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# The Callan-Symanzik equation of the electroweak Standard Model and its 1-loop functions

**Elisabeth Kraus**

*Physikalisches Institut, Universität Bonn*
*Nußallee 12, D–53115 Bonn, Germany*

and

**Georg Weiglein**

*Institut für Theoretische Physik, Universität Karlsruhe,*
*Postfach 6980, D–76128 Karlsruhe, Germany*

**Abstract**

We derive the Callan-Symanzik equation of the electroweak Standard Model in the QED-like on-shell parameterization. The various coefficient functions, the $\beta$-functions and anomalous dimensions, are determined in one-loop order in the most general linear gauge compatible with rigid symmetry. In this way the basic elements for a systematic investigation of higher-order leading logarithmic contributions in the Standard Model are provided. The one-loop $\beta$-function of the electromagnetic coupling turns out to be independent of mass ratios and it is QED-like in this sense. Besides the QED-contributions of fermions it contains non-abelian contributions from vectors and ghosts with negative sign, which overcompensate the contributions of the fermions if one restricts the latter to one fermion generation. We also compare our results with the symmetric theory and give relations between the $\beta$-functions of the spontaneously broken and the symmetric theory valid in one-loop order.

## 1. Introduction

The precision tests of the electroweak theory have reached such a high level of experimental accuracy [1] that in the perturbative evaluation of the theoretical predictions the incorporation of higher-order radiative corrections is indispensable (for a recent review see e.g. Ref. [2]). The theoretical predictions obtained in a fixed order of perturbation theory can be improved if it is known how the leading contributions can consistently be resummed to all orders. In this context one is often also interested in the large-momentum behavior of the contributing Green functions. In order to make the analysis of the large-momentum behavior meaningful, also contributions which depend on large ratios of different mass scales have to be considered.

Information of this kind can be obtained by studying the Callan-Symanzik (CS) equation [3] and the renormalization group (RG) [4] equation of the model under consideration. The CS equation describes the breaking of dilatations and contains information about the momentum structure of the theory. The RG equation on the other hand describes the invariance of the model under variations of the normalization point. Both equations can be systematically constructed in renormalized perturbation theory. Their importance is founded in the fact that RG invariance as well as the hard breaking of dilatations can be formulated as a partial differential equation. They both contain derivatives with respect to the independent parameters of the theory, which give rise to the CS and RG $\beta$-functions, and field differential operators, which are connected with the anomalous dimensions.

While these equations coincide in theories with unbroken symmetry and massless particles, in massive theories this is no longer the case. In theories with unbroken symmetry the large-momentum behavior can be related to the large-momentum behavior of the massless theory. In particular it can be shown that for asymptotic normalization conditions the $\beta$-functions and anomalous dimensions of the massive theory coincide with those of the massless theory [5]. This situation is changed drastically in theories with broken symmetries: In the physical on-shell schemes the CS equation has a different form than the RG equation, in particular it contains derivatives with respect to the physical masses of the theory. Moreover, solving both equations consistently it has been shown that the massless symmetric theory is not the asymptotic version of the spontaneously broken one, but contains large logarithms of the mass parameters of the broken theory [6]. Consequently it is not obvious how to interpret the solutions of the CS equation in terms of "running" couplings and masses. It is therefore not guaranteed that the results obtained from a RG-study using the symmetric parameterization of the theory (cf. Ref. [7] and Refs. therein and also [8]) are directly applicable to the Standard Model (SM) of electroweak interactions. Instead, modifications are to be expected beyond one-loop order.

As a first step towards a systematic analysis of large-momentum *and* mass-dependent higher-order contributions, we derive in the present paper the CS equation of the electroweak SM in the on-shell parameterization (see e.g. Refs. [9, 10, 11]) and determine its 1-loop coefficient functions. The benefits of working within an on-shell parameterization are founded not only in its transparency due to the formulation in terms of physical parameters, appropriate on-shell conditions for the mixing propagators involving massless particles are also important for ensuring decent infrared properties of higher-order Green functions. We evaluate the coefficient functions of the CS equation in the most general linear gauge compatible with rigid symmetry, providing them in this way in the form needed for higher-order investigations. As an explicit example, the quadratic logarithms in the asymptotic region are determined for the photon self-energy at two-loop order. We also compare our results to the symmetric theory and to QED of charged fermions. Concerning QED we find that the fermion contributions to the $\beta$-function of the electromagnetic coupling are the same as in the electroweak SM. However, the non-abelian contributions of ghosts and vectors enter with a negative sign. In particular it turns out that if one restricts the fermions of the SM to one generation, the one-loop $\beta$-function of the electromagnetic coupling has a different sign compared to the familiar QED $\beta$-function. In this context it should be noted that in QED there exist also partial differential equations with respect to variations of single fermion masses. Such equations cannot be derived in the electroweak SM. This gives rise to the fact that higher-order $\beta$-functions are not restricted from abstract analysis in their mass-parameter dependence.

Apart from the above-mentioned applications the CS equation is also an important object in the procedure of abstract renormalization. It allows to determine in a scheme-independent way the independent parameters of the theory.[1] The most important outcome of the present analysis in this context is the observation that the ghost mass ratio is an independent parameter of the model, i.e. it is renormalized independently from the vector mass ratio. In order to introduce it as an independent parameter we have to modify the BRS transformations in lowest order. Otherwise it is not possible to assign well-defined infrared power counting degrees to the neutral Faddeev-Popov fields and the off-shell infrared existence of higher-order Green functions is endangered (see Ref. [13]).

The plan of the paper is as follows: In section 2 we give the classical action of the electroweak SM including the gauge-fixing and ghost sector in the on-shell parameterization. The gauge fixing is constructed in such a way that it is compatible with the Ward identities of rigid symmetry and the local abelian $U(1)$ Ward identity. The latter identity is crucial for continuing the Gell-Mann Nishijima relation to higher orders. In section 3 we

[1] For an introduction to algebraic renormalization see Ref. [12].

give the Slavnov-Taylor identity in the tree approximation and show how the independent ghost mass ratio can be consistently included. In section 4 we derive the CS equation by constructing the invariant differential operators. In section 5 we apply the CS equation to different 1-loop vertices, calculating in this way the $\beta$-functions and anomalous dimensions in one-loop order in a general gauge. As an application at two-loop order, the leading logarithms of the photon self-energy are investigated. In section 6 we compare the results with the symmetric theory and with QED of charged fermions. In section 7 we give our conclusions. The appendix contains a list of free field propagators determined in the general linear $R_\xi$ gauge.

## 2. The classical action of the electroweak Standard Model

In order to set the general framework and to fix the notation we first give the classical action of the SM in the on-shell parameterization. In our conventions we follow closely the ones used in Ref. [11].

The Standard Model of electroweak interactions is a non-abelian gauge theory with the non-semisimple gauge group $SU(2) \times U(1)$. It comprises four vector fields $V_{\mu,a}, a = +,-,Z,A$: the charged bosons $V_{\mu,\pm} \equiv W_{\mu,\pm}$ with mass $M_W$ and electric charge $\pm 1$, the neutral boson $V_{\mu,Z} \equiv Z_\mu$ with mass $M_Z$ and the massless photon field $V_{\mu,A} \equiv A_\mu$. The masses of the vector bosons are generated by spontaneous symmetry breaking via the Higgs mechanism. The SM contains a complex scalar doublet

$$\Phi \equiv \begin{pmatrix} \phi^+(x) \\ \frac{1}{\sqrt{2}}(H(x) + i\chi(x)) \end{pmatrix} \qquad \tilde{\Phi} \equiv i\tau_2\Phi^* = \begin{pmatrix} \frac{1}{\sqrt{2}}(H(x) - i\chi(x)) \\ -\phi^-(x) \end{pmatrix}, \tag{2.1}$$

where $H$ is the physical Higgs field with mass $m_H$, and $\phi^+, \phi^-$ and $\chi$ are the unphysical would-be Goldstone bosons.

In the fermion sector there are the left-handed fermion doublets, the lepton and quark doublet

$$F^L_{\delta,i} \equiv F^L_{l,i}, F^L_{q,i} \qquad F^L_{l,i} = \begin{pmatrix} \nu^L_i \\ e^L_i \end{pmatrix} \qquad F^L_{q,i} = \begin{pmatrix} u^L_i \\ d^L_i \end{pmatrix} \tag{2.2}$$

and the right-handed singlets

$$f^R_i = e^R_i, u^R_i, d^R_i. \tag{2.3}$$

Here $i$ denotes the family index; $\nu_i$ stands for neutrinos, $e_i$ for charged leptons with mass $m_{e_i}$ and electric charge $Q_e = -1$, $u_i$ and $d_i$ for up and down- type quarks with mass $m_{u_i}$

and $m_{d_i}$ and electric charge $Q_u = \frac{2}{3}$ and $Q_d = -\frac{1}{3}$. Since we are mainly interested in the CS functions of the vector sector we do not consider mixing between different families, especially we assume CP-invariance throughout the paper.

For convenience we give the classical action of the SM as it arises after spontaneous breaking of the symmetry in terms of the physical fields, i.e. in mass and charge eigenstates. The free parameters are the masses of the fields given above and one coupling, which is chosen according to a QED-like parameterization:

$$M_W, M_Z, m_H, m_{f_i}, e. \tag{2.4}$$

We introduce the notation

$$\cos\theta_W = \frac{M_W}{M_Z}, \tag{2.5}$$

which relates the weak mixing angle to the mass ratio of the $W$- and $Z$-bosons. In higher orders the masses and also the field renormalizations have to be fixed by appropriate normalization conditions for the two-point functions. In a QED-like parameterization the coupling can be fixed as the interaction strength of the photon to the electromagnetic current in the Thompson limit, where it is determined by the fine structure constant.

$$\bar{u}(p)\Gamma_{eeA_\mu}(p,p,0)u(p)|_{p^2=m_e^2} = ie\bar{u}(p)\gamma_\mu u(p). \tag{2.6}$$

The classical action can be decomposed into a gauge-invariant part $\Gamma_{GSW}$ and the gauge-fixing and ghost part, which are constructed to be BRS-invariant. The gauge-invariant part of the action is given by:

$$\Gamma_{GSW} = \Gamma_{YM} + \Gamma_{scalar} + \Gamma_{ferm} \ , \tag{2.7}$$

$$\begin{aligned}
\Gamma_{YM} &= -\frac{1}{4}\int d^4x \; G_a^{\mu\nu}\tilde{I}_{aa'}G_{\mu\nu a'} && (2.8)\\
\Gamma_{scalar} &= \int d^4x \,\Big( (D^\mu(\Phi+\mathrm{v}))^\dagger D_\mu(\Phi+\mathrm{v}) - \frac{1}{8}\frac{m_H^2}{M_W^2}\frac{e^2}{\sin^2\theta_W}(\Phi^\dagger\Phi + \mathrm{v}^\dagger\Phi + \Phi^\dagger \mathrm{v})^2\Big) && (2.9)\\
\Gamma_{ferm} &= \sum_{i=1}^{N_F}\int d^4x \,\Big(\overline{F_{l,i}^L} i\not{D} F_{l,i}^L + \overline{F_{q,i}^L} i\not{D} F_{q,i}^L + \overline{f_i^R} i\not{D} f_i^R && (2.10)\\
&\qquad - \frac{e}{M_W\sqrt{2}\sin\theta_W}(m_{e_i}\overline{F_{l,i}^L}(\Phi+\mathrm{v})e_i^R + m_{u_i}\overline{F_{q,i}^L}(\Phi+\mathrm{v})u_i^R\\
&\qquad + m_{d_i}\overline{F_{q,i}^L}(\tilde{\Phi}+\tilde{\mathrm{v}})d_i^R + \mathrm{h.c.})\Big),
\end{aligned}$$

where $N_F$ is the number of fermion generations, and v denotes the shift of the scalar field doublet, which generates the masses of the particles:

$$\mathrm{v} = \begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}v \end{pmatrix} \quad \text{with} \quad v = \frac{2}{e}M_Z\cos\theta_W\sin\theta_W \ . \tag{2.11}$$

It has a component into the direction of the physical Higgs field. Assigning to the fields a definite transformation behavior under C,P and T the action can be shown to be CP-invariant. (A table of quantum numbers for all fields of the Standard Model can be found in Ref. [13].)

The field strength tensor and the covariant derivative have the form

$$
\begin{aligned}
G_a^{\mu\nu} &= \partial^\mu V_a^\nu - \partial^\nu V_a^\mu + \frac{e}{\sin\theta_W}\tilde{I}_{aa'} f_{a'bc} V_b^\mu V_c^\nu & (2.12)\\
D_\mu \Phi &= \partial_\mu \Phi - i\frac{e}{\sin\theta_W}\frac{\tau_a(G_s)}{2}\Phi V_{\mu a} & (2.13)\\
D_\mu F_{\delta,i}^L &= \partial_\mu F_{\delta,i}^L - i\frac{e}{\sin\theta_W}\frac{\tau_a(G_\delta)}{2} F_{\delta,i}^L V_{\mu a} \qquad \delta = l, q & (2.14)\\
D_\mu f_i^R &= \partial_\mu f_i^R + ieQ_f \frac{\sin\theta_W}{\cos\theta_W} f_i^R Z_\mu + ieQ_f f_i^R A_\mu \,. & (2.15)
\end{aligned}
$$

We use the summation convention for the roman indices $a, b, c$ with values $+, -, Z, A$ and have introduced convenient notations. The tensor

$$
f_{abc} = \begin{cases} f_{+-Z} = -i\cos\theta_W \\ f_{+-A} = i\sin\theta_W \end{cases} \tag{2.16}
$$

is completely antisymmetric and the matrices $\tau_a$ $(a = +, -, Z, A)$ form a representation of $SU(2) \times U(1)$ according to

$$
\left[\frac{\tau_a}{2}, \frac{\tau_b}{2}\right] = i f_{abc} \tilde{I}_{cc'} \frac{\tau_{c'}}{2} \,. \tag{2.17}
$$

They are explicitly given by ($\tau_i$, $i = 1, 2, 3$, are the Pauli matrices)

$$
\begin{aligned}
\tau_+ &= \tfrac{1}{\sqrt{2}}(\tau_1 + i\tau_2) \qquad \tau_Z(G) = \tau_3\cos\theta_W + G\mathbf{1}\sin\theta_W \\
\tau_- &= \tfrac{1}{\sqrt{2}}(\tau_1 - i\tau_2) \qquad \tau_A(G) = -\tau_3\sin\theta_W + G\mathbf{1}\cos\theta_W \,.
\end{aligned} \tag{2.18}
$$

These combinations depend on the abelian coupling $G$, which is not determined by the algebra. It is related to the weak hypercharge $Y_W$ and accordingly to the electric charge $Q_f$ of the particles:

$$
G_k = -Y_W^{(k)}\frac{\sin\theta_W}{\cos\theta_W} \qquad Y_W^{(k)} = \begin{cases} 1 & \text{for the scalar } (k = s) \\ \text{-1} & \text{for the lepton doublets } (k = l) \\ \frac{1}{3} & \text{for the quark doublets } (k = q)\,. \end{cases} \tag{2.19}
$$

The matrix $\tilde{I}_{aa'}$ guarantees the charge neutrality of the classical action

$$
\begin{aligned}
\tilde{I}_{+-} &= \tilde{I}_{-+} = \tilde{I}_{ZZ} = \tilde{I}_{AA} = 1 \\
\tilde{I}_{ab} &= 0 \text{ else.}
\end{aligned} \tag{2.20}
$$

The action $\Gamma_{GSW}$ is manifestly invariant under $SU(2)\times U(1)$ gauge transformations, if one includes the shift of the Higgs field into the transformation. The transformation behavior of the physical fields can be read off from the covariant derivatives.

In order to quantize the SM the gauge is fixed in such a way that renormalizability is guaranteed by power counting. Taking the usual linear $R_\xi$ gauges we choose the following gauge-fixing functions, which are the most general ones having definite transformation with respect to CP:

$$\begin{aligned} F_\pm &\equiv \partial_\mu W^\mu_\pm \mp i M_W \zeta_W \phi^\pm \\ F_Z &\equiv \partial_\mu Z^\mu - M_Z \zeta_Z \chi \\ F_A &\equiv \partial_\mu A^\mu - M_Z \zeta_A \chi . \end{aligned} \tag{2.21}$$

The mass terms of the would-be Goldstone fields are introduced in order to remove non-integrable infrared divergencies from the propagators. Coupling the gauge-fixing functions to a Lagrange multiplier field $B_a, a = +,-,Z,A$, with dimension 2 and odd under CP-transformations, the gauge fixing reads

$$\Gamma_{g.f.} = \int d^4x \left(\frac{1}{2}\xi_{ab} B_a B_b + B_a \tilde{I}_{ab} F_b\right). \tag{2.22}$$

It can be transformed into its usual form by eliminating the $B_a$-fields via their equations of motion:

$$\frac{\delta \Gamma}{\delta B_a} = \xi_{ab} B_b + \tilde{I}_{ab} F_b \stackrel{*}{=} 0. \tag{2.23}$$

The gauge fixing breaks gauge invariance and also its integrated version, the rigid $SU(2)\times U(1)$ symmetry, which is obtained by taking the infinitesimal transformation parameters of gauge transformations as constants. Therefore the unphysical fields, the longitudinal parts of the vectors and the would-be Goldstones, interact with the physical fields violating thereby unitarity. For this reason one has to introduce the Faddeev-Popov fields $c_a, a = +,-,Z,A$, with ghost charge 1 and the respective antighosts $\bar{c}_a, a = +,-,Z,A$, with ghost charge $-1$ and has to add the ghost part in such a way that the classical action is invariant under BRS transformations:

$$\begin{aligned} \mathrm{s}V_{\mu a} &= \partial_\mu c_a + \frac{e}{\sin\theta_W} \tilde{I}_{aa'} f_{a'bc} V_{\mu b} c_c \\ \mathrm{s}c_a &= -\frac{e}{2\sin\theta_W} \tilde{I}_{aa'} f_{a'bc} c_b c_c \\ \mathrm{s}\Phi &= i\frac{e}{\sin\theta_W}\frac{\tau_a(G_s)}{2}(\Phi + \mathrm{v}) c_a \\ \mathrm{s}F^L_\delta &= i\frac{e}{\sin\theta_W}\frac{\tau_a(G_\delta)}{2} F^L_\delta c_a \qquad \delta = l,q \\ \mathrm{s}f^R &= -ieQ_f \frac{\sin\theta_W}{\cos\theta_W} f^R c_Z - ieQ_f f^R c_A \end{aligned} \tag{2.24}$$

$$\begin{aligned} s\bar{c}_a &= B_a \\ sB_a &= 0. \end{aligned}$$

Having formulated the gauge fixing with the auxiliary fields $B_a$, the BRS transformations are nilpotent on all fields

$$s^2 = 0. \tag{2.25}$$

Requiring the classical action to be BRS-invariant

$$s\Gamma_{cl} = 0 \quad \text{with} \quad \Gamma_{cl} = \Gamma_{GSW} + \Gamma_{g.f.} + \Gamma_{ghost}, \tag{2.26}$$

the ghost action is determined

$$\Gamma_{ghost} = \int d^4x \, \left(-\bar{c}_a \tilde{I}_{ab} sF_b\right). \tag{2.27}$$

The bilinear terms are given explicitly by

$$\begin{aligned} \Gamma^{(bil)}_{ghost} &= \int d^4x \, \Big(-\bar{c}_a \Box \tilde{I}_{ab} c_b - \zeta_W M_W^2 (\bar{c}_+ c_- + \bar{c}_- c_+) \\ &\quad - \zeta_Z M_Z^2 \bar{c}_Z c_Z - \zeta_A M_Z^2 \bar{c}_A c_Z\Big). \end{aligned} \tag{2.28}$$

With the help of BRS invariance one is able to prove unitarity of the physical S-matrix in the tree approximation. It is therefore the relevant symmetry for quantization and renormalization because it fixes the interactions amongst the unphysical fields in such a way that the complete action is renormalizable and eventually the physical S-matrix unitary [14, 15].

The gauge-fixing parameters are not specified by BRS invariance. In general $\xi_{ab}$ is an arbitrary symmetric matrix and $\zeta_a$ are arbitrary parameters. They have to be restricted by normalization conditions on the ghost propagators or additional symmetries. A natural choice in the tree approximation is

$$\xi_{ab} = \xi \tilde{I}_{ab} \qquad \zeta_W = \zeta_Z \qquad \zeta_A = 0, \tag{2.29}$$

which makes the propagators of the longitudinal vectors and of the Faddeev-Popov ghosts diagonal. If we constrain the gauge fixing according to (2.29), rigid invariance is still broken by the mass terms and, moreover, the breaking transforms covariantly in such a way that it can be controlled by introducing an external scalar doublet

$$\hat{\Phi} = \begin{pmatrix} \hat{\phi}^+ \\ \frac{1}{\sqrt{2}}(\hat{H} + i\hat{\chi}) \end{pmatrix} \tag{2.30}$$

with the same quantum numbers as the scalar doublet $\Phi$, but which is BRS-transformed into an external doublet q with ghost charge 1 (cf. Refs. [13, 16])

$$s\hat{\Phi} = q \qquad sq = 0. \tag{2.31}$$

In the most general linear gauge fixing invariant with respect to rigid symmetry transformations the gauge parameters are restricted as follows

$$
\begin{aligned}
\xi_{ab} &= \xi \tilde{I}_{ab} + \hat{\xi} \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & \sin^2\theta_W & \sin\theta_W\cos\theta_W \\ 0 & 0 & \sin\theta_W\cos\theta_W & \cos^2\theta_W \end{pmatrix} && (2.32)\\
\zeta_W &= \zeta && (2.33)\\
\zeta_Z &= \zeta\cos\theta_W(\cos\theta_W - \hat{G}\sin\theta_W) && (2.34)\\
\zeta_A &= -\zeta\cos\theta_W(\sin\theta_W - \hat{G}\cos\theta_W). && (2.35)
\end{aligned}
$$

Then the gauge fixing (2.22) reads explicitly (including also the external scalars)

$$
\begin{aligned}
\Gamma_{g.f.} = \int d^4x \Big( & \frac{1}{2}\xi B_a \tilde{I}_{ab} B_b + \frac{1}{2}\hat{\xi}(\sin\theta_W B_Z + \cos\theta_W B_A)^2 + B_a \tilde{I}_{ab}\partial V_b \\
& - \frac{ie}{\sin\theta_W}\big((\hat{\Phi} + \zeta \mathrm{v})^\dagger \frac{\tau_a(\hat{G})}{2} B_a(\Phi + \mathrm{v}) - (\Phi + \mathrm{v})^\dagger \frac{\tau_a(\hat{G})}{2} B_a(\hat{\Phi} + \zeta\mathrm{v})\big)\Big), \quad (2.36)
\end{aligned}
$$

with v given in (2.11) and $\tau_a(G)$ in (2.18). Concerning the external scalar doublet $\hat{\Phi}$ and the quantum scalar doublet $\Phi$ this gauge-fixing term is equivalent to the one used in the background-field method [17, 18].

It is seen that the would-be Goldstone fields $\phi^\pm$ and $\chi$ as well as the massive Faddeev-Popov ghosts get their masses via the shift of the external scalar fields $\zeta$v. The gauge parameter $\hat{\xi}$ as well as the abelian coupling $\hat{G}$ are not determined by rigid symmetry, gauge invariance or BRS symmetry. The parameterization chosen in (2.29) reads now:

$$
\hat{\xi} = 0 \quad \text{and} \qquad \hat{G} = -\frac{\sin\theta_W}{\cos\theta_W}. \qquad (2.37)
$$

It turns out, however, that the minimal choice (2.37) is not stable under renormalization as will be seen from the Callan-Symanzik equation. In particular, the ghost mass ratio is independently renormalized from the vector mass ratio. In view of the investigation of higher-order contributions it is therefore important to work in the general gauge specified in (2.36).

## 3. Quantization

For a systematic treatment of quantization and renormalization one expresses invariance under BRS transformations (2.24) and rigid symmetry in the form of functional operators, the Slavnov-Taylor identity and the Ward identities of rigid symmetry.

Since the BRS transformations include non-linear field transformations in propagating fields, they have to be coupled to external fields. In order to avoid double definitions of the insertions $c_+c_-$ and $V_+c_- - V_-c_+$ one has to split off the linear $U(1)$-transformations explicitly [19, 13]. We introduce the external field action in the following form:

$$\begin{aligned}\Gamma_{ext.f.} &= \int d^4x \,\Big(\rho^\mu_+ \mathrm{s}W_{\mu,-} + \rho^\mu_- \mathrm{s}W_{\mu,+} + \rho^\mu_3(\cos\theta_W \mathrm{s}Z_\mu - \sin\theta_W \mathrm{s}A_\mu) \\ &\quad + \sigma_+ \mathrm{s}c_- + \sigma_- \mathrm{s}c_+ + \sigma_3(\cos\theta_W \mathrm{s}c_Z - \sin\theta_W \mathrm{s}c_A) \\ &\quad + Y^\dagger \mathrm{s}\Phi + (\mathrm{s}\Phi)^\dagger Y \\ &\quad + \sum_{i=1}^{N_F}\big(\sum_{\delta=l,q} \overline{\Psi^R_{\delta,i}} \mathrm{s}F^L_{\delta,i} + \sum_f \overline{\psi^L_{f,i}} \mathrm{s}f^R_i + \text{h.c.}\big)\Big).\end{aligned} \tag{3.1}$$

The external fields $\rho^\mu_\alpha$ and $\sigma_\alpha, \alpha = +, -, 3$, are $SU(2)$-triplets with ghost charge $-1$ and $-2$, respectively. The external field $Y$ is a complex scalar doublet with ghost charge $-1$, $\psi^L_{f,i}$ denotes external left-handed spinor singlets with ghost charge $-1$,

$$\psi^L_{f,i} \equiv \psi^L_{e,i},\ \psi^L_{u,i},\ \psi^L_{d,i}, \tag{3.2}$$

whereas $\Psi^R_{\delta,i}$ denotes external right-handed spinor doublets

$$\Psi^R_{\delta,i} \equiv \Psi^R_{l,i},\ \Psi^R_{q,i} \qquad \Psi^R_{l,i} = \begin{pmatrix} \psi^R_{\nu,i} \\ \psi^R_{e,i} \end{pmatrix} \qquad \Psi^R_{q,i} = \begin{pmatrix} \psi^R_{u,i} \\ \psi^R_{d,i} \end{pmatrix}. \tag{3.3}$$

The transformation under discrete symmetries is assigned in such a way that the external field part is neutral and CP-invariant.

The classical action corresponds to the lowest order of the perturbative expansion of 1PI Green functions and one can read off the respective functional operators of the defining symmetries in the tree approximation. Including the external field part (3.1) into the classical action (2.26) one is able to encode the BRS transformations (2.24) in the Slavnov-Taylor (ST) operator:

$$\begin{aligned}\mathcal{S}(\Gamma) &= \int d^4x \,\Big((\sin\theta_W \partial_\mu c_Z + \cos\theta_W \partial_\mu c_A)\big(\sin\theta_W \frac{\delta\Gamma}{\delta Z_\mu} + \cos\theta_W \frac{\delta\Gamma}{\delta A_\mu}\big) \\ &+ \frac{\delta\Gamma}{\delta\rho^\mu_3}\big(\cos\theta_W \frac{\delta\Gamma}{\delta Z_\mu} - \sin\theta_W \frac{\delta\Gamma}{\delta A_\mu}\big) + \frac{\delta\Gamma}{\delta\sigma_3}\big(\cos\theta_W \frac{\delta\Gamma}{\delta c_Z} - \sin\theta_W \frac{\delta\Gamma}{\delta c_A}\big) \\ &+ \frac{\delta\Gamma}{\delta\rho^\mu_+}\frac{\delta\Gamma}{\delta W_{\mu-}} + \frac{\delta\Gamma}{\delta\rho^\mu_-}\frac{\delta\Gamma}{\delta W_{\mu+}} + \frac{\delta\Gamma}{\delta\sigma_+}\frac{\delta\Gamma}{\delta c_-} + \frac{\delta\Gamma}{\delta\sigma_-}\frac{\delta\Gamma}{\delta c_+} + \frac{\delta\Gamma}{\delta Y^\dagger}\frac{\delta\Gamma}{\delta\Phi} + \frac{\delta\Gamma}{\delta\Phi^\dagger}\frac{\delta\Gamma}{\delta Y} \\ &+ \sum_{i=1}^{N_F}\big(\frac{\delta\Gamma}{\delta\overline{\psi^L_{f,i}}}\frac{\delta\Gamma}{\delta f^R_i} + \frac{\delta\Gamma}{\delta\overline{\Psi^R_{\delta,i}}}\frac{\delta\Gamma}{\delta F^L_{\delta,i}} + \text{h.c.}\big) \\ &+ B_a \frac{\delta\Gamma}{\delta\bar{c}_a} + \mathrm{q}\frac{\delta\Gamma}{\delta\hat{\Phi}} + \frac{\delta\Gamma}{\delta\hat{\Phi}^\dagger}\mathrm{q}^\dagger\Big).\end{aligned} \tag{3.4}$$

The ST identity of the tree approximation

$$\mathcal{S}(\Gamma_{cl}) = 0 \tag{3.5}$$

is fulfilled by construction.

Rigid symmetry can be formulated in terms of linear, integrated Ward operators which satisfy the $SU(2) \times U(1)$ algebra

$$\begin{aligned} [\mathcal{W}_\alpha, \mathcal{W}_\beta] &= \varepsilon_{\alpha\beta\gamma} \tilde{I}_{\gamma\gamma'} \mathcal{W}_{\gamma'} \\ [\mathcal{W}_\alpha, \mathcal{W}_4] &= 0. \end{aligned} \tag{3.6}$$

The Greek indices are $SU(2)$-group indices and run over $+, -, 3$, the Ward operator $\mathcal{W}_4$ corresponds to the transformation under the $U(1)$-group and commutes therefore with all Ward operators. The tensor $\varepsilon_{\alpha\beta\gamma}$ denotes the structure constants of charged $SU(2)$ and is completely antisymmetric:

$$\varepsilon_{+-3} = -i. \tag{3.7}$$

Vector fields, Faddeev-Popov ghosts and the auxiliary fields $B_a$ transform according to the adjoint representation, whereas all the scalars transform according to the fundamental representation. The external fields $\rho^\mu_\alpha$ and $\sigma_\alpha, \alpha = +, -, 3$, are only transformed under $SU(2)$. We thus arrive at

$$\mathcal{W}_\alpha \Gamma_{cl} = 0 \quad \text{and} \quad \mathcal{W}_4 \Gamma_{cl} = 0, \tag{3.8}$$

where the Ward operators of the tree approximation are given by

$$\begin{aligned} \mathcal{W}_\alpha = \tilde{I}_{\alpha\alpha'} \int d^4x \Big( & \Big( V^\mu_b \hat{\varepsilon}_{bc,\alpha'} \tilde{I}_{cc'} \frac{\delta}{\delta V^\mu_{c'}} + \{c, B, \bar{c}\} \Big) \\ & + \Big( \rho^\mu_\beta \varepsilon_{\beta\gamma\alpha'} \tilde{I}_{\gamma\gamma'} \frac{\delta}{\delta \rho^\mu_{\gamma'}} + \{\sigma\} \Big) \\ & + \Big( i(\Phi + \mathrm{v})^\dagger \frac{\tau_{\alpha'}}{2} \frac{\overrightarrow{\delta}}{\delta \Phi^\dagger} - i \frac{\overleftarrow{\delta}}{\delta \Phi} \frac{\tau_{\alpha'}}{2} (\Phi + \mathrm{v}) + \{Y, \hat{\Phi} + \zeta \mathrm{v}, q\} \Big) \\ & + \sum_{\delta, i} \Big( i \overline{F^L_{\delta,i}} \frac{\tau_{\alpha'}}{2} \frac{\overrightarrow{\delta}}{\delta \overline{F^L_{\delta,i}}} - i \frac{\overleftarrow{\delta}}{\delta F^L_{\delta,i}} \frac{\tau_{\alpha'}}{2} F^L_{\delta,i} + \{\Psi^R_{\delta,i}\} \Big) \Big) . \end{aligned} \tag{3.9}$$

Fields in curly brackets in (3.9) denote that these fields are transformed in the same way as the one explicitly given in the respective line of the formula. The matrices $\tau_\alpha = \tau_+, \tau_-, \tau_3$ are the Pauli matrices of the charged representation of $SU(2)$ (3.7). The tensor $\hat{\varepsilon}_{bc,\alpha}$, which governs the transformation of the vector fields, is antisymmetric in the first two indices. These indices are field indices and are generated by rotating the neutral $SU(2)$-fields and the abelian fields by the weak mixing angle $\theta_W$ into on-shell fields:

$$\hat{\varepsilon}_{bc,\alpha} = \begin{cases} \hat{\varepsilon}_{Z+,-} &= -i \cos \theta_W \\ \hat{\varepsilon}_{A+,-} &= i \sin \theta_W \\ \hat{\varepsilon}_{+-,3} &= -i \end{cases} . \tag{3.10}$$

In the electroweak Standard Model there are several rigid abelian operators $\mathcal{W}_4$ which commute with the $SU(2)$ operators (3.9). The respective symmetries of the classical action correspond to the conservation of electromagnetic charge ($\mathcal{W}_{em} - \mathcal{W}_3$) and conservation of lepton and quark family number ($\mathcal{W}_{l_i}$ and $\mathcal{W}_{q_i}$),

$$\mathcal{W}_4 \Gamma_{cl} = 0 \quad \text{where} \quad \mathcal{W}_4 \equiv (\mathcal{W}_{em} - \mathcal{W}_3) + \sum_i^{N_F} g_{l_i} \mathcal{W}_{l_i} + g_{q_i} \mathcal{W}_{q_i} . \tag{3.11}$$

Here $\mathcal{W}_{em}$ is the usual electromagnetic charge operator; explicit expressions are given in Ref. [13]. From the corresponding classically conserved currents only the electromagnetic current is gauged. When the gauge-fixing sector and ghost sector is constructed in accordance with rigid symmetry (2.36), then it is possible to establish the local abelian Ward identity corresponding to electromagnetic current conservation

$$\mathbf{w}_4^Q \Gamma_{cl} - \frac{1}{e} \cos\theta_W \Big( \sin\theta_W \partial \frac{\delta \Gamma_{cl}}{\delta Z} + \cos\theta_W \partial \frac{\delta \Gamma_{cl}}{\delta A} \Big) = \frac{1}{e} \cos\theta_W (\sin\theta_W \Box B_Z + \cos\theta_W \Box B_A) \,, \tag{3.12}$$

with

$$\mathbf{w}_4^Q = \mathbf{w}_{em} - \mathbf{w}_3 \,. \tag{3.13}$$

The local operators $\mathbf{w}_3$ and $\mathbf{w}_{em}$ are defined by taking away the integration from the rigid operators:

$$\mathcal{W}_3 = \int d^4x \; \mathbf{w}_3 \quad \text{and} \quad \mathcal{W}_{em} = \int d^4x \; \mathbf{w}_{em} \,. \tag{3.14}$$

The local Ward identity together with (3.13) is the functional generalization of the Gell-Mann Nishijima relation and allows to determine the weak hypercharges of fermion doublets and the electromagnetic charges of fermion singlets. In higher orders this local Ward identity plays an important role for a scheme-independent definition of the abelian charges (for details see Ref. [13]).[2]

In the procedure of renormalization and quantization one has to construct the Green functions in such a way that they satisfy simultaneously the Slavnov-Taylor identity, the Ward identities of rigid symmetry specified by the commutation relation (3.6), and the abelian local Ward identity (3.12). This problem has to be taken seriously due to the fact that the photon and the respective Faddeev-Popov ghosts are massless and these massless particles have non-abelian interactions. In order to ensure that all integrals are infrared convergent for non-exceptional momenta in higher orders of perturbation theory one has to supplement the usual normalization conditions, which fix the free parameters (2.4) and the wave function normalization, by the requirements that also the mixed 2-point

[2] In [20] the abelian charges are fixed by postulating an antighost equation. From there the local Ward identity is defined by using the consistency with the ST identity.

functions of massive and massless particles vanish at $p^2 = 0$:

$$\begin{aligned} \Gamma_{ZA}(p^2=0) = \Gamma_{AA}(p^2=0) &= 0 \\ \Gamma_{\bar{c}_A c_Z}(p^2=0) = \Gamma_{\bar{c}_Z c_A}(p^2=0) = \Gamma_{\bar{c}_A c_A}(p^2=0) &= 0. \end{aligned} \tag{3.15}$$

A careful analysis of higher orders shows that on-shell conditions which include (3.15) as well as corresponding conditions at the mass of the massive fields can be only fulfilled in agreement with the Slavnov-Taylor identity and the Ward identities if one takes into account higher-order corrections of the ST operator and the Ward operators [13]. The $\beta$-functions and anomalous dimensions of the 1-loop CS equation are independent of these higher-order corrections. But one cannot completely stick to the tree approximation as defined by the parameterization (2.37) and the BRS transformations (2.24) since the ghost mass ratio turns out to be an independent parameter of the Standard Model. This means that we have to keep the parameters of the gauge-fixing functions (2.36) $\xi, \hat{\xi}, \zeta$ and $\hat{G}$ as independent parameters. Keeping $\hat{G}$ arbitrarily and using at the same time the BRS transformation (2.24) brings about that the non-diagonal mass term $\bar{c}_A c_Z$ arises in the action (see (3.17)). Such a non-diagonal mass term leads in higher orders to off-shell infrared divergent contributions and prevents the introduction of definite infrared degrees of power counting for the ghosts. In order to remedy the situation one has to introduce a ghost angle into the BRS transformations already in the tree approximation, which allows consistently to remove the infrared divergent contributions $\bar{c}_A c_Z$ from the classical action for arbitrary ghost mass ratio and to derive the CS equation.

For the purpose of this paper we want to outline the definition of the ghost angle in the tree approximation, whereas a detailed analysis especially of higher orders is given in Ref. [13]. The linear $R_\xi$ gauge-fixing functions (2.21) are restricted by rigid symmetry as given in (2.32)–(2.35). They include only four free parameters:

$$\xi,\ \hat{\xi},\ \zeta,\ \hat{G}. \tag{3.16}$$

The parameters $\zeta$ and $\hat{G}$ are connected with the ghost masses:

$$-\zeta M_W \int d^4x \Big( M_W(\bar{c}_+ c_- + \bar{c}_- c_+) + M_Z \bar{c}_Z c_Z (\cos\theta_W - \hat{G}\sin\theta_W) - M_Z \bar{c}_A c_Z (\sin\theta_W + \hat{G}\cos\theta_W)\Big). \tag{3.17}$$

Introducing in analogy to (2.19) the notation

$$\hat{G} = -\frac{\sin\theta_G}{\cos\theta_G}, \tag{3.18}$$

the ratio of the ghost masses is determined by

$$\frac{\zeta_W M_W^2}{\zeta_Z M_Z^2} = \frac{\cos\theta_W \cos\theta_G}{\cos(\theta_W - \theta_G)}, \tag{3.19}$$

where $\zeta_W M_W^2$ is the mass of the charged ghosts and $\zeta_Z M_Z^2$ the mass of the massive neutral ghosts. The ghost angle $\theta_G$ is uniquely determined for arbitrary ghost masses. In order to be able to remove the non-diagonal ghost mass term for arbitrary ghost mass ratio from the action one has to redefine the neutral ghosts $c_Z$ and $c_A$ as well as the antighosts $\bar{c}_Z$ and $\bar{c}_A$ by a non-diagonal matrix $\hat{g}_{ab}$:

$$c_a \longrightarrow \hat{g}_{ab} c_b, \qquad \bar{c}_a \longrightarrow \bar{c}_b \hat{g}_{ba}^{-1} \,. \tag{3.20}$$

This procedure is analogous to the one which has to be carried out if one constructs mass eigenstates in the vector sector by introducing the weak mixing angle. The matrix $\hat{g}_{ab}$ is determined from the normalization conditions (3.15) up to two constants which we have fixed for convenience:

$$\begin{aligned} \hat{g}_{+-} &= 1 & \hat{g}_{-+} &= 1 \\ \hat{g}_{ZZ} &= \cos(\theta_W - \theta_G) & \hat{g}_{AZ} &= -\sin(\theta_W - \theta_G) \\ \hat{g}_{ZA} &= 0 & \hat{g}_{AA} &= 1. \end{aligned} \tag{3.21}$$

In matrix notation it reads:

$$\hat{g}_{ab} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \cos(\theta_W - \theta_G) & 0 \\ 0 & 0 & -\sin(\theta_W - \theta_G) & 1 \end{pmatrix}. \tag{3.22}$$

With these redefinitions the bilinear part of the Faddeev-Popov ghost action is diagonal also in the general gauge (2.36):

$$\Gamma^{(bil)}_{ghost} = \int d^4x \, \Big( -\bar{c}_a \Box \tilde{I}_{ab} c_b - \zeta_W M_W^2 (\bar{c}_+ c_- + \bar{c}_- c_+) - \zeta_Z M_Z^2 \bar{c}_Z c_Z \Big). \tag{3.23}$$

Consequently the ghost propagators are diagonal and allow to assign a well-defined infrared degree of power counting to $Z$ and $A$-ghosts. For arbitrary masses, however, the ghost angle enters the BRS transformations and the ST identity and eventually also the ghost–vector interactions. Explicitly the ST operator reads

$$\begin{aligned} \mathcal{S}(\Gamma) \;=\; & \int d^4x \, \Big( (\sin\theta_G \partial_\mu c_Z + \cos\theta_W \partial_\mu c_A) \Big( \sin\theta_W \frac{\delta\Gamma}{\delta Z_\mu} + \cos\theta_W \frac{\delta\Gamma}{\delta A_\mu} \Big) \\ & + \frac{\delta\Gamma}{\delta\rho_3^\mu} \Big( \cos\theta_W \frac{\delta\Gamma}{\delta Z_\mu} - \sin\theta_W \frac{\delta\Gamma}{\delta A_\mu} \Big) + \frac{\delta\Gamma}{\delta\sigma_3} \Big( \cos\theta_W \frac{\delta\Gamma}{\delta c_Z} - \sin\theta_G \frac{\delta\Gamma}{\delta c_A} \Big) \frac{1}{\cos(\theta_W - \theta_G)} \\ & + \Big( \cos(\theta_W - \theta_G) B_Z - \sin(\theta_W - \theta_G) B_A \Big) \frac{\delta\Gamma}{\delta\bar{c}_Z} + B_A \frac{\delta\Gamma}{\delta\bar{c}_A} \\ & + \frac{\delta\Gamma}{\delta\rho_+^\mu} \frac{\delta\Gamma}{\delta W_{\mu,-}} + \frac{\delta\Gamma}{\delta\rho_-^\mu} \frac{\delta\Gamma}{\delta W_{\mu,+}} + \frac{\delta\Gamma}{\delta\sigma_+} \frac{\delta\Gamma}{\delta c_-} + \frac{\delta\Gamma}{\delta\sigma_-} \frac{\delta\Gamma}{\delta c_+} + B_+ \frac{\delta\Gamma}{\delta\bar{c}_+} + B_- \frac{\delta\Gamma}{\delta\bar{c}_-} \\ & + \sum_{i=1}^{N_F} \Big( \sum_f \frac{\delta\Gamma}{\delta\overline{\psi^L_{f,i}}} \frac{\delta\Gamma}{\delta f_i^R} + \sum_{\delta=l,q} \frac{\delta\Gamma}{\delta\overline{\Psi^R_{\delta,i}}} \frac{\delta\Gamma}{\delta F^L_{\delta,i}} + \text{h.c.} \Big) + \Big( \frac{\delta\Gamma}{\delta Y^\dagger} \frac{\delta\Gamma}{\delta\Phi} + \text{q} \frac{\delta\Gamma}{S\delta\hat{\Phi}} + \text{h.c.} \Big) \Big) \end{aligned} \tag{3.24}$$

and the ST identity is fulfilled in the tree approximation for arbitrary ghost angle $\theta_G$:

$$\mathcal{S}(\Gamma_{cl}) = 0 \tag{3.25}$$

Establishing the ST identity in higher orders, $\theta_W$ and $\theta_G$ get independent higher-order corrections in the on-shell scheme. Their correct treatment is a necessary prerequisite for obtaining infrared convergent higher-order corrections for off-shell Green functions [13].

The ghost angle enters also the Ward identities of rigid $SU(2)$-transformations (3.8),(3.9) via the redefinitions (3.20). It is worth to note that the algebra (3.6) remains unchanged by such field redefinitions.

The usual formulation of the gauge-fixing sector without the $B_a$ fields is achieved if one eliminates the $B_a$ fields via their equations of motion from the gauge-fixing action (2.36) and inserts the result into the gauge fixing as well as in the ST identity (3.24). The above discussion concerning the ghost angle is independent from the formulation of the gauge fixing with or without $B_a$ fields.

## 4. The Callan-Symanzik equation

The Callan-Symanzik equation describes the response of the Green functions to the scaling of all momenta by an infinitesimal factor. The dilatational operator, which is just the scaling operator, acts on the 1PI Green functions in the same way as the differentiation with respect to all the mass parameters of the theory:

$$\begin{aligned} &\mathcal{W}^D\Gamma = -m\partial_m\Gamma \quad \text{with} \\ &m\partial_m \equiv M_W\partial_{M_W} + M_Z\partial_{M_Z} + m_H\partial_{m_H} + \sum_{i=1}^{N_F}\sum_f m_{f_i}\partial_{m_{f_i}} + \kappa\partial_\kappa \ . \end{aligned} \tag{4.1}$$

Here $\kappa$ is a normalization point which is introduced in order to fix the on-shell infrared divergent residua of charged particles off-shell without introducing a photon mass term.

In the SM dilatations are already broken in the tree approximation by the mass terms of the fields and the 3-dimensional interactions. Due to the spontaneous symmetry breaking all the masses of the physical fields are generated by the shift of the Higgs field. According to the construction of the gauge-fixing sector using rigid symmetry (2.36), the ghost masses are generated by the shift of the external Higgs field. In the tree approximation one gets therefore the expression

$$m\partial_m\Gamma_{cl} = \int d^4x \; v\Big(\frac{\delta\Gamma_{cl}}{\delta H} + \zeta\frac{\delta\Gamma_{cl}}{\delta\hat{H}}\Big) + \frac{m_H^2}{2}\Delta_{inv} \equiv \int d^4x \; v\Big(\frac{\delta}{\delta H} + \zeta\frac{\delta}{\delta\hat{H}} + \alpha_{inv}\frac{\delta}{\delta\hat{\varphi}_o}\Big)\,\Gamma_{cl}. \tag{4.2}$$

Here $\Delta_{inv}$ is the 2-dimensional BRS- and rigid-invariant scalar polynomial

$$\Delta_{inv} \equiv \int d^4x \; (2\phi^+\phi^- + \chi^2 + H^2 + 2vH) = \int d^4x \; (\Phi^\dagger\Phi + \mathrm{v}^\dagger\Phi + \Phi^\dagger\mathrm{v})\,, \qquad (4.3)$$

which we couple to an external invariant scalar $\hat{\varphi}_o$. For proceeding to higher orders it is important to note that the differential operators $m\partial_m$ as well as $\frac{\delta}{\delta H}$ and $\frac{\delta}{\delta \hat{H}}$ are BRS-symmetric operators and have a certain covariance with respect to rigid symmetry:

$$\big[\mathcal{W}_\alpha, m\partial_m\big] = \big[\mathcal{W}_\alpha, \int d^4x \; v\big(\frac{\delta}{\delta H} + \zeta\frac{\delta}{\delta \hat{H}}\big)\big], \qquad \alpha = +,-,3,4. \qquad (4.4)$$

The soft breaking is completely characterized by these symmetries and takes therefore the same form to all orders of perturbation theory, if one takes higher-order corrections of the shift and the coefficient $\alpha_{inv}$ into account:

$$v = \frac{2}{e}M_Z\cos\theta_W\sin\theta_W + O(\hbar), \qquad \alpha_{inv} = \frac{m_H^2}{2v} + O(\hbar). \qquad (4.5)$$

In higher orders the dilatations are not only broken by the soft mass terms but also by hard terms, the dilatational anomalies. The importance of the Callan-Symanzik equation is founded in the fact that these anomalies can be absorbed into differential operators with respect to fields and with respect to the independent parameters of the model. Their coefficients are the anomalous dimensions and the $\beta$-functions. In this way the CS equation determines the parameters that are independently renormalized in a scheme-independent way.

The dilatational anomalies at one-loop order are normalization-point-independent, but the differential operators introduced depend on the parameterization and the specific form of the breaking mechanism. They are essentially characterized by the symmetries of the tree approximation, i.e. the ST identity (3.24) and the Ward identities of rigid symmetry (3.8). To be more specific we want to outline the one-loop construction of the CS equation of the SM according to these symmetries.

Applying the quantum action principle [21, 22] one derives from (4.2) that the dilatations in 1-loop order are broken by

$$\Big(m\partial_m - \int d^4x \; v\big(\frac{\delta}{\delta H} + \zeta\frac{\delta}{\delta \hat{H}} - \alpha_{inv}\frac{\delta}{\delta \hat{\varphi}_o}\big)\Big)\Gamma = \Delta_m + O(\hbar^2)\,, \qquad (4.6)$$

where $\Delta_m$ is an integrated field polynomial in quantum and external fields compatible with ultraviolet dimension 4 and infrared dimension 2, neutral with respect to electric and ghost charge and CP-even. According to the fact that the l.h.s is BRS-symmetric and symmetric with respect to rigid symmetry (4.4), one gets

$$s_{\Gamma_{cl}}\Delta_m = 0, \qquad \mathcal{W}_\alpha\Delta_m = 0. \qquad (4.7)$$

Thereby $s_{\Gamma_{cl}}$ is the linearized version of the ST operator and acts on the quantum fields in the same way as the classical BRS transformations. We have therefore the task to find all independent field polynomials satisfying the above constraints and to express them in form of symmetric differential operators.

The 2- and 3-dimensional polynomials are already exhausted in the l.h.s, if one takes higher-order corrections of the shift and the parameter $\alpha_{inv}$ into account. The 4-dimensional polynomials are classified according to BRS variations, which are related to the anomalous dimensions, and non-variations, which are related to the $\beta$-functions. First we give a list of all symmetric field operators in the vector-ghost sector:

$$
\begin{aligned}
\mathcal{N}_V &= \int d^4x \Big( V_a \frac{\delta}{\delta V_a} - \rho_\alpha \frac{\delta}{\delta \rho_\alpha} \\
&\quad + \frac{1}{\cos(\theta_W - \theta_G)} (\sin\theta_G c_Z + \cos\theta_W c_A)(\sin\theta_W \frac{\delta}{\delta c_Z} + \cos\theta_G \frac{\delta}{\delta c_A}) \Big) \\
\hat{\mathcal{N}}_V &= \int d^4x \Big( (\sin\theta_W Z + \cos\theta_W A)(\sin\theta_W \frac{\delta}{\delta Z} + \cos\theta_W \frac{\delta}{\delta A}) \\
&\quad + \frac{1}{\cos(\theta_W - \theta_G)} (\sin\theta_G c_Z + \cos\theta_W c_A)(\sin\theta_W \frac{\delta}{\delta c_Z} + \cos\theta_G \frac{\delta}{\delta c_A}) \Big) \\
\mathcal{N}_B &= \int d^4x \Big( B_a \frac{\delta}{\delta B_a} + \bar{c}_a \frac{\delta}{\delta \bar{c}_a} \Big) \qquad (4.8) \\
\hat{\mathcal{N}}_B &= \int d^4x \Big( (\sin\theta_W B_Z + \cos\theta_W B_A)(\sin\theta_W \frac{\delta}{\delta B_Z} + \cos\theta_W \frac{\delta}{\delta B_A}) \\
&\quad + \frac{1}{\cos(\theta_W - \theta_G)} (\sin\theta_W \bar{c}_Z + \cos\theta_G \bar{c}_A)(\sin\theta_G \frac{\delta}{\delta \bar{c}_Z} + \cos\theta_W \frac{\delta}{\delta \bar{c}_A}) \Big) \\
\mathcal{N}_c &= \int d^4x \Big( c_+ \frac{\delta}{\delta c_+} + c_- \frac{\delta}{\delta c_-} \\
&\quad + \frac{1}{\cos(\theta_W - \theta_G)} (\cos\theta_G c_Z - \sin\theta_W c_A)(\cos\theta_W \frac{\delta}{\delta c_Z} - \sin\theta_G \frac{\delta}{\delta c_A}) \\
&\quad - \sigma_+ \frac{\delta}{\delta \sigma_+} - \sigma_- \frac{\delta}{\delta \sigma_-} - \sigma_3 \frac{\delta}{\delta \sigma_3} \Big).
\end{aligned}
$$

Some remarks are in order concerning the special form of these operators: According to rigid symmetry the counting operators of the charged fields are related to the ones of the neutral sector restricting the number of independent operators to two for the vectors and $B_a$ fields and one for the ghosts. Invariance under the ST identity relates the abelian field differential operators of ghosts and vectors. Furthermore it is seen that the abelian operator is not a BRS-variation and is related to the $\beta$-functions by the local abelian Ward identity (3.12), as it is usual in abelian gauge theories. The respective relation is given in (4.20). For completeness we have given the field operators for arbitrary ghost mass ratio (3.19); if one has introduced normalization conditions which set the ghost angle equal to the weak mixing angle, $\theta_G$ can be immediately replaced by $\theta_W$ in the above expressions.

The symmetric field operators of fermions can be split into the ones of the left-handed and right-handed fields. Due to the fact that we do not consider fermion mixing, we do not have to consider mixed operators between different fermion families:

$$\begin{aligned}
\mathcal{N}^L_{F_{\delta,i}} &= \int d^4x \,\Big(\overline{F^L_{\delta,i}}\frac{\delta}{\delta \overline{F^L_{\delta,i}}} - \overline{\Psi^R_{\delta,i}}\frac{\delta}{\delta \overline{\Psi^R_{\delta,i}}} + \frac{\delta}{\delta F^L_{\delta,i}}F^L_{\delta,i} - \frac{\delta}{\delta \Psi^R_{\delta,i}}\Psi^R_{\delta,i}\Big), \quad \delta = l, q \qquad (4.9)\\
\mathcal{N}^R_{f_i} &= \int d^4x \,\Big(\overline{f^R_i}\frac{\delta}{\delta \overline{f^R_i}} - \overline{\psi^L_{f,i}}\frac{\delta}{\delta \overline{\psi^L_{f,i}}} + \frac{\delta}{\delta f^R_i}f^R_i - \frac{\delta}{\delta \psi^L_{f,i}}\psi^L_{f,i}\Big), \qquad f_i = e_i, d_i, u_i.
\end{aligned}$$

The field operators of scalars comprise also the ones of the external scalars. They are symmetric with respect to the rigid operators, if one includes the shift of the Higgs field and the external Higgs field:

$$\begin{aligned}
\mathcal{N}_S + v\int d^4x\, \frac{\delta}{\delta H} &= \int d^4x\, \Big((\Phi + \mathrm{v})^\dagger \frac{\delta}{\delta \Phi^\dagger} + \frac{\delta}{\delta \Phi}(\Phi + \mathrm{v}) - \mathrm{Y}^\dagger \frac{\delta}{\delta \mathrm{Y}^\dagger} - \frac{\delta}{\delta \mathrm{Y}}\mathrm{Y}\Big) \quad (4.10)\\
\mathcal{N}_{\hat S} + \zeta v\int d^4x\, \frac{\delta}{\delta \hat H} &= \int d^4x\, \Big((\hat\Phi + \zeta\mathrm{v})^\dagger \frac{\delta}{\delta \hat\Phi^\dagger} + \frac{\delta}{\delta \hat\Phi}(\hat\Phi + \zeta\mathrm{v}) + \mathrm{q}^\dagger \frac{\delta}{\delta \mathrm{q}^\dagger} + \frac{\delta}{\delta \mathrm{q}}\mathrm{q}\Big).
\end{aligned}$$

Among the symmetric insertions there is one which mixes the external scalar with the quantum scalar. Defining the following mixed operator, which is symmetric with respect to rigid transformations,

$$\tilde{\mathcal{N}}_S + \zeta v\int d^4x\, \frac{\delta}{\delta H} = \int d^4x\, \Big((\hat\Phi + \zeta\mathrm{v})^\dagger \frac{\delta}{\delta \Phi^\dagger} + \frac{\delta}{\delta \Phi}(\hat\Phi + \zeta\mathrm{v})\Big), \qquad (4.11)$$

the corresponding BRS-symmetric insertion is given by:

$$\Big(\tilde{\mathcal{N}}_S + \zeta v\int d^4x\, \frac{\delta}{\delta H}\Big)\Gamma_{cl} + \int d^4x\, (\mathrm{q}^\dagger Y - Y^\dagger \mathrm{q}). \qquad (4.12)$$

The remaining symmetric insertions have to be generated by differentiating the classical action with respect to the free parameters; these are the coupling $e$, which is the perturbative expansion parameter, and furthermore the mass ratios, $\frac{M_W}{M_Z}$, for the weak interactions, $\frac{m_H}{M_Z}$ for the scalar interaction, and $\frac{m_{f_i}}{M_Z}$ for the Yukawa interactions. At this stage it is unavoidable to treat $\theta_G$, i.e. the ghost mass ratio (3.19), as an independent parameter, because its differentiation corresponds to an independent insertion in the gauge-fixing and ghost sector. Similarly it turns out that also the differentiations with respect to the two gauge parameters $\xi$ and $\hat\xi$ have to be included (cf. (2.36)).

Differentiations with respect to parameters which do not appear in the ST identity and the rigid Ward operators of the tree approximation directly correspond to symmetric insertions:

$$m_H\partial_{m_H},\; m_{f_i}\partial_{m_{f_i}},\; \xi\partial_\xi,\; \hat\xi\partial_{\hat\xi}\;. \qquad (4.13)$$

The differentiation with respect to the coupling $e$ is immediately symmetrized if one includes the shift; the operator

$$e\partial_e - e\partial_e v \int d^4x \,\Big(\frac{\delta}{\delta H} + \zeta\frac{\delta}{\delta \hat{H}}\Big) = e\partial_e + \frac{2}{e} M_Z \sin\theta_W \cos\theta_W \int d^4x \,\Big(\frac{\delta}{\delta H} + \zeta\frac{\delta}{\delta \hat{H}}\Big) \tag{4.14}$$

is BRS- and rigid symmetric. However, the differentiations with respect to the weak mixing angle and to the ghost angle

$$\partial_{\theta_W} = -M_Z \sin\theta_W \partial_{M_W}, \quad \partial_{\theta_G} \tag{4.15}$$

have to be supplemented by field differentiations in order to be BRS-symmetric:

$$\begin{aligned}
\tilde{\partial}_{\theta_W} \;&\equiv\; \partial_{\theta_W} + \int d^4x \,\Big(A\frac{\delta}{\delta Z} - Z\frac{\delta}{\delta A} + B_A\frac{\delta}{\delta B_Z} - B_Z\frac{\delta}{\delta B_A}\Big) \\
&+\frac{1}{\cos(\theta_W - \theta_G)} \int d^4x \; c_A\Big(\frac{\delta}{\delta c_Z} + \sin(\theta_W - \theta_G)\frac{\delta}{\delta c_A}\Big) \\
&-\frac{1}{\cos(\theta_W - \theta_G)} \int d^4x \,\Big(\bar{c}_Z + \sin(\theta_W - \theta_G)\bar{c}_A\Big)\frac{\delta}{\delta \bar{c}_A} \qquad (4.16)\\
\tilde{\partial}_{\theta_G} \;&\equiv\; \partial_{\theta_G} - \frac{1}{\cos(\theta_W - \theta_G)} \int d^4x \; c_Z\Big(\frac{\delta}{\delta c_A} + \sin(\theta_W - \theta_G)\frac{\delta}{\delta c_Z}\Big) \\
&+\frac{1}{\cos(\theta_W - \theta_G)} \int d^4x \,\Big(\sin(\theta_W - \theta_G)\bar{c}_Z + \bar{c}_A\Big)\frac{\delta}{\delta \bar{c}_Z}. \qquad (4.17)
\end{aligned}$$

These two operators are immediately symmetric with respect to the rigid transformations up to soft insertions corresponding to the shift. The shift depends in the tree approximation on the weak mixing angle and one has to enlarge $\tilde{\partial}_{\theta_W}$ by the differentiation with respect to the Higgs field and the external Higgs field. The operator

$$\tilde{\partial}_{\theta_W} - \partial_{\theta_W} v \int d^4x \,\Big(\frac{\delta}{\delta H} + \zeta\frac{\delta}{\delta \hat{H}}\Big) = \tilde{\partial}_{\theta_W} - \frac{2}{e} M_Z \cos 2\theta_W \int d^4x \,\Big(\frac{\delta}{\delta H} + \zeta\frac{\delta}{\delta \hat{H}}\Big) \tag{4.18}$$

is then also rigid symmetric.

Acting with the symmetric operators (4.8), (4.9), (4.10), (4.13), (4.14), (4.18) and (4.17) on the classical action one produces together with the polynomial (4.12) a complete basis for the breaking of the symmetric dilatational operator (4.6) in 1-loop order. Therefore it is possible to give the breaking in the form of a CS equation, i.e. as a linear combination of differential operators. Writing all the soft breakings produced by symmetrization with respect to the shift on the r.h.s we get in 1-loop order the CS equation of the SM:

$$\Big(\; m\partial_m + \beta_e e\partial_e - \beta_{M_W}\tilde{\partial}_{\theta_W} + \beta_{m_H} m_H \partial_{m_H} + \sum_{i=1}^{N_F}\sum_f \beta_{m_{f_i}} m_{f_i}\partial_{m_{f_i}} - \beta_{\theta_G}\tilde{\partial}_{\theta_G} \tag{4.19}$$

$$-\; \gamma_V \mathcal{N}_V - \gamma_B \mathcal{N}_B - \gamma_\xi \partial_\xi - \gamma_c \mathcal{N}_c - \hat{\gamma}_V \hat{\mathcal{N}}_V - \hat{\gamma}_B \hat{\mathcal{N}}_B - \gamma_{\hat{\xi}} \partial_{\hat{\xi}} - \gamma_S \mathcal{N}_S - \gamma_{\hat{S}} \mathcal{N}_{\hat{S}} - \tilde{\gamma}_S \tilde{\mathcal{N}}_S$$

$$
\begin{aligned}
&- \sum_{i=1}^{N_F}(\gamma_{F_{l,i}}\mathcal{N}^L_{F_{l,i}} + \gamma_{F_{q,i}}\mathcal{N}^L_{F_{q,i}} + \gamma_{e_i}\mathcal{N}^R_{e_i} + \gamma_{u_i}\mathcal{N}^R_{u_i} + \gamma_{d_i}\mathcal{N}^R_{d_i})\Big)\,\Gamma \\
&= \int d^4x\,\Big((1+\beta_e e\partial_e - \beta_{M_W}\partial_{\theta_W})v\Big(\frac{\delta\Gamma}{\delta H} + \zeta\frac{\delta\Gamma}{\delta\hat{H}}\Big) + v(\gamma_S + \tilde{\gamma}_S)\frac{\delta\Gamma}{\delta H} + \zeta v\gamma_{\hat{S}}\frac{\delta\Gamma}{\delta\hat{H}} + \alpha_{inv}\frac{\delta\Gamma}{\delta\hat{\varphi}_o}\Big) \\
&+ \int d^4x\;\tilde{\gamma}_S(q^\dagger Y - Y^\dagger q).
\end{aligned}
$$

Further information on the coefficient functions can be achieved by using the local Ward identity (3.12), which expresses gauge invariance of the classical action under the abelian transformation (3.13). Calculating the commutator of the CS operator and the local Ward operator one gets:

$$
\beta_e = \frac{\sin\theta_W}{\cos\theta_W}\beta_{M_W} + \gamma_V + \hat{\gamma}_V\;. \tag{4.20}
$$

Since we have used a linear gauge fixing in the propagating fields all the Green functions which include B-fields in the external legs do not get logarithmic higher-order corrections. The action of $m\partial_m$ on these Green functions is therefore trivial according to their canonical dimensions. Therefrom we get:

$$
\begin{array}{rclrcl}
\gamma_B &=& -\gamma_V & \hat{\gamma}_B &=& -\hat{\gamma}_V \\
\gamma_\xi &=& 2\xi\gamma_V & \gamma_{\hat{\xi}} &=& 2(\gamma_V + \hat{\gamma}_V)\hat{\xi} + 2\xi\hat{\gamma}_V \\
\beta_{\theta_G} &=& \sin\theta_G\cos\theta_G\gamma_V & \gamma_{\hat{S}} &=& \beta_e + \frac{\cos\theta_W}{\sin\theta_W}\beta_{M_W} + \gamma_V - \gamma_S\;.
\end{array} \tag{4.21}
$$

From the explicit 1-loop expressions it is seen that the choice $\theta_G = \theta_W$ and $\hat{\xi} = 0$ (2.37) is not stable under renormalization: Since the coefficients of the respective differential operators $\beta_{\theta_G}$ and $\gamma_{\hat{\xi}}$ are functions of the anomalous dimensions of vectors and for this reason non-vanishing (see (5.6)), the differentiation with respect to the independent parameters $\hat{\xi}$ and $\theta_G$ has to be included in order to be able to formulate the CS equation.

According to the derivation, the CS equation (4.19) and the relations (4.20) and (4.21) are valid in this form only in 1-loop order. Proceeding to higher orders of perturbation theory goes along the same lines as in the 1-loop order, especially the number of independent operators remains the same to all orders of perturbation theory. Their explicit form is modified order by order in such a way that they become symmetric with respect to the ST identity and rigid Ward identities valid for higher-order Green functions. Modifications essentially arise from establishing the normalization conditions for separating massless/massive fields at $p^2 = 0$ in addition to the usual on-shell conditions for the masses. It is interesting to note that the importance of these normalization conditions for the off-shell existence of Green functions can be already seen from the CS equation: Insertions with infrared degree 2 as $A^\mu A_\mu$ on the r.h.s. are not forbidden from infrared power counting, but those terms are mass terms for massless particles and endanger infrared existence off-shell and physical interpretation. In particular insertions of such field polynomials in higher-order Green functions are non-integrable and have to be proven to be absent. (A similar analysis has been carried out in a simple non-gauge model with spontaneously broken symmetry in Ref. [23].) The test with respect to the respective 2-point functions at $p^2 = 0$ shows that these terms vanish if the mass matrix of massless/massive fields is diagonal at $p^2 = 0$.

## 5. The 1-loop coefficient functions

Due to the spontaneous symmetry breaking mechanism the CS equation has an unconventional form compared to the symmetric $SU(2) \times U(1)$ gauge theory: It is an inhomogeneous equation with a soft mass insertion on the r.h.s., because dilatations are broken by the mass terms. The hard anomalies have to be absorbed in $\beta$-functions with respect to mass differentiations and cannot be expanded in power series of couplings according to the loop expansion. Moreover, since the $W$- and $Z$-bosons have different masses the anomalous dimensions are not purely leg counting operators in the neutral sector, but include field operators, which mix the neutral vectors and ghosts. Such field operators are not present in the renormalization group equation of the symmetric theory. The perturbative expansion parameter is the electromagnetic coupling, which gives rise to a $\beta$-function $\beta_e$.

We want to demonstrate how the $\beta$-functions and anomalous dimensions are determined from the CS equation by testing with respect to appropriate vertices. For this purpose we give that part of the CS equation which is relevant for the test with respect to vectors, quantum scalars and fermions in its explicit form, setting external fields and ghosts to zero:

$$\begin{aligned}
&\Big\{m\partial_m + \beta_e e\partial_e + \beta_{m_H} m_H \partial_{m_H} + \sum_{f_i} \beta_{m_{f_i}} m_{f_i} \partial_{m_{f_i}} - \beta_{M_W}\Big(\partial_{\theta_W} - \int d^4x\,(Z^\mu \frac{\delta}{\delta A^\mu} - A^\mu \frac{\delta}{\delta Z^\mu})\Big) \\
&-\gamma_V\Big(\int d^4x\, V_a^\mu \frac{\delta}{\delta V_a^\mu} + 2\xi\partial_\xi + 2\xi\partial_{\hat\xi} + \sin\theta_G\cos\theta_G\partial_{\theta_G}\Big) - \gamma_S \int d^4x\,(\Phi\frac{\delta}{\delta\Phi} + \frac{\delta}{\delta\Phi^\dagger}\Phi^\dagger) \qquad (5.1)\\
&-\hat\gamma^V\Big(\int d^4x\,(\sin\theta_W Z^\mu + \cos\theta_W A^\mu)(\sin\theta_W \frac{\delta}{\delta Z^\mu} + \cos\theta_W \frac{\delta}{\delta A^\mu}) + 2(\hat\xi + \xi)\partial_{\hat\xi}\Big) \\
&-\sum_{F_{\delta,i}} \gamma_{F_{\delta,i}} \int d^4x\,\Big(\overline{F^L_{\delta,i}}\frac{\delta}{\delta \overline{F^L_{\delta,i}}} + \frac{\delta}{\delta F^L_{\delta,i}} F^L_{\delta,i}\Big) - \sum_{f_i}\gamma_{f_i}\int d^4x\,\Big(\overline{f^R_i}\frac{\delta}{\delta\overline{f^R_i}} + \frac{\delta}{\delta f^R_i} f^R_i\Big)\Big\}\Gamma\Big|_{\substack{ext.f.=0\\ c_a,\bar c_a=0}} \\
&\quad = [\Delta_s]^3_3 \cdot \Gamma.
\end{aligned}$$

One is able to determine the $\beta$-functions from vertex functions of UV-dimension 4, using thereby that the soft insertion will vanish for asymptotic Euclidean momenta much larger than the mass of the heaviest particle involved. Similarly the anomalous dimensions are determined from the residua at asymptotic momentum. We evaluate the coefficient functions in the most general linear gauge fixing invariant with respect to rigid symmetry transformations as given in (2.36). For completeness, in the appendix we list the propagators of vector and scalar fields in the general linear $R_\xi$ gauge (2.22). From there the propagators compatible with rigid symmetry are obtained by assigning to the gauge-fixing parameters the values (2.32) – (2.35). In the computation of the CS coefficient functions only those parts of propagators of vector and scalar fields contribute which behave like $\frac{1}{p^2}$ in the expansion for asymptotically large $p^2$.

First we determine the $\beta$-functions of the electromagnetic coupling, $\beta_e$, and of the $W$-boson mass, $\beta_{M_W}$, and the anomalous dimensions of the vectors $\gamma_V$ and $\hat{\gamma}_V$. The anomalous dimensions of the vectors are calculated from the transverse parts of the 2-point functions:

$$\Gamma^{\mu\nu}_{V_aV_b}(p) \equiv -\left(\eta^{\mu\nu} - \frac{p^\mu p^\mu}{p^2}\right)\Gamma^T_{ab}(p^2) - \frac{p^\mu p^\mu}{p^2}\Gamma^L_{ab}(p^2). \tag{5.2}$$

In the tree approximation we have

$$\partial_{p^2}\Gamma^{T(0)}_{ab} = \tilde{I}_{ab}. \tag{5.3}$$

We find therefore in the asymptotic region (all functions involved are purely of 1-loop order)

$$\begin{aligned} m\partial_m\partial_{p^2}\Gamma^{T(1)}_{+-} &\stackrel{p^2\to-\infty}{=} 2\gamma^{(1)}_V \\ m\partial_m\partial_{p^2}\Gamma^{T(1)}_{ZZ} &\stackrel{p^2\to-\infty}{=} 2(\gamma^{(1)}_V + \sin^2\theta_W\hat{\gamma}^{(1)}_V) \\ m\partial_m\partial_{p^2}\Gamma^{T(1)}_{AA} &\stackrel{p^2\to-\infty}{=} 2(\gamma^{(1)}_V + \cos^2\theta_W\hat{\gamma}^{(1)}_V) \\ m\partial_m\partial_{p^2}\Gamma^{T(1)}_{ZA} &\stackrel{p^2\to-\infty}{=} 2\sin\theta_W\cos\theta_W\hat{\gamma}^{(1)}_V, \end{aligned} \tag{5.4}$$

which determines the high-energy logarithms of the one-loop self-energies, e.g.

$$\left(\partial_{p^2}\Gamma^{T(1)}_{+-}\right)_{\text{lead. log}} = -\gamma^{(1)}_V \ln\frac{|p^2|}{m^2}. \tag{5.5}$$

Accordingly, the anomalous dimensions $\gamma_V$ and $\hat{\gamma}_V$ are obtained by calculating the high-energy logarithms of two of the self-energies appearing in (5.4), while the leading logarithms of the other two self-energies are then already fixed. In the general linear gauge specified in (2.36) we get the following result:

$$\gamma^{(1)}_V = \frac{e^2}{4\pi^2\sin^2\theta_W}\Big(\frac{6\xi-25}{24} + \frac{1}{3}N_F\Big) \tag{5.6}$$

$$\hat{\gamma}^{(1)}_V = \frac{e^2}{4\pi^2}\Big(-\frac{6\xi-25}{24\sin^2\theta_W} + \frac{1}{24\cos^2\theta_W} + \frac{-3+8\sin^2\theta_W}{9\sin^2\theta_W\cos^2\theta_W}N_F\Big). \tag{5.7}$$

The fermion contributions to the coefficients of the high-energy logarithms are the same as those given in Ref. [18] in the framework of the background-field method, while the contributions of the vector bosons are gauge-parameter-dependent. The gauge-parameter dependence of the anomalous dimensions is the same as in usual $R_\xi$ gauges, i.e. they are independent of the abelian gauge parameter $\hat{\xi}$, the ghost mass parameter $\zeta$ and of the ghost angle (3.19). This is seen most easily by noting that the diagrams which contribute to the photon self-energy and to the Z-photon self-energy have only charged fields in internal lines and are not affected by non-diagonal propagators in the neutral sector nor by transformations of neutral ghosts (3.20). As before $N_F$ denotes the number of fermion generations.

The $\beta$-function $\beta_{M_W}$ can be determined from the neutrino–neutrino–photon vertex at high energies. Testing the CS equation (5.1) with respect to this vertex we get the following result:

$$m\partial_m \Gamma^{(1)}_{\bar{\nu}\nu A_\mu} \overset{p^2\to -\infty}{=} (\beta^{(1)}_{M_W} + \sin\theta_W \cos\theta_W \hat{\gamma}^{(1)}_V)\Gamma^{(0)}_{\bar{\nu}\nu Z_\mu} \, . \tag{5.8}$$

Since the photon does not couple to neutrinos in the tree approximation the contributions on the r.h.s. completely arise from the mixed field operators $A\frac{\delta}{\delta Z}$ present in the symmetric operators of $\beta^{(1)}_{\theta_W}$ and $\hat{\gamma}^{(1)}_V$. In the Feynman gauge ($\xi = 1, \hat{\xi} = 0$) one has

$$\beta^{(1)}_{M_W} + \sin\theta_W \cos\theta_W \hat{\gamma}^{(1)}_V = -\frac{e^2}{4\pi^2}\frac{\cos\theta_W}{\sin\theta_W}, \tag{5.9}$$

which gives the result

$$\beta^{(1)}_{M_W} = -\frac{e^2}{4\cdot 24\pi^2 \sin\theta_W \cos\theta_W}\Big((43 - 8N_F) - (42 - \tfrac{64}{3}N_F)\sin^2\theta_W\Big). \tag{5.10}$$

Applying the algebraic control of gauge-parameter dependence [24] to spontaneously broken theories [25] it can be derived that $\beta_{M_W}$ is gauge-parameter-independent.

The abelian relation (4.20) allows to determine $\beta^{(1)}_e$ without calculating further diagrams from the results obtained for $\gamma^{(1)}_V$, $\hat{\gamma}^{(1)}_V$ and $\beta^{(1)}_{M_W}$:

$$\beta^{(1)}_e = -\frac{e^2}{24\cdot 4\pi^2}\Big(42 - \tfrac{64}{3}N_F\Big). \tag{5.11}$$

Alternatively $\beta^{(1)}_e$ can of course also directly be obtained from the $W^+W^-A$ vertex at asymptotic momenta

$$m\partial_m \Gamma^{(1)}_{W^\mu_+ W^\nu_- A^\rho} \longrightarrow (3\gamma^{(1)}_V - \beta^{(1)}_e - \frac{\cos\theta_W}{\sin\theta_W}\beta^{(1)}_{M_W})\Gamma^{(0)}_{W^\mu_+ W^\nu_- A^\rho} \tag{5.12}$$

or from the $\bar{e}eA$ vertex (cf. (5.18)). We have explicitly checked that this indeed results in $\beta^{(1)}_e$ as given in (5.11). As can be seen in (5.11), the $\beta$-function of the electromagnetic coupling is QED-like in the sense that it only depends on the electromagnetic coupling $e^2$ but not on $\sin^2\theta_W$. However, due to non-abelian interactions of the photon with the $W$-bosons it receives contributions with negative sign. This leads to the fact that $\beta_e^{(1)}$ in the SM has a negative sign if one includes only one fermion family. In section 6.2 we compare it to the QED $\beta$-function.

We now turn to the anomalous dimensions of the fermions, which are needed e.g. for the independent determination of $\beta_e^{(1)}$ from the electron–electron–photon vertex. Splitting the fermion self-energy into left- and right-handed parts and into the scalar mass contribution,

$$\Gamma_{\bar{f}f} = \not{p} - m_f + \not{p}\frac{1}{2}(1-\gamma^5)\Sigma_f^L + \not{p}\frac{1}{2}(1+\gamma^5)\Sigma_f^R + m\Sigma_f^m\ , \tag{5.13}$$

one is able to calculate $\gamma_{F_{\delta,i}}$ from left-handed and $\gamma_{f_i}$ from right-handed contributions:

$$\begin{aligned} m\partial_m\Sigma_{e_i}^{L(1)} &\stackrel{p^2\to-\infty}{=} 2\gamma_{F_{l,i}}^{(1)}\ , \quad m\partial_m\Sigma_{d_i}^{L(1)} \stackrel{p^2\to-\infty}{=} 2\gamma_{F_{q,i}}^{(1)}\ , \\ m\partial_m\Sigma_{f_i}^{R(1)} &\stackrel{p^2\to-\infty}{=} 2\gamma_{f_i}^{(1)}\ . \end{aligned} \tag{5.14}$$

Calculating the high-energy logarithms of the fermion self-energy contributions one gets the following result:

$$\gamma_{F_{l,i}}^{(1)} = \frac{e^2}{16\pi^2}\frac{1}{\sin^2 2\theta_W}\left[\left(3-2\sin^2\theta_W\right)\xi + \sin^2\theta_W\hat{\xi} + \frac{m_{e_i}^2}{M_Z^2}\right] \tag{5.15}$$

$$\gamma_{F_{q,i}}^{(1)} = \frac{e^2}{16\pi^2}\frac{1}{\sin^2 2\theta_W}\left[\left(3-\frac{26}{9}\sin^2\theta_W\right)\xi + \frac{1}{9}\sin^2\theta_W\hat{\xi} + \frac{m_{u_i}^2+m_{d_i}^2}{M_Z^2}\right] \tag{5.16}$$

$$\gamma_{f_i}^{(1)} = \frac{e^2}{16\pi^2}\frac{2}{\sin^2 2\theta_W}\left[2Q_f^2\sin^2\theta_W\left(\xi+\hat{\xi}\right) + \frac{m_{f_i}^2}{M_Z^2}\right]. \tag{5.17}$$

For the asymptotic behavior of the electron–electron–photon vertex we obtain:

$$\begin{aligned} m\partial_m\Gamma_{\bar{e}eA_\mu}^{(1)} \stackrel{p^2\to-\infty}{=}\ & e\Big(\gamma_V^{(1)} + \hat{\gamma}_V^{(1)} - \beta_e^{(1)} + 2\gamma_e^{(1)} + \beta_{M_W}^{(1)}\frac{\sin\theta_W}{\cos\theta_W}\Big)\gamma^\mu \\ & + e\Big(2\gamma_{F_l}^{(1)} - 2\gamma_e^{(1)} - \frac{1}{\sin 2\theta_W}(\beta_{M_W}^{(1)} + \sin\theta_W\cos\theta_W\hat{\gamma}_V^{(1)})\Big)\gamma^\mu\frac{1}{2}(1-\gamma^5). \end{aligned} \tag{5.18}$$

In this formula we have already inserted the tree vertices

$$\begin{aligned} \Gamma_{\bar{e}eA_\mu}^{(0)} &= e\gamma^\mu \\ \Gamma_{\bar{e}eZ_\mu}^{(0)} &= -e\frac{1}{\sin 2\theta_W}\gamma^\mu\frac{1}{2}(1-\gamma^5) + e\frac{\sin\theta_W}{\cos\theta_W}\gamma^\mu. \end{aligned} \tag{5.19}$$

As mentioned above, using (5.18) and the results of (5.15) one can check the abelian relation we have used to determine $\beta_e$. It is seen in (5.18) that the parity non-violating contribution satisfies an analogous relation as in QED: The high-energy logarithms of the electron–electron–photon vertex are completely related to the anomalous dimensions of (right-handed) electrons. Due to the non-abelian contributions there are however parity-violating high-energy logarithms for the off-shell Green functions.

For calculating the remaining $\beta$-functions of fermion masses and the Higgs mass one first has to determine the anomalous dimensions of the scalars. We obtain

$$\gamma_S^{(1)} = \frac{e^2}{8\pi^2}\frac{1}{\sin^2 2\theta_W}\left[\sum_i \frac{m_{e_i}^2+3m_{d_i}^2+3m_{u_i}^2}{M_Z^2} + \frac{1}{2}\left(3-2\sin^2\theta_W\right)(\xi-3)+\frac{1}{2}\sin^2\theta_W\hat{\xi}\right]. \tag{5.20}$$

The $\beta$ functions $\beta_{m_{f_i}}^{(1)}$ and $\beta_{m_H}^{(1)}$ are determined from the high-energy logarithms according to the following formulas:

$$\begin{aligned}
m\partial_m\Gamma^{(1)}_{\bar{f}_i f_i H} &\overset{p^2\to-\infty}{\doteq} \Big(-\beta_e^{(1)} - 2\frac{\cos 2\theta_W}{\sin 2\theta_W}\beta_{M_W}^{(1)} - \beta_{m_{f_i}}^{(1)} + \gamma_S^{(1)} + \gamma_{F_{\delta,i}}^{(1)} + \gamma_{f_i}^{(1)}\Big)\Gamma^{(0)}_{\bar{f}_i f_i H} \\
m\partial_m\Gamma^{(1)}_{HHHH} &\overset{p^2\to-\infty}{\doteq} 2\Big(-\beta_e^{(1)} - 2\frac{\cos 2\theta_W}{\sin 2\theta_W}\beta_{M_W}^{(1)} - \beta_{m_H}^{(1)} + 2\gamma_S^{(1)}\Big)\Gamma^{(0)}_{HHHH}\,.
\end{aligned} \tag{5.21}$$

Therefrom we derive the result:

$$\begin{aligned}
\beta_{m_{e_i}}^{(1)} &= \frac{e^2}{24\pi^2\sin^2 2\theta_W}\Big\{\frac{9}{2}\frac{m_{e_i}^2}{M_Z^2} + 3\sum_{f_j}\frac{m_{e_j}^2+3m_{u_j}^2+3m_{d_j}^2}{M_Z^2} \\
&\quad +(\frac{59}{2}-8N_F)-(95-16N_F)\sin^2\theta_W+(42-64\frac{N_F}{3})\sin^4\theta_W\Big\}
\end{aligned} \tag{5.22}$$

$$\begin{aligned}
\beta_{m_{u_i}}^{(1)} &= \frac{e^2}{24\pi^2\sin^2 2\theta_W}\Big\{\frac{9}{2}\frac{m_{u_i}^2-m_{d_i}^2}{M_Z^2} + 3\sum_{f_j}\frac{m_{e_j}^2+3m_{u_j}^2+3m_{d_j}^2}{M_Z^2} \\
&\quad +(\frac{59}{2}-8N_F)-(81-16N_F)\sin^2\theta_W+(42-64\frac{N_F}{3})\sin^4\theta_W\Big\}
\end{aligned} \tag{5.23}$$

$$\begin{aligned}
\beta_{m_{d_i}}^{(1)} &= \frac{e^2}{24\pi^2\sin^2 2\theta_W}\Big\{\frac{9}{2}\frac{m_{d_i}^2-m_{u_i}^2}{M_Z^2} + 3\sum_{f_j}\frac{m_{e_j}^2+3m_{u_j}^2+3m_{d_j}^2}{M_Z^2} \\
&\quad +(\frac{59}{2}-8N_F)-(75-16N_F)\sin^2\theta_W+(42-64\frac{N_F}{3})\sin^4\theta_W\Big\}
\end{aligned} \tag{5.24}$$

$$\begin{aligned}
\beta_{m_H}^{(1)} &= \frac{e^2}{24\pi^2\sin^2 2\theta_W}\Big\{9\frac{m_H^2}{M_Z^2} + 6\sum_{f_j}\frac{m_{e_j}^2+3m_{u_j}^2+3m_{d_j}^2}{M_Z^2} \\
&\quad -12\sum_{f_j}\frac{m_{e_j}^4+3m_{d_j}^4+3m_{u_j}^4}{M_Z^2 m_H^2} \\
&\quad +(16-8N_F)-(68-16N_F)\sin^2\theta_W+(42-64\frac{N_F}{3})\sin^4\theta_W \\
&\quad +\frac{M_Z^2}{m_H^2}\Big(27-36\sin^2\theta_W+18\sin^4\theta_W\Big)\Big\}.
\end{aligned} \tag{5.25}$$

From considerations of gauge-parameter dependence it is seen that these $\beta$-functions are gauge-parameter-independent [24, 25]. As mentioned above, the same holds for the $\beta$-functions of the electromagnetic coupling and the vector-boson mass ratio.

Finally, by testing with respect to the ghost self-energy one finds the following result for the anomalous dimension of the ghosts, $\gamma_c$:

$$\gamma_c^{(1)} = \frac{e^2}{4\pi^2 \sin^2\theta_W}\left(\frac{\xi}{2} - \frac{43}{24} + \frac{1}{3}N_F\right). \tag{5.26}$$

In the Landau gauge ($\xi = 0$) the anomalous dimension of the Faddeev-Popov ghosts is equal to the $\beta$-function of the non-abelian gauge coupling $g_2$ in (6.5):

$$\gamma_c^{(1)}\Big|_{\xi=0} = \beta_e^{(1)} + \frac{\cos\theta_W}{\sin\theta_W}\beta_{M_W}^{(1)}. \tag{5.27}$$

This coincidence is not accidental, but is derived from the existence of an integrated antighost equation in the Landau gauge.

Having determined the 1-loop $\beta$-functions and anomalous dimensions, it is possible to determine the high-energy logarithms of any 1-loop vertex function of the Standard Model in an analogous way as shown, for instance, in (5.12) for the W-boson–photon vertex and in (5.18) for the electron–electron–photon vertex. Since we have also calculated the anomalous dimensions of ghosts (5.26), this is also possible for the external field vertices appearing in (3.1), which determine the higher-order corrections to the BRS transformations.

In their applications the importance of the CS and RG equation is founded in the fact that from the knowledge of the equations at 1-loop order one can draw conclusions for the asymptotic behavior of the vertex functions in higher orders. In particular, if the 1-loop coefficient functions of the CS equation are given in a general gauge as has been worked out above, one is able to determine the quadratic (leading) logarithms of 2-loop order for any vertex function of the Standard Model. For illustration we evaluate the CS equation for the photon self-energy in 2-loop order at an asymptotically large momentum:

$$\begin{aligned} m\partial_m \partial_{p^2}\Gamma_{AA}^{T(2)} \overset{p^2\to-\infty}{=} & \Big(2\gamma_V^{(1)} + 2\hat\gamma_V^{(1)}\cos^2\theta_W - \beta_e^{(1)} e\partial_e + \beta_{M_W}^{(1)}\partial_{\theta_W} + 2\gamma_V^{(1)}\xi\partial_\xi\Big)\partial_{p^2}\Gamma_{AA}^{T(1)} \\ & + (2\hat\gamma_V^{(1)}\cos\theta_W\sin\theta_W + 2\beta_{M_W}^{(1)})\partial_{p^2}\Gamma_{ZA}^{T(1)} + \mathrm{Const.}^{(2)}. \end{aligned} \tag{5.28}$$

In Const.$^{(2)}$ all terms are included which approach constants if we take the limit of asymptotically large Euclidean momentum $p^2$. They give rise to linear logarithmic contributions. Contributions to these terms arise from three different sources:

1. Applying the CS equation (4.19) with 2-loop coefficient functions to the tree vertices in analogy to the 1-loop case (5.4), one gets constant contributions with 2-loop coefficient functions which have to be determined by testing with respect to appropriate vertex functions. In the example above the 2-loop coefficient functions read $2(\gamma^{(2)} + \hat\gamma^{(2)}\cos^2\theta_W)$.

2. Since the ST operator and the Ward operators of rigid symmetry are renormalized in the on-shell schemes, the symmetric operators which build up the CS equation (see section 4) get higher-order corrections. In 2-loop order these corrections depend on the 1-loop corrections to the ST and Ward operators and on the CS coefficient functions of 1-loop order. These contributions depend strongly on the normalization conditions, but can be determined by a 1-loop calculation.

3. In 2-loop order for asymptotic momenta not only logarithms arise from the 1-loop vertex functions, but also constants, i.e. in the example above

$$\Gamma^{T(1)}_{ZA} \stackrel{p^2\to-\infty}{=} -\sin\theta_W\cos\theta_W\hat{\gamma}^{(1)}_V \ln\frac{|p^2|}{m^2} + C^{(1)}_{ZA}\,. \tag{5.29}$$

   The finite constant $C^{(1)}_{ZA}$ is determined from the normalization conditions for diagonalizing the mass matrix of photon and $Z$-boson on-shell and depends on the mass parameters of the Standard Model. In general such finite constants as shown in the above example are specific for on-shell normalization conditions of spontaneously broken theories.

All these constant terms contribute to the single logarithms of the photon self-energy in 2-loop order and, of course, the computation of all single logarithms of 2-loop order demands a 2-loop calculation. However, in the list above we have given also such constant contributions which can be determined from a 1-loop calculation. They all depend strongly on the normalization conditions. In Ref. [6] such 1-loop induced logarithmic contributions with large mass-dependent logarithmic coefficients have been found in the spontaneously broken Yukawa-Higgs model. They arise from normalization-dependent 1-loop contributions as discussed above and have to be separated from the mass-parameter independent logarithmic contributions to the 2-loop order. In Ref. [6] this has been achieved by using the consistency equation between the CS equation and the RG equation. For further applications it is certainly very interesting to completely single out these large 1-loop induced contributions by a self-consistent construction of higher-order solutions to the CS equation or by using the RG equation in a similar way as in Ref. [6].

Focusing now on the quadratic logarithms of 2-loop order, we are able to evaluate (5.28) by inserting the asymptotic 1-loop results (5.4)

$$\begin{aligned}\left(\partial_{p^2}\Gamma^{T(1)}_{AA}\right)_{\text{lead. log}} &= -(\gamma^{(1)}_V + \cos^2\theta_W\hat{\gamma}^{(1)}_V)\ln\frac{|p^2|}{m^2} \\ \left(\partial_{p^2}\Gamma^{T(1)}_{ZA}\right)_{\text{lead. log}} &= -\sin\theta_W\cos\theta_W\hat{\gamma}^{(1)}_V\ln\frac{|p^2|}{m^2}.\end{aligned} \tag{5.30}$$

Further simplification can be achieved by eliminating $\beta^{(1)}_e$ using the abelian relation (4.20) and by noting that the coefficient of the leading logarithm of the photon self-energy depends only on the electromagnetic coupling in 1-loop order. Then we end up with

$$\partial_{p^2}\Gamma^{T(2)}_{AA} \stackrel{p^2\to-\infty}{=} -\frac{1}{2}(\hat\gamma^{(1)}_V \sin\theta_W\cos\theta_W + \beta^{(1)}_{M_W})\frac{\sin\theta_W}{\cos\theta_W}\gamma^{(1)}_V \ln^2\frac{|p^2|}{m^2} \\ +\frac{1}{2}\gamma^{(1)}_V\xi\partial_\xi(\gamma^{(1)}_V+\cos^2\theta_W\hat\gamma^{(1)}_V)\ln^2\frac{|p^2|}{m^2}+\mathcal{O}(\ln\frac{|p^2|}{m^2}). \tag{5.31}$$

Since the anomalous dimensions of vectors are not gauge-parameter-independent, the derivative with respect to the gauge parameter contributes to the 2-loop order leading logarithms and underlines the significance of having calculated 1-loop coefficient functions in a general gauge. Inserting the explicit expressions of 1-loop order, (5.6) and (5.10), we finally get

$$\begin{aligned}\partial_{p^2}\Gamma^{T(2)}_{AA} &\stackrel{p^2\to-\infty}{=} \frac{e^2}{4\pi^2}\gamma^{(1)}_V\ln^2\frac{|p^2|}{m^2}\frac{1}{2}\left(\frac{\xi}{2}+\frac{3}{4}\right)+\mathcal{O}(\ln\frac{|p^2|}{m^2}) \\ &= \frac{e^4}{16\pi^4}\ln^2\frac{|p^2|}{m^2}\frac{1}{2\sin^2\theta_W}\left(\frac{\xi}{2}+\frac{3}{4}\right)\left(\frac{6\xi-25}{24}+\frac{1}{3}N_F\right)+\mathcal{O}(\ln\frac{|p^2|}{m^2}).\end{aligned} \tag{5.32}$$

Here $m$ denotes the largest mass parameter of the Standard Model. As a result, for asymptotic momenta much larger than all masses of the theory the photon self-energy includes quadratic logarithms in 2-loop order. This is in contrast to pure QED and is caused by the non-abelian interaction of the photon with the W-bosons and the Faddeev-Popov ghosts.

Further applications as well as a detailed consideration of the above-mentioned 1-loop induced large logarithms in 2-loop order will be given elsewhere.

# 6. Comparison with the massless symmetric theory and QED

## 6.1. Symmetric theory

From pure power-counting arguments it has been reasoned that the divergence structure of the symmetric theory is related to the one of the corresponding spontaneously broken theory [26]. The divergence structure corresponds to the appearance of high-energy logarithms order by order in perturbation theory. In 1-loop order the high-energy logarithms of the spontaneously broken theory are the same as the ones of the symmetric theory, since they arise from diagrams with only 4-dimensional vertices which are not affected by spontaneous breaking of the theory. The low-energy structure of the spontaneously broken theory is summarized in the r.h.s. of the CS equation, which vanishes if one goes to non-exceptional momenta much larger than the masses of the theory. As we have shown in the previous section, the $\beta$-functions of the CS equation are related to the asymptotic logarithms of the model. For this reason there exist in 1-loop order simple relations between the $\beta$-functions of the electroweak Standard Model and the corresponding $SU(2) \times U(1)$ massless symmetric theory. These relations reflect the tree relations between the parameters of the symmetric and the spontaneously broken theory. Since these relations get higher-order corrections it is expected that the higher-order $\beta$-functions get different higher-order contributions in the spontaneously broken theory than in the symmetric one. Indeed the constants which appear as a consequence of on-shell conditions in the asymptotic limit (see (5.29)) enter these higher-order corrections and affect – as already pointed out above – the coefficients of 2-loop single logarithms. A detailed computation of mass effects due to spontaneous breaking of the theory and due to on-shell conditions has been carried out in Ref. [6]. In order to specify these contributions one has to find self-consistent solutions of the CS equation in higher orders in the on-shell schemes. In the present context we restrict ourselves to the 1-loop relations between the $\beta$-functions of the symmetric and spontaneously broken theory.

As usually the independent parameters of the massless symmetric theory are parameterized with the $U(1)$-coupling $g_1$, the $SU(2)$-coupling $g_2$, the Yukawa coupling $G_{f_i}$ and the Higgs self-coupling $\lambda$. With the conventions of Ref. [7] the parameters of the spontaneously broken theory are related to the couplings of the symmetric theory in the tree approximation by

$$\begin{aligned} g_1 &= \frac{e}{\cos\theta_W} + \mathcal{O}(\hbar) \\ g_2 &= \frac{e}{\sin\theta_W} + \mathcal{O}(\hbar) \\ G_{f_i} &= \frac{\sqrt{2}m_{f_i}}{M_Z \sin 2\theta_W} + \mathcal{O}(\hbar) \\ \lambda &= e^2 \frac{4m_H^2}{M_Z^2 \sin^2(2\theta_W)} + \mathcal{O}(\hbar). \end{aligned} \tag{6.1}$$

In the massless theory one has to introduce a scale parameter, the normalization point $\kappa$, for fixing the coupling constants. Usually there is introduced only one normalization point $\kappa$, its variation $\kappa\partial_\kappa$ expresses at the same time renormalization group invariance and breaking of dilatations. The corresponding partial differential equation is then valid to all orders of perturbation theory,

$$\Big\{\kappa\partial_\kappa + \beta_{g_1}\partial_{g_1} + \beta_{g_2}\partial_{g_2} + \beta_\lambda\partial_\lambda + \sum_{f_i} \beta_{G_{f_i}}\partial_{G_{f_i}} \tag{6.2}$$

$$
\begin{aligned}
&-\gamma_1^V\Big(\int d^4x\, B^\mu \frac{\delta}{\delta B^\mu} - \xi_1\partial_{\xi_1}\Big) - \gamma_2^V\Big(\int d^4x\,\big(W_\alpha^\mu \frac{\delta}{\delta W_\alpha^\mu} - \bar{c}_\alpha \frac{\delta}{\delta \bar{c}_\alpha}\big) - \xi_2\partial_{\xi_2}\Big)\\
&-\sum_{i=1}^{N_F}\Big[\sum_\delta \gamma^L_{F_{\delta,i}} \int d^4x\,\big(\overline{F^L_{\delta,i}}\frac{\delta}{\delta \overline{F^L_{\delta,i}}} + \frac{\delta}{\delta F^L_{\delta,i}} F^L_{\delta,i}\big) - \sum_f \gamma^R_{f_i}\int d^4x\,\big(\overline{f^R_i}\frac{\delta}{\delta \overline{f^R_i}} + \frac{\delta}{\delta f^R_i} f^R_i\big)\Big]\\
&-\gamma^S\int d^4x\,\big(\Phi\frac{\delta}{\delta\Phi} + \frac{\delta}{\delta\Phi^\dagger}\Phi^\dagger\big) - \gamma^g\int d^4x\, c_\alpha\frac{\delta}{\delta c_\alpha}\Big\}\Gamma\Big|_{ext.f.=0} = 0. \qquad (6.3)
\end{aligned}
$$

Here $W^\mu_\alpha, \alpha = 1,2,3$, are the $SU(2)$-gauge fields and $B_\mu$ is the abelian gauge field. The abelian relation in the symmetric theory reads:

$$
\beta_{g_1} = \gamma_1^V. \qquad (6.4)
$$

Comparing (6.2) with the CS equation of the Standard Model it is seen that we did not have to introduce the external scalar doublet. In addition we have left out the abelian ghosts, since they are free fields in the symmetric theory. Comparing the CS equation of the spontaneously broken theory to the one of the symmetric theory for different high-energy vertex functions one gets the following 1-loop relations by inserting the tree relations (6.1) into the $\beta$-functions of the symmetric theory

$$
\begin{aligned}
\beta^{(1)}_{g_1} &= \beta^{(1)}_e - \frac{\sin\theta_W}{\cos\theta_W}\beta^{(1)}_{M_W}\\
\beta^{(1)}_{g_2} &= \beta^{(1)}_e + \frac{\cos\theta_W}{\sin\theta_W}\beta^{(1)}_{M_W}\\
\lambda^{-1}\beta^{(1)}_\lambda &= 2\Big(\beta^{(1)}_e + 2\frac{\cos 2\theta_W}{\sin 2\theta_W}\beta^{(1)}_{M_W} + \beta^{(1)}_{m_H}\Big)\\
\beta^{(1)}_{G_{f_i}} &= \beta^{(1)}_e + 2\frac{\cos 2\theta_W}{\sin 2\theta_W}\beta^{(1)}_{M_W} + \beta^{(1)}_{m_{f_i}}. \qquad (6.5)
\end{aligned}
$$

These relations can be verified using the explicit form of the $\beta$-functions for the spontaneously broken theory given above and the $\beta$-functions in the symmetric parameterization from Ref. [7]. Similar relations occur if renormalization constants introduced for the parameters of the symmetric theory are expressed in terms of the renormalization constants of the electric charge and the particle masses of the spontaneously broken theory (see e.g. [10]). The simple relations (6.5) between the $\beta$-functions are not expected to hold beyond one-loop order since scheme-dependent corrections enter in higher orders.

### 6.2. QED

Considering only the charged fermions and conservation of electromagnetic charge one is able to construct usual QED as it is embedded in the classical action of the Standard Model. Among the vector bosons it only includes the photon which is now an abelian gauge field by construction. Since all fermion masses are invariant under QED transformations, one does not have to introduce a Higgs field into the theory. Charged scalars are not added because they are unphysical particles in the Standard Model and even more their interaction with fermions is only well-defined if we include the non-abelian symmetries of weak interactions.

The QED-action takes then the usual form

$$\Gamma_{cl}^{QED} = \int d^4x \, \Big(-\frac{1}{4}F^{\mu\nu}F_{\mu\nu} + \sum_{f_i}(i\bar{f}_i\gamma^\mu D_\mu f_i - m_{f_i}\bar{f}_i f_i) - \frac{1}{2\xi}\,(\partial_\mu A^\mu)^2\Big), \tag{6.6}$$

and

$$D_\mu f_i = \partial_\mu f_i - ieQ_f A_\mu f_i. \tag{6.7}$$

The action and the charges are defined by the QED Ward identity

$$\Big(e\mathbf{w}_{em} - \partial^\mu \frac{\delta}{\delta A^\mu}\Big)\Gamma = \frac{1}{\xi}\Box\partial A. \tag{6.8}$$

The CS equation is given to all orders by:

$$\Big\{m\partial_m + \beta_e e\partial_e - \gamma_A\Big(\mathcal{N}_A - \xi\partial_\xi\Big) - \sum_{f_i}\gamma_{f_i}\mathcal{N}_{f_i}\Big\}\Gamma = \sum_{f_i} m_{f_i}\int d^4x \,\frac{\delta\Gamma}{\delta\hat{\varphi}_{f_i}}, \tag{6.9}$$

with

$$\begin{aligned} m\partial_m &= \kappa\partial_\kappa + \sum_{f_i} m_{f_i}\partial m_{f_i} \\ \mathcal{N}_A &= \int d^4x\, A^\mu \frac{\delta}{\delta A^\mu} \\ \mathcal{N}_{f_i} &= \int d^4x\,(\bar{f}_i\frac{\delta}{\delta \bar{f}_i} - \frac{\delta}{\delta f_i}f_i). \end{aligned} \tag{6.10}$$

The $\hat{\varphi}_{f_i}$ are *external* scalar fields, which are introduced for defining the soft breaking of dilatations. In pure QED the $\beta$-function is related to the anomalous dimension of the photon field

$$\beta_e = \gamma_A. \tag{6.11}$$

The 1-loop contributions to the $\beta$-function are exactly the same as the ones which contribute to the $\beta$-function of the electromagnetic coupling in the electroweak Standard Model from fermions (5.11):

$$\beta_e^{(1)} = \frac{e^2}{4\pi^2}\frac{1}{3}N_F(Q_e^2 + 3Q_u^2 + 3Q_d^2). \tag{6.12}$$

The Standard Model $\beta$-function in addition includes contributions from unphysical scalars $\phi^\pm$ and especially non-abelian contributions from charged vector bosons and charged ghosts with negative sign, which sum up to a negative sign if one considers only one family. In QED one defines the effective coupling by the characteristic equation

$$\frac{\partial e}{\partial t} = e\beta_e \quad \text{with} \quad t = \ln\left|\frac{p^2}{\kappa^2}\right| . \tag{6.13}$$

Its solution is interpreted as the momentum and scale dependence of the interaction strength in the high-energy region. Due to the relation between the anomalous dimension of the photon field and the $\beta$-function (6.11) the solution can also be identified with the complete Dyson-summed photon propagator. The behavior of the 1-loop effective coupling of the Standard Model as solution of the corresponding characteristic equation (6.13) differs from the QED-behavior by the additional non-abelian contributions from ghosts and vector bosons. It approaches zero if one includes only one family, and goes much more flat to infinity if one takes into account two or three families of fermions. Moreover, since the abelian relation of QED (6.11) is replaced by the relation (4.20) an interpretation of the running coupling in terms of 2-point photon Green functions is not clear in the Standard Model.

For evaluating the CS equation it is important to control the $\beta$-functions of higher orders. In particular it has to be shown that the result which one obtains by integrating the CS equation is meaningful if one includes only lowest order $\beta$-functions. In this context it is important to mention that in pure QED there exist also equations for the differentiation with respect to single fermion masses. The fermion-mass equations read

$$\Big(m_{f_i}\partial_{m_{f_i}} + \beta_e^{f_i} e\partial_e - \gamma_A^{f_i}(\mathcal{N}_A - 2\xi\partial_\xi) - \sum_{f_j}\gamma_{f_j}^{f_i}\mathcal{N}_{f_j}\Big)\Gamma = m_{f_i}\int d^4x \frac{\delta}{\delta\hat{\varphi}_{f_i}}\Gamma. \tag{6.14}$$

The Ward identity (6.8) relates the $\beta$-function and anomalous dimension of the photon for any of these equations,

$$\beta_e^{f_i} = \gamma_A^{f_i}. \tag{6.15}$$

The $\beta$-functions of these equations depend strongly on the normalization condition imposed for the photon residuum, e.g.

$$\partial_{p^2}\Gamma^T_{AA}\Big|_{p^2=\kappa^2} = 1. \tag{6.16}$$

They all vanish to all orders if the residuum of the photon is normalized at a normalization point at infinity ($\kappa^2 \to -\infty$),

$$\lim_{\kappa^2\to-\infty}\beta_e^{f_i} = 0. \tag{6.17}$$

Taking the normalization point at zero momentum they are given in 1-loop order by

$$\beta_e^{e_i}(\kappa^2=0) = \frac{e^2}{4\pi^2}\frac{1}{3}Q_e^2 + O(\hbar^2) \qquad \beta_e^{q_i}(\kappa^2=0) = \frac{e^2}{4\pi^2}\frac{1}{3}3Q_q^2 + O(\hbar^2)\,, \quad q = u,d. \tag{6.18}$$

In higher orders the consistency equations of the fermion-mass equation (6.14) with the CS equation (6.9) in QED give important restrictions on the mass dependence of the various $\beta$-functions. The fermion-mass equations together with the consistency equations are the main ingredients for being able to formulate the running of the effective coupling from the low-energy to the high-energy region in the simplified version introducing step functions. Eventually they also allow to study decoupling of fermions and the construction of effective low-energy theories.

Contrary to QED none of these fermion-mass differential equations exists in the Standard Model, since differentiations with respect to single mass parameters produce hard insertions of Yukawa interactions. As a consequence the CS $\beta$-functions can in principle depend on mass ratios in an arbitrary way. The appearance of such a logarithmic mass dependence in theories with spontaneously broken symmetry has been demonstrated in the simple Higgs-Yukawa-model [6], and the respective analysis has to be continued to the Standard Model by a systematic construction of one-loop induced higher-order contributions (cf. the discussion at the end of section 5).

## 7. Conclusions

In theories with spontaneously broken symmetry the CS equation plays a crucial role for a systematic investigation of the large-momentum behavior of higher-order contributions. It is furthermore an important instrument within the framework of abstract renormalization allowing to determine the independent parameters of the theory in a scheme-independent way. In this paper we have derived the CS equation for the electroweak Standard Model in the on-shell parameterization and evaluated all its coefficient functions in one-loop order.

As a direct application, we have shown that the ghost mass ratio is an independent parameter of the model. It is renormalized independently from the vector-boson mass ratio, and consequently the choice of setting these parameters equal in lowest order is not stable under renormalization.

We have compared the CS equation of the Standard Model with the ones of the symmetric $SU(2) \times U(1)$ theory and of QED. While the one-loop $\beta$-function of the electromagnetic coupling depends only on the coupling itself and is QED-like in this sense, due to non-abelian interactions it receives contributions with negative sign, which dominate over the contributions of the fermions if only one family of fermions is considered. The one-loop $\beta$-functions in the on-shell parameterization can be related to the $\beta$-functions of the symmetric theory in a simple way. These simple relations are not expected to hold anymore beyond one-loop order, since the higher-order $\beta$-functions in the on-shell parameterization will contain logarithms of the masses which are absent in the symmetric theory.

With the CS equation and its one-loop coefficient functions we have provided the basic tools necessary for an investigation of one-loop induced higher-order contributions in the electroweak Standard Model, as e.g. the leading logarithms. Since a restricted choice of the gauge fixing will in general not be stable under renormalization, we have given the explicit form of all one-loop coefficient functions in the most general linear gauge compatible with rigid symmetry transformations. As an example we have determined the leading quadratic logarithms of the photon self-energy in 2-loop order. Contrary to QED it is seen that the quadratic logarithms of the photon self-energy in the asymptotic region are non-vanishing. In this context we have also discussed the possible sources for the appearance of large mass-dependent logarithms in 2-loop order. All these contributions strongly depend on the normalization conditions imposed for fixing the free parameters of the Standard Model. If the Standard Model is renormalized in the on-shell schemes, these contributions are expected to be present and to depend logarithmically on the different mass ratios. Due to the presence of massless particles on-shell conditions which allow to diagonalize the mass-matrix of the neutral massive/massless particles on-shell are crucial for obtaining off-shell infrared-finite Green functions in higher orders. A systematic analysis of mass-dependent higher-order contributions is needed for an improvement of the perturbative series on the basis of a summation of large higher-order terms by using the CS or RG equation and its 1-loop $\beta$-functions. This issue is the subject of further investigations.

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## Appendix

In this appendix we give the propagators of the free fields in the general linear gauge defined in (2.22) and (2.21). The propagators of vector and scalar fields are non-diagonal in the vector/scalar and in the neutral vector fields. We omit propagators with $B_a$ fields, since they do not contribute in loops and are not relevant for determining the coefficient functions of the Callan-Symanzik equation. The free field propagators determined from the action with $B_a$ fields are equivalent to the ones determined in usual $R_\xi$ gauges where $B_a$ fields are eliminated via their equations of motion (see (2.23)).

For determining the $\beta$-functions and anomalous dimensions we have taken the choice (2.32) – (2.35), which is compatible with rigid symmetry. Moreover, from boson propagators only those terms contribute to the CS coefficient functions in 1-loop order which behave like $\frac{1}{p^2}$ for asymptotically large $p^2$, the other terms contribute as soft mass insertions on the r.h.s. of the CS equation.

We have taken the following definitions for determining the free field propagators of vector and scalar fields:

$$\sum_l \int d^4z \Gamma^{(0)}_{\varphi_k\varphi_l}(x,z)\Delta_{\varphi_l\varphi_m}(z,y) = i\delta_{km}\delta^4(x-y). \tag{A.1}$$

Here $\varphi_k$ denotes all vector and scalar fields of the Standard Model, and the index $k$ is understood to include field indices as well as Lorentz indices:

$$\varphi_k = (W^+_\mu, W^-_\mu, Z_\mu, A_\mu, \phi^+, \phi^-, H, \chi). \tag{A.2}$$

The $\Gamma^{(0)}_{\varphi_k\varphi_l}$ denote the lowest-order vertex functions derived from the generating functional of 1PI Green functions,

$$\Gamma^{(0)}_{\varphi_k\varphi_l}(x,y) \equiv \left.\frac{\delta^2\Gamma_{cl}}{\delta\varphi_k(x)\delta\varphi_l(y)}\right|_{all\ fields\ =\ 0}. \tag{A.3}$$

The free field propagators $\Delta_{\varphi_k\varphi_l}(x,y)$ are the time ordered vacuum expectation values of free fields:

$$\Delta_{\varphi_k\varphi_l}(x,y) = \langle 0|T\,\varphi_k(x)\,\varphi_l(y)|0\rangle^{(0)}. \tag{A.4}$$

The Fourier transformed propagators are defined according to the conventions:

$$\begin{aligned} \Delta_{\varphi_k\varphi_l}(x,y) &= \int \frac{d^4p}{(2\pi)^4}\Delta_{\varphi_k\varphi_l}(p,-p)e^{-ip(x-y)} && \text{(A.5)} \\ (2\pi)^4\delta^4(p+q)\Delta_{\varphi_k\varphi_l}(p,q) &= \int d^4x d^4y \Delta_{\varphi_k\varphi_l}(x,y)e^{i(px+qy)}. && \text{(A.6)} \end{aligned}$$

1. Free field propagators of the charged vector and scalar fields
   Starting from the general gauge-fixing action (2.22) we find with the notation

   $$\xi_W \equiv \xi_{+-} = \xi_{-+} \tag{A.7}$$

   the following expressions:

   $$\Delta_{\phi^+\phi^-}(p^2) = \frac{i}{p^2-\zeta_W M_W^2}\left(1-\frac{(\xi_W-\zeta_W)M_W^2}{p^2-\zeta_W M_W^2}\right) \tag{A.8}$$

$$\Delta^L_{\phi^+W_-}(p^2) = \frac{i(\xi_W - \zeta_W)M_W}{(p^2 - \zeta_W M_W^2)^2} \tag{A.9}$$

$$\Delta^T_{W_+W_-}(p^2) = \frac{i}{p^2 - M_W^2} \tag{A.10}$$

$$\Delta^L_{W_+W_-}(p^2) = \frac{i}{p^2 - \zeta_W M_W^2}\left(\xi_W + \frac{(\xi_W - \zeta_W)\zeta_W M_W^2}{p^2 - \zeta_W M_W^2}\right). \tag{A.11}$$

Here we have defined the longitudinal and transverse parts of vector propagators by

$$\Delta_{V_a^\mu V_b^\nu}(p,-p) = -\left(\eta^{\mu\nu} - \frac{p^\mu p^\mu}{p^2}\right)\Delta^T_{V_aV_b}(p^2) - \frac{p^\mu p^\mu}{p^2}\Delta^L_{V_aV_b}(p^2). \tag{A.12}$$

Similarly we have split off the 4-momentum $p^\mu$ from the scalar/vector propagator:

$$\Delta_{\phi_a V_b^\mu}(p,-p) = p^\mu \Delta^L_{\phi_a V_b}(p^2). \tag{A.13}$$

The remaining propagators are obtained by complex conjugation:

$$\Delta^*_{\varphi_k\varphi_l}(p,-p) = -\tilde{I}_{kk'}\tilde{I}_{ll'}\Delta_{\varphi_{k'}\varphi_{l'}}(-p,p). \tag{A.14}$$

The matrix $\tilde{I}$ is defined in analogy to eq. (2.20), and (A.14) means in particular:

$$\Delta^*_{\phi^+W_-^\mu}(p,-p) = -\Delta_{\phi^-W_+^\mu}(-p,p) = \Delta_{\phi^-W_+^\mu}(p,-p). \tag{A.15}$$

2. Free field propagators of the neutral scalar and vector fields
With the notation

$$\xi_Z \equiv \xi_{ZZ} \qquad \xi_A \equiv \xi_{AA} \tag{A.16}$$

for the arbitrary gauge parameters of (2.22) one obtains

$$\Delta_{HH}(p^2) = \frac{i}{p^2 - m_H^2} \tag{A.17}$$

$$\Delta_{\chi\chi}(p^2) = \frac{i}{p^2 - \zeta_Z M_Z^2}\left(1 - \frac{(\xi_Z - \zeta_Z)M_Z^2}{p^2 - \zeta_Z M_Z^2}\right) \tag{A.18}$$

$$\Delta^L_{\chi Z}(p^2) = \frac{-(\xi_Z - \zeta_Z)M_Z}{(p^2 - \zeta_Z M_Z^2)^2} \tag{A.19}$$

$$\Delta^L_{\chi A}(p^2) = \frac{M_Z}{p^2(p^2 - \zeta_Z M_Z^2)}\left(\zeta_A - \xi_{AZ} - \frac{(\xi_Z - \zeta_Z)\zeta_A M_Z^2}{p^2 - \zeta_Z M_Z^2}\right) \tag{A.20}$$

$$\Delta^T_{ZZ}(p^2) = \frac{i}{p^2 - M_Z^2} \tag{A.21}$$

$$\Delta^T_{ZA}(p^2) = 0 \tag{A.22}$$

$$\Delta^T_{AA}(p^2) = \frac{i}{p^2} \tag{A.23}$$

$$\Delta^L_{ZZ}(p^2) = \frac{i}{p^2 - \zeta_Z M_Z^2}\left(\xi_Z + \frac{(\xi_Z - \zeta_Z)\zeta_Z M_Z^2}{p^2 - \zeta_Z M_Z^2}\right) \tag{A.24}$$

$$\Delta^L_{ZA}(p^2) = \frac{i}{p^2-\zeta_Z M_Z^2}\left(\xi_{ZA} + \frac{(\xi_Z-\zeta_Z)\zeta_A M_Z^2}{p^2-\zeta_Z M_Z^2}\right) \tag{A.25}$$

$$\Delta^L_{AA}(p^2) = \frac{i}{p^2}\left(\xi_A + \frac{(2\xi_{AZ}-\zeta_A)\zeta_A M_Z^2}{p^2-\zeta_Z M_Z^2} + \frac{(\xi_Z-\zeta_Z)\zeta_A M_Z^4}{(p^2-\zeta_Z M_Z^2)^2}\right). \tag{A.26}$$

Non-diagonal propagators that are not given in the above list vanish identically because of CP-invariance of the free field action.

3. Free field propagators of the Faddeev-Popov fields

   The free field propagators of the Faddeev-Popov fields are derived from the bilinear part of the ghost action (3.23). They are diagonal according to the construction outlined in section 3, eqs. (3.16) – (3.24). For this reason they have their conventional form:

$$\Delta_{c_+\bar{c}_-}(p^2) = \frac{i}{p^2-\zeta_W M_W^2} \tag{A.27}$$

$$\Delta_{c_Z\bar{c}_Z}(p^2) = \frac{i}{p^2-\zeta_Z M_Z^2} \tag{A.28}$$

$$\Delta_{c_A\bar{c}_A}(p^2) = \frac{i}{p^2} \tag{A.29}$$

$$\Delta_{c_A\bar{c}_Z}(p^2) = \Delta_{c_Z\bar{c}_A}(p^2) = 0. \tag{A.30}$$

   They are derived from the classical action in an equivalent way to (A.1),

$$\sum_d \int d^4z \Gamma^{(0)}_{c_a\bar{c}_d}(x,z)\Delta_{c_b\bar{c}_d}(y,z) = i\delta_{ab}\delta^4(x-y), \qquad \Gamma^{(0)}_{c_a\bar{c}_b}(x,y) \equiv \frac{\delta^2\Gamma_{cl}}{\delta c_a(x)\bar{c}_b(y)}, \tag{A.31}$$

   and are related to the time ordered vacuum expectation values of free fields by

$$\Delta_{c_a\bar{c}_b}(x,y) = \langle 0|T\, c_a(x)\, \bar{c}_b(y)|0\rangle^{(0)}. \tag{A.32}$$

   Fourier transformation is defined as in (A.5), (A.6).

4. Free field propagators of fermions

   For completeness we also give the free field propagator of a Dirac fermion:

$$\Delta_{f\bar{f}}(p,-p) = \frac{i(\not{p}+m_f)}{p^2-m_f^2}. \tag{A.33}$$

   It is determined from the classical action by

$$\sum_\beta \int d^4z \Gamma^{(0)}_{\bar{f}_\alpha f_\beta}(x,z)\Delta_{f_\beta\bar{f}_\gamma}(z,y) = i\delta_{\alpha\gamma}\delta^4(x-y) \tag{A.34}$$

   with

$$\Gamma^{(0)}_{\bar{f}f}(x,y) \equiv \frac{\overrightarrow{\delta}}{\delta\bar{f}(x)}\Gamma_{cl}\frac{\overleftarrow{\delta}}{\delta f(y)}. \tag{A.35}$$

Differentiation with respect to the adjoint spinor $\bar{f}$ is applied from the left, whereas differentiation with respect to the spinor $f$ is applied from the right, $\alpha, \beta, \gamma$ are spinor indices. The free field propagator is related to the time ordered vacuum expectation value of free fields by

$$\Delta_{f\bar{f}}(x,y) = \langle 0|T\, f(x)\, \bar{f}(y)|0\rangle^{(0)}. \tag{A.36}$$

Fourier transformation is defined as in (A.5), (A.6).

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