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Reconciling the Two-Loop Diagrammatic and Effective Field Theory Computations of the Mass of the Lightest $\mathcal{CP}$-even Higgs Boson in the MSSM

M. Carena [§,†], **H.E. Haber** [♯], **S. Heinemeyer** [‡],

W. Hollik [¶], **C.E.M. Wagner** [†,*,♮] **and G. Weiglein** [†]

[§] FERMILAB, Batavia, IL 60510-0500 USA
[†] CERN, TH Division, CH–1211 Geneva 23, Switzerland
[♯] Santa Cruz Inst. for Part. Phys., Univ. of California, Santa Cruz, CA 95064 USA
[‡] DESY Theorie, Notkestrasse 85, 22603 Hamburg, Germany
[¶] Institut für Theoretische Physik, Univ. of Karlsruhe, 76128 Karlsruhe, Germany
[*] High Energy Physics Division, Argonne National Lab., Argonne, IL 60439 USA
[♮] Enrico Fermi Institute, Univ. of Chicago, 5640 Ellis, Chicago, IL 60637 USA

Abstract

The mass of the lightest $\mathcal{CP}$-even Higgs boson of the minimal supersymmetric extension of the Standard Model (MSSM) has previously been computed including $\mathcal{O}(\alpha\alpha_s)$ two-loop contributions by an on-shell diagrammatic method, while approximate analytic results have also been obtained via renormalization-group-improved effective potential and effective field theory techniques. Initial comparisons of the corresponding two-loop results revealed an apparent discrepancy between terms that depend logarithmically on the supersymmetry-breaking scale, and different dependences of the non-logarithmic terms on the squark mixing parameter, X_t. In this paper, we determine the origin of these differences as a consequence of different renormalization schemes in which both calculations are performed. By re-expressing the on-shell result in terms of $\overline{\mathrm{MS}}$ parameters, the logarithmic two-loop contributions obtained by the different approaches are shown to coincide. The remaining difference, arising from genuine non-logarithmic two-loop contributions, is identified, and its effect on the maximal value of the lightest $\mathcal{CP}$-even Higgs boson mass is discussed. Finally, we show that in a simple analytic approximation to the Higgs mass, the leading two-loop radiative corrections can be absorbed to a large extent into an effective one-loop expression by evaluating the running top quark mass at appropriately chosen energy scales.

1 Introduction

In the minimal supersymmetric extension of the Standard Model (MSSM), the mass of the lightest $\mathcal{CP}$-even Higgs boson, m_h, is calculable as a function of the MSSM parameters. At tree-level, m_h is a function of the $\mathcal{CP}$-odd Higgs boson mass, m_A, and the ratio of vacuum expectation values, $\tan\beta$. Moreover, the tree-level value of m_h is bounded by $m_h \leq m_Z|\cos 2\beta|$, which is on the verge of being ruled out by the LEP Higgs search [1]. When radiative corrections are taken into account, m_h depends in addition on the MSSM parameters that enter via virtual loops. The radiatively corrected value of m_h depends most sensitively on the parameters of the top-squark (stop) sector: the average squared-mass of the two stops, M_S^2, and the off-diagonal stop squared-mass parameter, $m_t X_t$. The stop mixing parameter is $X_t \equiv A_t - \mu\cot\beta$, where A_t is the coefficient of the soft-supersymmetry-breaking stop-Higgs boson tri-linear interaction term and μ is the supersymmetric Higgs mass parameter. The radiatively-corrected value of m_h^2 is enhanced by a factor of $G_F m_t^4$ and grows logarithmically as M_S increases [2]. In particular, the upper bound for m_h (which is achieved when $m_A \gg m_Z$ and $\tan\beta \gg 1$) is significantly increased beyond its tree-level upper bound of m_Z.

The complete one-loop diagrammatic computation of m_h has been carried out in refs. [3–5]. However, for $M_S \gg m_t$, the logarithmically enhanced terms are significant (in particular, the most significant logarithmic terms are those that are enhanced by the $G_F m_t^4$ pre-factor noted above), in which case leading-logarithmic corrections from higher-loop contributions must be included. These terms can be summed via renormalization group techniques. The result of these corrections is to reduce the one-loop upper bound on m_h. For $M_S \lesssim \mathcal{O}(1\text{ TeV})$, it is found that $m_h \lesssim 125$ GeV, where the maximum is reached at large m_A and $\tan\beta$ when M_S is maximal and $X_t \simeq \pm\sqrt{6}M_S$ (the so-called "maximal-mixing" value for stop mixing). However, sub-leading two-loop corrections may not be negligible, and a more complete two-loop computation is required.

The full diagrammatic calculations lead to very complicated expressions for the radiatively corrected value of m_h. Effective potential and effective field theory techniques have been developed which can extract the dominant contributions to the Higgs mass radiative corrections (when M_S is large), resulting in a simpler analytic expression for m_h. These methods also provide a natural setting for renormalization group improvement. Although the exact solution of the renormalization group equations (RGEs) must be obtained numerically, the iterative solution of the RGEs can easily yield simple analytic expressions for the one-loop and two-loop leading logarithmic contributions to m_h. These leading logarithms can also be obtained by expanding the complete diagrammatic results in the limit of $M_S \gg m_t$, and this serves as an important check of the various computations.

The effective potential method [6] provides an important tool for evaluating the Higgs mass beyond the tree-level. It can be used to provide a short-cut for the calculation of certain combinations of Higgs boson two-point functions that arise in the diagrammatic computation. The effective field theory (EFT) approach [7] provides a powerful method for isolating the leading terms of the Higgs mass radiative corrections when $M_S \gg m_t$. In this formalism, one matches the full supersymmetric theory above M_S to an effective Standard Model with supersymmetric particles decoupled below M_S.

The Standard Model couplings in the $\overline{\text{MS}}$ scheme are fixed at M_S by supersymmetric matching conditions. Standard Model RGEs are then used to evolve these couplings down to the electroweak scale (either m_t or m_Z). While the stops are decoupled at M_S, the stop mixing (so-called "threshold") effects are incorporated by modifying the matching conditions at M_S. With the use of effective potential techniques, the EFT formalism and the iteration of the RGEs to two-loops, the leading contributions to the radiatively-corrected Higgs mass was obtained in analytic form in refs. [8–11] These results included the full one-loop leading logarithmic corrections and one-loop leading squark-mixing threshold corrections, the two-loop leading double-logarithmic corrections and the two-loop leading logarithmic squark-mixing threshold corrections up to $\mathcal{O}(h_t^2\alpha_s)$ and $\mathcal{O}(h_t^4)$, where h_t is the Higgs–top quark Yukawa coupling.

In order to extend the above results, genuine two-loop computations are required. The first two-loop diagrammatic computation was performed in ref. [12] in the limit of $m_A \gg m_Z$ and $\tan\beta \gg 1$ (where m_h attains its maximal bound), where only terms of $\mathcal{O}(h_t^4)$ and $\mathcal{O}(h_t^2\alpha_s)$ were evaluated, and all squark mixing effects were neglected. More recently, a more complete two-loop diagrammatic computation of the dominant contributions at $\mathcal{O}(\alpha\alpha_s)$ to the neutral $\mathcal{CP}$-even Higgs boson masses has been performed [13–15]. This result was obtained for arbitrary values of m_A, $\tan\beta$ and the stop-mixing parameter X_t. This two-loop result, which had been obtained first in the on-shell scheme, was subsequently combined [14, 15] with the complete diagrammatic one-loop on-shell result of ref. [4] and the leading two-loop Yukawa corrections of $\mathcal{O}(h_t^4)$ obtained by the EFT approach [8–11]. The resulting two-loop expression was then expressed in terms of the top-quark mass in the $\overline{\text{MS}}$ scheme. By comparing the final expression with the results obtained in refs. [8–11], it was shown that the upper bound on the lightest Higgs mass was shifted upwards by up to 5 GeV, an effect that is more pronounced in the low $\tan\beta$ region.

Besides the shift in the upper bound of m_h, apparent deviations between the explicit diagrammatic two-loop calculation and the results of the EFT computation were observed in the dependence of m_h on the stop-mixing parameter X_t. While the value of X_t that maximizes the lightest $\mathcal{CP}$-even Higgs mass is $(X_t)_{\max} \simeq \pm\sqrt{6}\,M_S \approx \pm 2.4\,M_S$ in the results of refs. [8–11], the corresponding on-shell two-loop diagrammatic computation found a maximal value for m_h at $(X_t)_{\max} \approx 2M_S$.[1] Moreover, in the results of refs. [8–11], m_h is symmetric under $X_t \to -X_t$ and has a (local) minimum at $X_t = 0$. In contrast, the two-loop diagrammatic computation yields m_h values for positive and negative X_t that differ significantly from each other and the local minimum in m_h is shifted slightly away from $X_t = 0$ [16]. A similar conclusion was reached in ref. [17], which used an effective potential calculation to extend the results of ref. [12] to the case of non-zero stop mixing.

A closer comparison of results of the EFT computation of the radiatively-corrected Higgs mass and the two-loop diagrammatic computation at first sight revealed a surprising discrepancy. Namely, the two-loop leading logarithmic squark-mixing threshold corrections at $\mathcal{O}(h_t^2\alpha_s)$ of the former do not appear to match the results of the latter. In this paper, we shall show that this apparent discrepancy in the leading-logarithmic

[1] A local maximum for m_h is also found for $X_t \simeq -2M_S$, although the corresponding value of m_h at $X_t \simeq +2M_S$ is significantly larger [15, 16].

contributions is caused by the different renormalization schemes employed in the two approaches. While the original two-loop diagrammatic computations of ref. [13] were performed in an on-shell scheme, the results of the EFT approach are most naturally carried out in the $\overline{\rm MS}$ scheme. In comparing results obtained within different renormalization schemes in terms of the (not directly observable) parameters X_t and M_S, one has to take into account the fact that these parameters are renormalization-scheme dependent. The effect of this scheme dependence first enters into the calculation of m_h at the two-loop level. In order to allow a detailed comparison between the results of the different approaches we derive relations between these parameters in the two different schemes. We apply these relations to re-express the diagrammatic on-shell result in terms of $\overline{\rm MS}$ parameters. In this way we show that the leading logarithmic two-loop contributions in the two approaches in fact coincide. The remaining numerical difference between the diagrammatic calculation in the $\overline{\rm MS}$ scheme and the result obtained by the EFT approach can thus be identified with new threshold effects due to non-logarithmic two-loop terms contained in the diagrammatic result. We furthermore show that in the analytic approximation employed in this paper, the dominant numerical contribution of these terms can be absorbed into an effective one-loop expression by choosing an appropriate scale for the running top-quark mass in different terms of the expression.

This paper is organized as follows. To simplify the analysis, we focus completely on the radiatively corrected Higgs squared-mass in the "leading m_t^4 approximation" (in which only the dominant loop corrections proportional to $m_t^2 h_t^2 \sim G_F m_t^4$ are kept). In addition, we choose a very simple form for the stop squared-mass matrix, which significantly simplifies the subsequent analysis while maintaining the most important features of the general result. In section 2 we sketch the derivation of the EFT result for the radiatively-corrected Higgs mass of refs. [8–11] at $\mathcal{O}(m_t^2 h_t^2 \alpha_s)$ in the limit of $m_A \gg m_Z$. The corresponding result of the two-loop diagrammatic computation, under the same set of approximations, is outlined in section 3. In order to compare the two results, we must convert on-shell quantities to $\overline{\rm MS}$ quantities. In section 4 we derive the relations between the on-shell and the $\overline{\rm MS}$ values of the parameters m_t, X_t and M_S in the limit of large M_S. Details of the exact calculation are given in Appendix A, while explicit relations between the on-shell and the $\overline{\rm MS}$ parameters up to $\mathcal{O}(m_t^4/M_S^4)$ are given in Appendix B. In section 5 the diagrammatic on-shell result is expressed in terms of $\overline{\rm MS}$ parameters and compared to the result of the EFT computation of section 2. The logarithmic contributions are shown to coincide, and the remaining difference caused by non-logarithmic two-loop terms is analyzed. We argue that the remaining difference can be minimized by improving the EFT computation by taking into account the stop-mixing threshold contribution to the running top-quark mass. In addition, we demonstrate that in a simple analytic approximation to the Higgs mass, one can absorb the dominant two-loop contributions into an effective one-loop expression. In section 6, we summarize our results and discuss suggestions for future improvements.

2 Effective Field Theory Approach

At the tree level, the mass matrix of the neutral $\mathcal{CP}$-even Higgs bosons in the basis of weak eigenstates of definite hypercharge -1 and $+1$ respectively can be expressed in terms of m_Z, m_A and $\tan\beta \equiv v_2/v_1$ as follows:

$$\mathcal{M}_H^{2,\text{tree}} = \begin{pmatrix} m_A^2 \sin^2\beta + m_Z^2 \cos^2\beta & -(m_A^2 + m_Z^2)\sin\beta\cos\beta \\ -(m_A^2 + m_Z^2)\sin\beta\cos\beta & m_A^2\cos^2\beta + m_Z^2\sin^2\beta \end{pmatrix}. \tag{1}$$

Diagonalizing this mass matrix yields the tree-level prediction for the lightest neutral $\mathcal{CP}$-even Higgs-boson mass

$$m_h^{2,\text{tree}} = \tfrac{1}{2}\left[m_A^2 + m_Z^2 - \sqrt{(m_A^2 + m_Z^2)^2 - 4m_Z^2 m_A^2 \cos^2 2\beta}\right]. \tag{2}$$

For simplicity, we shall consider the limit of $m_A \gg m_Z$ and large supersymmetry-breaking masses characterized by a scale M_S. Then, at energy scales below M_S, the effective low-energy theory consists of the Standard Model with one Higgs doublet. The corresponding Higgs squared-mass at tree-level is given by $m_h^{2,\text{tree}} = m_Z^2\cos^2 2\beta$. The dominant contributions to the radiatively-corrected Higgs mass within the EFT approach is based on the evaluation of the effective quartic Higgs self-coupling, λ, evaluated at the scale $Q = \overline{m}_t$. The value of $\lambda(M_S)$ is fixed by the supersymmetric boundary condition, although this value is slightly modified by one-loop threshold effects (denoted below by $\Delta_{\text{th}}\lambda$), due to the decoupling of squarks at M_S with non-zero mixing. One then employs the Standard Model RGEs to obtain $\lambda(Q)$. Finally, the Higgs mass is obtained via $m_h^2 = 2\lambda(\overline{m}_t)v^2(\overline{m}_t)$ [where $v = 174$ GeV is the Higgs vacuum expectation value]. The mass $\overline{m}_t$ denotes the running top-quark mass in the $\overline{\text{MS}}$ scheme at the scale m_t. It is related to the on-shell (or pole) top-quark mass $M_t \equiv m_t^{\text{OS}}$ by the following relation

$$\overline{m}_t \equiv m_{t,\text{SM}}^{\overline{\text{MS}}}(m_t) = \frac{M_t}{1 + \frac{4}{3\pi}\alpha_s(M_t)}, \tag{3}$$

where we have only included the QCD corrections to leading order in α_s. In eq. (3), the subscript 'SM' indicates that the running mass is defined in the usual way, *i.e.* in terms of the pure Standard Model (gluonic) contributions in the (modified) minimally subtracted dimensional regularization (DREG) scheme [18].

The most important contributions to the mass of the lightest $\mathcal{CP}$-even Higgs boson arise from the t–$\tilde{t}$ sector of the MSSM, which is characterized by the following squared-mass matrix

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} M_{\tilde{t}_L}^2 + m_t^2 + \cos 2\beta(\frac{1}{2} - \frac{2}{3}s_W^2)m_Z^2 & m_t X_t \\ m_t X_t & M_{\tilde{t}_R}^2 + m_t^2 + \frac{2}{3}\cos 2\beta s_W^2 m_Z^2 \end{pmatrix}, \tag{4}$$

where $X_t \equiv A_t - \mu\cot\beta$. The $\tilde{t}$-masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ and the mixing angle $\theta_{\tilde{t}}$ are determined at tree-level by diagonalizing $\mathcal{M}_{\tilde{t}}^2$. Neglecting the numerically small contributions proportional to m_Z^2 in the stop squared-mass matrix and setting

$$M_{\tilde{t}_L} = M_{\tilde{t}_R} \equiv M_{\text{SUSY}}, \quad M_S^2 \equiv M_{\text{SUSY}}^2 + m_t^2 \tag{5}$$

leads to the simplified mass matrix

$$\mathcal{M}_{\tilde{t}}^2 = \begin{pmatrix} M_S^2 & m_t X_t \\ m_t X_t & M_S^2 \end{pmatrix} . \tag{6}$$

In this approximation, the $\tilde{t}$-masses and the mixing angle are given by

$$\begin{aligned} m_{\tilde{t}_1}^2 &= M_S^2 - |m_t X_t|, \\ m_{\tilde{t}_2}^2 &= M_S^2 + |m_t X_t|, \end{aligned} \tag{7}$$

$$\theta_{\tilde{t}} = \begin{cases} \frac{\pi}{4} & \text{for } X_t < 0 \\ -\frac{\pi}{4} & \text{for } X_t > 0 \,, \end{cases} \tag{8}$$

where by definition, $m_{\tilde{t}_1} \leq m_{\tilde{t}_2}$.

The one-loop threshold corrections to the quartic Higgs self-coupling, induced by the decoupling of stops, lead to a change of the effective quartic Higgs self-coupling at the scale M_S,

$$\lambda(M_S) = \tfrac{1}{4}(g^2 + g'^2)\cos^2 2\beta + \Delta_{\rm th}\lambda, \tag{9}$$

where the first term is the tree-level value of the quartic Higgs self-coupling in the effective low-energy Standard Model and the second term is the effect of the one-loop threshold corrections at the scale M_S [7,10],

$$\Delta_{\rm th}\lambda = \frac{3}{8\pi^2} h_t^2 \left\{ \left[h_t^2 - \tfrac{1}{8}(g^2 + g'^2) \right] \left(\frac{X_t^2}{M_S^2} \right) - \tfrac{1}{12} h_t^2 \left(\frac{X_t^4}{M_S^4} \right) \right\} + \cdots \,, \tag{10}$$

where all couplings in eq. (10) should be evaluated at the scale M_S. The running Higgs–top quark Yukawa coupling is related to the $\overline{\rm MS}$ running top quark mass:

$$\overline{m}_t(\mu) = h_t(\mu) v(\mu) \,, \tag{11}$$

where the running Higgs vacuum expectation value, $v^2(M_S) = v^2(\overline{m}_t)\,\xi^{-2}(\overline{m}_t)$, is governed by the Higgs field anomalous dimension

$$\xi(\overline{m}_t) = 1 + \frac{3}{32\pi^2} h_t^2(\overline{m}_t) \ln\left(\frac{M_S^2}{\overline{m}_t^2} \right) . \tag{12}$$

In obtaining eq. (10), an expansion in the variable

$$\Delta_{\tilde{t}} \equiv \frac{|m_t X_t|}{M_S^2} = \frac{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2}, \quad 0 \leq \Delta_{\tilde{t}} < 1, \tag{13}$$

has been performed. Terms not explicitly exhibited in eq. (10) denote the contributions from higher powers in m_t/M_S and X_t^2/M_S^2, which arise from the contributions of the t–$\tilde{t}$ sector. Contributions from other supersymmetric-breaking sectors have been omitted for simplicity of the presentation. These contributions typically contribute no more than a few GeV to the radiatively-corrected Higgs mass.

As it was shown in refs. [10] and [11], one can obtain the two-loop leading-logarithmic correction by expanding the parameter λ up to order $[\ln(M_S^2/\overline{m}_t^2)]^2$,

$$\begin{aligned}\lambda(\overline{m}_t) &= \lambda(M_S) - \beta_\lambda(M_S)t + \tfrac{1}{2}\beta'_\lambda(\overline{m}_t)t^2 + \cdots \\ &= \lambda(M_S) - \beta_\lambda(\overline{m}_t)t - \tfrac{1}{2}\beta'_\lambda(\overline{m}_t)t^2 + \cdots \end{aligned} \tag{14}$$

where $\lambda(M_S)$ is given by eq. (9) and

$$t \equiv \ln\frac{M_S^2}{\overline{m}_t^2}\,. \tag{15}$$

Following ref. [10], we define $\beta_\lambda = a_\lambda\lambda + b_\lambda$. Therefore

$$\lambda(\overline{m}_t) = \lambda(M_S)\,[1 - a_\lambda(\overline{m}_t)\,t] - b_\lambda(\overline{m}_t)\,t\,\;[1 - a_\lambda(\overline{m}_t)\,t] - \tfrac{1}{2}\beta'_\lambda(\overline{m}_t)t^2\,. \tag{16}$$

Here, $1 - a_\lambda(\overline{m}_t)\,t = \xi^{-4}(\overline{m}_t)$, where ξ is the Higgs field anomalous dimension [eq. (12)]. Multiplying eq. (16) by $2v^2(\overline{m}_t)$, we obtain an equation for the Higgs squared-mass in the low-energy theory, which takes the following form:

$$m_h^2(\overline{m}_t) = m_h^2(M_S)\,\xi^{-2}(\overline{m}_t) + \Delta_{\rm rad}\,m_h^2(\overline{m}_t)\,, \tag{17}$$

which defines the quantity $\Delta_{\rm rad}\,m_h^2(\overline{m}_t)$. In eq. (17),

$$m_h^2(M_S) = 2\lambda(M_S)v^2(M_S)\,, \tag{18}$$

where $\lambda(M_S)$ is given in eq. (9) with all couplings and masses evaluated at the scale M_S.

In the present analysis, we are working in the approximation of $h_b = g = g' = 0$. That is, we focus only on the Higgs–top quark Yukawa and QCD coupling effects. The relevant β-functions for λ, g_3^2 and h_t^2 at scales below the scale M_S are given by [19]

$$16\pi^2\beta_\lambda \equiv 6(\lambda^2 + \lambda - h_t^4) + \frac{h_t^4}{8\pi^2}\left(15h_t^2 - 16g_3^2\right)\,, \tag{19}$$

$$16\pi^2\beta_{h_t^2} \equiv h_t^2\left(\tfrac{9}{2}h_t^2 - 8g_3^2\right)\,, \tag{20}$$

$$16\pi^2\beta_{g_3^2} \equiv \left(-11 + \tfrac{2}{3}N_f\right)g_3^4\,, \tag{21}$$

where $\beta_X \equiv dX/d\ln Q^2$ and N_f is the number of quark flavors with masses less than Q (*e.g.*, $N_f = 6$ for scales between m_t and M_S). Observe that we have included the dominant strong gauge coupling two-loop contribution to the β function of the quartic Higgs self-coupling, since it will contribute once we include all two-loop leading-logarithmic corrections.

Using the above expressions, it is simple to find an approximate formula for the lightest $\mathcal{CP}$-even Higgs mass in the large m_A limit. First, one obtains

$$\Delta_{\rm rad}\,m_h^2(\overline{m}_t) = \frac{3}{4\pi^2}\frac{\overline{m}_t^4}{v^2(\overline{m}_t)}\,t\left[1 + \frac{1}{16\pi^2}\left(\tfrac{3}{2}h_t^2 - 32\pi\alpha_s\right)t\right]\,, \tag{22}$$

where all couplings in eq. (22) are evaluated at the scale $Q^2 = \overline{m}_t^2$. To complete the computation of $m_h^2(\overline{m}_t)$ [eq. (17)], one must evaluate $m_h^2(M_S)$ [see eq. (18)] in terms of low-energy parameters. This is accomplished by using one-loop renormalization group evolution to relate $\lambda(M_S)v^2(M_S)$ to $\lambda(\overline{m}_t)v^2(m_t)$. In this way, one finally arrives at the expression[2]

$$m_h^2 = m_h^{2,\text{tree}} + \frac{3}{4\pi^2}\frac{\overline{m}_t^4}{v^2}\left\{t + \frac{X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)\right. \\ \left. + \frac{1}{16\pi^2}\left(\frac{3}{2}\frac{\overline{m}_t^2}{v^2} - 32\pi\alpha_s\right)\left[\frac{2X_t^2}{M_S^2}\left(1 - \frac{X_t^2}{12M_S^2}\right)t + t^2\right] + \left(\frac{4\alpha_s}{3\pi} - \frac{5h_t^2}{16\pi^2}\right)t\right\}. \quad (23)$$

The last two terms in eq. (23) reflect the two-loop single logarithmic dependence induced by the two-loop β-function contribution to the running of the quartic Higgs self-coupling. It is interesting to note that these two terms are numerically close in size, and they tend to cancel each other in the computation of the Higgs mass. Eq. (23) differs from the one presented in ref. [9] only in the inclusion of these terms, which although sub-dominant compared to the remaining terms, should be kept for comparison with the diagrammatic result.

The full two-loop corrections to m_h^2 at $\mathcal{O}(m_t^2 h_t^4)$ have not yet been calculated in the diagrammatic approach; thus we neglect terms of this order in what follows.[3] With a slight rewriting of eq. (23) we finally obtain the expression that will be compared with the diagrammatic result in the following sections:

$$m_h^2 = m_h^{2,\text{tree}} + \frac{3}{2}\frac{G_F\sqrt{2}}{\pi^2}\overline{m}_t^4\left\{-\ln\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right) + \frac{\overline{X}_t^2}{\overline{M}_S^2}\left(1 - \frac{1}{12}\frac{\overline{X}_t^2}{\overline{M}_S^2}\right)\right\} \\ - 3\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\overline{m}_t^4\left\{\ln^2\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right) + \left[\frac{2}{3} - 2\frac{\overline{X}_t^2}{\overline{M}_S^2}\left(1 - \frac{1}{12}\frac{\overline{X}_t^2}{\overline{M}_S^2}\right)\right]\ln\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right)\right\}, \quad (24)$$

where we have introduced the notation $\overline{M}_S, \overline{X}_t$ to emphasize that the corresponding quantities are $\overline{\text{MS}}$ parameters, which are evaluated at the scale $\mu = M_S$:

$$\overline{M}_S \equiv M_S^{\overline{\text{MS}}}(M_S), \quad \overline{X}_t \equiv X_t^{\overline{\text{MS}}}(M_S), \quad (25)$$

and $\overline{m}_t \equiv m_{t,\text{SM}}^{\overline{\text{MS}}}(m_t)$ as defined in eq. (3).

3 Diagrammatic calculation

In the diagrammatic approach the masses of the $\mathcal{CP}$-even Higgs bosons are obtained by evaluating loop corrections to the h, H and hH-mixing propagators. The masses of the two $\mathcal{CP}$-even Higgs bosons, m_h and m_H, are determined as the poles of this propagator matrix, which are given by the solution of

$$\left[q^2 - m_h^{2,\text{tree}} + \hat{\Sigma}_{hh}(q^2)\right]\left[q^2 - m_H^{2,\text{tree}} + \hat{\Sigma}_{HH}(q^2)\right] - \left[\hat{\Sigma}_{hH}(q^2)\right]^2 = 0, \quad (26)$$

[2]In the "leading m_t^4 approximation" that is employed here, there is no distinction between $m_h(\overline{m}_t)$ and the on-shell (or pole) Higgs mass, m_h.

[3]As noted below eq. (23), terms of $\mathcal{O}(m_t^2 h_t^4)$ can be as numerically important as terms of $\mathcal{O}(m_t^2 h_t^2 \alpha_s)$. Hence, in a complete phenomenological analysis, one should not neglect terms of the former type.

where $\hat{\Sigma}_{hh}(q^2)$, $\hat{\Sigma}_{HH}(q^2)$, $\hat{\Sigma}_{hH}(q^2)$ denote the renormalized Higgs boson self-energies. In ref. [13] the dominant two-loop contributions to the masses of the $\mathcal{CP}$-even Higgs bosons of $\mathcal{O}(\alpha\alpha_s)$ have been evaluated. These corrections, obtained in the on-shell scheme, have been combined in refs. [14] and [15] with the complete one-loop on-shell result of ref. [4] and the two-loop corrections of $\mathcal{O}(m_t^2 h_t^4)$ given in refs. [9–11].

The diagrammatic two-loop calculation of ref. [13] involves a renormalization in the Higgs sector up to the two-loop level and a renormalization in the stop sector up to $\mathcal{O}(\alpha_s)$. In the on-shell scheme, the renormalization in the stop sector is performed such that the $\tilde{t}$-masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ correspond to the poles of the propagators, *i.e.*

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{t}_1\tilde{t}_1}(m_{\tilde{t}_1}^2) = 0, \quad \mathrm{Re}\,\hat{\Sigma}_{\tilde{t}_2\tilde{t}_2}(m_{\tilde{t}_2}^2) = 0 \tag{27}$$

for the renormalized self-energies. In Ref. [13] the renormalization condition

$$\mathrm{Re}\,\hat{\Sigma}_{\tilde{t}_1\tilde{t}_2}(m_{\tilde{t}_1}^2) = 0 \tag{28}$$

has been chosen to define the stop mixing angle.[4] In ref. [16] a compact analytic approximation has been derived from the rather complicated diagrammatic two-loop result by performing an expansion in $\Delta_{\tilde{t}}$ [eq. (13)] of the t–$\tilde{t}$ sector contributions.

The diagrammatic two-loop corrections to the Higgs mass also depend non-trivially on the gluino mass, which is a free input parameter of the supersymmetric model. In the EFT approach described in section 2, the gluino is decoupled at the same scale as the stops. Thus, in order to compare the results of the EFT and diagrammatic approaches, one must take $m_{\tilde{g}} \simeq \mathcal{O}(M_S)$. In this paper, we have chosen

$$m_{\tilde{g}} = M_{\mathrm{SUSY}} = \sqrt{M_S^2 - m_t^2}\,. \tag{29}$$

For the one-loop contributions from the other sectors of the MSSM the leading logarithmic approximation has been used [7, 11]. In this approximation, the momentum dependence in eq. (26) is neglected everywhere. The resulting expression can thus be written as a correction to the tree-level mass matrix [eq. (1)]. The expression for m_h^2 in this approximation is obtained by diagonalizing the loop-corrected mass matrix. The compact analytic expression derived in this way, which is valid for arbitrary values of m_A, has been shown to approximate the full diagrammatic result for m_h rather well, typically within about 2 GeV for most parts of the MSSM parameter space [16].

In the following we will restrict ourselves to the contribution of the t–$\tilde{t}$ sector. In order to perform a simple comparison with the EFT approach of section 2, we only consider the dominant one-loop and two-loop terms of $\mathcal{O}(m_t^2 h_t^2)$ and $\mathcal{O}(m_t^2 h_t^2 \alpha_s)$, respectively. We focus on the case $m_A \gg m_Z$, for which the result for m_h^2 can be expressed in a particularly compact form,

$$m_h^2 = m_h^{2,\mathrm{tree}} + m_h^{2,\alpha} + m_h^{2,\alpha\alpha_s}, \tag{30}$$

[4] In this paper, our analysis is presented in a simplified model of stop mixing, where the tree-level stop squared-mass matrix given by eq. (6). In this case, $\tilde{t}_1$ and $\tilde{t}_2$ are states of definite parity at tree-level. Since parity is preserved to all orders in α_s, it follows that $\hat{\Sigma}_{\tilde{t}_1\tilde{t}_2}(p^2) = 0$ (when electroweak corrections are neglected) and eq. (28) is trivially satisfied.

and neglect the non-leading terms of $\mathcal{O}(m_Z^2/m_A^2)$. Moreover, assuming that $M_S \gg M_t$ and neglecting the non-leading terms of $\mathcal{O}(M_t/M_S)$ and $\mathcal{O}(m_Z^2/M_t^2)$, one obtains the following simple result for the one-loop and two-loop contributions

$$m_h^{2,\alpha} = \frac{3}{2}\frac{G_F\sqrt{2}}{\pi^2}M_t^4\left\{-\ln\left(\frac{M_t^2}{M_S^2}\right)+\frac{X_t^2}{M_S^2}\left(1-\frac{1}{12}\frac{X_t^2}{M_S^2}\right)\right\}, \tag{31}$$

$$m_h^{2,\alpha\alpha_s} = -3\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}M_t^4\left\{\ln^2\left(\frac{M_t^2}{M_S^2}\right)-\left(2+\frac{X_t^2}{M_S^2}\right)\ln\left(\frac{M_t^2}{M_S^2}\right)-\frac{X_t}{M_S}\left(2-\frac{1}{4}\frac{X_t^3}{M_S^3}\right)\right\}. \tag{32}$$

The corresponding formulae, in which terms up to $\mathcal{O}(M_t^4/M_S^4)$ are kept, can be found in Appendix B [see eqs. (B.1) and (B.2)].

In eqs. (31) and (32) the parameters M_t, M_S, X_t are on-shell quantities. Using eq. (3), the on-shell result for m_h^2 [eqs. (30)–(32)] can easily be rewritten in terms of the running top-quark mass $\overline{m}_t$. While this reparameterization does not change the form of the one-loop result, it induces an extra contribution at $\mathcal{O}(\alpha\alpha_s)$. Keeping again only terms that are not suppressed by powers of $\overline{m}_t/M_S$, the resulting expressions read

$$m_h^{2,\alpha} = \frac{3}{2}\frac{G_F\sqrt{2}}{\pi^2}\overline{m}_t^4\left\{-\ln\left(\frac{\overline{m}_t^2}{M_S^2}\right)+\frac{X_t^2}{M_S^2}\left(1-\frac{1}{12}\frac{X_t^2}{M_S^2}\right)\right\}, \tag{33}$$

$$\begin{aligned} m_h^{2,\alpha\alpha_s} &= -3\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\overline{m}_t^4\left\{\ln^2\left(\frac{\overline{m}_t^2}{M_S^2}\right)+\left(\frac{2}{3}-\frac{X_t^2}{M_S^2}\right)\ln\left(\frac{\overline{m}_t^2}{M_S^2}\right)\right. \\ &\quad \left. +\frac{4}{3}-2\frac{X_t}{M_S}-\frac{8}{3}\frac{X_t^2}{M_S^2}+\frac{17}{36}\frac{X_t^4}{M_S^4}\right\}, \end{aligned} \tag{34}$$

in accordance with the formulae given in ref. [16].

We now compare the diagrammatic result expressed in terms of the parameters $\overline{m}_t$, M_S, X_t [eqs. (33) and (34)] with the EFT result [eq. (24)] which is given in terms of the $\overline{\rm MS}$ parameters $\overline{m}_t$, $\overline{M}_S$, $\overline{X}_t$ [eq. (25)]. While the X_t–independent logarithmic terms are the same in both the diagrammatic and EFT results, the corresponding logarithmic terms at two-loops that are proportional to powers of X_t and $\overline{X}_t$, respectively, are different. Furthermore, eq. (34) does not contain a logarithmic term proportional to X_t^4, while the corresponding term proportional to $\overline{X}_t^4$ appears in eq. (24). To check whether these results are consistent, one must relate the on-shell and $\overline{\rm MS}$ definitions of the parameters M_S and X_t.

Finally, we note that the non-logarithmic terms contained in eq. (34) correspond to genuine two-loop contributions that are not present in the EFT result of eq. (24). They can be interpreted as a two-loop finite threshold correction to the quartic Higgs self-coupling in the EFT approach. In particular, note that eq. (34) contains a term that is linear in X_t This is the main source of the asymmetry in the two-loop corrected Higgs mass under $X_t \to -X_t$ obtained by the diagrammatic method. The non-logarithmic terms in eq. (34) give rise to a numerically significant increase of the maximal value of m_h of about 5 GeV in this approximation.

4 On-shell and $\overline{\mathrm{MS}}$ definitions of M_S and X_t

Since the parameters $p = \{m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}, \theta_{\tilde{t}}, m_t\}$ of the t–$\tilde{t}$ sector are renormalized differently in different schemes, the parameters M_S and X_t also have a different meaning in these schemes. In order to derive the relation between these parameters in the $\overline{\mathrm{MS}}$ and in the on-shell scheme we start from the observation that at lowest order the parameters p are the same in both schemes, *i.e.* $p = p^{\mathrm{OS}} = p^{\overline{\mathrm{MS}}}$ in lowest order. Expressing the bare parameters in terms of the renormalized parameters and the counterterms leads to

$$p^{\overline{\mathrm{MS}}} + \delta p^{\overline{\mathrm{MS}}} = p^{\mathrm{OS}} + \delta p^{\mathrm{OS}}. \tag{35}$$

Here δp^{OS} is the on-shell counterterm in $D \equiv 4 - 2\epsilon$ dimensions, and according to the $\overline{\mathrm{MS}}$ prescription $\delta p^{\overline{\mathrm{MS}}}$ is given just by the pole part of δp^{OS}, *i.e.* the contribution proportional to $1/\epsilon - \gamma_{\mathrm{E}} + \ln 4\pi$, where γ_{E} is Euler's constant. The $\overline{\mathrm{MS}}$ parameters are thus related to the on-shell parameters by

$$p^{\overline{\mathrm{MS}}} = p^{\mathrm{OS}} + \Delta p, \tag{36}$$

where $\Delta p \equiv \delta p^{\mathrm{OS}} - \delta p^{\overline{\mathrm{MS}}}$ is finite in the limit $D \to 4$ and contains the $\overline{\mathrm{MS}}$ scale μ which can be chosen appropriately. In this paper, we only need to know Δp to $\mathcal{O}(\alpha_s)$ one-loop accuracy. In the following we will compare the result for m_h expressed in terms of the on-shell parameters p^{OS} with results for m_h in terms of the corresponding $\overline{\mathrm{MS}}$ parameters $p^{\overline{\mathrm{MS}}}$, which are related to p^{OS} as in eq. (36).

In the EFT approach, the parameters M_S and X_t are running parameters evaluated at the scale $\mu = M_S$ [eq. (25)]. In the simplified model for the stop squared-mass matrix given by eq. (6), the relations between the parameters X_t and M_S in the on-shell and $\overline{\mathrm{MS}}$ scheme are obtained using

$$m^{2,\mathrm{OS}}_{\tilde{t}_1} = M^{2,\mathrm{OS}}_S \mp M_t X^{\mathrm{OS}}_t\,, \qquad m^{2,\mathrm{OS}}_{\tilde{t}_2} = M^{2,\mathrm{OS}}_S \pm M_t X^{\mathrm{OS}}_t\,, \tag{37}$$

$$m^{2,\overline{\mathrm{MS}}}_{\tilde{t}_1} = \overline{M}^2_S \mp \overline{m}_t(M_S)\overline{X}_t\,, \qquad m^{2,\overline{\mathrm{MS}}}_{\tilde{t}_2} = \overline{M}^2_S \pm \overline{m}_t(M_S)\overline{X}_t\,, \tag{38}$$

where we have written $\overline{m}_t(M_S) \equiv m_t^{\overline{\mathrm{MS}}}(M_S)$ and $M_t \equiv m_t^{\mathrm{OS}}$ as in section 2. In both eqs. (37) and (38), the upper and lower signs refer to $X_t^{\mathrm{OS}} > 0$ and $X_t^{\mathrm{OS}} < 0$, respectively. In the model of stop mixing under consideration, there is no shift in the scalar top mixing angle to all orders in α_s, from which it follows that $|\theta^{\mathrm{OS}}_{\tilde{t}}| = |\theta^{\overline{\mathrm{MS}}}_{\tilde{t}}|$.

Inserting the relation between $m^2_{\tilde{t}_1}, m^2_{\tilde{t}_2}$ in the on-shell and the $\overline{\mathrm{MS}}$ scheme into eqs. (37) and (38) yields up to first order in α_s

$$\overline{M}^2_S = M^{2,\mathrm{OS}}_S + \tfrac{1}{2}\left(\Delta m^2_{\tilde{t}_1} + \Delta m^2_{\tilde{t}_2}\right), \tag{39}$$

$$\overline{X}_t = X^{\mathrm{OS}}_t \frac{M_t}{\overline{m}_t(M_S)} \pm \frac{1}{2m_t}\left(\Delta m^2_{\tilde{t}_2} - \Delta m^2_{\tilde{t}_1}\right)\,, \tag{40}$$

where again the upper and lower sign in the last equation refers to $X_t^{\mathrm{OS}} > 0$ and $X_t^{\mathrm{OS}} < 0$, respectively. In the second term of eq. (40) it is not necessary to distinguish (at one-loop) between $\overline{m}_t(M_S)$ and M_t, since $\Delta m^2_{\tilde{t}_1}$, $\Delta m^2_{\tilde{t}_2}$ are $\mathcal{O}(\alpha_s)$ quantities; hence, the generic symbol m_t is used here.

In Appendix A, we have obtained explicit results for $\Delta m^2_{\tilde{t}_1}$, $\Delta m^2_{\tilde{t}_2}$ and $M_t/\overline{m}_t(M_S)$. Inserting the appropriate expressions for these quantities into eq. (40), one observes that the functional form for $\overline{X}_t$ is the same for $X_t^{\rm OS} > 0$ and $X_t^{\rm OS} < 0$ [*i.e.*, the sign difference in eq. (40) is compensated by the term $(\Delta m^2_{\tilde{t}_2} - \Delta m^2_{\tilde{t}_1})$]. As a result, it is no longer necessary to distinguish between these two cases. The case $X_t^{\rm OS} = 0$ (which formally would have to be treated separately) is understood as being included in eq. (40).

Using the expansions given in Appendix A and setting the gluino mass according to eq. (29), we obtain to leading order in m_t/M_S

$$\overline{M}_S^2 = M_S^{2,{\rm OS}} - \frac{8}{3}\frac{\alpha_s}{\pi} M_S^2\,, \tag{41}$$

$$\overline{X}_t = X_t^{\rm OS}\frac{M_t}{\overline{m}_t(M_S)} + \frac{8}{3}\frac{\alpha_s}{\pi} M_S\,. \tag{42}$$

As previously noted, it is not necessary to specify the definition of the parameters that appear in the $\mathcal{O}(\alpha_s)$ terms. Thus, we use the generic symbol M_S^2 in the $\mathcal{O}(\alpha_s)$ terms of eqs. (41)–(42). The corresponding results including terms up to $\mathcal{O}\left(m_t^4/M_S^4\right)$ can be found in Appendix B.

Finally, we need to evaluate the ratio $M_t/\overline{m}_t(M_S)$. The relevant expression is given in eq. (A.14). Using the expansions given at the end of Appendix A, we find to leading order in m_t/M_S

$$\overline{m}_t(M_S) = \overline{m}_t\left[1 + \frac{\alpha_s}{\pi}\ln\left(\frac{m_t^2}{M_S^2}\right) + \frac{\alpha_s}{3\pi}\frac{X_t}{M_S}\right]\,, \tag{43}$$

where $\overline{m}_t \equiv m_{t,{\rm SM}}^{\overline{\rm MS}}(m_t)$ is given in terms of M_t by eq. (3). The corresponding formula, where terms up to $\mathcal{O}\left(m_t^4/M_S^4\right)$ are kept, can be found at the end of Appendix B. Note that the term in eq. (43) that is proportional to X_t is a threshold correction due to the supersymmetry-breaking stop-mixing effect. Inserting the result of eq. (43) into eq. (42) yields:

$$\overline{X}_t = X_t^{\rm OS} + \frac{\alpha_s}{3\pi} M_S\left[8 + \frac{4X_t}{M_S} - \frac{X_t^2}{M_S^2} - \frac{3X_t}{M_S}\ln\left(\frac{m_t^2}{M_S^2}\right)\right]\,. \tag{44}$$

It is interesting to note that $\overline{X}_t \neq 0$ when $X_t^{\rm OS} = 0$. Moreover, it is clear from eq. (44) that the relation between X_t defined in the on-shell and the $\overline{\rm MS}$ schemes includes a leading logarithmic effect, which has to be taken into account in a comparison of the leading logarithmic contributions in the EFT and the two-loop diagrammatic results.

The above results are relevant for calculations in the full theory in which the effects of the supersymmetric particles are fully taken into account. However, in effective field theory below M_S, one must decouple the supersymmetric particles from the loops and compute with the Standard Model spectrum. Thus, it will be useful to define a running $\overline{\rm MS}$ top-quark mass in the effective Standard Model, $m_{t,{\rm SM}}^{\overline{\rm MS}}(\mu)$, which to $\mathcal{O}(\alpha_s)$ is given by:

$$m_{t,{\rm SM}}^{\overline{\rm MS}}(\mu) = \overline{m}_t\left[1 + \frac{\alpha_s}{\pi}\ln\left(\frac{m_t^2}{\mu^2}\right)\right]\,. \tag{45}$$

At the scale M_S, we must match this result onto the expression for $\overline{m}_t(M_S)$ as computed in the full theory [eq. (43)]. The matching is discontinuous at $\mu = M_S$ due to the threshold corrections arising from stop mixing effects.

In comparing with the EFT results of refs. [8–11], it should be noted that the threshold correction in eq. (43) were omitted. This is relevant, since $\overline{m}_t(M_S)$ [or the related quantity $h_t(M_S)$, see eq. (11)] appears in the threshold correction to the quartic Higgs self-coupling $\lambda(M_S)$ [eqs. (9) and (10)]. In refs. [8–11], $\overline{m}_t(M_S)$ is re-expressed in terms of $\overline{m}_t(m_t)$ by using eq. (45) rather than eq. (43). As a result, a two-loop non-logarithmic term proportional to X_t is missed in the computation of m_h. Such a term is of the same order as the two-loop threshold correction to the quartic Higgs self-coupling, which were also neglected in refs. [8–11]. However, in this work we do not neglect the latter. Hence, it would be incorrect to use eq. (45) in the evaluation of $\overline{m}_t(M_S)$. In section 5 we will apply $\overline{m}_t(\mu)$ with different choices of μ for the X_t–independent and X_t–dependent contributions to m_h^2, which will prove useful for absorbing numerically large two-loop contributions into an effective one-loop result. In the spirit of EFT, we will argue that for $\mu = M_S$, one should use the results of eq. (43) while for $\mu < M_S$, one should use eq. (45).

A remark on the regularization scheme is in order here. In effective field theory, the running top-quark mass at scales below M_S is the SM running coupling [eq. (45)], which is calculated in dimensional regularization. This is matched onto the running top-quark mass as computed in the full supersymmetric theory. One could argue that the appropriate regularization scheme for the latter should be dimensional reduction (DRED) [20], which is usually applied in loop calculations in supersymmetry.[5] The result of such a change would be to modify slightly the two-loop non-logarithmic contribution to m_h that is proportional to powers of X_t. Of course, the physical Higgs mass is independent of scheme. One is free to re-express eqs. (31) and (32) [which depend on the on-shell parameters M_t, M_S, X_t] in terms of parameters defined in any other scheme. In this paper, we find $\overline{\text{MS}}$–renormalization via DREG to be the most convenient scheme for the comparison of the diagrammatic and EFT results for m_h.

5 Comparing the EFT and diagrammatic results

In order to directly compare the two-loop diagrammatic and EFT results, we must convert from on-shell to $\overline{\text{MS}}$ parameters. Inserting eqs. (41) and (44) into eqs. (33) and (34), one finds

$$m_h^{2,\alpha} = \frac{3}{2}\frac{G_F\sqrt{2}}{\pi^2}\overline{m}_t^4\left\{-\ln\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right) + \frac{\overline{X}_t^2}{\overline{M}_S^2}\left(1 - \frac{1}{12}\frac{\overline{X}_t^2}{\overline{M}_S^2}\right)\right\}, \tag{46}$$

$$\begin{aligned} m_h^{2,\alpha\alpha_s} = -3\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\overline{m}_t^4\Bigg\{&\ln^2\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right) + \left[\frac{2}{3} - 2\frac{\overline{X}_t^2}{\overline{M}_S^2}\left(1 - \frac{1}{12}\frac{\overline{X}_t^2}{\overline{M}_S^2}\right)\right]\ln\left(\frac{\overline{m}_t^2}{\overline{M}_S^2}\right) \\ &+ \frac{\overline{X}_t}{\overline{M}_S}\left(\frac{2}{3} - \frac{7}{9}\frac{\overline{X}_t^2}{\overline{M}_S^2} + \frac{1}{36}\frac{\overline{X}_t^3}{\overline{M}_S^3} + \frac{1}{18}\frac{\overline{X}_t^4}{\overline{M}_S^4}\right)\Bigg\} + \mathcal{O}\left(\frac{\overline{m}_t}{\overline{M}_S}\right). \end{aligned} \tag{47}$$

Comparing eq. (47) with eq. (24) shows that the logarithmic contributions of the diagrammatic result expressed in terms of the $\overline{\text{MS}}$ parameters $\overline{m}_t$, $\overline{M}_S$, $\overline{X}_t$ agree with

[5] In order to obtain the corresponding DRED result, one simply has to replace the term $4\alpha_s/3\pi$ in the denominator of eq. (3) by $5\alpha_s/3\pi$.

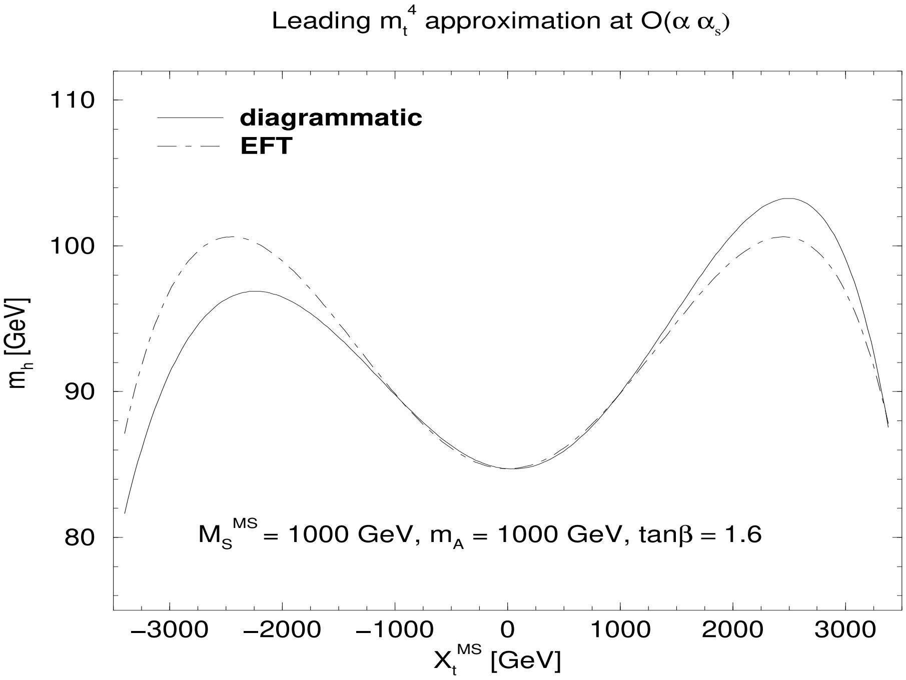
the logarithmic contributions obtained by the EFT approach. The differences in the logarithmic terms observed in the comparison of eqs. (33) and (34) with eq. (24) have thus been traced to the different renormalization schemes applied in the respective calculations. The fact that the logarithmic contributions obtained within the two approaches agree after a proper rewriting of the parameters of the stop sector is an important consistency check of the calculations. In addition to the logarithmic contributions, eq. (47) also contains non-logarithmic contributions, which are numerically sizable.

In fig. 1, we compare the diagrammatic result for m_h in the leading m_t^4 approximation to the results obtained in section 2 by EFT techniques, for two different values of $\tan\beta$. However, as noted at the end of section 4, in the derivation of the EFT result of eq. (24) the supersymmetric threshold corrections to $\overline{m}_t(M_S)$ were neglected. Thus, $\overline{X}_t$, which appears in eqs. (46) and (47), is not precisely the same as the $\overline{X}_t$ parameter appearing in the EFT result of eq. (24) due to the difference in the definition of $\overline{m}_t(M_S)$ [eq. (43)] and $m_{t,\mathrm{SM}}^{\overline{\mathrm{MS}}}$ [eq. (45)]. Taking this difference into account in eq. (42), it follows that the $\overline{X}_t$ parameter that appears in eq. (24) is given by $\overline{X}_t' \equiv \overline{X}_t [1 + (\alpha_s/3\pi)(X_t/M_S)]$. It can be easily checked that the change from $\overline{X}_t$ to $\overline{X}_t'$ does not affect the comparison of two-loop logarithmic terms between eqs. (24) and (47). Moreover, the difference between $\overline{X}_t$ and $\overline{X}_t'$ is numerically small. In fig. 1, the diagrammatic result for m_h is plotted versus $\overline{X}_t$, while the EFT result is plotted versus $\overline{X}_t'$.

While the diagrammatic result expressed in terms of $\overline{m}_t$, $\overline{M}_S$, $\overline{X}_t$ agrees well with the EFT result in the region of no mixing in the stop sector, sizable deviations occur for large mixing. In particular, the non-logarithmic contributions give rise to an asymmetry under the change of sign of the parameter $\overline{X}_t$, while the EFT result is symmetric under $\overline{X}_t \to -\overline{X}_t$. In the approximation considered here, the maximal value for m_h in the diagrammatic result lies about 3 GeV higher than the maximal value of the EFT result for $\tan\beta = 1.6$. The differences are slightly smaller for $\tan\beta = 30$. In addition, as previously noted, the maximal-mixing point $(\overline{X}_t)_{\max}$ [where the radiatively corrected value of m_h is maximal] is equal to its one-loop value, $(\overline{X}_t)_{\max} \simeq \pm\sqrt{6}M_S$, in the EFT result of eq. (24), while it is shifted in the two-loop diagrammatic result. However, fig. 1 illustrates that the shift in $(\overline{X}_t)_{\max}$ from its one-loop value, while significant in the two-loop on-shell diagrammatic result, is largely diminished when the latter is re-expressed in terms of $\overline{\mathrm{MS}}$ parameters.

The differences between the diagrammatic and EFT results shown in fig. 1 can be attributed to non-negligible non-logarithmic terms proportional to powers of X_t. Clearly, the EFT technique can be improved to incorporate these terms. As previously discussed, one can account for such terms in the EFT approach by: (i) including one-loop finite threshold effects in the definition of $\overline{m}_t(M_S)$, and (ii) including two-loop finite threshold effects to $\Delta_{\rm th}\lambda$ [eq. (10)] in the matching condition for $\lambda(M_S)$.[6] Although both effects are nominally of the same order, it is interesting to investigate what fraction of the terms proportional to powers of $\overline{X}_t$ in eq. (47) can be obtained by including the threshold effects in the definition of $\overline{m}_t(M_S)$.

[6] An additional two-loop $\mathcal{O}(h_t^2\alpha_s)$ correction to $\lambda(M_S)$ can arise because the Higgs self-coupling in the $\overline{\mathrm{MS}}$ scheme does not precisely satisfy the supersymmetric relation $[\lambda(M_S) = \frac{1}{4}(g^2+g'^2)\cos^2 2\beta]$ in the supersymmetric limit. This correction corresponds to a matching of the $\overline{\mathrm{MS}}$ and $\overline{\mathrm{DR}}$ couplings at the scale M_S [21]. We will count this as part of the two-loop finite threshold effects.



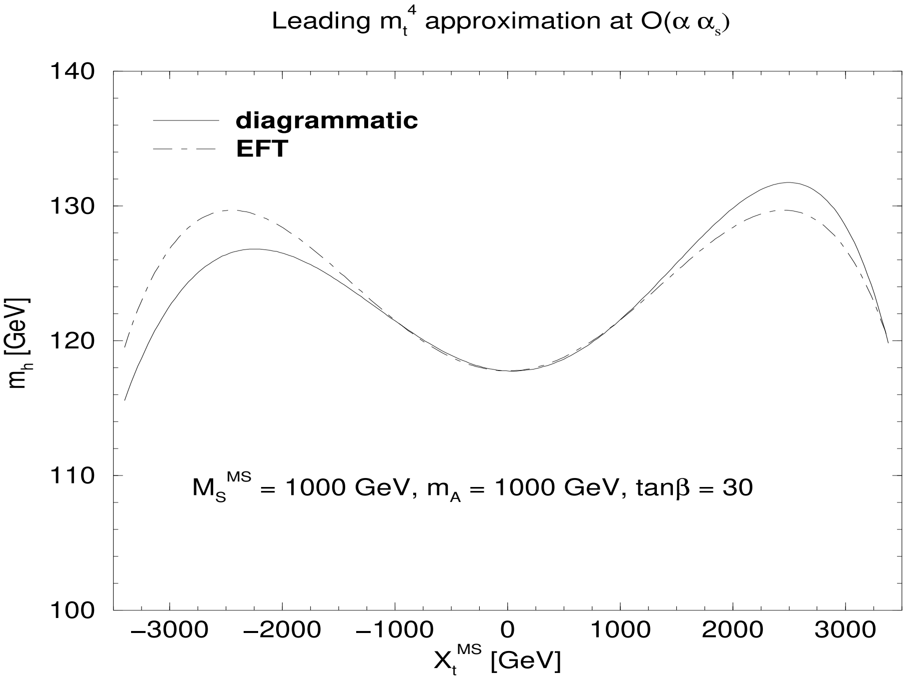


Figure 1. Comparison of the diagrammatic two-loop $\mathcal{O}(m_t^2 h_t^2 \alpha_s)$ result for m_h, to leading order in $\overline{m}_t/\overline{M}_S$ [eqs. (46) and (47)] with the EFT result of eq. (24). Note that the latter omits the one-loop threshold corrections due to stop mixing in the evaluation of $\overline{m}_t(M_S)$. Since this quantity enters in the definition of $\overline{X}_t$ [see eq. (42)], the meaning of $X_t^{\rm MS}$ plotted along the x-axis is slightly different for the diagrammatic curve, where $X_t^{\rm MS} = \overline{X}_t$, and the EFT curve, where $X_t^{\rm MS} = \overline{X}_t\,[1 + (\alpha_s/3\pi)(X_t/M_S)]$. See text for further details. The two graphs above are plotted for $\overline{M}_S = m_A = (m_{\tilde{g}}^2 + \overline{m}_t^2)^{1/2} = 1$ TeV for the cases of $\tan\beta = 1.6$ and $\tan\beta = 30$, respectively.

In ref. [11], it was shown that the leading two-loop contributions to m_h^2 given by the EFT result of eq. (24) could be absorbed into an effective one-loop expression. This was accomplished by considering separately the X_t–independent leading double logarithmic term (the "no-mixing" contribution) and the leading single logarithmic term that is proportional to powers of $\overline{X}_t$ (the "mixing" contribution) at $\mathcal{O}(m_t^2 h_t^2 \alpha_s)$. Both terms can be reproduced by an effective one-loop expression, where $\overline{m}_t$ in eq. (46), which appears in the no-mixing and mixing contributions, is replaced by the *running* top-quark mass evaluated at the scales μ_t and $\mu_{\tilde{t}}$, respectively:

$$\text{no mixing: } \mu_t \equiv (\overline{m}_t \overline{M}_S)^{1/2}\,, \qquad \text{mixing: } \mu_{\tilde{t}} \equiv \overline{M}_S\,. \tag{48}$$

That is, at $\mathcal{O}(m_t^2 h_t^2 \alpha_s)$, the leading double logarithmic term is precisely reproduced by the single-logarithmic term at $\mathcal{O}(m_t^2 h_t^2)$, by replacing $\overline{m}_t$ with $\overline{m}_t(\mu_t)$, while the leading single logarithmic term at two-loops proportional to powers of $\overline{X}_t$ is precisely reproduced by the corresponding non-logarithmic terms proportional to $\overline{X}_t$ at $\mathcal{O}(m_t^2 h_t^2)$, by replacing $\overline{m}_t$ with $\overline{m}_t(\overline{M}_S)$.

Applying the same procedure to eq. (47) and rewriting it in terms of the running top-quark mass at the corresponding scales as specified in eq. (48), we obtain

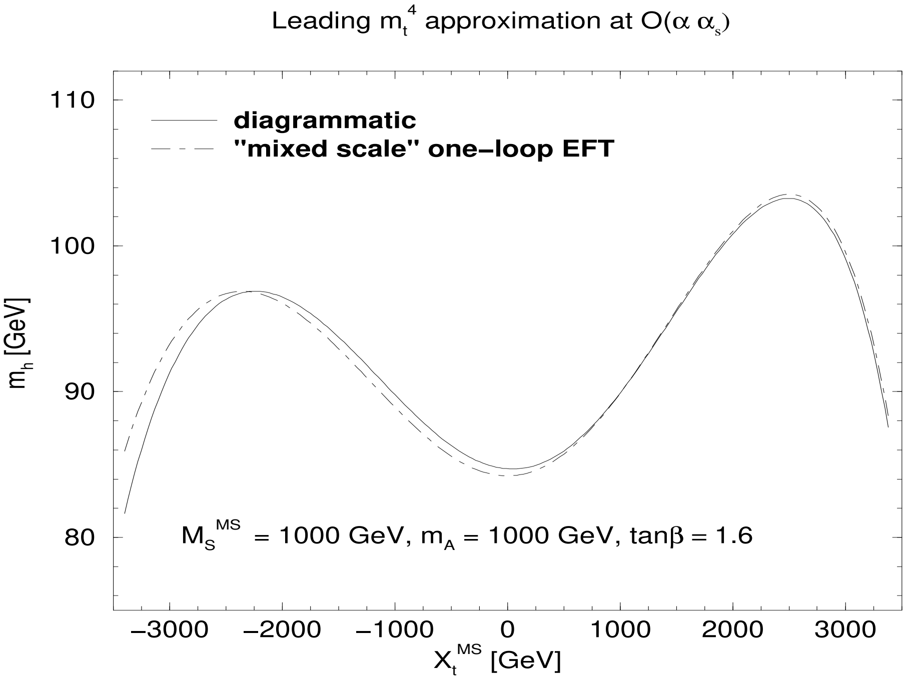
$$m_h^{2,\alpha} = \frac{3}{2}\frac{G_F\sqrt{2}}{\pi^2}\left\{-\overline{m}_t^4(\mu_t)\ln\left(\frac{\overline{m}_t^2(\mu_t)}{\overline{M}_S^2}\right) + \overline{m}_t^4(M_S)\frac{\overline{X}_t^2}{\overline{M}_S^2}\left(1-\frac{\overline{X}_t^2}{12\overline{M}_S^2}\right)\right\}, \tag{49}$$

$$m_h^{2,\alpha\alpha_s} = -3\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}m_t^4\left\{\frac{1}{6}\ln\left(\frac{m_t^2}{M_S^2}\right) + \frac{X_t}{M_S}\left(\frac{2}{3} - \frac{1}{9}\frac{X_t^2}{M_S^2} + \frac{1}{36}\frac{X_t^3}{M_S^3}\right)\right\}. \tag{50}$$

Indeed, the X_t–independent leading double logarithmic term and the leading single logarithmic term that is proportional to powers of $\overline{X}_t$ have disappeared from the two-loop expression [eq. (50)], having been absorbed into an effective one-loop result [eq. (49)] (denoted henceforth as the "mixed-scale" one-loop EFT result). Of the terms that remain [eq. (50)], there is a subleading one-loop logarithm at two-loops which is a remnant of the no-mixing contribution. But, note that the magnitude of the coefficient (1/6) has been reduced from the corresponding coefficients that appear in eqs. (32) and (47) [-2 and 2/3, respectively]. In addition, the remaining leftover two-loop non-logarithmic terms are also numerically insignificant. We conclude that the "mixed-scale" one-loop EFT result provides a very good approximation to m_h^2, in which the most significant two-loop terms have been absorbed into an effective one-loop expression.

To illustrate this result, we compare in fig. 2 the diagrammatic two-loop result expressed in terms of $\overline{\text{MS}}$ parameters [eqs. (46) and (47)] with the "mixed-scale" one-loop EFT result [eq. (49)] as a function of $\overline{X}_t$. In order to make a fair comparison of two-loop expressions, we first evaluate eq. (49) as a perturbation expansion which is truncated beyond the $\mathcal{O}(\alpha_s^2)$ term.[7] It is this result that is plotted as a dashed line in fig. 2. Note that by construction, the sum of the two-loop truncated version of eq. (49) and the leftover two-loop term given by eq. (50) is equal to the sum of eqs. (46) and (47). That is, the difference between the solid and dashed lines of fig. 2 is precisely

[7] For example, using eq. (45), we would write $\overline{m}_t^4(\mu_t) = \overline{m}_t^4\left[1 + 4(\alpha_s/\pi)\ln(m_t^2/\mu_t^2)\right]$ and insert this into eq. (49).



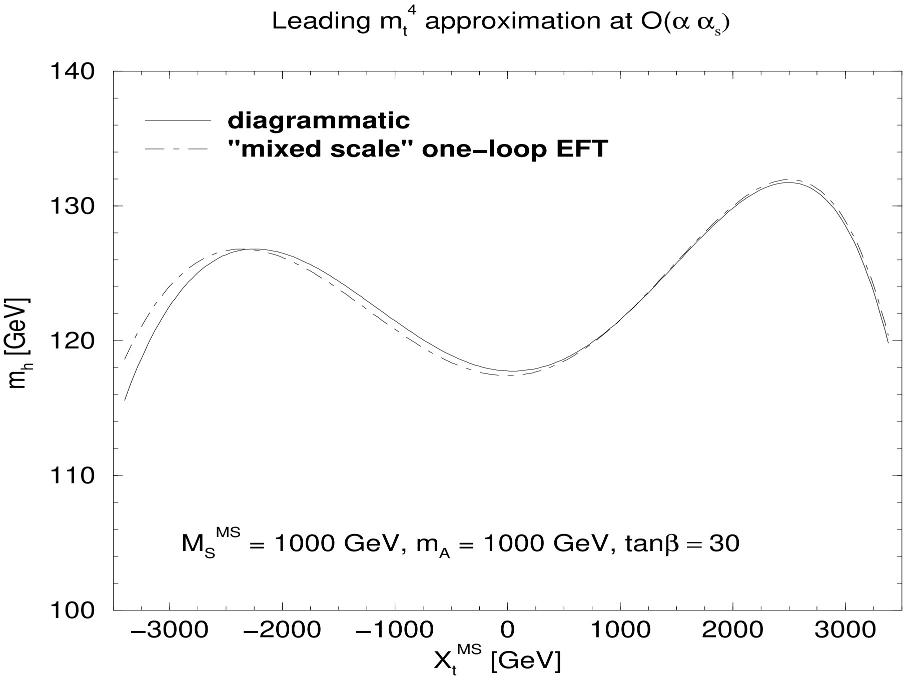


Figure 2. Comparison of the diagrammatic two-loop $\mathcal{O}(m_t^2 h_t^2 \alpha_s)$ result for m_h, to leading order in $\overline{m}_t/\overline{M}_S$ [eqs. (46) and (47)] with the "mixed-scale" one-loop EFT result [eq. (49)]. Note that the latter now includes the threshold corrections due to stop mixing in the evaluation of $\overline{m}_t(M_S)$ in contrast to the EFT results depicted in fig. 1. "Mixed-scale" indicates that in the no-mixing and mixing contributions to the one-loop Higgs mass, the running top quark mass is evaluated at different scales according to eq. (48). See text for further details. The two graphs above are plotted for $\overline{M}_S = m_A = (m_{\tilde{g}}^2 + \overline{m}_t^2)^{1/2} = 1$ TeV for the cases of $\tan\beta = 1.6$ and $\tan\beta = 30$, respectively.

equal to the leftover two-loop term given by eq. (50), which is seen to be numerically small. Hence, within the simplifying framework under consideration (*i.e.*, only leading t–$\widetilde{t}$ sector-contributions are taken into account assuming a simplified stop squared-mass matrix [eq. (6)], with $\overline{M}_S$, $m_A \gg \overline{m}_t$ and $m_{\tilde{g}} = M_{\rm SUSY}$), we see that the "mixed-scale" one-loop result for m_h provides a very good approximation to a more complete two-loop result for all values of $\overline{X}_t$.[8] In the EFT picture this means that, once re-expressed in terms of the appropriate $\overline{\rm MS}$ running parameters, the dominant contributions to the lightest $\mathcal{CP}$-even Higgs mass arising from the two-loop threshold corrections induced by the decoupling of the stops, have their origin in the one-loop threshold corrections to the Higgs–top quark Yukawa coupling.

In the present analysis we have focused on the leading contributions of the t–$\widetilde{t}$ sector of the MSSM. However, these contributions alone are not sufficient to provide an accurate determination of the Higgs mass (and can be off by 5 GeV or more in certain regions of the MSSM parameter space). In any realistic phenomenological analysis of the properties of the Higgs sector, one must include sub-leading contributions of the t–$\widetilde{t}$ sector as well as contributions from other particle/superpartner sectors. Such contributions have been obtained within the EFT approach in refs. [10] and [11], which incorporates one-loop leading logarithmic terms from all partner/superpartner sectors, plus single and double logarithmic two-loop contributions from the t–$\widetilde{t}$ and b–$\widetilde{b}$ sectors. The full one-loop diagrammatic result is known [3–5], and this along with the diagrammatic two-loop result from the t–$\widetilde{t}$ sector at $\mathcal{O}(\alpha\alpha_s)$ are included in the Fortran code *FeynHiggs* [22]. The impact of the additional contributions to the radiatively-corrected Higgs mass on phenomenological studies have been investigated in refs. [15, 23].

In the large $\tan\beta$ regime, the b–$\widetilde{b}$ sector is especially important. Here the corrections induced by the bottom Yukawa coupling become relevant, and one should correspondingly include the bottom mass corrections originating from the decoupling of the supersymmetric particles. These corrections are enhanced by a large $\tan\beta$ factor and hence can have a sizable impact on the phenomenology of the Higgs sector. Some of the most relevant consequences of these corrections have been recently discussed in refs. [24–26].

6 Discussion and Conclusions

In this work, we have compared the results for the lightest $\mathcal{CP}$-even Higgs-boson mass obtained from the two-loop $\mathcal{O}(\alpha\alpha_s)$ diagrammatic calculation in the on-shell scheme with the results of an effective field theory approach. In the latter, the two-loop $\mathcal{O}(\alpha\alpha_s)$ terms are generated via renormalization group running of the effective low-energy parameters from the supersymmetry-breaking scale, M_S to the scale m_t. We have focused on the leading $\mathcal{O}(m_t^2 h_t^2 \alpha_s)$ two-loop contributions to m_h^2 in the limit of large m_A and M_S. In this case, the effective field theory below M_S is the one-Higgs-doublet Standard Model, which greatly simplifies the calculation. In addition, the gluino mass was set to $m_{\tilde{g}} = M_{\rm SUSY} \equiv (M_S^2 - m_t^2)^{1/2}$. The resulting transparent analytic expressions for the

[8] Strictly speaking, the analytic approximations of this paper break down when $\overline{m}_t \overline{X}_t \sim \overline{M}_S^2$. Thus, one does not expect an accurate result for the corresponding formulae when $\overline{X}_t$ is too large [9, 11, 16]. In practice, one should not trust the accuracy of the analytic formulae once $\overline{X}_t > (\overline{X}_t)_{\rm max}$.

radiatively-corrected Higgs mass were well suited for investigating the basic relations between the various approaches. In order to compare the on-shell diagrammatic and effective field theory approaches, one must note two important facts. First, the two calculations are performed in different renormalization schemes. Hence, the resulting expressions actually depend on soft-supersymmetry-breaking parameters whose definitions differ at the one-loop level. Second, the diagrammatic calculation includes genuine non-logarithmic two-loop corrections to the lightest $\mathcal{CP}$-even Higgs-boson mass. In the effective field theory approach, these would correspond to two-loop threshold corrections resulting from the decoupling of the two heavy top squarks in the low energy effective theory.

Previous comparisons of the corresponding two-loop results revealed an apparent discrepancy between terms that depend logarithmically on M_S, and different dependences of the non-logarithmic terms on X_t. However, after re-expressing the one-loop and the two-loop terms of the on-shell diagrammatic result in terms of $\overline{\rm MS}$ parameters, *i.e.*, applying the same renormalization scheme for both approaches, we have shown that the discrepancy in the logarithmic dependence on M_S of both expressions disappears. This constitutes an important consistency check of the calculations. There remain, however, genuine non-logarithmic two-loop contributions in the diagrammatic result. They give rise to an asymmetry under $X_t \to -X_t$, while the effective field theory computations that neglected the two-loop threshold corrections due to stop mixing only yield results that are symmetric under the change of sign of X_t. Moreover, the non-logarithmic two-loop contributions of the on-shell diagrammatic computation, in the approximations considered in this paper, can *increase* the predicted value of m_h by as much as 3 GeV.[9] Finally, they induce a shift of the value of X_t where m_h is maximal relative to the corresponding one-loop value $(X_t)_{\rm max} \simeq \pm\sqrt{6}M_S$. It is interesting to note that this shift is more (less) pronounced when m_h is expressed in terms of X_t in the on-shell ($\overline{\rm MS}$) scheme. The effect of the leading non-logarithmic two-loop contributions can be taken into account in the effective field theory method by incorporating the $\mathcal{O}(h_t^2\alpha_s)$ X_t–dependent corrections into the boundary conditions of the effective quartic Higgs self-coupling at the scale M_S and performing a proper one-loop $\mathcal{O}(\alpha_s)$ matching of the running Higgs–top Yukawa coupling at M_S.

In ref. [11], it was shown that the leading two-loop contributions to m_h^2 could be absorbed into an effective one-loop expression by the following procedure. The running top-quark mass that appears in the X_t–independent one-loop expression for m_h^2 is evaluated at the scale $\mu_t = (M_S m_t)^{1/2}$. In the corresponding X_t–dependent terms, the running top quark mass is evaluated at the scale $\mu_{\tilde{t}} = M_S$. The result, which we call the *mixed-scale* one-loop EFT expression neatly incorporates the leading two-loop effects. In this paper, we have extended this result by explicitly including stop mixing effects in evaluating the running parameters $\overline{m}_t(M_S)$ and $h_t(M_S)$. By doing so, we are able to incorporate some portion of the genuine leading non-logarithmic two-loop contributions. The remaining terms at this order would then be identified with two-loop threshold corrections to the effective Higgs quartic coupling at the scale M_S. Remarkably, the latter turn out to be numerically small. This means that the mixed-scale one-loop EFT

[9] If only the on-shell top-quark mass is re-expressed in terms of the corresponding $\overline{\rm MS}$ parameter, the resulting increase in m_h can be as much as 5 GeV.

expression for m_h provides a rather accurate estimate of the radiatively-corrected mass of the lightest $\mathcal{CP}$-even Higgs boson of the MSSM.

The above results have been obtained in a rather simple setting. A special choice for the stop squared-mass matrix was made [eq. (6)] to simplify our analysis. The gluino mass was fixed to a value of order M_S. The leading $\mathcal{O}(m_t^2 h_t^4)$ corrections were neglected. Subleading terms of $\mathcal{O}(m_Z^2 h_t^2 \alpha_s)$ and $\mathcal{O}(m_Z^2 h_t^4)$ terms were also neglected. For example, consider the effect of varying the gluino mass. The two-loop diagrammatic results of refs. [13–16] showed that the value of m_h changed by as much as ± 2 GeV as a function of $m_{\tilde{g}}$, for $m_t \lesssim m_{\tilde{g}} \lesssim M_S$. The gluino mass dependence can be treated in the EFT approach as follows. Let us assume that M_S characterizes the scale of the squark masses, and $m_{\tilde{g}} < M_S$. Then, at scales below M_S one integrates out the squarks but keeps the gluino as part of the low-energy effective theory. However, since the gluino *always* appears with squarks in diagrams contributing to m_h^2 at two-loops, once the squarks are integrated out, they no longer affect the running of any of the relevant low-energy parameters below M_S. However, the gluino mass does affect the value of $h_t(M_S)$ and $\overline{m}_t(M_S)$ [the relevant formulae are given in Appendix A]. Thus, in the EFT approach, gluino mass dependence enters via the threshold corrections to the Higgs–top quark Yukawa couplings.

The case of a more general stop squared-mass matrix can be treated using the same methods outlined in Appendix A.[10] Here the computations are more complicated since there is now one-loop mixing between $\tilde{t}_1$ and $\tilde{t}_2$. In the EFT approach, one must decouple separately the two stops, and include the most general stop-mixing effects in the determination of the relation between the on-shell and $\overline{\text{MS}}$ parameters. The leading $\mathcal{O}(m_t^2 h_t^4)$ corrections in the EFT approach can be incorporated as in section 2 [10,11] by extending the computations of Appendix A to include the one-loop $\mathcal{O}(h_t^2)$ corrections to the running top-quark mass and stop sector parameters. However, at present, one cannot check these results against an $\mathcal{O}(m_t^2 h_t^4)$ two-loop diagrammatic computation, since the latter does not yet appear in the literature in full generality.

Going beyond the approximations made in this paper, the next step is to incorporate the above improvements, as well as the next subleading contributions of $\mathcal{O}(m_Z^2 h_t^2 \alpha_s)$ and $\mathcal{O}(m_Z^2 h_t^4)$ into the computation of m_h^2. One might then hope to show that a complete mixed-scale one-loop EFT result, suitably generalized, provides a very good approximation to the radiatively-corrected $\mathcal{CP}$-even Higgs mass of the MSSM. Such an analysis could be used to organize the most significant non-leading one-loop and two-loop contributions to m_h^2 and provide some insight regarding the magnitude of the unknown higher-order corrections, thus reducing the theoretical uncertainty in the prediction for m_h. This would have a significant impact on the physics of the lightest CP-even Higgs boson at LEP2, the upgraded Tevatron and the LHC.

[10] The transformation from on-shell to $\overline{\text{MS}}$ input parameters for the case of the most general stop squared-mass matrix will be included in the new version of the program *FeynHiggs*.

Note Added

After this work was completed, we received a paper [27] which discusses many of the same issues that we address in this work. In ref. [27], the two-loop effective potential of the MSSM is employed, including renormalization group resummation of logarithmic terms, and the leading non-logarithmic two-loop terms of $\mathcal{O}(\alpha\alpha_s)$. The relation between on-shell and $\overline{\mathrm{MS}}$ parameters are also taken into account. The end results are qualitatively similar to the ones obtained here.

Acknowledgments

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Appendix A: Relations between on-shell and $\overline{\mathbf{MS}}$ definitions of m_t, M_S and X_t

In this appendix, we derive the relations between the on-shell and $\overline{\mathrm{MS}}$ definitions of m_t, M_S and X_t. We have checked that these results agree with similar results given in ref. [5]. These results are derived in a model where the stop mass-squared matrix is given by eq. (6). The corresponding stop squared-masses and mixing angle are given by eqs. (7) and (8). Note that in this model, the top-squark mass eigenstates, $\widetilde{t}_1$ and $\widetilde{t}_2$, are states of definite parity in their interactions with gluons and gluinos. The corresponding Feynman rules are shown in fig. 3.

Consider first the one-loop contribution to the top-quark two-point function at $\mathcal{O}(\alpha_s)$ due to: (i) the top-quark/gluon loop [fig. 4(a)] and (ii) the stop/gluino loop [fig. 4(b)]. Divergences are regulated by dimensional regularization in $D \equiv 4 - 2\epsilon$ dimensions and removed by minimal subtraction. Including the tree-level contribution (which is equal to the negative of the inverse tree-level propagator), the end result is[11]

$$\begin{aligned}\Gamma^{(2)}(p) = i[\not{p} - \overline{m}_t(\mu)] - \frac{iC_F\alpha_s}{4\pi}\Big\{&\not{p}[1 + 2\mathcal{B}_1(p^2; m_t^2, 0)] - 2m_t[1 - 2\mathcal{B}_0(p^2; m_t^2, 0)] \\ &+ \not{p}[\mathcal{B}_1(p^2; m_{\tilde{g}}^2, M_S^2 - m_tX_t) + \mathcal{B}_1(p^2; m_{\tilde{g}}^2, M_S^2 + m_tX_t)] \\ &-m_{\tilde{g}}[\mathcal{B}_1(p^2; m_{\tilde{g}}^2, M_S^2 - m_tX_t) - \mathcal{B}_1(p^2; m_{\tilde{g}}^2, M_S^2 + m_tX_t)]\Big\}\,, \qquad \text{(A.1)}\end{aligned}$$

where $T^aT^a = C_F\mathbf{1}$ is the SU(3) quadratic Casimir operator in the fundamental repre-

[11] All results in this section are given in the $\overline{\mathrm{MS}}$ subtraction scheme. In the $\overline{\mathrm{DR}}$ scheme, all the formulae of this appendix still apply except in the case of the top-quark gluon loop. To obtain the corresponding $\overline{\mathrm{DR}}$ result, simply remove the additive factors of 1 in the two occurences in eq. (A.1).

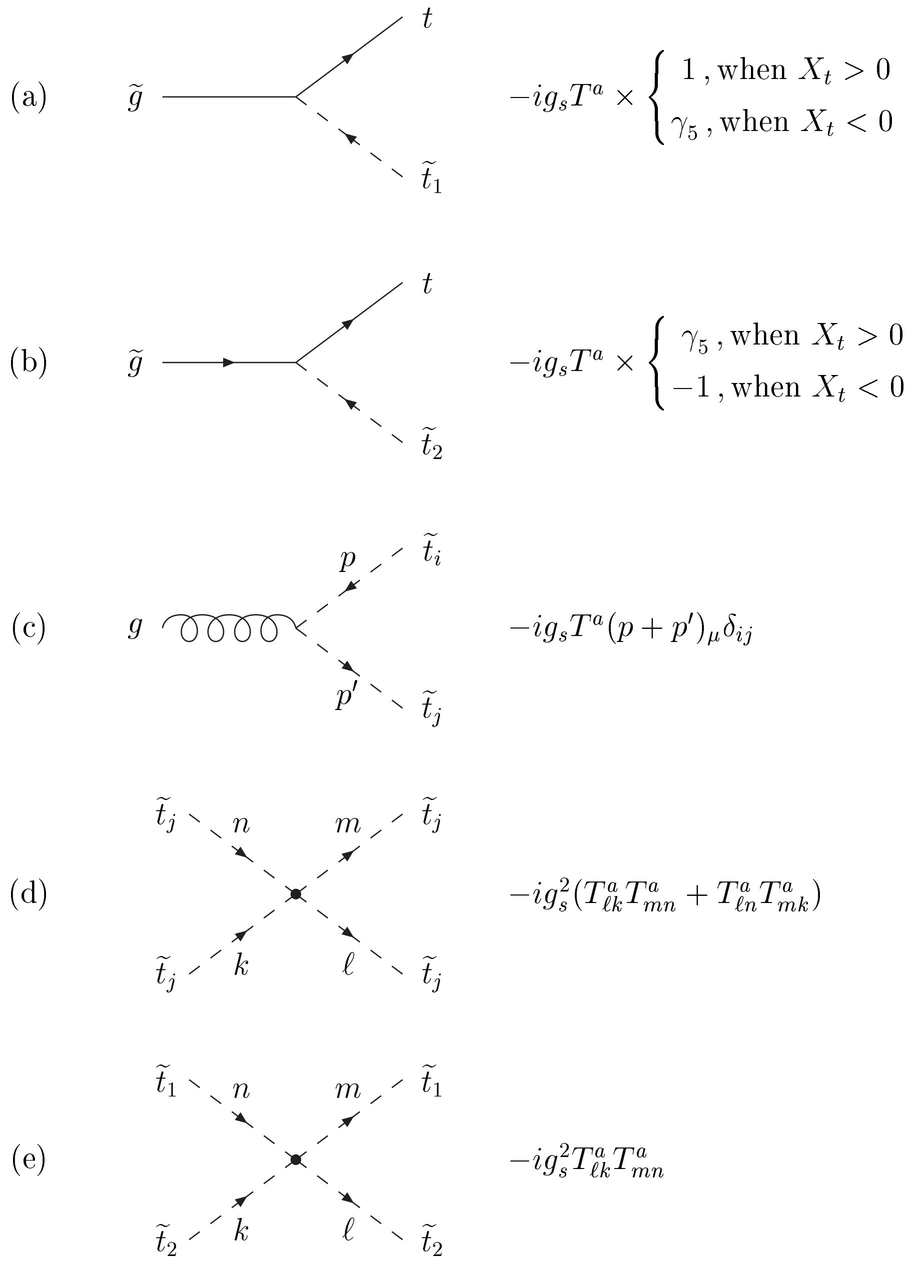

Figure 3. Feynman rules for top-squark interactions in a model where the stop mass-squared matrix is given by eq. (6).

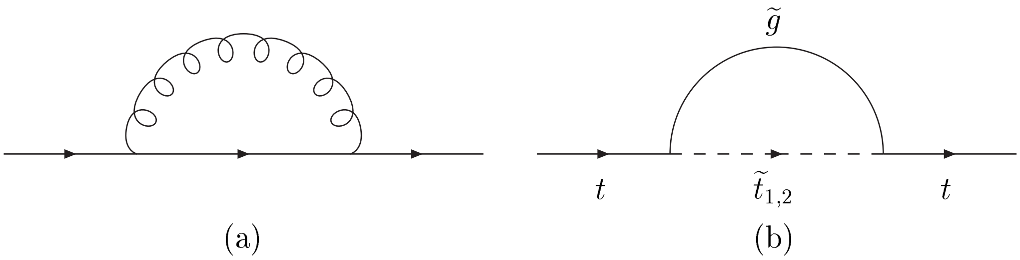

Figure 4. One-loop contributions to the top quark mass.

sentation, $C_F = 4/3$, and

$$\mathcal{B}_n(p^2; m_1^2, m_2^2) \equiv (-1)^{n+1} \int_0^1 dy\, y^n \ln\left(\frac{m_2^2 y + m_1^2(1-y) - p^2 y(1-y)}{\mu^2}\right). \quad \text{(A.2)}$$

The $\mathcal{B}_n$ (n = 0,1) are related to the standard two-point loop functions that arise in one-loop computations [28]:

$$\begin{aligned} B_0(p^2; m_1^2, m_2^2) &\equiv \Delta + \mathcal{B}_0(p^2; m_1^2, m_2^2)\,, \\ B_1(p^2; m_1^2, m_2^2) &\equiv -\tfrac{1}{2}\Delta + \mathcal{B}_1(p^2; m_1^2, m_2^2)\,, \end{aligned} \quad \text{(A.3)}$$

where all occurrences of $\Delta \equiv (4\pi)^\epsilon \Gamma(\epsilon)$ are removed in the minimal subtraction procedure. Note that $B_0(p^2; m_1^2, m_2^2)$ is invariant under the interchange of m_1^2 and m_2^2, whereas

$$B_1(p^2; m_2^2, m_1^2) = -B_1(p^2; m_1^2, m_2^2) - B_0(p^2; m_1^2, m_2^2)\,. \quad \text{(A.4)}$$

In eq. (A.1), μ is the arbitrary mass parameter of the $\overline{\text{MS}}$–scheme. The on-shell top-quark mass, M_t, is defined by $\Gamma^{(2)}(\not{p} = M_t) = 0$. It follows that

$$\begin{aligned} M_t = \overline{m}_t(\mu) + \frac{C_F \alpha_s m_t}{4\pi} \Bigg\{ & 4\mathcal{B}_0(m_t^2; m_t^2, 0) + 2\mathcal{B}_1(m_t^2; m_t^2, 0) - 1 \\ & + \mathcal{B}_1(m_t^2; m_{\tilde{g}}^2, M_S^2 - m_t X_t) + \mathcal{B}_1(m_t^2; m_{\tilde{g}}^2, M_S^2 + m_t X_t) \\ & - \frac{m_{\tilde{g}}}{m_t} \Big[\mathcal{B}_0(m_t^2; m_{\tilde{g}}^2, M_S^2 - m_t X_t) - \mathcal{B}_0(m_t^2; m_{\tilde{g}}^2, M_S^2 + m_t X_t)\Big] \Bigg\}. \end{aligned} \quad \text{(A.5)}$$

In the $\mathcal{O}(\alpha_s)$ terms above, we simply use the generic notation m_t for the top-quark mass, since to one-loop accuracy one need not distinguish between $\alpha_s M_t$ and $\alpha_s m_t(\mu)$. As previously noted, the relation between the top-quark mass in the on-shell and $\overline{\text{DR}}$ schemes is obtained by dropping the -1 (which does not multiply a loop-function) in eq. (A.5).

Two of the loop functions are easily evaluated: $\mathcal{B}_0(m_t^2; m_t^2, 0) = 2 - \ln(m_t^2/\mu^2)$ and $\mathcal{B}_1(m_t^2; m_t^2, 0) = -\frac{1}{2}[3 - \ln(m_t^2/\mu^2)]$. We note the following curious fact. In the supersymmetric limit ($m_{\tilde{t}_1} = m_{\tilde{t}_2} = m_t$ and $m_{\tilde{g}} = 0$) at one-loop, the relation between the on-shell and $\overline{\text{DR}}$ running top-quark mass evaluated at $\mu = m_t$ is precisely the same as the corresponding relation between the on-shell and $\overline{\text{MS}}$ running top-quark mass in non-supersymmetric QCD.

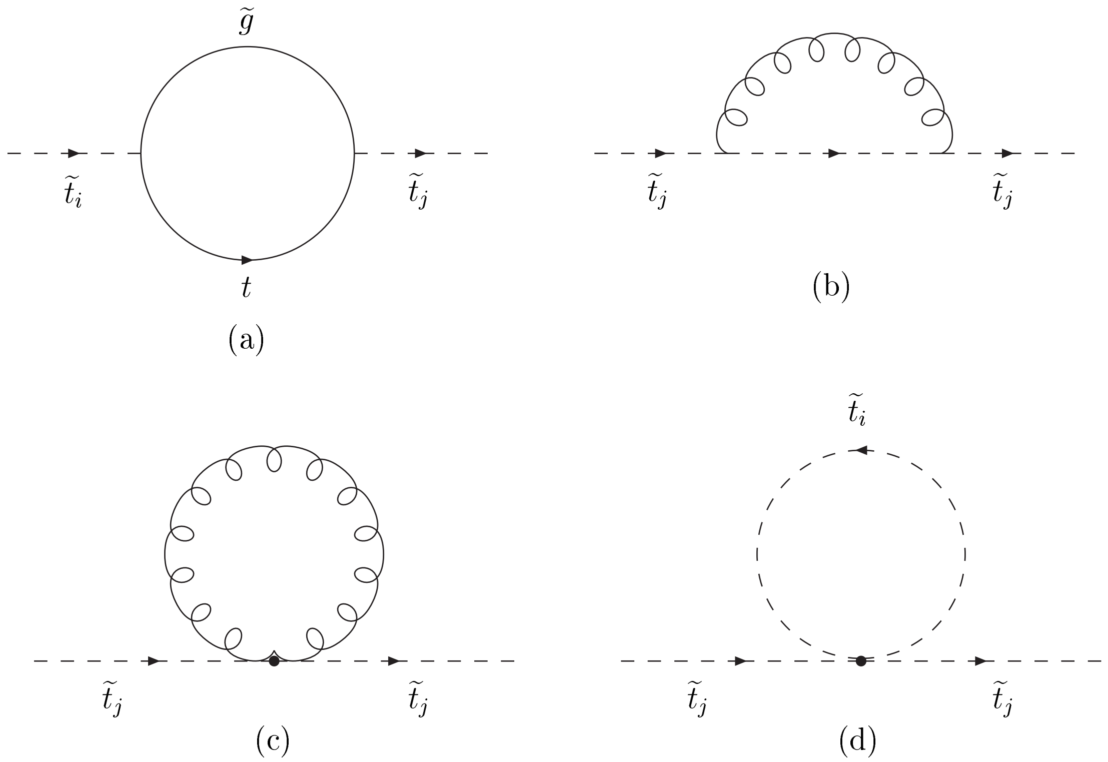

Figure 5. One-loop contributions to the top squark mass.

Next, we examine the one-loop contributions to the top-squark two-point function at $\mathcal{O}(\alpha_s)$. The contributing graphs are shown in fig. 5. We immediately note that graph (c) of fig. 5 vanishes in dimensional regularization. Moreover, since $\tilde{t}_1$ and $\tilde{t}_2$ are states of definite parity in a model where the stop mass-squared matrix is given by eq. (6), the one-loop mixing of $\tilde{t}_1$ and $\tilde{t}_2$ vanishes to all orders in α_s. Including the tree-level contribution, the final result for the top squark two-point function is[12]

$$\begin{aligned}\tilde{\Gamma}^{(2)}_{jj}(p^2) = i(p^2 - m^2_{\tilde{t}_j}) - \frac{iC_F\alpha_s}{\pi}\Big[\mathcal{A}_0(m^2_{\tilde{g}}) + m_t^2\mathcal{B}_0(p^2; m_t^2, m^2_{\tilde{g}}) \\ +p^2\mathcal{B}_1(p^2; m_t^2, m^2_{\tilde{g}}) - (-1)^j m_{\tilde{g}} m_t \mathcal{B}_0(p^2; m_t^2, m^2_{\tilde{g}}) - p^2\mathcal{B}_1(p^2; m^2_{\tilde{t}_j}, 0)\Big]\,, \qquad \text{(A.6)}\end{aligned}$$

for $j = 1$, 2, where

$$\mathcal{A}_0(m^2) \equiv m^2\left[1 - \ln\left(\frac{m^2}{\mu^2}\right)\right] \qquad \text{(A.7)}$$

is related to the standard one-point loop function $A_0(m^2) = m^2\Delta + \mathcal{A}_0(m^2)$.

There is no distinction in this calculation between the $\overline{\rm MS}$ and $\overline{\rm DR}$ schemes. The on-shell stop squared-masses are defined by $\tilde{\Gamma}^{(2)}_{jj}(p^2 = (m^{\rm OS}_{\tilde{t}_j})^2) = 0$. Noting the form for the tree-level squared-masses [eq. (7)], it follows that:

$$M_S^{2,\rm OS} \mp M_t X_t^{\rm OS} = \overline{M}^2_S(\mu) \mp \overline{m}_t(\mu)\overline{X}_t(\mu) + \frac{C_F\alpha_s}{\pi}\left[f(M_S^2 \mp m_t X_t) \pm g(M_S^2 \mp m_t X_t)\right]\,, \qquad \text{(A.8)}$$

where M_t is the on-shell top quark mass and

$$\begin{aligned}f(p^2) &\equiv \mathcal{A}_0(m^2_{\tilde{g}}) + m_t^2\mathcal{B}_0(p^2; m_t^2, m^2_{\tilde{g}}) + p^2\mathcal{B}_1(p^2; m_t^2, m^2_{\tilde{g}})\,, \\ g(p^2) &\equiv m_{\tilde{g}} m_t \mathcal{B}_0(p^2; m_t^2, m^2_{\tilde{g}})\,. \qquad \text{(A.9)}\end{aligned}$$

It is then straightforward to solve for $M_S^{2,\rm OS}$ and $M_t X_t^{\rm OS}$ in terms of the corresponding $\overline{\rm MS}$ quantities evaluated at the scale $\mu = M_S$. Using the notation of eq. (25), we obtain

$$\begin{aligned}M_S^{2,\rm OS} &= \overline{M}^2_S + \frac{C_F\alpha_s}{2\pi}\Big[f(M_S^2 + m_t X_t) + f(M_S^2 - m_t X_t) \\ &\qquad -g(M_S^2 + m_t X_t) + g(M_S^2 + m_t X_t)\Big]\,, \qquad \text{(A.10)} \\ M_t X_t^{\rm OS} &= \overline{m}_t(M_S)\overline{X}_t + \frac{C_F\alpha_s}{2\pi}\Big[f(M_S^2 + m_t X_t) - f(M_S^2 - m_t X_t) \\ &\qquad -g(M_S^2 + m_t X_t) - g(M_S^2 - m_t X_t)\Big]\,, \qquad \text{(A.11)}\end{aligned}$$

where all loop functions in eqs. (A.10) and (A.11) are evaluated at $\mu = M_S$. Dividing eq. (A.11) by M_t, and using eq. (A.5) to evaluate $\overline{m}_t(M_S)/M_t$, one obtains a direct relation between $X_t^{\rm OS}$ and $\overline{X}_t$.

To obtain the expansions derived in Appendix B, we consider the case of $m^2_{\tilde{g}} = M^2_{\rm SUSY} = M_S^2 - m_t^2$. We introduce the following notation:

$$x_t \equiv \frac{X_t}{M_S}\,, \qquad z \equiv \frac{M_t}{M_S}\,. \qquad \text{(A.12)}$$

[12] In deriving eq. (A.6), we note that the contribution of fig. 5(d) [only the case $i = j$ yields a non-zero contribution, which is equal to $(iC_F\alpha_s/4\pi)\mathcal{A}_0(m_{\tilde{t}_j})$] cancels a similar term that arises in the evaluation of fig. 5(b).

We are interested in the limit of $z \ll 1$ and $x_t \lesssim 1$. First, consider the relation between the on-shell and running top quark mass. Using eq. (A.2), we must evaluate the following integrals:

$$J_n^{(\pm)} = \int_0^1 dy\, y^n \ln\left[1 \pm x_t z y - z^2(1-y^2)\right], \tag{A.13}$$

for $n = 0$, 1. Eq. (A.5) then yields:

$$\overline{m}_t(M_S) = M_t\left\{1 + \frac{C_F\alpha_s}{4\pi}\left[-4 + 6\ln z - J_1^{(+)} - J_1^{(-)} + \frac{1}{z}(1-z^2)^{1/2}(J_0^{(+)} - J_0^{(-)})\right]\right\}. \tag{A.14}$$

Expanding out the logarithm in the integrand of $J_n^{(\pm)}$ in a double power series in x_t and z and integrating term by term, one readily obtains the result given in eq. (B.5).

Second, consider the relation between the on-shell and $\overline{\rm MS}$ definitions of M_S and X_t obtained in eq. (A.10). Using the integral expressions for the loop functions that appear in eq. (A.9), one must evaluate the following integrals:

$$I_n = \int_0^1 dy\, y^n \ln\left[y^2(1-zx_t) + yz(x_t - 2z) + z^2\right], \tag{A.15}$$

for $n = 0$, 1. In this case, one cannot simply expand the logarithms about $x_t = z = 0$, since the integration range extends down to $y = 0$. Instead, we have used *Mathematica* to evaluate the integral exactly, and then perform the double expansion in x_t and z. The result for I_0 up to $\mathcal{O}(x_t^2 z^4)$ is

$$\begin{aligned} I_0 = {} & -2 + \pi z + z^2(2\ln z - 1) - \tfrac{1}{2}\pi z^3 + \tfrac{1}{2}z^4 \\ & + x_t\left[-z - z\ln z + \pi z^2 + z^3(2\ln z - \tfrac{1}{2}) - \tfrac{1}{2}\pi z^4\right] \\ & + x_t^2\left[-\tfrac{1}{8}\pi z - z^2(\ln z + 1) + \tfrac{15}{16}\pi z^3 + z^4(2\ln z - \tfrac{1}{2})\right] \\ & + x_t^3\left[\tfrac{1}{12}z - \tfrac{1}{8}\pi z^2 - z^3(\ln z + \tfrac{11}{12}) + \tfrac{15}{16}\pi z^4\right] \\ & + x_t^4\left[-\tfrac{1}{128}\pi z + \tfrac{1}{12}z^2 - \tfrac{35}{256}\pi z^3 - z^4(\ln z + \tfrac{11}{12})\right]. \end{aligned} \tag{A.16}$$

We have checked the validity of this expansion using numerical integration. One can derive a similar expression for I_1 either directly, or by noting that:

$$\begin{aligned} I_1 = {} & \frac{1}{2(1-zx_t)}\left[z^2(1-2\ln z) - (1-z^2)[1-\ln(1-z^2)] - z(x_t-2z)I_0\right] \\ = {} & -\tfrac{1}{2} - z^2(\ln z + \tfrac{3}{2}) + \pi z^3 + z^4(2\ln z - \tfrac{3}{4}) \\ & + x_t\left[\tfrac{1}{2}z - \tfrac{1}{2}\pi z^2 - z^3(3\ln z + 2) + \tfrac{9}{4}\pi z^4\right] \\ & + x_t^2\left[\tfrac{1}{2}z^2(\ln z + 2) - \tfrac{9}{8}\pi z^3 - z^4(5\ln z + \tfrac{11}{4})\right] \\ & + x_t^3\left[\tfrac{1}{16}\pi z^2 + z^3(\ln z + \tfrac{19}{12}) - \tfrac{55}{32}\pi z^4\right] \\ & + x_t^4\left[-\tfrac{1}{24}z^2 + \tfrac{15}{128}\pi z^3 + z^4(\tfrac{3}{2}\ln z + \tfrac{17}{8})\right]. \end{aligned} \tag{A.17}$$

Inserting these results into eqs. (A.10) and (A.11), and expanding out the remaining factors [*e.g.*, $m_{\tilde g} m_t = M_S^2 z(1-z^2)^{1/2}$, *etc.*], one ends up with the results given in eqs. (B.3) and (B.4).

Appendix B: Results up to $\mathcal{O}\left(m_t^4/M_S^4\right)$

We list the necessary formulae in order to derive the results given in sections 3 and 4 up to terms of $\mathcal{O}\left(m_t^4/M_S^4\right)$. As before, we define $x_t \equiv X_t/M_S$ and $z \equiv m_t/M_S$. In the approximation discussed in section 3, we obtain expansions that are valid in the limit of $z \ll 1$ and $x_t \lesssim 1$.

The diagrammatic result in the on-shell scheme for the one-loop and two-loop contributions to m_h^2, in the approximations used in this paper, is given up to $\mathcal{O}(z^4)$ by

$$m_h^{2,\alpha} = \frac{3}{2}\frac{G_F\sqrt{2}}{\pi^2}M_t^4\left\{-2\ln z + x_t^2\left[1-\tfrac{1}{12}x_t^2 - z^2(\tfrac{1}{2}-\tfrac{1}{3}x_t^2) - \tfrac{1}{4}z^4x_t^2\right]\right\} \tag{B.1}$$

$$\begin{aligned}
m_h^{2,\alpha\alpha_s} = -3\frac{G_F\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}M_t^4\Bigg\{ & 4\ln^2 z + \Bigg[-4 - x_t^2 + \tfrac{1}{9}z^2\left(6+42x_t+33x_t^2-26x_t^3-18x_t^4\right) \\
& \qquad + \tfrac{1}{9}z^4\left(2-41x_t-10x_t^2+91x_t^3+45x_t^4\right)\Bigg]\ln z \\
& - \tfrac{1}{4}x_t(8-x_t^3) + \tfrac{1}{36}\pi z x_t\left(48+24x_t-14x_t^2-7x_t^3\right) \\
& + \tfrac{1}{36}z^2x_t\left(60-6x_t-106x_t^2-61x_t^3\right) - \tfrac{1}{72}\pi z^3x_t\left(192+72x_t-236x_t^2-111x_t^3\right) \\
& + \tfrac{1}{36}z^4x_t\left(57+34x_t+95x_t^2+20x_t^3\right)\Bigg\}.
\end{aligned} \tag{B.2}$$

Eq. (B.2) is a generalization of the corresponding formula given in ref. [16].

The relations between the $\overline{\rm MS}$ parameters $\overline{M}_S$ and $\overline{X}_t$ [eq. (25)] and the corresponding on-shell parameters $M_S^{\rm OS}$, $X_t^{\rm OS}$ can be obtained from eqs. (A.10) and (A.11) using the expansions of eqs. (A.16) and (A.17). The end result up to $\mathcal{O}(z^4)$ is given by

$$\begin{aligned}
\overline{M}_S^2 = M_S^{2,\rm OS}\Bigg\{1+\frac{2\alpha_s}{3\pi}\Bigg[& -\tfrac{4}{3} - \tfrac{1}{36}z^2\left(12+24x_t+6x_t^2-2x_t^3-x_t^4\right) \\
& + \tfrac{1}{24}\pi z^3x_t(16+8x_t-2x_t^2-x_t^3) + \tfrac{1}{36}z^4(6-6x_t^2-23x_t^3-10x_t^4) \\
& \left[z^2(2-2x_t-x_t^2) + z^4x_t(5+2x_t-2x_t^3-x_t^3)\right]\ln z\Bigg]\Bigg\},
\end{aligned} \tag{B.3}$$

$$\begin{aligned}
\overline{X}_t = X_t^{\rm OS}\frac{m_t^{\rm OS}}{m_t^{\overline{\rm MS}}(M_S)} + \frac{2\alpha_s}{3\pi}M_S\Bigg\{ & 4 - \tfrac{1}{64}\pi z(128+64x_t-16x_t^2-8x_t^3-x_t^4) \\
& + \tfrac{1}{6}z^2x_t(6+12x_t+4x_t^2-x_t^3) + \tfrac{1}{64}\pi z^3(128+32x_t-128x_t^2-60x_t^3+17x_t^4) \\
& - \tfrac{1}{12}z^4(30+6x_t-6x_t^3-23x_t^4) \\
& + \left[z^2(2+x_t)(-2+x_t^2) + z^4(2-5x_t^2-2x_t^3+2x_t^4)\right]\ln z\Bigg\}.
\end{aligned} \tag{B.4}$$

To complete the evaluation of $\overline{X}_t$ we need to find a relation between the on-shell top-quark mass, M_t, and the running top-quark mass, $\overline{m}_t(M_S)$, evaluated at the scale M_S. Using eq. (A.14), one obtains the following expansion:

$$\begin{aligned}
\overline{m}_t(M_S) = M_t\Bigg\{1+\frac{\alpha_s}{3\pi}\Big[& -4+6\ln z + x_t + z^2\left(\tfrac{1}{2}+\tfrac{1}{4}x_t^2+\tfrac{1}{6}x_t^3+\tfrac{1}{12}x_t^4\right) \\
& + z^4\left(\tfrac{1}{6}-\tfrac{1}{24}x_t+\tfrac{1}{6}x_t^2+\tfrac{1}{12}x_t^3+\tfrac{1}{12}x_t^4+\tfrac{1}{15}x_t^5\right)\Big]\Bigg\}.
\end{aligned} \tag{B.5}$$

Eq. (B.5) was derived using DREG. In order to obtain the corresponding formula using DRED (which yields a formula for the top-quark mass in the $\overline{\rm DR}$ scheme), simply replace -4 with -5 in the first term of eq. (B.5) after the left square bracket.

Note that eq. (B.5) provides a connection between $\overline{m}_t(M_S)$ and the on-shell mass M_t in the full supersymmetric theory. In the limit of large M_S with fixed X_t/M_S, the threshold correction arising from the stop mixing effects does not vanish. On the other hand, the $\overline{\rm MS}$ top-quark mass, $\overline{m}_t \equiv m_{t,\rm SM}^{\overline{\rm MS}}(M_t)$ is defined in the low-energy (non-supersymmetric) effective theory via eq. (3). Thus, in eq. (B.5), we may replace M_t with $\overline{m}_t$ simply by removing the factor of $-4\alpha_s/3\pi$. To leading order in m_t/M_S, one immediately obtains eq. (43).

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