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# Precise Calculations for the Neutral Higgs-Boson Masses in the MSSM [a]

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**Abstract**

We review the comparison of the results for the neutral $\mathcal{CP}$-even Higgs-boson masses recently obtained within the Feynman-diagrammatic (FD) approach with the previous results based on the renormalization group (RG) approach. We show that the results differ by new genuine two-loop contributions present in the FD calculation. The numerical effect of these terms on the result for $m_{\rm h}$ is briefly discussed.

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# PRECISE CALCULATIONS FOR THE NEUTRAL HIGGS-BOSON MASSES IN THE MSSM

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We review the comparison of the results for the neutral $\mathcal{CP}$-even Higgs-boson masses recently obtained within the Feynman-diagrammatic (FD) approach with the previous results based on the renormalization group (RG) approach. We show that the results differ by new genuine two-loop contributions present in the FD calculation. The numerical effect of these terms on the result for $m_{\rm h}$ is briefly discussed.

## 1 Theoretical basis

Within the Minimal Supersymmetric Standard Model (MSSM) the masses of the $\mathcal{CP}$-even neutral Higgs bosons are calculable in terms of the other MSSM parameters. The mass of the lightest Higgs boson, $m_{\rm h}$, has been of particular interest as it is bounded from above at the tree level to be smaller than the Z-boson mass. This bound, however, receives large radiative corrections. The one-loop results [1–3] have been supplemented in the last years with the leading two-loop corrections, performed in the renormalization group (RG) approach [4], in the effective potential approach [5] and most recently in the Feynman-diagrammatic (FD) approach [6]. These calculations predict an upper bound on $m_{\rm h}$ of about $m_{\rm h} \lesssim 135$ GeV.

The dominant radiative corrections to $m_{\rm h}$ arise from the top and scalar top sector of the MSSM, with the input parameters $m_{\rm t}$, $M_{\rm SUSY}$ and $X_{\rm t}$. Here we assume the soft SUSY breaking parameters in the diagonal entries of the scalar top mixing matrix to be equal for simplicity, $M_{\rm SUSY} = M_{\tilde{t}_L} = M_{\tilde{t}_R}$. The off-diagonal entry of the mixing matrix in our conventions (see Ref. [6]) reads $m_{\rm t} X_{\rm t} = m_{\rm t}(A_t - \mu \cot\beta)$. We furthermore use the short-hand notation $M_{\rm S}^2 := M_{\rm SUSY}^2 + m_{\rm t}^2$.

Within the RG approach, $m_{\rm h}$ is calculated from the effective renormalized Higgs quartic coupling at the scale $Q = m_{\rm t}$. The RG improved leading logarithmic approximation is obtained by applying the one-loop RG running of this coupling from the high scale $Q = M_{\rm S}$ to the scale $Q = m_{\rm t}$ and including the one-loop threshold effects from the decoupling of the supersymmetric particles at $M_{\rm S}$ [4]. This approach relies on using the $\overline{\rm MS}$ renormalization scheme. The parameters in terms of which the RG result for $m_{\rm h}$ is expressed are thus $\overline{\rm MS}$ parameters.

In the FD approach, the masses of the $\mathcal{CP}$-even Higgs bosons are determined by the poles of the corresponding propagators. The corrections to the masses $m_{\rm h}$ and $m_{\rm H}$ are thus obtained by evaluating loop corrections to the $h$, $H$ and $hH$-mixing propagators. The poles of the corresponding propagator matrix are given by the

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solutions of

$$\left[q^2 - m^2_{\mathrm{h,tree}} + \hat{\Sigma}_{hh}(q^2)\right] \left[q^2 - m^2_{\mathrm{H,tree}} + \hat{\Sigma}_{HH}(q^2)\right] - \left[\hat{\Sigma}_{hH}(q^2)\right]^2 = 0, \quad (1)$$

where $\hat{\Sigma}_{hh}(q^2)$, $\hat{\Sigma}_{HH}(q^2)$, $\hat{\Sigma}_{hH}(q^2)$ denote the renormalized Higgs-boson self-energies. In Ref. [6] the dominant two-loop contributions to the masses of the $\mathcal{CP}$-even Higgs bosons of $\mathcal{O}(\alpha\alpha_s)$ have been evaluated. These corrections, obtained in the on-shell scheme, have been combined with the complete one-loop on-shell result [3] and the sub-dominant two-loop corrections of $\mathcal{O}(G_\mu^2 m_{\mathrm{t}}^6)$ [4]. The corresponding results have been implemented into the Fortran code *FeynHiggs* [7].

## 2 Leading two-loop contributions in the FD approach

In Ref. [8] the leading contributions have been extracted via a Taylor expansion from the rather complicated diagrammatic two-loop result obtained in Ref. [6] and a compact expression for the dominant contributions has been derived. Restricting to the leading terms in $m_{\mathrm{t}}/M_{\mathrm{S}}$, $M_{\mathrm{Z}}^2/m_{\mathrm{t}}^2$ and $M_{\mathrm{Z}}^2/M_{\mathrm{A}}^2$, the expression up to $\mathcal{O}(\alpha\alpha_s)$ reduces to the simple form

$$m^2_{\mathrm{h,FD}} = m_{\mathrm{h}}^{2,\mathrm{tree}} + \Delta m^{2,\alpha}_{\mathrm{h,FD}} + \Delta m^{2,\alpha\alpha_s}_{\mathrm{h,FD}}, \quad (2)$$

where the one-loop correction is given by

$$\Delta m^{2,\alpha}_{\mathrm{h,FD}} = \frac{3}{2}\frac{G_\mu\sqrt{2}}{\pi^2}\overline{m}_{\mathrm{t}}^4 \left\{ -\ln\left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\right) + \frac{X_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\left(1 - \frac{1}{12}\frac{X_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\right)\right\}. \quad (3)$$

The two-loop contribution reads

$$\Delta m^{2,\alpha\alpha_s}_{\mathrm{h,FD}} = \Delta m^{2,\alpha\alpha_s}_{\mathrm{h,log}} + \Delta m^{2,\alpha\alpha_s}_{\mathrm{h,non-log}}, \quad (4)$$

$$\Delta m^{2,\alpha\alpha_s}_{\mathrm{h,log}} = -\frac{G_\mu\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_{\mathrm{t}}^4 \left[3\ln^2\left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\right) + \ln\left(\frac{\overline{m}_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\right)\left(2 - 3\frac{X_{\mathrm{t}}^2}{M_{\mathrm{S}}^2}\right)\right], \quad (5)$$

$$\Delta m^{2,\alpha\alpha_s}_{\mathrm{h,non-log}} = -\frac{G_\mu\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_{\mathrm{t}}^4 \left[4 - 6\frac{X_{\mathrm{t}}}{M_{\mathrm{S}}} - 8\frac{X_{\mathrm{t}}^2}{M_{\mathrm{S}}^2} + \frac{17}{12}\frac{X_{\mathrm{t}}^4}{M_{\mathrm{S}}^4}\right], \quad (6)$$

in which the leading logarithmic and the non-logarithmic terms have been given separately. The parameter $\overline{m}_{\mathrm{t}}$ in eqs. (3)–(6) denotes the running top-quark mass at the scale $m_{\mathrm{t}}$, which is related to the pole mass $m_{\mathrm{t}}$ in $\mathcal{O}(\alpha_s)$ via

$$\overline{m}_{\mathrm{t}} \equiv \overline{m}_{\mathrm{t}}(m_{\mathrm{t}}) = \frac{m_{\mathrm{t}}}{1 + \frac{4}{3\,\pi}\alpha_s(m_{\mathrm{t}})}, \quad (7)$$

while $M_{\mathrm{S}}$ and $X_{\mathrm{t}}$ are on-shell parameters.

The one-loop correction, eq. (3), as well as the dominant logarithmic two-loop contributions, eq. (5), are seen to be symmetric with respect to the sign of $X_{\mathrm{t}}$. The non-logarithmic two-loop contributions, on the other hand, give rise to an asymmetry in the $X_{\mathrm{t}}$ dependence through the term in eq. (6) which is linear in $X_{\mathrm{t}}/M_{\mathrm{S}}$. The numerical effect of the non-logarithmic two-loop terms is investigated in Fig. 1. The

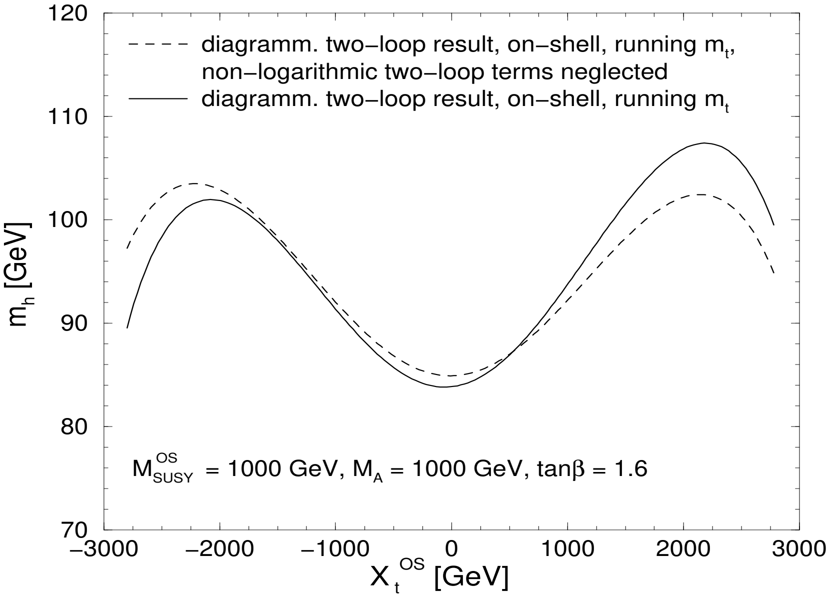


Figure 1: The dominant one-loop and two-loop contributions to $m_{\rm h}$ evaluated in the FD approach are shown as a function of (the on-shell parameter) $X_{\rm t}$ for $\tan\beta = 1.6$. The full curve shows the result including the new genuine two-loop contributions, eq. (6), while the dashed curve shows the result where these non-logarithmic two-loop corrections have been neglected.

result for the dominant contributions to $m_{\rm h}$ of eqs. (2)–(6) is compared to the result where the non-logarithmic contributions of eq. (6) are omitted. The numerical effect of the non-logarithmic genuine two-loop contributions is seen to be sizable. Besides a considerable asymmetry in $X_{\rm t}$ the non-logarithmic two-loop terms in particular lead to an increase in the maximal value of $m_{\rm h}$ of about 5 GeV.

## 3 Comparison between the FD and the RG approach

The results for the dominant contributions derived by FD methods can be compared with the explicit expressions obtained within the RG approach which have been given in Refs. [4]. At the two-loop level, the RG methods applied in Refs. [4] lead to the following result in terms of the $\overline{\rm MS}$ parameters $\overline{m}_{\rm t}$, $\overline{M}_{\rm S}$, $\overline{X}_{\rm t}$

$$\Delta m^{2,\alpha\alpha_s}_{\rm h,RG} = -\frac{G_\mu\sqrt{2}}{\pi^2}\frac{\alpha_s}{\pi}\,\overline{m}_{\rm t}^4\left\{3\ln^2\left(\frac{\overline{m}_{\rm t}^2}{\overline{M}_{\rm S}^2}\right) + \left[2 - 6\frac{\overline{X}_{\rm t}^2}{\overline{M}_{\rm S}^2}\left(1 - \frac{1}{12}\frac{\overline{X}_{\rm t}^2}{\overline{M}_{\rm S}^2}\right)\right]\ln\left(\frac{\overline{m}_{\rm t}^2}{\overline{M}_{\rm S}^2}\right)\right\}, \tag{8}$$

which solely consists of leading logarithmic contributions. The one-loop result for the dominant contributions in the RG approach has the same form as eq. (3), where the parameters $M_{\rm S}$ and $X_{\rm t}$ have to be replaced by $\overline{M}_{\rm S}$ and $\overline{X}_{\rm t}$, respectively.

The one-loop RG-improved effective potential expression eq. (8) does not contain non-logarithmic contributions. In the viewpoint of the RG approach these genuine two-loop contributions are interpreted as two-loop finite threshold corrections to the quartic Higgs couplings.

For a direct comparison of the FD result given in the last section with the RG result of eq. (8), one has to take into account that $M_{\rm S}$ and $X_{\rm t}$ in the FD result are on-shell parameters, while the corresponding parameters in the RG result, $\overline{M}_{\rm S}$ and $\overline{X}_{\rm t}$, are $\overline{\rm MS}$ quantities. The relations between these parameters are given in leading order by [9]

$$\overline{M}_{\rm S}^2 = M_{\rm S}^2 - \frac{8}{3}\frac{\alpha_s}{\pi} M_{\rm S}^2, \qquad \overline{X}_{\rm t} = X_{\rm t} + \frac{\alpha_s}{3\pi} M_{\rm S} \left( 8 + 4\frac{X_{\rm t}}{M_{\rm S}} - 3\frac{X_{\rm t}}{M_{\rm S}} \ln\left(\frac{m_{\rm t}^2}{M_{\rm S}^2}\right)\right). \quad (9)$$

Applying these relations for rewriting the FD result given in eqs. (2)–(6) in terms of the $\overline{\rm MS}$ parameters $\overline{m}_{\rm t}$, $\overline{M}_{\rm S}$, $\overline{X}_{\rm t}$ one finds that the leading logarithmic contributions in the two approaches in fact coincide [9], as it should be as a matter of consistency. The FD result, however, contains further non-logarithmic genuine two-loop contributions which are not present in the RG result. The effect of these extra terms within the $\overline{\rm MS}$ parameterization considered here is qualitatively the same as discussed in the preceding section. They lead to an asymmetry in the dependence of $m_{\rm h}$ on $X_{\rm t}$ and to an increase in the maximal value of $m_{\rm h}$ compared to the RG result.

The analysis above has been performed for the dominant contributions only. Further deviations between the RG and the FD result arise from non-leading one-loop and two-loop contributions, in which the results differ, and from varying the gluino mass, $m_{\tilde{\rm g}}$, in the FD result, which does not appear as a parameter in the RG result. Changing the value of $m_{\tilde{\rm g}}$ in the interval $0 \leq m_{\tilde{\rm g}} \leq 1$ TeV shifts the FD result relative to the RG result within $\pm 2$ GeV [6].


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