KA-TP-11-1998
hep-ph/9807222

Results for Precision Observables in the Electroweak Standard Model at Two-Loop Order and Beyond

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Abstract

Higher-order contributions to the precision observables Δr, $\sin^2\theta_{\rm eff}$ and Γ_l are discussed. The Higgs-mass dependence of the observables is investigated at the two-loop level, and exact results are derived for the Higgs-dependent two-loop corrections associated with fermions. The top-quark corrections are compared with the results obtained by an expansion in the top-quark mass up to next-to-leading order. For the pure fermion-loop contributions to Δr results up to four-loop order are derived. They allow to investigate the validity of the commonly used resummation of the leading fermionic contributions to Δr.

RESULTS FOR PRECISION OBSERVABLES IN THE ELECTROWEAK STANDARD MODEL AT TWO-LOOP ORDER AND BEYOND*

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(Received July 2, 1998)

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PACS numbers: 12.15.-y, 12.15.Lk, 13.10.+q, 13.35.Bv

1. Introduction

Confronting the predictions of the electroweak Standard Model (SM) with the precision data [1] allows to derive indirect constraints on the mass of the Higgs boson, which is the last missing ingredient of the minimal SM, and also sets the basis for the investigation of possible effects of new physics via radiative corrections. The bounds on the Higgs-boson mass, $M_{\rm H}$, obtained from fitting the SM predictions to the data, are strongly affected by the theoretical uncertainty due to unknown higher-order corrections. In particular the two-loop electroweak corrections have to be under control in order to obtain reliable bounds. At this order, the resummations of the leading one-loop contributions are known [2], the leading and next-to-leading term in an expansion for asymptotically large values of the top-quark mass, $m_{\rm t}$, have been evaluated [3, 4, 5], and also the leading term of an asymptotic expansion in the Higgs-boson mass has been obtained [6]. The terms in the

* Presented at the Zeuthen Workshop on Elementary Particle Physics, Loops and Legs in Gauge Theories, Rheinsberg, April 19–24, 1998.

m_t expansion were found to be sizable, and the next-to-leading term turned out to be about equally large as the (formally) leading term [4]. Exact results have been derived for the Higgs-mass dependence of the fermionic two-loop corrections to muon decay [7]. In the present paper the Higgs-mass dependence of Δr (which is derived from muon decay [8]), of the effective weak mixing angle at the Z-boson resonance, $\sin^2\theta_{\rm eff}$, and of the leptonic width of the Z boson, Γ_l, is briefly discussed. The results are compared with those obtained by the expansion in the top-quark mass. Furthermore the pure fermion-loop contributions, which constitute the leading corrections in one-loop order, are analyzed in higher orders. Exact results for these contributions to Δr are derived up to four-loop order. They are compared with the resummations of the leading one-loop terms according to Ref. [2].

2. Higgs-mass dependence of Δr, $\sin^2\theta_{\rm eff}$ and Γ_l at two-loop order

In order to study the Higgs-mass dependence of the precision observables, we consider subtracted quantities of the form

$$a_{\rm subtr}(M_{\rm H}) = a(M_{\rm H}) - a(M_{\rm H}^0), \quad \text{where } a = \Delta r, \sin^2\theta_{\rm eff}, \Gamma_l, \qquad (1)$$

and Δr is defined by $M_{\rm W}^2(1 - M_{\rm W}^2/M_{\rm Z}^2) = (\pi\alpha)/(\sqrt{2}G_\mu)(1+\Delta r)$. The subtracted quantities $a_{\rm subtr}(M_{\rm H})$ indicate the shift in the precision observables caused by varying the Higgs-boson mass between $M_{\rm H}^0$ and $M_{\rm H}$. In the analysis below we will consider values of $M_{\rm H}$ in the interval $65\,\text{GeV} \leq M_{\rm H} \leq 1\,\text{TeV}$. Allowing for such a wide range of Higgs-boson mass values may seem somewhat over-conservative regarding the fact that the SM fits favor a light Higgs-boson with a mass smaller than about 300 GeV [1]. However, the form of the fit curve and thus the value of the upper bound is affected also by the SM predictions for larger $M_{\rm H}$-values (and also for $M_{\rm H}$-values below the current experimental lower bound of $M_{\rm H}$). Thus accurate theoretical predictions for the observables are needed in a rather wide range of $M_{\rm H}$-values in order to obtain a reliable fit result.

Potentially large $M_{\rm H}$-dependent contributions are the ones associated with the top quark due to the large Yukawa coupling $\rm t\bar{t}H$, and the corrections proportional to $\Delta\alpha$. Further contributions are the ones of the light fermions (except the corrections already contained in $\Delta\alpha$), and purely bosonic contributions. The latter are expected to be rather small [7]. We therefore have focussed on the Higgs-dependent fermionic contributions to the precision observables, for which we have obtained exact two-loop results [7, 9].

The methods used for these calculations have been outlined in Ref. [10]. The generation of the diagrams and counterterm contributions is performed with the help of computer-algebra tools [11]. For the renormalization the

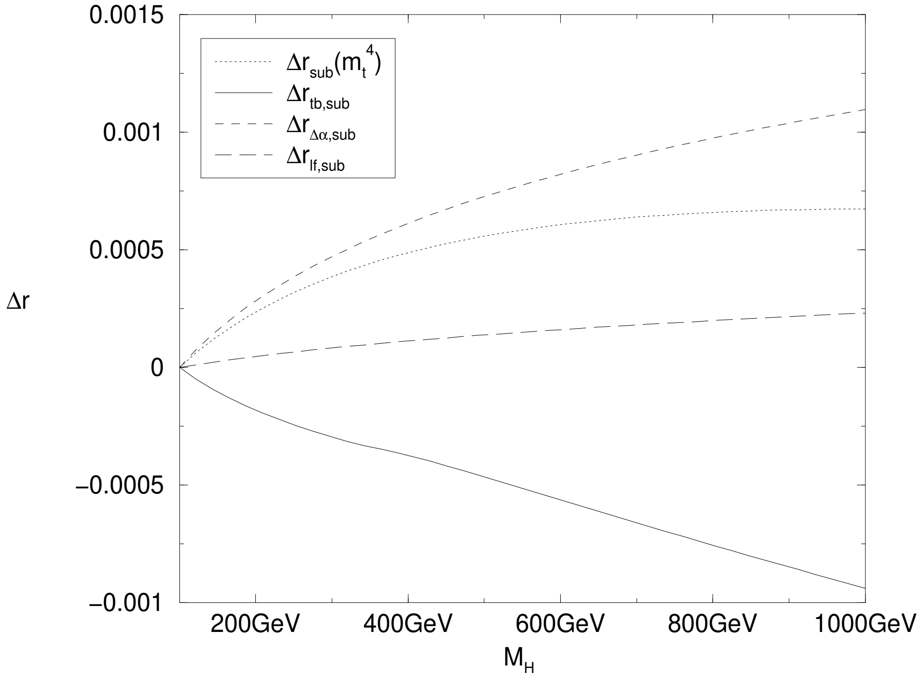

Fig. 1. Higgs-mass dependent fermionic contributions to Δr at two-loop order. The different curves show the contribution from the diagrams involving the top/bottom doublet ($\Delta r_{\rm tbh}$), the contribution proportional to $\Delta\alpha$ ($\Delta r_{\Delta\alpha}$), the contribution of the light fermions ($\Delta r_{\rm lfh}$), and the approximation of the top/bottom correction by the leading term proportional to $m_{\rm t}^4$ ($\Delta r(m_{\rm t}^4)$).

complete on-shell scheme (with the conventions as in Ref. [12]) has been used, i.e. physical parameters are used throughout. The two-loop scalar integrals are evaluated numerically with one-dimensional integral representations [13].

Fig. 1 shows the Higgs-mass dependence of the two-loop corrections to Δr associated with the top/bottom doublet, with $\Delta\alpha$, and with the light fermions. The dotted line furthermore indicates the Higgs-mass dependence of the leading $m_{\rm t}^4$-term [3] in the top/bottom contribution. The top/bottom contribution gives rise to a shift in the W-boson mass of $\Delta M_{\rm W,subtr,(2)}^{\rm top}(M_{\rm H} = 1000\,{\rm GeV}) \sim 16$ MeV, which is about 10% of the one-loop contribution. Its Higgs-mass dependence turns out to be very poorly approximated by the leading $m_{\rm t}^4$-term; the contribution of the latter even enters with a different sign. It can be seen from Fig. 1 that the two-loop contributions to a large extent cancel each other. The contribution of the light fermions yields a shift in $M_{\rm W}$ of up to $\Delta M_{\rm W,subtr,(2)}^{\rm lf}(M_{\rm H} = 1000\,{\rm GeV}) \lesssim 4$ MeV. In total the

$M_{\mathrm{H}}/$ GeV	$M_{\mathrm{W,subtr}}^{\mathrm{top},\Delta\alpha}/$ MeV	$M_{\mathrm{W,subtr}}^{\mathrm{top},\Delta\alpha,\mathrm{DGS}}/$ MeV	$\Delta M_{\mathrm{W}}/$ MeV
65	0	0	0
100	−22	−23	1
300	−93	−98	5
600	−145	−152	7
1000	−183	−191	8

Table 1. The Higgs-mass dependence of M_{W} based on the exact result for the contribution of the top/bottom doublet (left column) and on the result of the expansion in m_{t} [5] (right column).

two-loop contributions lead to a slight increase in the sensitivity of Δr to the Higgs-boson mass compared to the one-loop case.

In Tab. 1 the Higgs-mass dependence of M_{W} based on the exact result for the top/bottom doublet is compared with the results of the expansion in the top-quark mass up to $\mathcal{O}(G_\mu^2 m_{\mathrm{t}}^2 M_{\mathrm{Z}}^2)$ given in Ref. [5] (with the input parameters as in Ref. [5]). Besides the difference in the two-loop results also higher-order effects due to differences in the renormalization procedure and in the treatment of non-leading higher-order corrections enter the comparison (in the comparison performed in Ref. [7] the QCD corrections have been omitted). A discussion about different options for treating these higher-order corrections will be given in Ref. [9]. Over the range of Higgs-mass values from 65 GeV to 1 TeV the difference between the results amounts to about 8 MeV, which corresponds to 50% of the two-loop top/bottom contribution. The difference in the Higgs-mass dependence of 8 MeV is slightly larger than the theoretical uncertainty quoted in Ref. [14] of 2 MeV from electroweak and 5 MeV from QCD corrections. The contribution of the light fermions has not been included in the comparison in Tab. 1, since it is not contained in the result of Ref. [5]. As mentioned above, its Higgs-mass dependence gives rise to a further shift of up to 4 MeV in the considered range of M_{H}-values.

We have performed an analogous analysis also for $\sin^2\theta_{\mathrm{eff}}$ and Γ_l. The detailed results will be presented in a forthcoming publication [9]. For the shift in $\sin^2\theta_{\mathrm{eff}}$ induced by varying M_{H} from 65 GeV to 1 TeV we find (leaving out again the light-fermion contribution) $\sin^2\theta_{\mathrm{eff,subtr}}^{\mathrm{top},\Delta\alpha}(M_{\mathrm{H}} = 1\,\mathrm{TeV}) = 13.5\cdot 10^{-4}$, which differs from the value given in Ref. [5] by $1.0\cdot 10^{-4}$. This difference amounts to about 80% of the two-loop top/bottom contribution and is larger than the uncertainty quoted in Ref. [14] of $4\cdot 10^{-5}$ (electroweak) and $3\cdot 10^{-5}$ (QCD). It turns out that the difference in the predictions for $\sin^2\theta_{\rm eff}(M_{\rm H})$ is mainly induced by the difference in the prediction for $M_{\rm W}(M_{\rm H})$. We have calculated $\sin^2\theta^{\rm top,\Delta\alpha}_{\rm eff,subtr}(M_{\rm H}=1\,{\rm TeV})$ using the value for $M_{\rm W}(M_{\rm H}=1\,{\rm TeV})$ from Ref. [5] instead of our result given above, and found a value for $\sin^2\theta^{\rm top,\Delta\alpha}_{\rm eff,subtr}(M_{\rm H}=1\,{\rm TeV})$ that differs from the one given in Ref. [5] only by $2\cdot 10^{-5}$. The Higgs-mass dependence of the light-fermion contributions, which is not included in Ref. [5], amounts to a shift in $\sin^2\theta_{\rm eff}$ of up to $2\cdot 10^{-5}$. For the Higgs-mass dependence of the leptonic width of the Z boson, Γ_l, we find good agreement with the result of the expansion in $m_{\rm t}$ [15]. The difference amounts to about 15% of the two-loop top/bottom contribution.

3. Fermion-loop corrections

The dominant contributions to the electroweak precision observables at one-loop order are the fermion-loop corrections $\Delta\alpha$ and $\Delta\rho = N_C\frac{G_\mu m_{\rm t}^2}{8\pi^2\sqrt{2}}$, which arise from the renormalization of the electric charge and the weak mixing angle, respectively. One of the main issues in improving the one-loop predictions has therefore been to find a proper resummation of these leading one-loop contributions. In the case of Δr, for instance, it could be shown [2] that the resummation

$$1+\Delta r \to \frac{1}{(1-\Delta\alpha)(1+\frac{c_{\rm W}^2}{s_{\rm W}^2}\Delta\rho)-\Delta{\rm r}_{\rm rem}} \tag{2}$$

correctly takes into account the terms of the form $(\Delta\alpha)^2$, $(\Delta\rho)^2$, $(\Delta\alpha\Delta\rho)$, $\Delta\alpha\Delta r_{\rm rem}$ at two-loop order. However, even with the knowledge of the recently evaluated terms of $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$ [4, 5] different treatments of contributions whose correct resummation is not known (i.e. non-leading two-loop terms and the higher-order terms except the leading term in $\Delta\alpha$) may result in predictions for $M_{\rm W}$ that differ from each other by several MeV.

We have investigated the pure fermion-loop contributions to the precision observables, i.e. contributions containing n fermion loops at n-loop order (which in the case of quark loops are therefore proportional to the nth power of the color factor N_C).[1] The relevant diagrams in a given loop order are reducible diagrams of vacuum-polarization type, which can easily be taken into account by Dyson summation, but also graphs with counterterm insertions into fermion-loop diagrams (see Fig. 2). We have derived recursive relations which allow to express the n-loop results in terms of one-loop one- and two-point functions [16].

[1] These contributions obviously constitute an UV-finite and gauge-invariant subset.

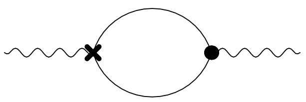

Fig. 2. A one-loop counterterm insertion into a one-loop diagram contributing to the fermion-loop corrections at two-loop order.

Using the on-shell renormalization scheme we have worked out explicit results for the fermion-loop contributions to the precision observables up to four-loop order [16]. We have compared our result for the two-loop contribution to Δr with the $\overline{\rm MS}$ result derived in Ref. [17], and found very good agreement within less than 1 MeV for the $M_{\rm W}$ prediction. Our exact results allow to investigate the validity of the resummation eq. (2) for the non-leading two-loop and the higher-order contributions. We found that already the first non-leading two-loop term of $\mathcal{O}(\Delta\rho \log(m_{\rm t}^2/M_{\rm Z}^2))$ is not correctly produced by the resummation eq. (2). The terms of the form $\Delta\rho\Delta r_{\rm top,rem}$ and $(\Delta r_{\rm top,rem})^2$ generated by eq. (2) give rise to a deviation in $M_{\rm W}$ of about 7 MeV compared to the exact result for the top/bottom doublet. It turns out, however, that large numerical cancellations occur between the non-leading two-loop terms that are not correctly resummed in eq. (2). While separately these terms differ from the corresponding terms in the exact result by several MeV, their sum approximates the full result remarkably well, up to less than 1 MeV.

From our exact result for the three- and four-loop contributions we find that eq. (2) in fact correctly produces the leading terms in $\Delta\alpha$, $\Delta\rho$ of the form $(\Delta\alpha)^a(\Delta\rho)^b$, where $a + b = n$, and n is the number of loops. The three-loop term of the form $(\Delta\alpha)^2\,\Delta r^{\alpha}_{\rm ferm,rem}$ is also correctly generated, while the three-loop term $\Delta\alpha\Delta r^{\alpha^2}_{\rm ferm,rem}$ is not correctly produced. In the full fermion-loop contribution at three-loop order again accidental numerical cancellations occur, which give rise to the fact that the total contribution is much smaller than the individual terms and affects the prediction for $M_{\rm W}$ by less than 1 MeV.

In total we find that due to accidental numerical cancellations the resummation of the fermion-loop contributions according to eq. (2) yields a very good numerical approximation of the complete fermion-loop result. The exact result is approximated within about 2 MeV. The results for the pure fermion-loop contributions to $\sin^2\theta_{\rm eff}$ and Γ_l will be discussed in Ref. [16].

The author thanks T. Riemann and the other organizers of the Zeuthen Workshop for their kind hospitality and the pleasant atmosphere during the workshop. I am grateful to my collaborators S. Bauberger, W. Hollik,

B. Krause and A. Stremplat, with whom the results presented here have been worked out, and to G. Degrassi, P. Gambino and A. Sirlin for useful discussions.

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