $\Delta\alpha$. It reads

$$\begin{aligned}\Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}}(M_{\mathrm{H}}) &= 2\Delta\alpha \left[\frac{\Sigma^{\mathrm{W}}_{(1),\mathrm{H}}(0) - \mathrm{Re}\,\Sigma^{\mathrm{W}}_{(1),\mathrm{H}}(M_{\mathrm{W}}^2)}{M_{\mathrm{W}}^2} - 2\frac{\delta s_{\mathrm{W},(1),\mathrm{H}}}{s_{\mathrm{W}}}\right]_{\mathrm{subtr}} \\ &= 2\Delta\alpha\, \Delta r_{(1),\mathrm{subtr}}(M_{\mathrm{H}}), \end{aligned} \tag{7}$$

and can easily be obtained by a proper resummation of one-loop terms [6].

The remaining fermionic contribution, $\Delta r^{\mathrm{lf}}_{(2),\mathrm{subtr}}$, is the one of the light fermions, i.e. of the leptons and of the quark doublets of the first and second generation, which is not contained in $\Delta\alpha$. Its structure is analogous to (4), but due to the negligible coupling of the light fermions to the Higgs boson much less diagrams contribute.

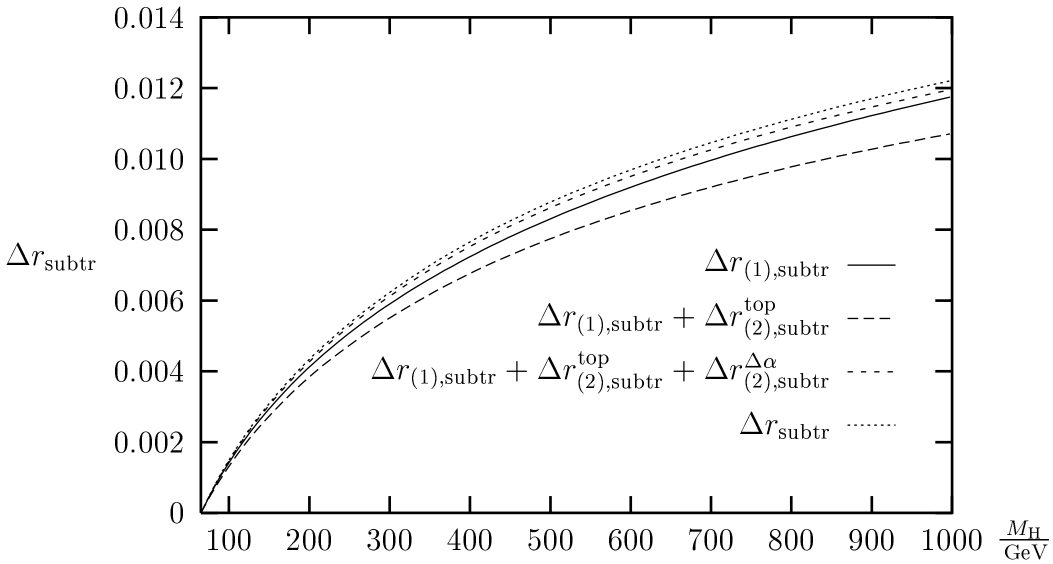

Figure 2: One-loop and two-loop contributions to Δr subtracted at $M_{\mathrm{H}} = 65\,\mathrm{GeV}$. $\Delta r_{\mathrm{subtr}}$ is the result for the full one-loop and fermionic two-loop contributions to Δr, as defined in the text.

The total result for the one-loop and fermionic two-loop contributions to Δr, subtracted at $M_{\mathrm{H}} = 65\,\mathrm{GeV}$, reads

$$\Delta r_{\mathrm{subtr}} \equiv \Delta r_{(1),\mathrm{subtr}} + \Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}} + \Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}} + \Delta r^{\mathrm{lf}}_{(2),\mathrm{subtr}}. \tag{8}$$

It is shown in Fig. 2, where separately also the one-loop contribution $\Delta r_{(1),\mathrm{subtr}}$, as well as $\Delta r_{(1),\mathrm{subtr}} + \Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}$, and $\Delta r_{(1),\mathrm{subtr}} + \Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}} + \Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}}$ are shown for

$m_t = 175.6\,\mathrm{GeV}$. It can be seen that the higher-order contributions $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H})$ and $\Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}}(M_\mathrm{H})$ are of about the same size and to a large extent cancel each other. If the one-loop result had only been supplemented by the contribution (7), which is accessible by resummation of one-loop quantities, but not by the contribution of the irreducible two-loop diagrams, the result for the Higgs-mass dependence would have been misleading. The light-fermion contributions which are not contained in $\Delta\alpha$ add a relatively small correction. Over the full range of the Higgs-boson mass it amounts to about $4\,\mathrm{MeV}$. In total, the inclusion of the higher-order contributions discussed here leads to a slight increase in the sensitivity to the Higgs-boson mass compared to the pure one-loop result.

Regarding the remaining Higgs-mass dependence of Δr at the two-loop level, there are only purely bosonic corrections left, which contain no specific source of enhancement. They can be expected to yield a contribution to $\Delta r_{(2),\mathrm{subtr}}(M_\mathrm{H})$ of about the same size as $\left(\Delta r^{\mathrm{bos}}_{(1)}(M_\mathrm{H})\right)^2\Big|_{\mathrm{subtr}}$, where $\Delta r^{\mathrm{bos}}_{(1)}$ denotes the bosonic contribution to Δr at the one-loop level. The contribution of $\left(\Delta r^{\mathrm{bos}}_{(1)}(M_\mathrm{H})\right)^2\Big|_{\mathrm{subtr}}$ amounts to only about 10% of $\Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}}(M_\mathrm{H})$ corresponding to a shift of about $2\,\mathrm{MeV}$ in the W-boson mass.

2.3 Comparison with an expansion in m_t

The result for $\Delta r^{\mathrm{top},\Delta\alpha}_{\mathrm{subtr}} \equiv \Delta r_{(1),\mathrm{subtr}} + \Delta r^{\mathrm{top}}_{(2),\mathrm{subtr}} + \Delta r^{\Delta\alpha}_{(2),\mathrm{subtr}}$ can be compared to the result obtained via an expansion in m_t up to next-to-leading order, i.e. $\mathcal{O}(G_\mu^2 m_t^2 M_Z^2)$ [13, 14]. From this expansion the results for M_W as a function of M_H read (without QCD corrections; $m_t = 175.6$) [26]

$M_\mathrm{H}/\,\mathrm{GeV}$	65	100	300	600	1000
$M_\mathrm{W}/\,\mathrm{GeV}$	80.4819	80.4584	80.3837	80.3294	80.2901

. (9)

Extracting from (9) the corresponding values of Δr and subtracting at $M_\mathrm{H} = 65\,\mathrm{GeV}$ yields the values $\Delta r^{\mathrm{top},\Delta\alpha,\mathrm{expa}}_{\mathrm{subtr}}(M_\mathrm{H})$ as results of the expansion in m_t. The comparison with the exact result $\Delta r^{\mathrm{top},\Delta\alpha}_{\mathrm{subtr}}(M_\mathrm{H})$ reads

$M_\mathrm{H}/\,\mathrm{GeV}$	$\Delta r^{\mathrm{top},\Delta\alpha}_{\mathrm{subtr}}/10^{-3}$	$\Delta r^{\mathrm{top},\Delta\alpha,\mathrm{expa}}_{\mathrm{subtr}}/10^{-3}$	$\delta M_\mathrm{W}/\,\mathrm{MeV}$
65	0	0	0
100	1.48	1.52	0.6
300	6.16	6.32	2.5
600	9.56	9.79	3.6
1000	12.0	12.3	4.1

, (10)

where in the last column the approximate shift in $M_{\rm W}$ is given that corresponds to the difference between exact result and expansion. The results agree within about 30% of $\Delta r^{\rm top}_{\rm subtr}(M_{\rm H})$, which amounts to a difference in $M_{\rm W}$ of up to about 4 MeV.

3 QCD corrections to the ρ parameter in the MSSM

The leading contributions to the ρ parameter can be written in terms of the transverse parts of the W- and Z-boson self-energies at zero momentum-transfer,

$$\Delta\rho = \frac{\Sigma^{\rm ZZ}(0)}{M_{\rm Z}^2} - \frac{\Sigma^{\rm WW}(0)}{M_{\rm W}^2} \; . \tag{11}$$

In the SM, the contribution of a fermion isodoublet (u, d) to $\Delta\rho$ reads at one-loop order

$$\Delta\rho_0^{\rm SM} = \frac{N_c G_\mu}{8\sqrt{2}\pi^2} F_0\left(m_u^2, m_d^2\right) \; , \tag{12}$$

with the color factor N_c and the function F_0 given by

$$F_0(x, y) = x + y - \frac{2xy}{x-y}\log\frac{x}{y} \; . \tag{13}$$

The function $F_0\left(m_u^2, m_d^2\right)$ vanishes if the u- and d-type quarks are degenerate in mass: $F_0(m_q^2, m_q^2) = 0$; in the limit of large quark mass splitting it is proportional to the heavy quark mass squared: $F_0(m_q^2, 0) = m_q^2$. Therefore, in the SM the only relevant contribution is due to the top/bottom weak isodoublet. Because $m_{\rm t} \gg m_{\rm b}$, one obtains $\Delta\rho_0^{\rm SM} = 3G_\mu m_{\rm t}^2/(8\sqrt{2}\pi^2)$. The two-loop QCD corrections in the SM read [7]:

$$\Delta\rho_1^{\rm SM} = -\Delta\rho_0^{\rm SM}\frac{2}{3}\frac{\alpha_s}{\pi}(1 + \frac{\pi^2}{3}). \tag{14}$$

In SUSY theories, the scalar partners of each SM quark will induce additional contributions. The current eigenstates, $\tilde{q}_L$ and $\tilde{q}_R$, mix to give the mass eigenstates. The mixing angle is proportional to the quark mass and therefore is important only in the case of the third generation scalar quarks [27]. In particular, due to the large value of $m_{\rm t}$, the mixing angle $\theta_{\tilde{\rm t}}$ between $\tilde{t}_L$ and $\tilde{t}_R$ can be very large and lead to a scalar top quark $\tilde{t}_1$ much lighter than the top quark and all the scalar partners of the light quarks [27]. The mixing in the bottom-quark sector can be sizable only in a small area of the SUSY parameter space.

Similarly to the SM case, the contribution of a scalar quark doublet $(\tilde{u}, \tilde{d})$ vanishes if all masses are degenerate. This means that in most SUSY scenarios, where the scalar partners of the light quarks are almost mass degenerate, only the third generation will contribute. Neglecting the mixing in the $\tilde{b}$ sector, $\Delta\rho$ is given at one-loop order by the simple expression [16]

$$\begin{aligned}\Delta\rho_0^{\rm SUSY} = \frac{3G_\mu}{8\sqrt{2}\pi^2}&\Big[-\sin^2\theta_{\tilde{\rm t}}\cos^2\theta_{\tilde{\rm t}} F_0\left(m_{\tilde{\rm t}_1}^2, m_{\tilde{\rm t}_2}^2\right) \\ &+\cos^2\theta_{\tilde{\rm t}} F_0\left(m_{\tilde{\rm t}_1}^2, m_{\tilde{\rm b}_L}^2\right) + \sin^2\theta_{\tilde{\rm t}} F_0\left(m_{\tilde{\rm t}_2}^2, m_{\tilde{\rm b}_L}^2\right)\Big] \, .\end{aligned} \tag{15}$$

In a large area of the parameter space, the stop mixing angle is either very small, $\theta_{\rm t} \sim 0$, or maximal, $\theta_{\rm t} \sim -\pi/4$. The contribution $\Delta\rho_0^{\rm SUSY}$ is shown in Fig. 3 as a function of the common scalar mass $m_{\tilde{q}} = m_{\tilde{\rm t}_{L,R}} = m_{\tilde{\rm b}_L}$ (see e.g. Ref. [28]) for these two scenarios. The contribution can be at the level of a few per mille and therefore within the range of the experimental observability. Relaxing the assumption of a common scalar quark mass, the corrections can become even larger [16].

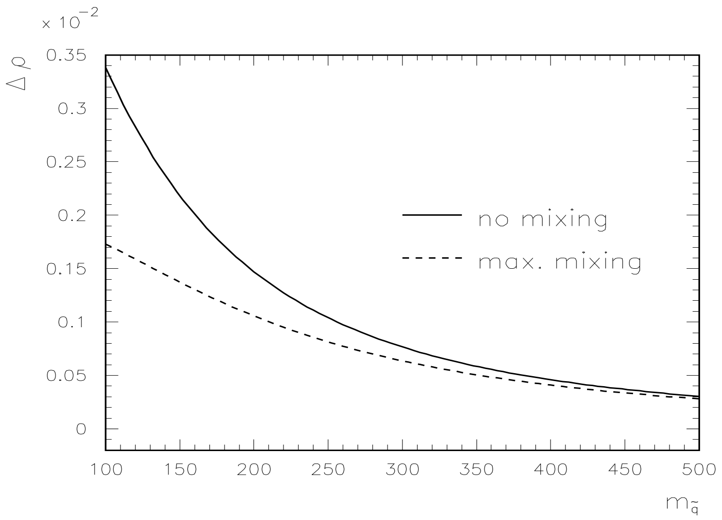

Figure 3: One-loop contribution of the $(\tilde{t}, \tilde{b})$ doublet to $\Delta\rho$ as a function of the common mass $m_{\tilde{q}}$, for $\theta_{\tilde{\rm t}} = 0$ and $\theta_{\tilde{\rm t}} \sim -\pi/4$ (with $\tan\beta = 1.6$ and $m_{\rm LR} = 0$ and 200 GeV, respectively, where $m_{\rm LR}$ is the off-diagonal term in the $\tilde{t}$ mass matrix).

At $\mathcal{O}(\alpha\alpha_s)$, the two-loop Feynman diagrams contributing to the ρ parameter in the MSSM (see Fig. 4) consist of two sets which, at vanishing external momentum and after the inclusion of the counterterms, are separately ultraviolet finite and gauge-invariant. The first one contains diagrams involving only gluon exchange, Fig. 4a; in this case the calculation is similar to the SM, although technically more complicated due to the larger number of diagrams and the presence of $\tilde{q}$ mixing. The diagrams involving the quartic scalar-quark interaction in Fig. 4a either contribute only to the longitudinal component of the self-energies or can be absorbed into the squark mass and mixing-angle renormalization. The renormalization of the mixing-angle is performed in such a way that all transitions from $\tilde{q}_i \leftrightarrow \tilde{q}_j$ which do not depend on the loop-momenta in the two-loop diagrams are canceled; therefore the contribution of the pure scalar quark diagrams in Fig. 4a is completely canceled by the renormalization. The second set of graphs consists of diagrams involving scalar quarks, gluinos as well as quarks, Fig. 4b; in this case the calculation becomes much more complicated due to the even larger number of diagrams and to the presence of up to 5 particles with different masses in the loops.

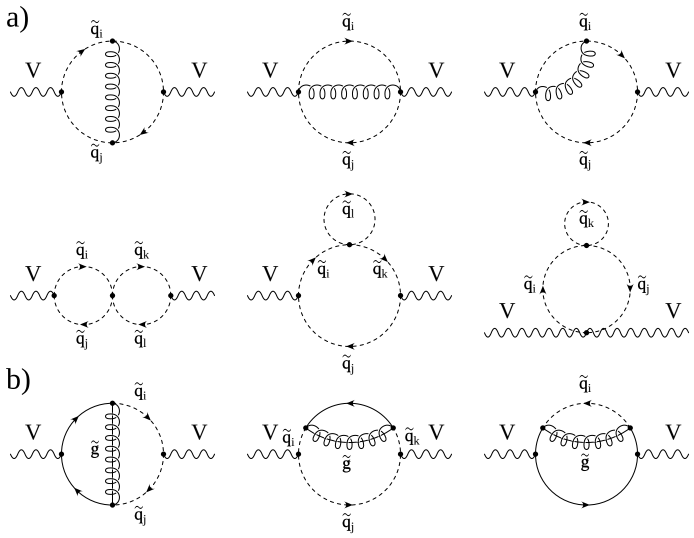

Figure 4: Typical Feynman diagrams for the contribution of scalar quarks and gluinos to the W/Z-boson self-energies at the two-loop level.

In order to discuss our results, let us first concentrate on the contribution of the gluonic corrections, Fig. 4a, and the corresponding counterterms. At the two-loop level, the results for the electroweak gauge-boson self-energies at zero momentum-transfer have very simple analytical expressions. In the case of an isodoublet $(\tilde{u}, \tilde{d})$ where general mixing is allowed, the structure is similar to the one-loop case:

$$\begin{aligned} \Sigma^{\rm WW}(0) &= -\frac{G_\mu M_{\rm W}^2 \alpha_s}{4\sqrt{2}\pi^3} \sum_{i,j=1,2} \left(a_i^{\tilde{u}} a_j^{\tilde{d}}\right)^2 F_1\left(m_{\tilde{u}_i}^2, m_{\tilde{d}_j}^2\right), \\ \Sigma^{\rm ZZ}(0) &= -\frac{G_\mu M_{\rm Z}^2 \alpha_s}{8\sqrt{2}\pi^3} \sum_{\substack{\tilde{q}=\tilde{u},\tilde{d} \\ i,j=1,2}} (a_i^{\tilde{q}} a_j^{\tilde{q}})^2 F_1\left(m_{\tilde{q}_i}^2, m_{\tilde{q}_j}^2\right), \end{aligned} \tag{16}$$

where the factors $a_i^{\tilde{q}}$ are given in terms of the squark mixing angle $\theta_{\tilde{q}}$ as $a_1^{\tilde{q}} = \cos\theta_{\tilde{q}}$ and $a_2^{\tilde{q}} = \sin\theta_{\tilde{q}}$. The two-loop function $F_1(x,y)$ is given in terms of dilogarithms by

$$\begin{aligned} F_1(x,y) &= x + y - 2\frac{xy}{x-y}\log\frac{x}{y}\left[2 + \frac{x}{y}\log\frac{x}{y}\right] \\ &\quad + \frac{(x+y)x^2}{(x-y)^2}\log^2\frac{x}{y} - 2(x-y)\mathrm{Li}_2\left(1 - \frac{x}{y}\right). \end{aligned} \tag{17}$$

This function is symmetric in the interchange of x and y. As in the case of the one-loop function F_0, it vanishes for degenerate masses, $F_1(x,x) = 0$, while in the case of large mass splitting it increases with the heavy scalar quark mass squared: $F_1(x,0) = x(1 + \pi^2/3)$.

From the previous expressions, the contribution of the $(\tilde{t}, \tilde{b})$ doublet to the ρ parameter, including the two-loop gluon exchange and pure scalar quark diagrams, are obtained straightforwardly. In the case where the $\tilde{b}$ mixing is neglected, the SUSY two-loop contribution is given by an expression similar to (15):

$$\Delta\rho_1^{\rm SUSY} = \frac{G_\mu \alpha_s}{4\sqrt{2}\pi^3}\left[-\sin^2\theta_{\tilde{t}}\cos^2\theta_{\tilde{t}} F_1\left(m_{\tilde{t}_1}^2, m_{\tilde{t}_2}^2\right) \right.$$
$$\left. + \cos^2\theta_{\tilde{t}} F_1\left(m_{\tilde{t}_1}^2, m_{\tilde{b}_L}^2\right) + \sin^2\theta_{\tilde{t}} F_1\left(m_{\tilde{t}_2}^2, m_{\tilde{b}_L}^2\right)\right]. \quad (18)$$

The two-loop gluonic SUSY contribution to $\Delta\rho$ is shown in Fig. 5 as a function of the common scalar mass $m_{\tilde{q}}$ for the two scenarios discussed previously: $\theta_{\tilde{t}} = 0$ and $\theta_{\tilde{t}} \simeq -\pi/4$. As can be seen, the two-loop contribution is of the order of 10 to 15% of the one-loop result. Contrary to the SM case (and to many QCD corrections to electroweak processes in the SM, see Ref. [29] for a review) where the two-loop correction screens the one-loop contribution, $\Delta\rho_1^{\rm SUSY}$ has the same sign as $\Delta\rho_0^{\rm SUSY}$. For instance, in the case of degenerate stops with masses $m_{\tilde{t}} \gg m_{\tilde{b}}$, the result is the same as the QCD correction to the (t, b) contribution in the SM, but with opposite sign. The gluonic correction to the contribution of scalar quarks to the ρ parameter will therefore enhance the sensitivity in the search of the virtual effects of scalar quarks in high-precision electroweak measurements.

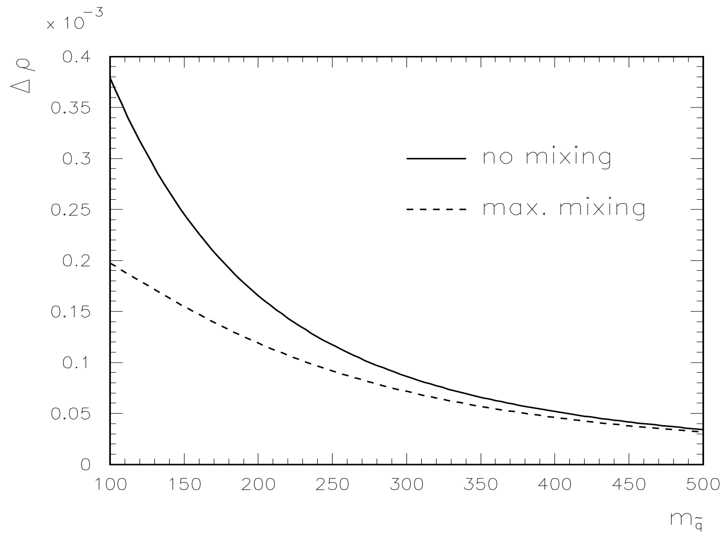

Figure 5: Gluon exchange contribution to the ρ parameter at two-loop order as a function of $m_{\tilde{q}}$ for the scenarios of Fig. 3.

The analytical expressions of the contribution of the two-loop diagrams with gluino exchange, Fig. 4b, to the electroweak gauge-boson self-energies are very complicated even at zero momentum-transfer. Besides the fact that the squark mixing leads to a large number of contributing diagrams, this is mainly due to the presence

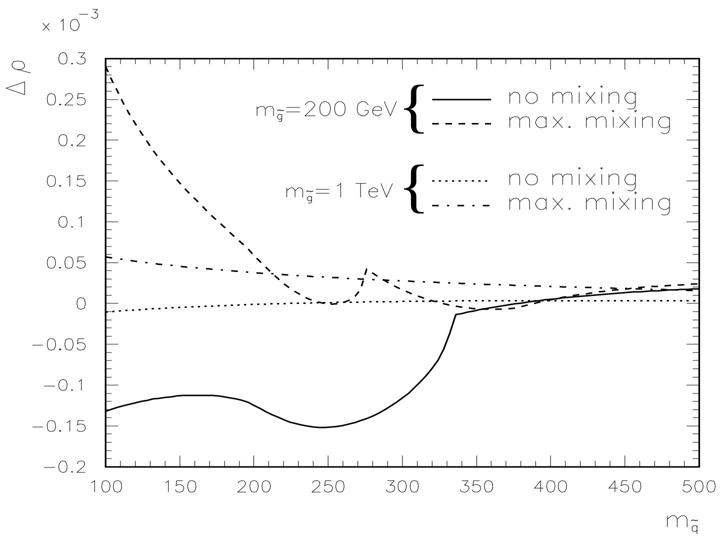

Figure 6: Contribution of the gluino exchange diagrams to $\Delta\rho_1^{\rm SUSY}$ for two values of $m_{\tilde{\rm g}}$ in the scenarios of Fig. 3.

of up to five particles with different masses in the loops. The lengthy expressions will be given elsewhere [30]. It turned out that in general the gluino exchange diagrams give smaller contributions compared to gluon exchange. Only for gluino and squark masses close to the experimental lower bounds they compete with the gluon exchange contributions. In this case, the gluon and gluino contributions add up to $\sim 30\%$ of the one-loop value for maximal mixing (see Fig. 6). For larger values of $m_{\tilde{\rm g}}$, the contribution decreases rapidly since the gluinos decouple for high masses. For vanishing gluino mass, in the limit of exact SUSY, the gluino exchange contribution reads $-\Delta\rho_0^{\rm SM}\frac{8}{3}\frac{\alpha_s}{\pi}$, while as mentioned above in the SUSY limit the gluon exchange contribution of the scalar quarks cancels the one of the quarks.

4 Conclusions

In this article higher-order contributions to the relation between the vector-boson masses in the SM and the MSSM have been discussed. In the SM, the Higgs-mass dependence of the two-loop contribution to Δr has been analyzed. Exact results have been given for the $M_{\rm H}$-dependent corrections associated with the fermions, i.e. no expansion in $m_{\rm t}$, $M_{\rm H}$ and the gauge-boson masses has been made. The size of the contribution associated with the top quark was found to be about 10% of the one-loop result and roughly the same as of the higher-order contributions proportional to $\Delta\alpha$, which enter with opposite sign. These results have been compared with the result of an expansion up to next-to-leading order in $m_{\rm t}$. Agreement within about 30% of the two-loop top-quark correction has been found, which corresponds to a difference in

M_W of about 4 MeV in the range 65 GeV $\leq M_\mathrm{H} \leq$ 1 TeV of the Higgs-boson mass. The Higgs-dependence of the light-fermion contributions leads to a shift of M_W of up to 4 MeV. The remaining Higgs-dependent corrections are purely bosonic and have been estimated to give a relatively small contribution of up to about 2 MeV in the W-boson mass.

In the MSSM, the two-loop $\mathcal{O}(\alpha_s)$ corrections to the squark-loop contributions to the weak gauge-boson self-energies at zero momentum-transfer have been calculated and the QCD correction to the ρ parameter has been derived. The gluonic corrections are of $\mathcal{O}(10\%)$: they are positive and increase the sensitivity in the search for scalar quarks through their virtual effects in high-precision electroweak observables. The gluino contributions are in general smaller except for relatively light gluinos and scalar quarks; the contribution vanishes for large gluino masses.

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