KA-TP-06-1998
hep-ph/9806250

TWO-LOOP RESULTS FOR THE MASSES OF THE NEUTRAL CP-EVEN HIGGS BOSONS IN THE MSSM[1]

S. Heinemeyer, W. Hollik and G. Weiglein
Institut für Theoretische Physik, Universität Karlsruhe,
D–76128 Karlsruhe, Germany

Abstract

Diagrammatic two-loop results are presented for the leading QCD corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the Minimal Supersymmetric Standard Model (MSSM). The results are valid for arbitrary values of the parameters of the Higgs and scalar top sector of the MSSM. Their impact on a precise prediction for the mass of the lightest Higgs boson is briefly discussed.

The MSSM offers a theoretically very appealing extension of the Standard Model (SM) of the electroweak and strong interactions. While for the predicted superpartners of the SM particles present-day accelerators can cover only a very limited part of the MSSM parameter space, the Higgs sector of the model provides the opportunity for a stringent direct test in requiring the existence of a light neutral Higgs boson. At tree level its mass, m_h, is predicted to be lower than the one of the Z boson. However, the one-loop corrections are known to be huge [1, 2] and shift the upper bound to about $150\ \mathrm{GeV}$. Beyond one-loop order renormalization group methods have been applied in order to include higher-order leading and next-to-leading logarithmic contributions [3, 4, 5]. A diagrammatic calculation is available for the dominant two-loop contributions in the limiting case of vanishing $\tilde{t}$-mixing and infinitely large M_A and $\tan\beta$ [6]. These results indicate that the two-loop corrections considerably reduce the predicted value of m_h. A precise prediction for m_h in terms of the relevant SUSY parameters is crucial in order to determine the discovery and exclusion potential of LEP2 and the upgraded Tevatron and also for physics at the LHC, where eventually a high-precision measurement of m_h might be possible.

Diagrammatic two-loop results which take into account virtual particle effects without restrictions of their masses and mixing are therefore very desirable. We have performed a Feynman diagrammatic calculation of the leading two-loop QCD corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM [7]. The calculation has been performed in the on-shell scheme. The results are valid for arbitrary values of the parameters of the Higgs and scalar top sector of the MSSM.

The Higgs sector of the MSSM contains two Higgs doublets,

$$H_1 = \begin{pmatrix} H_1^1 \\ H_1^2 \end{pmatrix} = \begin{pmatrix} v_1 + (\phi_1^0 + i\chi_1^0)/\sqrt{2} \\ \phi_1^- \end{pmatrix}, \quad H_2 = \begin{pmatrix} H_2^1 \\ H_2^2 \end{pmatrix} = \begin{pmatrix} \phi_2^+ \\ v_2 + (\phi_2^0 + i\chi_2^0)/\sqrt{2} \end{pmatrix}, \quad (1)$$

[1]Talk given by G. Weiglein at the XXXIIIrd Rencontres de Moriond, Electroweak Interactions and Unified Theories, Les Arcs, March 14–21 1997, to appear in the proceedings.

and can be described using the two input parameters $\tan\beta = v_2/v_1$ and $M_A^2 = -m_{12}^2(\tan\beta + \cot\beta)$, where M_A is the mass of the $\mathcal{CP}$-odd A boson.

The tree-level mass predictions are affected by large corrections at one-loop order through terms proportional to $G_F m_t^4 \ln(m_{\tilde{t}_1} m_{\tilde{t}_2}/m_t^2)$ [1]. These dominant one-loop contributions can be obtained by evaluating the contribution of the $t - \tilde{t}$-sector to the $\phi_{1,2}$ self-energies at zero external momentum from the Yukawa part of the theory (neglecting the gauge couplings). Accordingly, the one-loop corrected Higgs masses are derived by diagonalizing the mass matrix (given in the $\phi_1 - \phi_2$ basis; $m_{\phi_1}^2, m_{\phi_1\phi_2}^2, m_{\phi_2}^2$ are the tree-level entries)

$$M_{\rm Higgs}^2 = \begin{pmatrix} m_{\phi_1}^2 - \hat{\Sigma}_{\phi_1}(0) & m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}(0) \\ m_{\phi_1\phi_2}^2 - \hat{\Sigma}_{\phi_1\phi_2}(0) & m_{\phi_2}^2 - \hat{\Sigma}_{\phi_2}(0) \end{pmatrix}. \tag{2}$$

Here the $\hat{\Sigma}$ denote the Yukawa contributions of the $t - \tilde{t}$-sector to the renormalized one-loop $\phi_{1,2}$ self-energies. By comparison with the full one-loop result [2] it has been shown that these contributions indeed contain the bulk of the one-loop corrections. They typically approximate the full one-loop result up to about 5 GeV.

In order to derive the leading two-loop contributions to the masses of the neutral $\mathcal{CP}$-even Higgs bosons we have evaluated the QCD corrections to eq. (2), which because of the large value of the strong coupling constant are expected to be the most sizable ones (see also Ref. [6]). This requires the evaluation of the renormalized $\phi_{1,2}$ self-energies at the two-loop level. The renormalization has been performed in the on-shell scheme. The counterterms in the Higgs sector are derived from the Higgs potential. The renormalization conditions for the tadpole counterterms have been chosen in such a way that they cancel the tadpole contributions in one- and two-loop order. The renormalization in the $t - \tilde{t}$-sector has been performed in the same way as in Ref. [8]. For the present calculation the one-loop counterterms δm_t, $\delta m_{\tilde{t}_1}$, $\delta m_{\tilde{t}_2}$ for the top-quark and scalar top-quark masses and $\delta\theta_{\tilde{t}}$ for the mixing angle contribute, which enter via the subloop renormalization. The appearance of the $\tilde{t}$ mixing angle $\theta_{\tilde{t}}$ reflects the fact that the current eigenstates, $\tilde{t}_L$ and $\tilde{t}_R$, mix to give the mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$. Since the non-diagonal entry in the scalar quark mass matrix is proportional to the quark mass the mixing is particularly important in the case of the third generation scalar quarks. In deriving our results we have made strong use of computer-algebra tools [9]. The calculations have been performed using Dimensional Reduction [10], which preserves the relevant SUSY relations. As results we have obtained analytical expressions for the two-loop $\phi_{1,2}$ self-energies in terms of the SUSY parameters $\tan\beta$, M_A, μ, $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$, $\theta_{\tilde{t}}$, and $m_{\tilde{g}}$.

Inserting the one-loop and two-loop $\phi_{1,2}$ self-energies into eq. (2), the predictions for the masses of the neutral $\mathcal{CP}$-even Higgs bosons are derived by diagonalizing the two-loop mass matrix. For the numerical evaluation we have chosen two values for $\tan\beta$ which are favored by SUSY-GUT scenarios [11]: $\tan\beta = 1.6$ for the $SU(5)$ scenario and $\tan\beta = 40$ for the $SO(10)$ scenario. The scalar top masses and the mixing angle are derived from the parameters $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$ and M_t^{LR} of the $\tilde{t}$ mass matrix, where $M_t^{LR} = A_t - \mu\cot\beta$ (our conventions are the same as in Ref. [8]). In the figures below we have chosen $m_{\tilde{q}} \equiv M_{\tilde{t}_L} = M_{\tilde{t}_R}$.

The plot in Fig. 1 shows m_h as a function of $M_t^{LR}/m_{\tilde{q}}$, where $m_{\tilde{q}}$ is fixed to 500 GeV. A minimum is reached for $M_t^{LR} = 0$ GeV which we refer to as 'no mixing'. A maximum in the two-loop result for m_h is reached for about $M_t^{LR}/m_{\tilde{q}} \approx 2$ in the $\tan\beta = 1.6$ scenario as well as in the $\tan\beta = 40$ scenario. This case we refer to as 'maximal mixing'. Note that the maximum is shifted compared to its one-loop value of about $M_t^{LR}/m_{\tilde{q}} \approx 2.4$.

In Fig. 2 the two scenarios with $\tan\beta = 1.6$ and $\tan\beta = 40$ are analyzed. The tree-level, the one-loop and the two-loop results for m_h are shown as a function of $m_{\tilde{q}}$ for no mixing and maximal mixing (the curves for the maximal-mixing case start at higher values of $m_{\tilde{q}}$

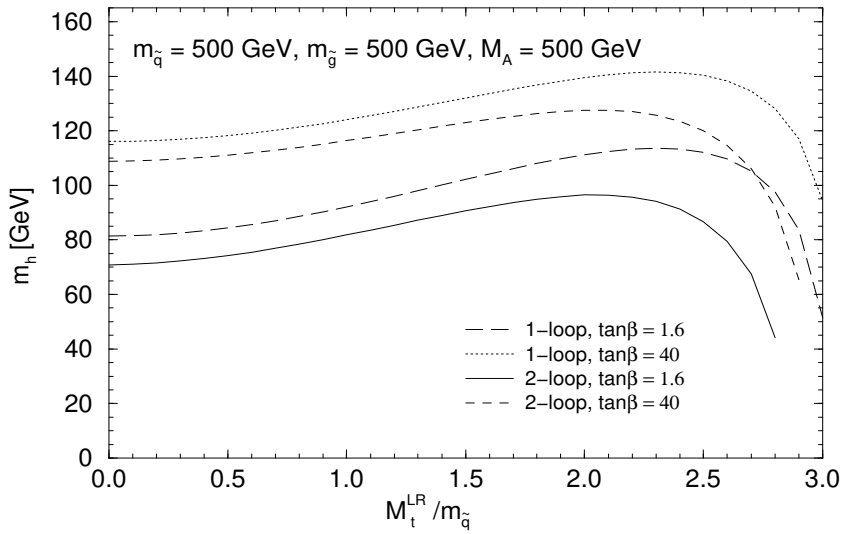

Figure 1: One- and two-loop results for m_h as a function of $M_t^{LR}/m_{\tilde{q}}$ for two values of $\tan\beta$.

than those for the no-mixing case, since below these values of $m_{\tilde{q}}$ the resulting $\tilde{t}$-masses are unphysical or experimentally excluded). The plots show that the one-loop on-shell result for m_h is in general considerably reduced by the inclusion of the two-loop corrections. For the low-$\tan\beta$ scenario the difference between the one-loop and two-loop result amounts to up to about 18 GeV for $m_{\tilde{q}} = 1$ TeV in the no-mixing case, and up to about 25 GeV for $m_{\tilde{q}} = 1$ TeV in the maximal-mixing case. For the high-$\tan\beta$ scenario the reduction of the one-loop result is slightly smaller than for $\tan\beta = 1.6$. The variation of our results with $m_{\tilde{g}}$ is of the order of few GeV.

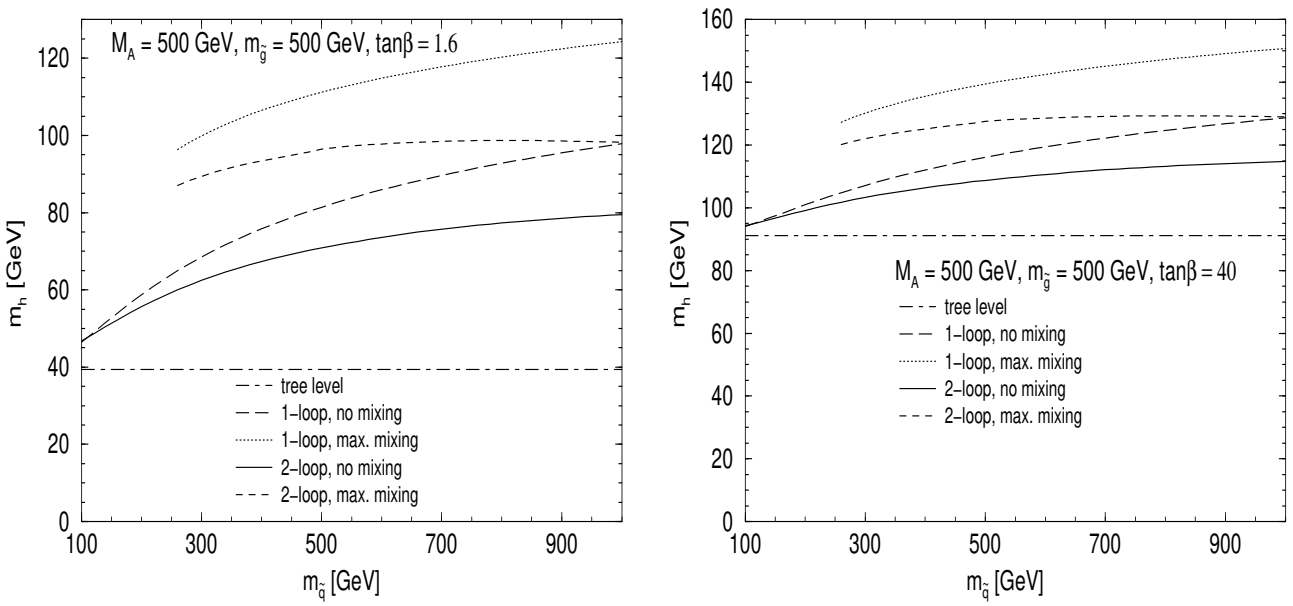

Figure 2: The mass of the lightest Higgs boson for the two scenarios with $\tan\beta = 1.6$ and $\tan\beta = 40$. The tree-, the one- and the two-loop results for m_h are shown as a function of $m_{\tilde{q}}$ for the no-mixing and the maximal-mixing case.

Supplementing our results for the leading $\mathcal{O}(\alpha\alpha_s)$ corrections with the leading higher-

order Yukawa term of $\mathcal{O}(\alpha^2 m_t^6)$ given in Ref. [4] leads to an increase in the prediction of m_h of up to about 3 GeV. A similar shift towards higher values of m_h emerges if at the two-loop level the running top-quark mass, $m_t = 166.5$ GeV, is used instead of the pole mass, $m_t = 175$ GeV, thus taking into account leading higher-order effects beyond the two-loop level. We have compared our results with the results of a renormalization group improvement of the leading one-loop contributions given in Ref. [5]. We find good agreement for the case of no $\tilde{t}$-mixing, while for larger $\tilde{t}$-mixing sizable deviations exceeding 5 GeV occur. In particular, the value of $M_t^{LR}/m_{\tilde{q}}$ for which m_h becomes maximal is shifted from $M_t^{LR}/m_{\tilde{q}} \approx 2.4$ in the one-loop case to $M_t^{LR}/m_{\tilde{q}} \approx 2$ when our diagrammatic two-loop results are included (see Fig. 1). In the results based on renormalization group methods [3, 5], on the other hand, the maximal value of m_h is obtained for $M_t^{LR}/m_{\tilde{q}} \approx 2.4$, i.e. at the one-loop value.

In summary, we have diagrammatically calculated the leading $\mathcal{O}(\alpha\alpha_s)$ corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM. We have applied the on-shell scheme and have imposed no restrictions on the parameters of the Higgs and scalar top sector of the model. The two-loop correction leads to a considerable reduction of the prediction for the mass of the lightest Higgs boson compared to the one-loop value. The reduction turns out to be particularly important for low values of $\tan\beta$. Compared to the results obtained via renormalization group methods we find sizable deviations for large mixing in the $\tilde{t}$-sector.

G.W. thanks the organizers of the "XXXIIIrd Rencontres de Moriond" for the pleasant atmosphere enjoyed at Les Arcs.

References

[1] H. Haber, R. Hempfling, *Phys. Rev. Lett.* **66** (1991) 1815; Y. Okada, M. Yamaguchi, T. Yanagida, *Prog. Theor. Phys.* **85** (1991) 1; J. Ellis, G. Ridolfi, F. Zwirner, *Phys. Lett.* **B 257** (1991) 83; *Phys. Lett.* **B 262** (1991) 477; R. Barbieri, M. Frigeni, *Phys. Lett.* **B 258** (1991) 395.

[2] P. Chankowski, S. Pokorski and J. Rosiek, *Nucl. Phys.* **B 423** (1994) 437;
A. Dabelstein, *Nucl. Phys.* **B 456** (1995) 25; *Z. Phys.* **C 67** (1995) 495;
J. Bagger, K. Matchev, D. Pierce and R. Zhang, *Nucl. Phys.* **B 491** (1997) 3.

[3] J. Casas, J. Espinosa, M. Quirós and A. Riotto, *Nucl. Phys.* **B 436** (1995) 3, E: *ibid.* **B 439** (1995) 466; M. Carena, M. Quirós and C. Wagner, *Nucl. Phys.* **B 461** (1996) 407.

[4] M. Carena, J. Espinosa, M. Quirós and C. Wagner, *Phys. Lett.* **B 355** (1995) 209.

[5] H. Haber, R. Hempfling and A. Hoang, *Z. Phys.* **C 75** (1997) 539.

[6] R. Hempfling and A. Hoang, *Phys. Lett.* **B 331** (1994) 99.

[7] S. Heinemeyer, W. Hollik and G. Weiglein, KA-TP-2-1998, hep-ph/9803277.

[8] A. Djouadi, P. Gambino, S. Heinemeyer, W. Hollik, C. Jünger and G. Weiglein, *Phys. Rev. Lett.* **78** (1997) 3626; *Phys. Rev.* **D 57** (1998) 4179.

[9] J. Küblbeck, M. Böhm and A. Denner, *Comp. Phys. Comm.* **60** (1990) 165;
G. Weiglein, R. Scharf and M. Böhm, *Nucl. Phys.* **B416** (1994) 606.

[10] W. Siegel, *Phys. Lett.* **B 84** (1979) 193;
D. Capper, D. Jones, P. van Nieuwenhuizen, *Nucl. Phys.* **B 167** (1980) 479.

[11] W. de Boer et al., *Z. Phys.* **C 71** (1996) 415.