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Higher-order Results for Precision Observables in the Standard Model and the MSSM[a]

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Abstract

Recent higher-order results for precision observables in the Standard Model (SM) and the Minimal Supersymmetric Standard Model (MSSM) and for the neutral $\mathcal{CP}$-even Higgs-boson masses of the MSSM are summarized. Furthermore a brief discussion of the technical aspects of evaluating higher-order corrections in the electroweak theory is given. In the SM, results for the Higgs-mass dependence of the precision observables are analyzed. The exact two-loop results for the Higgs-mass dependence of the fermionic contributions are compared with the results of an expansion in the top-quark mass up to next-to-leading order. In the MSSM, results for the leading two-loop contributions to the precision observables and to the masses of the neutral $\mathcal{CP}$-even Higgs bosons are discussed. The latter are compared with results obtained by renormalization group calculations.

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HIGHER-ORDER RESULTS FOR PRECISION OBSERVABLES IN THE STANDARD MODEL AND THE MSSM

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Recent higher-order results for precision observables in the Standard Model (SM) and the Minimal Supersymmetric Standard Model (MSSM) and for the neutral $\mathcal{CP}$-even Higgs-boson masses of the MSSM are summarized. Furthermore a brief discussion of the technical aspects of evaluating higher-order corrections in the electroweak theory is given. In the SM, results for the Higgs-mass dependence of the precision observables are analyzed. The exact two-loop results for the Higgs-mass dependence of the fermionic contributions are compared with the results of an expansion in the top-quark mass up to next-to-leading order. In the MSSM, results for the leading two-loop contributions to the precision observables and to the masses of the neutral $\mathcal{CP}$-even Higgs bosons are discussed. The latter are compared with results obtained by renormalization group calculations.

1 Introduction

By comparing the electroweak Standard Model (SM) and its extensions, most notably the Minimal Supersymmetric Standard Model (MSSM), with the precision data [1] it is possible to test the theory at its quantum level, where all parameters of the model enter. In this way one is able within the SM to infer indirect constraints on the mass of the Higgs boson, which is the last missing ingredient of the SM. Within the MSSM the comparison with the precision data allows to constrain mainly the parameters of the scalar quark sector. Eventually the precision tests of the electroweak theory could hint towards a distinction between the SM and Supersymmetry via their respective virtual effects. At present the bounds that can be obtained from this analysis on the parameters of the SM and the MSSM are still relatively weak [1]. In order to improve this situation, a very high accuracy both of the measurements and the theoretical predictions is needed.

While QCD corrections to Δr and the Z-boson observables $\sin^2\theta_{\rm eff}$ and Γ_l are known within the SM at $\mathcal{O}(\alpha\alpha_{\rm s})$ [2] and $\mathcal{O}(\alpha\alpha_{\rm s}^2)$ [3], only partial information is available about the electroweak two-loop contributions. Beyond one-loop order the resummations of the leading one-loop contributions are known [4,5], the leading and next-to-leading term in an expansion for asymptotically large values of the top-quark mass, $m_{\rm t}$, have been evaluated [6,7,8], and also the leading term of an asymptotic expansion in the Higgs-boson mass, $M_{\rm H}$, has been derived [9]. The terms obtained in the m_t-expansion were found to be numerically sizable, and the next-to-leading term turned out to be about equally large as the (formally) leading term [7]. Exact results have been derived for the Higgs-mass dependence of the fermionic two-loop corrections to the precision observables [10,5,11].

In order to treat the MSSM at the same level of accuracy as the SM, the one-loop results for Δr and the Z-boson observables [12] have to be supplemented by higher-order contributions. Recently the QCD corrections to the ρ parameter in the MSSM have been evaluated [13], which incorporates the leading two-loop contribution to the precision observables.

Besides the indirect tests of the MSSM via the precision observables, there exists a very stringent direct test of the model, since it predicts the existence of a light Higgs boson, which at the tree level is restricted to be lighter than the Z boson. A precise prediction for the mass of the lightest Higgs boson, m_h, in terms of the relevant SUSY parameters is crucial in order to determine the discovery and exclusion potential of LEP2 and the upgraded Tevatron. If the Higgs boson exists, it will be accessible at the LHC and future linear colliders, where then a high-precision measurement of the mass of this particle will become feasible. A precise knowledge of the mass of the heavier $\mathcal{CP}$-even Higgs boson, m_H, is important for resolving the mass splitting between the $\mathcal{CP}$-even and -odd Higgs-boson masses.

The tree-level bound on m_h is strongly affected by the inclusion of radiative corrections, which yield an upper bound of about 130 GeV. In order to go beyond the one-loop results [14,15,16,17], renormalization group (RG) methods have been applied for obtaining leading logarithmic higher-order contributions [18,19]. In the effective potential approach the leading QCD corrections have been calculated [20]. Up to now phenomenological analyses have been based either on the results of the RG improved one-loop effective potential approach [19] or on the complete one-loop on-shell results [15,16], which differ by up to 20 GeV in m_h. Recently a Feynman-diagrammatic calculation of the leading QCD corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons has been performed [21,22]. The result obtained in this way contains the leading two-loop corrections, the full diagrammatic one-loop on-shell result [16], and further improvements taking into account leading electroweak two-loop and leading QCD corrections beyond $\mathcal{O}(\alpha\alpha_s)$.

In this paper some recent higher-order results in the SM and the MSSM are summarized. Before discussing these results in some detail, a brief overview is given over technical issues involved in the calculation of two-loop corrections in the electroweak theory.

2 Techniques for calculating higher-order corrections in the Standard Model and the MSSM

The evaluation of higher-order corrections in the electroweak theory to the precision observables and to the predictions for the MSSM Higgs-boson masses involves as a main technical obstacle the evaluation of two-loop 2-point functions. In order to express the results for the precision observables in terms of the physical masses M_{W} and M_{Z} of the gauge bosons, the respective 2-point functions have to be evaluated on-shell, i.e. at non-zero momentum transfer, while vertex and box contributions in processes with light external particles can often be reduced to vacuum integrals. As a consequence of the many different mass scales present in the electroweak theory, the evaluation of diagrams at the two-loop level is in general very complicated.

Besides the large number of contributing Feynman diagrams, problems encountered in such a calculation are due to the complicated tensor structure of the diagrams and to the fact that the scalar two-loop integrals are in general not expressible in terms of polylogarithmic functions [23]. Further complications are related to the need for an adequate regularization and for a renormalization at the two-loop level, which has not yet been worked out in full detail.

Concerning the regularization, two schemes are commonly applied in calculations within the electroweak theory, namely Dimensional Regularization (DREG) [24,25] and Dimensional Reduction(DRED) [26]. In DREG the regularization is performed by analytically continuing the space-time dimension from 4 to D. This prescription preserves the Lorentz and the gauge invariance of the theory, apart from problems related to the treatment of γ_5 in dimensions other than 4. In Supersymmetric theories, however, a D-dimensional treatment of vector fields leads to a mismatch between the fermionic and bosonic degrees of freedom, which gives rise to a breaking of the Supersymmetric relations. This led to the development of DRED. In this scheme only the momentum integrals are treated D-dimensional, while the fields and the Dirac algebra are kept 4-dimensional. DRED involves potential ambiguities related to the treatment of γ_5, and its application to non-supersymmetric theories turns out to be problematic (for a recent review see [27]). In a naive application of both regularization schemes (without an appropriate shift in the couplings which relates the two schemes to each other) at one-loop order differences in the finite parts of the Feynman diagrams (proportional to $(D-4)^0$) arise, while at two-loop order both the finite parts and the divergent part proportional to $(D-4)^{-1}$ are different.

As already mentioned, a problem closely related to the one of finding an adequate regularization for the considered calculation is the treatment of γ_5

in D dimensions. The consistent prescription according to [25,28] leads to a breaking of the Ward identities. As in all cases where one works with a non-invariant regulator, this breaking has to be compensated by the introduction of additional counterterms that restore the symmetries. In order to investigate the invariance properties of the regulators in Supersymmetry, a detailed study of the supersymmetric Ward identities is necessary. A new approach that could be helpful for applications in the electroweak theory is differential renormalization [29].

The results presented in this paper have been obtained using several computer-algebraic tools [30]. The generation of the diagrams and counterterm contributions has been done with the help of the computer-algebra program *FeynArts* [31]. For the calculations in the MSSM, the relevant part of the MSSM Lagrangian has been implemented into *FeynArts*. The program inserts propagators and vertices into the graphs in all possible ways and creates the amplitudes including all symmetry factors. The calculation of the two-loop diagrams and counterterms was performed with the package *TwoCalc* [32]. It is based on a general algorithm for the tensor reduction of two-loop 2-point functions, and reduces the amplitudes to a minimal set of standard scalar integrals. Properties like the gauge-parameter dependence or the validity of Ward identities can directly be read off from the algebraic result obtained with *TwoCalc*.

For the calculations in the SM, DREG has been applied, while the results within the MSSM presented below have been obtained using DRED. For the calculations described in this paper the use of an anticommuting γ_5 in D dimensions was possible without encountering inconsistencies. The renormalization has been carried out using the on-shell scheme. Within the SM a two-loop renormalization was necessary in the gauge-boson sector. In the MSSM a two-loop renormalization in the Higgs-boson sector and a one-loop renormalization of the gauge-boson and scalar quark sector had to be performed. Those two-loop scalar integrals for which no analytic expression in terms of polylogarithmic functions can be derived have been evaluated numerically with one-dimensional integral representations [33]. These allow a very fast calculation of the integrals with high precision without any approximation in the masses.

3 Higgs-mass dependence of precision observables in the Standard Model at two-loop order

In order to study the Higgs-mass dependence of the precision observables Δr, $\sin^2\theta_{\rm eff}$ and Γ_l at two-loop order, we consider subtracted quantities of the form

$$a_{\rm subtr}(M_{\rm H}) = a(M_{\rm H}) - a(M_{\rm H}^0), \quad \text{where } a = \Delta r, \sin^2\theta_{\rm eff}, \Gamma_l, \tag{1}$$

which indicate the shift in the precision observables caused by varying the Higgs-boson mass between M_{H}^0 and M_{H}. In the analysis below the Higgs-boson mass is varied in the interval 65 GeV $\leq M_{\mathrm{H}} \leq 1$ TeV. The quantity Δr, which determines the relation between the vector-boson masses in terms of the Fermi constant, is derived from muon decay [34]. It is defined according to

$$M_{\mathrm{W}}^2 \left(1 - \frac{M_{\mathrm{W}}^2}{M_{\mathrm{Z}}^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_\mu} (1 + \Delta r) . \qquad (2)$$

The leptonic effective weak mixing angle and the leptonic width of the Z boson are determined from the effective couplings of the neutral current at the Z-boson resonance, $J_\mu^{\mathrm{NC}} = \left(\sqrt{2}G_\mu M_{\mathrm{Z}}^2\right)^{1/2} \left[g_V^f \gamma_\mu - g_A^f \gamma_\mu \gamma_5\right]$, according to

$$\sin^2\theta_{\mathrm{eff}} = \frac{1}{4|Q_f|}\left(1 - \frac{\mathrm{Re}\, g_V^f}{\mathrm{Re}\, g_A^f}\right), \quad \Gamma_l = \frac{\alpha M_{\mathrm{Z}}}{12 s_{\mathrm{W}}^2 c_{\mathrm{W}}^2}\left(1 + \frac{3\alpha}{4\pi}\right)\left(|g_V^f|^2 + |g_A^f|^2\right) . \qquad (3)$$

Potentially large M_{H}-dependent contributions are the ones associated with the top quark due to the large Yukawa coupling $\mathrm{t\bar{t}H}$, and the corrections proportional to $\Delta\alpha$. Further contributions are the ones of the light fermions (except the corrections already contained in $\Delta\alpha$), and purely bosonic contributions. The latter are expected to be relatively small [10]. We therefore have focussed on the Higgs-dependent fermionic contributions to the precision observables, for which we have obtained exact two-loop results [10,11].

Fig. 1 shows the Higgs-mass dependence of the two-loop corrections to Δr associated with the top/bottom doublet, with $\Delta\alpha$, and with the light fermions. The dotted line furthermore indicates the Higgs-mass dependence of the leading m_{t}^4-term [6] in the top/bottom contribution. The two-loop top/bottom contribution gives rise to a shift in the W-boson mass of $\Delta M_{\mathrm{W,subtr,(2)}}^{\mathrm{top}}(M_{\mathrm{H}} = 1000\,\mathrm{GeV}) \approx 16$ MeV, which is about 10% of the one-loop contribution. The Higgs-mass dependence of this contribution turns out to be very poorly approximated by the leading m_{t}^4-term; the contribution of the latter even enters with a different sign. It can be seen from Fig. 1 that the two-loop contributions to a large extent cancel each other. The contribution of the light fermions yields a shift in M_{W} of up to 4 MeV. In total the two-loop contributions lead to a slight increase in the sensitivity of Δr to the Higgs-boson mass compared to the one-loop case. Similarly as for Δr, also for the Higgs-mass dependence of $\sin^2\theta_{\mathrm{eff}}$ and Γ_l large cancellations occur between the two-loop contributions. For these observables the higher-order contributions decrease the sensitivity to the Higgs-boson mass [11]. The Higgs-mass dependence of M_{W}, $\sin^2\theta_{\mathrm{eff}}$ and Γ_l is shown in Fig. 2 together with the experimental values of the observables. The theoretical values for $M_{\mathrm{H}}^0 = 65$ GeV are taken from [8].

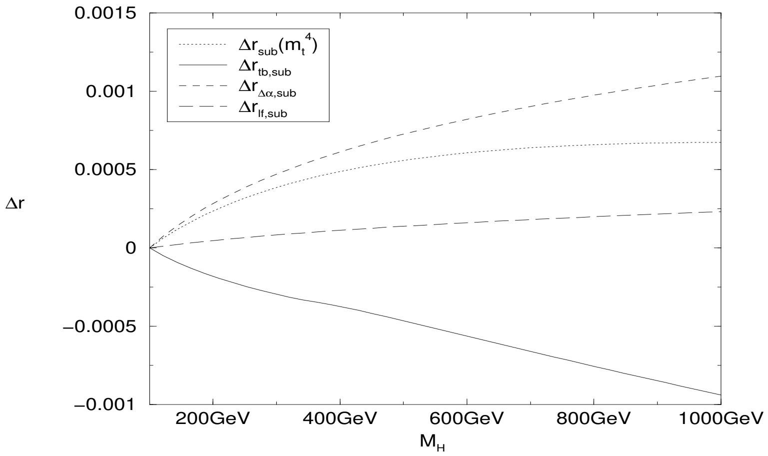

Figure 1: Higgs-mass dependent fermionic contributions to Δr at two-loop order. The different curves show the contribution from the diagrams involving the top/bottom doublet ($\Delta r_{\rm tb}$), the contribution proportional to $\Delta\alpha$ ($\Delta r_{\Delta\alpha}$), the contribution of the light fermions ($\Delta r_{\rm lf}$), and the approximation of the top/bottom correction by the leading term proportional to $m_{\rm t}^4$ ($\Delta r(m_{\rm t}^4)$).

In Tab. 1 the Higgs-mass dependence of $M_{\rm W}$ and $\sin^2\theta_{\rm eff}$ based on the exact result for the fermionic contributions is compared with the results of the expansion in the top-quark mass up to $\mathcal{O}(G_\mu^2 m_{\rm t}^2 M_{\rm Z}^2)$ given in [8] (with the input parameters as in [8]).[b] Over the range of Higgs-mass values from 65 GeV to 1 TeV the difference between the results amounts to about 4 MeV for $M_{\rm W}$ and to about $7\cdot10^{-5}$ for $\sin^2\theta_{\rm eff}$. The difference in the prediction for $\sin^2\theta_{\rm eff}$ turns out to be mainly induced by the difference in the prediction for $M_{\rm W}(M_{\rm H})$. Evaluating $\sin^2\theta_{\rm eff,subtr}(M_{\rm H} = 1\,{\rm TeV})$ using the value for $M_{\rm W}(M_{\rm H} = 1\,{\rm TeV})$ from [8] instead of our result (see Tab. 1) yields a value for $\sin^2\theta_{\rm eff,subtr}(M_{\rm H} = 1\,{\rm TeV})$ that differs from the one given in [8] only by about $2\cdot10^{-5}$.

In the comparison of Tab. 1 besides the difference between the exact treatment of the top-quark contributions and the expansion up to next-to-leading order in $m_{\rm t}$ further effects enter. In [8] the Higgs-mass dependence of light-fermion contributions only arises from reducible contributions that are generated by Dyson summation, while in our results above also the Higgs-mass dependence of the light-fermion contributions is taken into account exactly.

[b] It should be noted that in contrast to the numbers given in [5] the contribution of the light fermions is included here.

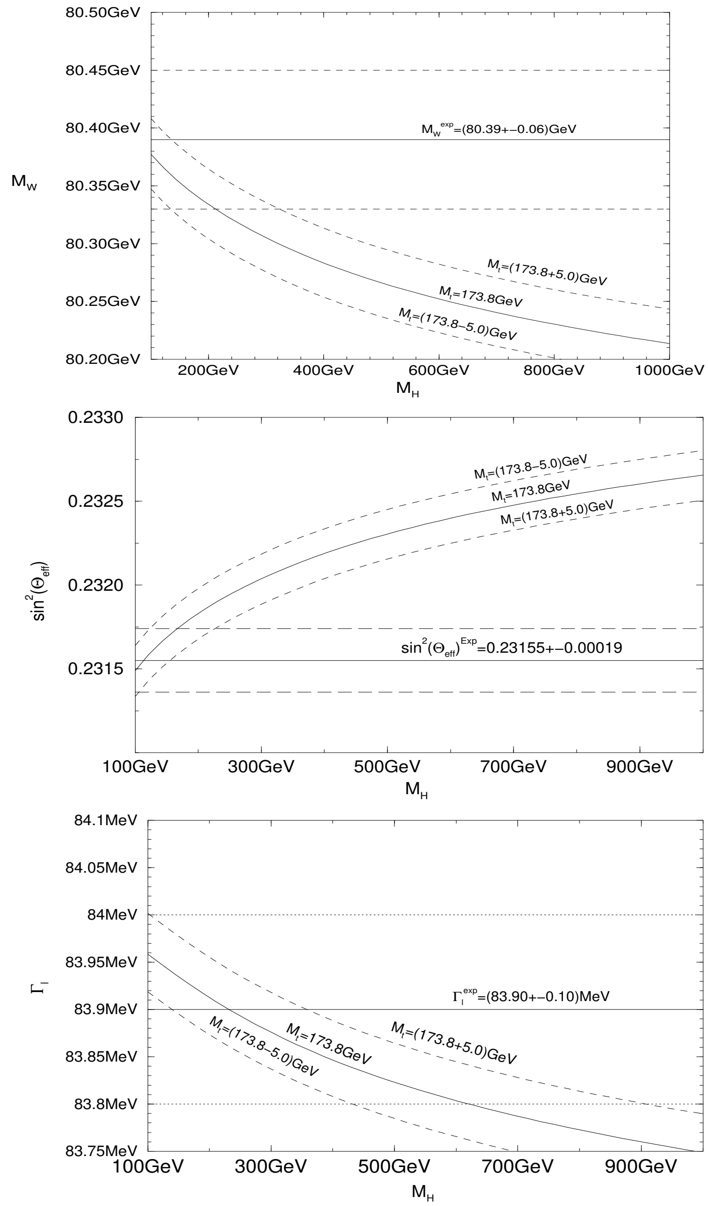

Figure 2: The Higgs-mass dependence of the prediction for M_{W}, $\sin^2\theta_{\mathrm{eff}}$ and Γ_l is shown for different values of m_{t}. The experimental values of the observables are also indicated.

Table 1: The Higgs-mass dependence of M_W and $\sin^2\theta_\mathrm{eff}$ based on the exact result for the fermionic contribution (left column) and on the result of the expansion in m_t (right column).

M_H/ GeV	$M_\mathrm{W,subtr}^\mathrm{ferm}$/ MeV	$M_\mathrm{W,subtr}^{\mathrm{top},\Delta\alpha,\mathrm{DGS}}$/ MeV	ΔM_W/ MeV
65	0	0	0
100	−23	−23	0
300	−95	−98	3
600	−148	−152	4
1000	−187	−191	4

M_H/ GeV	$\sin^2\theta_\mathrm{eff,subtr}^\mathrm{ferm}$	$\sin^2\theta_\mathrm{eff,subtr}^{\mathrm{top},\Delta\alpha,\mathrm{DGS}}$	$\Delta\sin^2\theta_\mathrm{eff}$
65	0	0	0
100	0.00020	0.00021	0.00001
300	0.00076	0.00080	0.00004
600	0.00112	0.00118	0.00006
1000	0.00138	0.00145	0.00007

Further differences are due to a different treatment of higher-order QCD and electroweak corrections. In [35] the effect solely caused by the difference between the exact treatment of the top-quark contributions and the expansion in m_t has been analyzed in detail, and it has been found that over the full range of Higgs-boson masses this effect alone amounts to a difference of about 2 MeV in M_W and $3\cdot10^{-5}$ in $\sin^2\theta_\mathrm{eff}$. In the case of M_W the three other sources of differences mentioned above, namely the light-fermion contribution and the treatment of higher-order QCD and electroweak corrections, all individually give rise to a difference of $2-3$ MeV over the full range of Higgs-boson masses, which combine with the effect of the m_t-expansion to the difference of 4 MeV listed in the last entry of the first table in Tab. 1.

4 QCD corrections to precision observables in the MSSM

The leading two-loop corrections to the electroweak precision observables in the MSSM enter via the quantity $\Delta\rho$, which is given in terms of the transverse parts of the two-loop Z-boson and W-boson self-energies as

$$\Delta\rho = \frac{\Sigma^{\rm Z}(0)}{M_{\rm Z}^2} - \frac{\Sigma^{\rm W}(0)}{M_{\rm W}^2}. \tag{4}$$

The contributions to $\Delta\rho$ of $\mathcal{O}(\alpha\alpha_s)$ have been evaluated in [13]. They can be separated into the contribution of diagrams with gluon exchange, which dominates in general (it is of the order of 10–15% of the one-loop result) and for which a very compact analytic result can be derived[13], and into the gluino-exchange contribution, which goes to zero for large gluino mass.

The results of[13] have recently been extended by including also the effects of mixing in the scalar bottom sector, which previously had been neglected, and by evaluating the gluon-exchange contributions to the precision observables Δr, $\sin^2\theta_{\rm eff}$ and Γ_l, thus going beyond the $\Delta\rho$ approximation[36]. In Fig. 3 the gluino-exchange contribution is shown for $\tan\beta\ = 40$ and large mixing in the scalar bottom sector as a function of the common scalar mass parameter

$$m_{\tilde{q}} \equiv M_{\tilde{t}_L} = M_{\tilde{t}_R} = M_{\tilde{b}_L} = M_{\tilde{b}_R}. \tag{5}$$

The $M_{\tilde{q}_{L/R}}$ are the soft SUSY breaking parameters in the diagonal entries of the mass matrices of the scalar top and bottom quarks. The off-diagonal entries read $m_q M_q^{LR}$ with $M_t^{LR} = A_t - \mu\cot\beta$, $M_b^{LR} = A_b - \mu\tan\beta$, see [13]. The full result is compared in Fig. 3 to the case where mixing in the scalar bottom sector is neglected ($m_{\rm b} = 0$). The figure shows that effects of $\tilde{b}$ mixing can be sizable for large $\tan\beta$ and small gluino masses.

The leading contribution to Δr in the MSSM can be approximated by the contribution to $\Delta\rho$ according to $\Delta r \approx -c_{\rm W}^2/s_{\rm W}^2\Delta\rho$, where $c_{\rm W}^2 = 1 - s_{\rm W}^2 = M_{\rm W}^2/M_{\rm Z}^2$. The gluon-exchange contribution to $\Delta r^{\rm SUSY}$ of a squark doublet is given by

$$\Delta r_{\rm gluon}^{\rm SUSY} = \Pi^\gamma(0) - \frac{c_{\rm W}^2}{s_{\rm W}^2}\left(\frac{\delta M_{\rm Z}^2}{M_{\rm Z}^2} - \frac{\delta M_{\rm W}^2}{M_{\rm W}^2}\right) + \frac{\Sigma^{\rm W}(0) - \delta M_{\rm W}^2}{M_{\rm W}^2}, \tag{6}$$

where $\delta M_{\rm W}^2 = {\rm Re}\,\Sigma^{\rm W}(M_{\rm W}^2)$, $\delta M_{\rm Z}^2 = {\rm Re}\,\Sigma^{\rm Z}(M_{\rm Z}^2)$, and Π^γ, $\Sigma^{\rm W}$, and $\Sigma^{\rm Z}$ denote the transverse parts of the two-loop gluon-exchange contributions to the photon vacuum polarization and the W-boson and Z-boson self-energies, respectively, which all are understood to contain the subloop renormalization. This result is shown together with the $\Delta\rho$ approximation in Fig. 4. The two-loop contribution leads to a shift in the W-boson mass of up to 20 MeV for low values of $m_{\tilde{q}}$ in the no-mixing case. If the parameter M_t^{LR} is made large or the relation eq. (5) is relaxed, much bigger effects are possible[13]. As can be seen in Fig. 4, the $\Delta\rho$ contribution approximates the full result rather well. The two results agree within 10–15%.

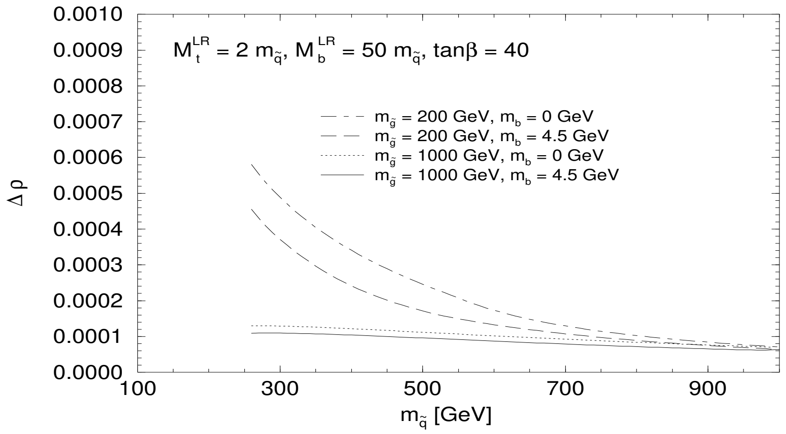

Figure 3: Contribution of the gluino-exchange diagrams to $\Delta\rho$ for $\tan\beta\ = 40$ and large mixing in the scalar bottom sector. The full result is compared to the case where mixing in the scalar bottom sector is neglected ($m_{\rm b} = 0$).

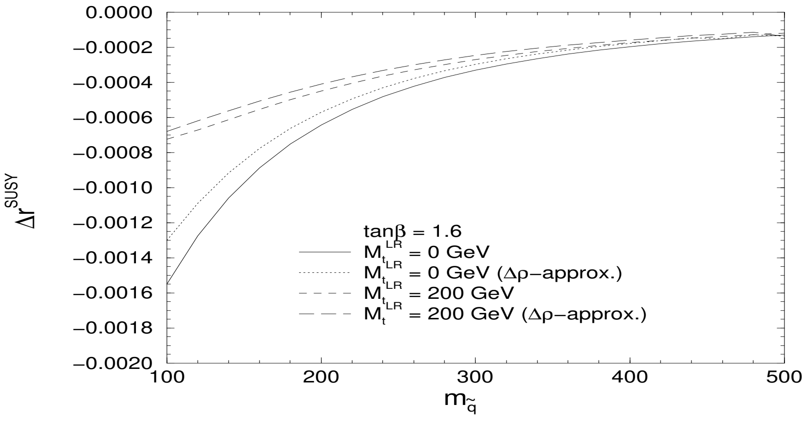

Figure 4: Contribution of the gluon-exchange diagrams to $\Delta r^{\rm SUSY}$ as a function of the common scalar mass parameter $m_{\tilde{q}}$ for the scenario of no mixing ($M_t^{LR} = 0$) and large mixing ($M_t^{LR} = 200$ GeV) in the $\tilde{t}$ sector. The exact result is compared with the approximation derived from the contribution of $\Delta\rho$.

5 Diagrammatic two-loop results for the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM

In [21,22] a Feynman-diagrammatic calculation of the leading QCD corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons has been performed. The result

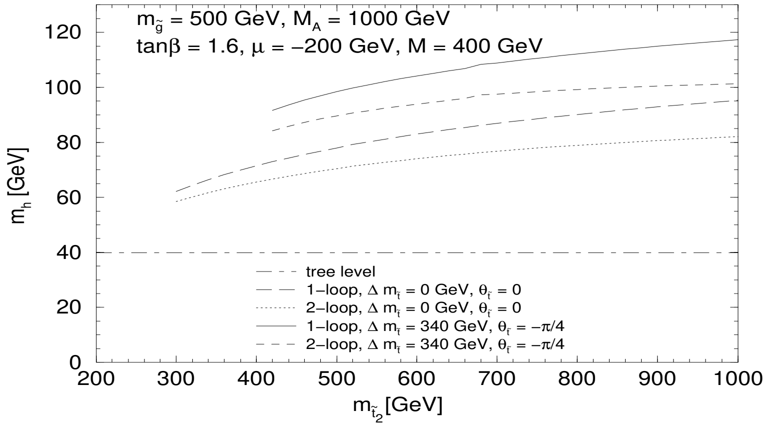

Figure 5: The mass of the lightest Higgs boson in the MSSM in terms of the physical parameters $m_{\tilde{t}_1}, m_{\tilde{t}_2}$ and $\theta_{\tilde{t}}$, where $\Delta m_{\tilde{t}} \equiv m_{\tilde{t}_2} - m_{\tilde{t}_1}$. The scenarios $\Delta m_{\tilde{t}} = 0$ GeV, $\theta_{\tilde{t}} = 0$ (no mixing) and $\Delta m_{\tilde{t}} = 340$ GeV, $\theta_{\tilde{t}} = -\pi/4$ (maximal mixing) are shown.

obtained in this way contains the leading two-loop corrections, the complete diagrammatic one-loop on-shell result [16], and further improvements taking into account leading electroweak two-loop and leading QCD corrections beyond $\mathcal{O}(\alpha\alpha_s)$. The calculation of $m_{\rm h}$ and $m_{\rm H}$ has been carried out for arbitrary values of the parameters in the Higgs and scalar top sector of the MSSM. The results have been implemented into the Fortran program *FeynHiggs* [37].

In Fig. 5 the mass of the lightest Higgs boson is shown as a function of the mass of the heavier scalar top quark, $m_{\tilde{t}_2}$, for different values of $m_{\tilde{t}_1}$ and the mixing angle in the scalar top sector, $\theta_{\tilde{t}}$. The tree-level, the one-loop and the two-loop results for $m_{\rm h}$ are given for the case of no mixing ($\theta_{\tilde{t}} = 0$) and degenerate $\tilde{t}$ masses ($\Delta m_{\tilde{t}} \equiv m_{\tilde{t}_2} - m_{\tilde{t}_1} = 0$), as well as for maximal mixing ($\theta_{\tilde{t}} = -\pi/4$) and a mass difference $\Delta m_{\tilde{t}} \approx 340$ GeV that gives rise to the largest values of $m_{\rm h}$ (the corresponding values for $\Delta\rho^{\rm SUSY}$ lie within the experimentally favored [38] region $\Delta\rho^{\rm SUSY} \lesssim 1.3 \cdot 10^{-3}$). As can be seen from the figure, in both cases the two-loop corrections give rise to a large reduction of the one-loop on-shell result of up to 15 GeV.

In order to derive an upper bound for $m_{\rm h}$ within the MSSM as a function of $\tan\beta$, we have performed a scan over the other parameters such that the corresponding value of $m_{\rm h}$ is maximized. Fig. 6 shows the maximally possible value for $m_{\rm h}$ in the region $\tan\beta \leq 5$ for $m_{\tilde{q}} = 1000$ GeV and three values of the top-quark mass, $m_{\rm t} = 173.8$ GeV, 178.8 GeV, 183.8 GeV (the current

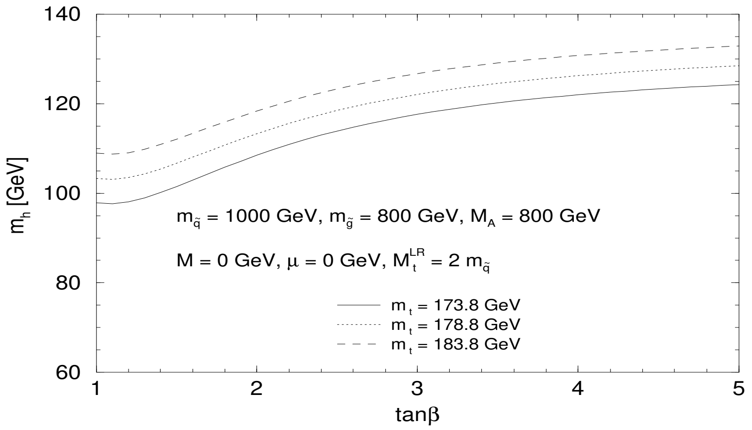

Figure 6: The maximally possible value for $m_{\rm h}$ as a function of $\tan\beta$ for $m_{\tilde{q}} = 1000$ GeV and three values of $m_{\rm t}$.

experimental value of $m_{\rm t}$ and values one and two standard deviations above it). It can be seen that the maximal values of $m_{\rm h}$ in the particularly interesting region of $\tan\beta \lesssim 2$ are at the edge of the LEP2 reach, which is expected to be roughly 105 GeV. The question whether a full coverage of this region is possible depends sensitively on the range of $m_{\rm t}$ values employed for evaluating $m_{\rm h}$.

In Fig. 7 the results of the Feynman-diagrammatic calculation for $m_{\rm h}$ are compared with the results obtained by RG methods [19]. The mass of the lightest Higgs boson is shown as a function of $M_t^{LR}/m_{\tilde{q}}$ for the two scenarios with $\tan\beta = 1.6$ and $\tan\beta = 40$. Good agreement is found for vanishing mixing in the scalar top sector ($M_t^{LR} = 0$). Sizable deviations occur, however, when mixing in the $\tilde{t}$ sector is taken into account. They reach about 5 GeV for moderate mixing and become very large for $|M_t^{LR}/m_{\tilde{q}}| \gtrsim 2.5$. The maximal value for $m_{\rm h}$ in the diagrammatic approach is reached for $M_t^{LR}/m_{\tilde{q}} \approx \pm 2$, whereas the RG results have a maximum at $M_t^{LR}/m_{\tilde{q}} \approx \pm 2.4$, which corresponds to the maximum in the one-loop result. In the case of positive M_t^{LR}, the maximal values for $m_{\rm h}$ reached in the diagrammatic calculation are up to 5 (3) GeV larger than the ones of the RG method for $\tan\beta = 1.6$ (40). A comparison of the diagrammatic and the RG results in terms of the physical parameters $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$ has been performed in [22]. It yields qualitatively the same results. An additional source of deviation between the Feynman-diagrammatic and the RG result is the dependence of the diagrammatic result on the mass of the gluino, which does not appear in the RG result. Its variation gives rise to a shift of

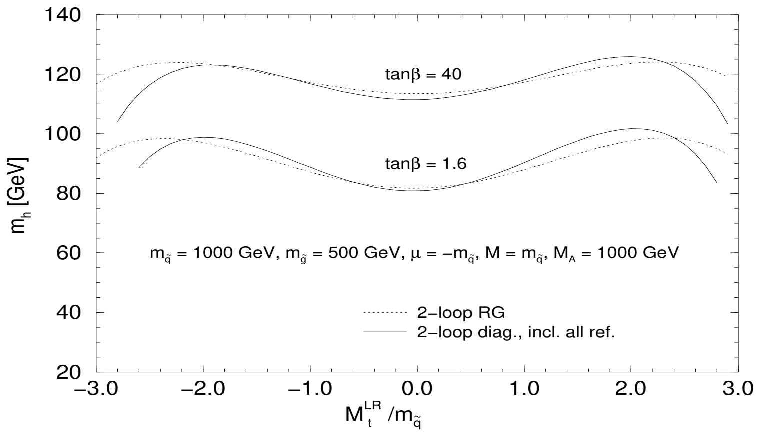

Figure 7: Comparison between the Feynman-diagrammatic calculation and the results obtained by renormalization group methods. The mass of the lightest Higgs boson is shown for the two scenarios with $\tan\beta\ = 1.6$ and $\tan\beta\ = 40$ as a function of $M_t^{LR}/m_{\tilde{q}}$.

the diagrammatic result relative to the RG result of up to ± 2 GeV.

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