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Dipole Moments in Supersymmetry*

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The one–loop MSSM contributions to the weak dipole moments of the τ lepton and the b quark (at the Z peak) as well as the electromagnetic and weak dipole form factors of the t quark (at arbitrary $s > 4m_t^2$) are reviewed. Emphasis is given to the relevance of the t–quark CP–violating weak and electric dipole form factors as a part of the full one–loop correction to the process $e^+e^- \to t\bar{t}$ in the MSSM.

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1. Dipole moments

The most general Lorentz structure of the vertex function describing the interaction of a neutral vector boson with on–shell fermions is

$$\Gamma_\mu^{Vff} = \mathrm{i}\left\{\gamma_\mu\left[F_\mathrm{V}^V - F_\mathrm{A}^V\gamma_5\right] + \left[\mathrm{i}F_\mathrm{S}^V + F_\mathrm{P}^V\gamma_5\right]p_\mu + \left[\mathrm{i}F_\mathrm{M}^V + F_\mathrm{E}^V\gamma_5\right]\sigma_{\mu\nu}p^\nu\right\} \quad (1)$$

with p being the incoming momentum of the vector boson. The form factors F_i account for the dynamics of the interaction and are functions of the invariant $s = p^2$. The vector and axial–vector form factors are the only ones that can receive a contribution at tree level in a renormalizable theory while the others are given by quantum corrections. The effects of the scalar and pseudoscalar form factors, F_S^V and F_P^V, vanish for on–shell vector bosons and are negligible for the physical processes of interest (typically proportional to the electron mass).

The magnetic and electric form factors, F_M^V and F_E^V, are related to the usual anomalous (weak) magnetic dipole form factor [A(W)MDFF] and (weak) electric dipole form factors [(W)EDFF] for $V = \gamma, Z$, respectively, through

$$a_f^V(s) = \frac{2m_f}{e}F_\mathrm{M}^V(s) \ \text{ and } \ d_f^V(s) = -F_\mathrm{E}^V(s). \quad (2)$$

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The electromagnetic properties of a fermion, namely the charge and the magnetic and electric dipole moments, are given by the static (classical) limit ($s = 0$). In our convention for the covariant derivative they are

$$\text{charge} = -F_{\rm V}^{\gamma}(0) = eQ_f, \tag{3a}$$

$$\text{MDM} = \frac{F_{\rm V}^{\gamma}(0)}{2m_f} + F_{\rm M}^{\gamma}(0) = \frac{e}{2m_f}\left[-Q_f + a_f^{\gamma}(0)\right], \tag{3b}$$

$$\text{EDM} = d_f^{\gamma}(0). \tag{3c}$$

The anomalous magnetic dipole moment is AMDM$\equiv a_f^{\gamma}(0) \equiv -Q_f(g_f - 2)/2$ and g_f is the gyromagnetic ratio.[1]

Similarly, the weak dipole moments are defined from the Zff vertex at $s = M_Z^2$. They do not have a classical analogy. One refers to them as

$$\text{AWMDM} = a_f^w \equiv a_f^Z(M_Z^2), \tag{4a}$$

$$\text{WEDM} = d_f^w \equiv d_f^Z(M_Z^2). \tag{4b}$$

At the Z resonance they are gauge invariant and contribute to physical processes with (almost) no interference with the electromagnetic form factors.

The (W)EDMs are CP violating what makes their study of particular interest. Unlike the vector and axial–vector form factors, the dipole form factors are chirality flipping and therefore they must be proportional to a fermion mass (either in the loop or in the external legs). The heavier fermions are hence the best candidates to have larger dipoles.

Here we review the MSSM contributions to the weak dipole moments of the τ lepton and the b quark (the heaviest fermions to which an on–shell Z boson can decay) of relevance for LEP. The t quark dipole form factors, both weak and electromagnetic, are of interest for the NLC but at $s > M_Z^2$ the physical observables depend on more form factors than the ones given in the vertices γff and Zff. For consistency, we briefly present the MSSM predictions for the CP violating t (W)EDFFs and compare their influence on some CP–odd observables in the context of the full calculation of the process $e^+e^- \to t\bar{t}$ to one loop.

2. One–loop weak dipole moments of τ and b in the MSSM

The γff and Zff one–loop vertex corrections can be classified into six classes of topologies. Expressions in terms of generic couplings and three–point integrals for the contribution of a renormalizable theory to the dipole

[1] A massless neutral fermion might have a magnetic moment, by quantum corrections (3b). Massless neutrinos have zero magnetic moment in the SM to all orders but this is not necessarily true in a different theory.

form factors in the 't Hooft–Feynman gauge can be found in Refs. [1, 4]. The integrals involved are all IR and UV finite. The contributions to the CP–violating (W)EDFFs are proportional to the imaginary part of combinations of couplings.

2.1. The magnetic dipole moments

The one–loop SM prediction for the AWMDM of the τ lepton and the t quark were first calculated in [2]. The only free parameter, the SM Higgs boson mass, does not significantly affect the electroweak contribution to $a_\tau^w = (2.10 + 0.61\ i) \times 10^{-6}$ but it is more important for the real part of $a_b^w = [(1.1; 2.0; 2.4) - 0.2\ i] \times 10^{-6}$, with $M_{H^0} = M_Z, 2M_Z, 3M_Z$ respectively. The QCD contribution (a gluon exchange diagram) in the case of the b quark dramatically enlarges the result to $a_b^w = (-2.96 + 1.56\ i) \times 10^{-4}$.

In the MSSM, the different Higgs sector consists of a constrained 2HDM mainly controlled by the pseudoscalar Higgs boson mass M_A, the μ parameter and the ratio of VEVs $\tan\beta$. Besides, several soft–SUSY–breaking terms must be introduced. A simplified set is given by the assumption of R–parity conservation, universal scalar mass terms $m_{\tilde{q}}$ for squarks and $m_{\tilde{l}}$ for sleptons, trilinear terms A_τ, A_b and A_t (for the third family) and gaugino mass terms related by the GUT constraint: $\alpha M_3 = \alpha_s s_W^2 M_2 = 3/5\ \alpha_s c_W^2 M_1$. In Ref. [1] a complete scan of the SUSY parameter space was performed with the following results:

- The Higgs sector can provide the only contribution to the imaginary part, of the order of the SM contribution, assuming the present experimental limits on the masses of the superpartners. The real part is typically negative and not very large.
- The neutralino contribution to the real part is also small and has opposite sign than μ in most of the parameter space.
- The chargino contribution is the dominant one, being real and with the same sign as μ: $|\mathrm{Re}(a_\tau^w)| \lesssim 0.2(7) \times 10^{-6}$ and $|\mathrm{Re}(a_b^w)| \lesssim 1(30) \times 10^{-6}$ for $\tan\beta = 1.6(50)$ respectively.
- The gluinos compete in importance with the charginos for the b: $|\mathrm{Re}(a_b^w)| \lesssim 2(40) \times 10^{-6}$.

The sum of the MSSM contributions can amount to $|\mathrm{Re}(a_\tau^w)| \lesssim 0.5(7) \times 10^{-6}$ and $|\mathrm{Re}(a_b^w)| \lesssim 2(50) \times 10^{-6}$ for not so extreme and not excluded regions of the SUSY parameter space. Decoupling is observed for large values of the parameters.

2.2. The electric dipole moments

In the SM there is only one source of CP violation, the $\delta_{\rm CKM}$ phase of the Cabibbo-Kobayashi-Maskawa (CKM) mixing matrix for quarks (appart from $\theta_{\rm QCD}$ that will be ignored). The SM contribution to the (W)EDM comes at three loops and hence it is as small as $\sim eG_F m_f \alpha^2 \alpha_s J/(4\pi)^5$, with $J \equiv c_1 c_2 c_3 s_1^2 s_2 s_3 s_\delta$ (an invariant under reparametrizations of the CKM matrix). That means (W)EDM$\sim 3\times 10^{-34}$ (10^{-33}) $e$cm for τ (b), respectively

In the MSSM there appear new physical phases, provided by the soft–breaking terms, whose effects show up already at the one-loop level. We restrict ourselves to R–parity preserving and generation–diagonal trilinear soft-breaking terms and assume the unification of the soft–breaking gaugino masses at the GUT scale. In such a constrained framework the following SUSY parameters can be complex: the μ parameter, the gaugino masses, the bilinear mixing mass parameter m_{12}^2 and the trilinear soft-SUSY-breaking parameters. Not all of these phases are physical: the MSSM has two additional U(1) symmetries for vanishing μ and soft–breaking terms, the Peccei–Quinn and the R–symmetry, that can be used to absorb two of the phases by redefinition of the fields [3]. In addition, the GUT constraint leads to only one common phase for the gaugino mass terms. Our choice of CP violating physical phases is: $\varphi_\mu \equiv \arg(\mu)$, $\varphi_{\tilde f} \equiv \arg(m_{\rm LR}^f)$ $(f = \tau,\ t,\ b)$ with $m_{\rm LR}^t \equiv A_t - \mu^* \cot\beta$ and $m_{\rm LR}^{\tau,b} \equiv A_{\tau,b} - \mu^* \tan\beta$. The scan of the SUSY parameter space [4] leads to numerical results of the same order as the AWMDM. As a summary:

- The MSSM Higgs sector is CP conserving and therefore it does not contribute to the (W)EDM.
- The diagrams with neutralinos involve φ_μ and $\varphi_{\tilde\tau}$ $(\varphi_{\tilde b})$ for the τ (b) case and its contribution is maximal for these phases being $\pi/2$.
- The chargino diagrams only involve φ_μ in the τ case (there is no scalar neutrino mixing) and also $\varphi_{\tilde t}$ for the b. The former contribution is enhanced for $\varphi_\mu = \pi/2$ and the two phases conspire in the latter to yield a maximum effect for $\varphi_\mu = \pi/2$ and $\varphi_{\tilde t} = \pi$.
- The gluinos contribute maximally to the b WEDM for $\varphi_{\tilde b} = \pi/2$.

The total one–loop predictions of the MSSM are $|{\rm Re}(d_\tau^w)| \lesssim 0.3(12) \times 10^{-21}$ $e$cm, $|{\rm Re}(d_b^w)| \lesssim 1.4(35) \times 10^{-21}$ $e$cm.

3. One–loop dipole form factors of the t quark in the MSSM

The t quark weak dipole moments ($s = M_Z^2$) cannot be defined and it is not conceivable to measure the static electromagnetic properties ($s = 0$) of such a short–living particle. Only the electromagnetic and weak form factors at $s > 4m_t^2$ can contribute to actual processes involving top quarks.

Since no gauge bosons (neither ghosts or Goldstone bosons) are involved in the one–loop contribution to the (W)EDFFs, they are gauge independent in the MSSM even for off–shell photon and Z bosons. The same is not true for the A(W)MDFFs.

The MSSM predictions for the electric and weak electric dipole form factors of the t quark have been evaluated independently in Ref. [5]. After a full scan of the SUSY space, a set of parameters can be found [6] that maximizes the value of the t (W)EDFFs. The choice of the optimal set closely depends on the value of the top–pair invariant mass, due to the possibility of threshold effects from supersymmetric particles running in the loop. For instance, the reference set of SUSY parameters given by

$$\tan\beta = 1.6,\ M_2 = |\mu| = m_{\tilde{q}} = |m_{LR}^t| = |m_{LR}^b| = 200 \text{ GeV} \tag{5a}$$

$$\text{and the CP–phases } \varphi_\mu = -\varphi_{\tilde{t}} = -\varphi_{\tilde{b}} = \pi/2 \tag{5b}$$

leads to high value of the (W)EDFFs in the neighbourhood of $\sqrt{s} = 500$ GeV (Fig. 1). Such a set of parameters yields the following masses (in GeV) for the supersymmetric particles: $m_{\tilde{b}_1} = 201.36$, $m_{\tilde{b}_2} = 213.18$, $m_{\tilde{t}_1} = 186.31$, $m_{\tilde{t}_2} = 323.60$, $m_{\tilde{\chi}_1^+} = 153.47$, $m_{\tilde{\chi}_2^+} = 263.36$, $m_{\tilde{\chi}_1^0} = 91.79$, $m_{\tilde{\chi}_2^0} = 158.93$, $m_{\tilde{\chi}_3^0} = 202.34$, $m_{\tilde{\chi}_4^0} = 266.83$, $m_{\tilde{g}} = 753.22$. Up to now, none of them have been ruled out by experimental searches. As the MSSM Higgs sector is irrelevant for CP–violation the Higgs boson masses have no impact on the result.

We assume $\sqrt{s} = 500$ GeV in the following, for which

$$d_t^\gamma(s) \;=\; (1.515, -1.481) \times 10^{-3}\ \mu_t, \tag{6a}$$

$$d_t^Z(s) \;=\; (0.682, -2.171) \times 10^{-3}\ \mu_t \tag{6b}$$

in t magnetons $\mu_t \equiv e/2m_t = 8.65 \times 10^{-4}\ \text{GeV}^{-1} = 5.64 \times 10^{-17}\ e\text{cm}$.

4. CP violation in $e^+e^- \to t\bar{t}$

Consider the pair–production of polarized tops $e^+(\mathbf{p}_+) + e^-(\mathbf{p}_-) \to t(\mathbf{k}_+, \mathbf{s}_+) + \bar{t}(\mathbf{k}_-, \mathbf{s}_-)$. A list of CP–odd spin observables classified according to their CPT properties is given in Table 1. The directions of polarization of top/antitop $\mathbf{a}$, $\mathbf{b}$ are taken normal (N), transversal (T) to the scattering

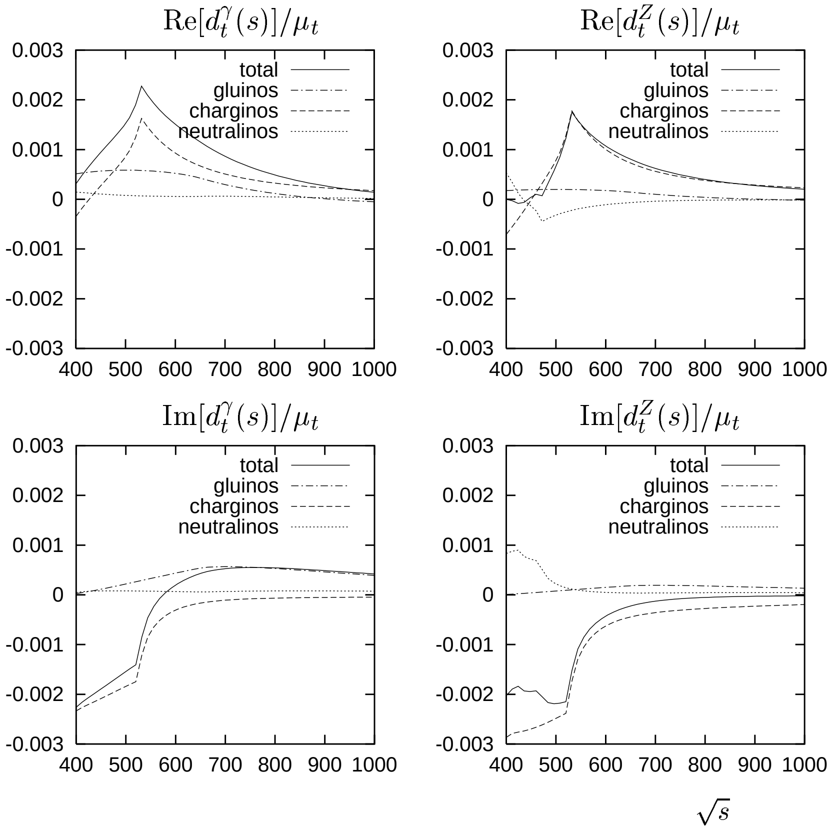

Fig. 1. The different contributions to the top (W)EDFFs for the reference set of SUSY parameters (5) [$\mu_t = 5.64 \times 10^{-17}$ $e$cm].

plane or longitudinal (L). They can be either paralell (↑) or antiparalell (↓) to the axes defined by $\hat{z} = \hat{\mathbf{k}}_+$, $\hat{y} = \hat{\mathbf{k}}_+ \times \hat{\mathbf{p}}_+ / |\hat{\mathbf{k}}_+ \times \hat{\mathbf{p}}_+|$ and $\hat{x} = \hat{y} \times \hat{z}$. A well known asymmetry is $\langle \mathcal{O}_5 \rangle = [N(t_L\bar{t}_L) - N(t_R\bar{t}_R)]/[N(t_L\bar{t}_L) + N(t_R\bar{t}_R)]$.

The one–loop MSSM correction to the differential cross section for top–pair production [7] has been extended to accommodate complex parameters. Two types of box graphs are involved: one with vector boson exchange containing only (CP–even) SM contributions[2] ($[ZeZt]$, $[W\nu_e Wb]$) and one CP–violating, purely supersymmetric ($[\tilde{e}\tilde{\chi}^0\tilde{\chi}^0\tilde{t}]$, $[\tilde{\nu}\tilde{\chi}^\pm\tilde{\chi}^\pm\tilde{b}]$). In Fig. 2 we compare the contributions of dipoles and boxes to the spin observables for the reference set of parameters (5) as a function of the polar angle of the t quark. The plots show that the MSSM box graphs contribute in general to

[2] The box graphs with Higgs–boson exchange are proportional to the electron mass and are neglected.

Table 1. Expectation values of the integrated spin observables and the ratio $r = \langle\mathcal{O}\rangle/\sqrt{\langle\mathcal{O}^2\rangle}$ at $\sqrt{s}$ = 500 GeV given the reference set of SUSY parameters (5). The left column includes only the t (W)EDFF corrections and the right one comes from the complete one–loop cross section for $e^+e^- \to t\bar{t}$.

i	CPT	$\mathcal{O}_i$	**a**	**b**	$\langle\mathcal{O}\rangle_{\mathbf{ab}}$ (10^{-3})		r (10^{-3})	
1	even	$(\mathbf{s_1^*}-\mathbf{s_2^*})_y$	T↑	T↓	1.832	−0.347	1.219	−0.230
2	even	$(\mathbf{s_1^*}\times\mathbf{s_2^*})_x$	T↑	L↑	−0.747	−0.363	−0.747	−0.363
3	even	$(\mathbf{s_1^*}\times\mathbf{s_2^*})_z$	N↑	T↑	1.171	0.465	1.171	0.465
4	odd	$(\mathbf{s_1^*}-\mathbf{s_2^*})_x$	N↑	N↓	−1.644	−0.313	−1.608	−0.309
5	odd	$(\mathbf{s_1^*}-\mathbf{s_2^*})_z$	L↑	L↓	2.722	0.287	3.263	0.344
6	odd	$(\mathbf{s_1^*}\times\mathbf{s_2^*})_y$	L↑	T↓	−2.036	−0.126	−2.036	0.126

CP violation in the process $e^+e^- \to t\bar{t}$ by roughly the same amount and with a different profile than the MSSM (W)EDFFs. The sum of both effects can generate either suppressions or enhancements of the CP signal depending on the observables and also on the set of SUSY parameters employed. For our choice there happens to be always a suppression. The shape of the solid and dashed curves is the same; their different size is due to the contributions to the normalization factors coming from self–energies, A(W)MFFs and other CP–even vertex corrections.

In Table 1 we show the expectation values for the integrated spin observables and the ratio $r \equiv \langle\mathcal{O}\rangle/\sqrt{\langle\mathcal{O}^2\rangle}$ ($|r|\sqrt{N}$ provides the statistical significance of the signal of CP violation for a sample of N events). There we compare the result for (i) the assumption of tree level cross section including only the (W)EDFFs (left column) to (ii) the complete one–loop calculation (right column) provided the reference set of SUSY parameters (5). The MSSM dipole and box graphs contribute with similar importance to the CP–odd observables and with a significance not far form the NLC capabilities (also for more realistic observables [6]).

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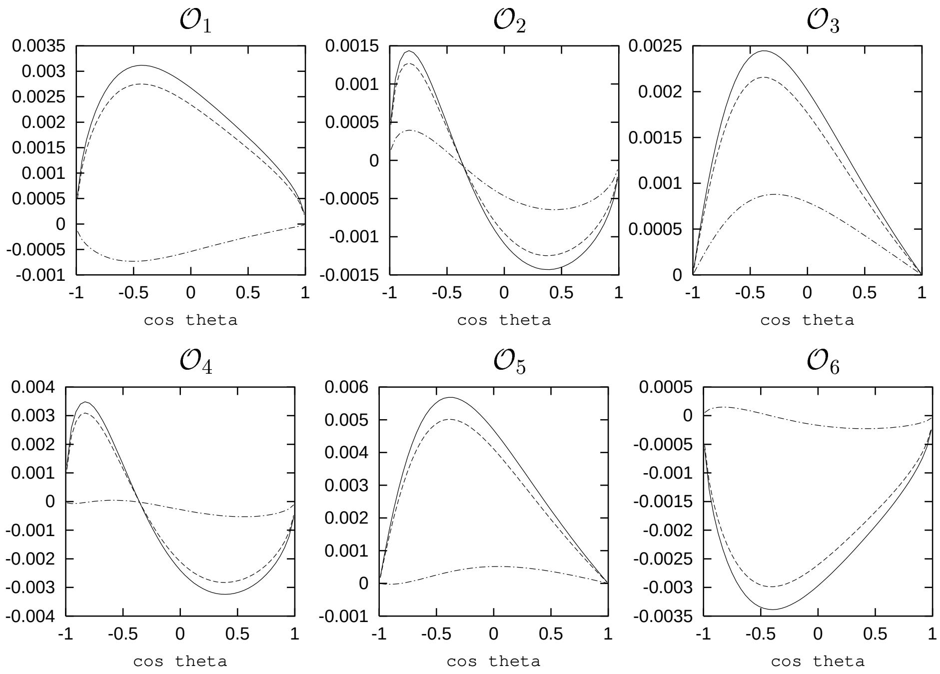

Fig. 2. The expectation value of the spin observables for the reference set of SUSY parameters, assuming for the cross section: (i) tree level plus contribution from (W)EDFFs only (solid line); (ii) one loop including all the vertex corrections and the self–energies (dashed line); (iii) complete one loop (dot–dashed line).

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