Loop Calculations with *FeynArts*, *FormCalc*, and *LoopTools*

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Three programs are presented for automatically generating and calculating Feynman diagrams: the diagrams are generated with *FeynArts*, algebraically simplified with *FormCalc*, and finally evaluated numerically using the *LoopTools* package. The calculations are performed analytically as far as possible, with results given in a form well suited for numerical evaluation. The latter is then straightforward using the implementations of the one-loop integrals in *LoopTools*.

1. Introduction

With the increasing accuracy of experimental data, one-loop calculations have long since become indispensible. Doing such calculations by hand is arduous and error-prone and in some cases simply impossible. So for some time already, software packages have been developed to automate these calculations (*e.g.* [1, 2]). Yet one remaining obstacle is that these packages generally tackle only part of the problem, and there is still considerable work left in making them work together.

In this paper three packages, *FeynArts*, *FormCalc*, and *LoopTools*, are presented which work hand in hand. The user has to supply only small driver programs whose main purpose is to specify the necessary input parameters. This makes the whole system very "open" in the sense that the results are returned as *Mathematica* expressions which can easily be manipulated, *e.g.* to select or modify terms.

FormCalc can work either in dimensional regularization or in constrained differential renormalization [3], the latter of which is equivalent at the one-loop level to regularization by dimensional reduction [4]. This makes *FormCalc* suitable *e.g.* for calculations in supersymmetric models.

Since one-loop calculations can range anywhere from a handful to several hundreds of diagrams (particularly so in models with many particles like the MSSM), speed is an issue, too. *FormCalc*, the program which does the algebraic simplification, therefore uses *FORM* [5] for the time-consuming parts of the calculation. Owing to *FORM*'s speed, *FormCalc* can process, for example, the 1000-odd one-loop diagrams of W–W scattering in the Standard Model [6] in about 5 minutes on an ordinary Pentium PC.

The following table summarizes the steps in a one-loop calculation and the distribution of tasks among the programs *FeynArts*, *FormCalc*, and *LoopTools*:

Diagram generation	• Create the topologies • Insert fields • Apply the Feynman rules • Paint the diagrams	*FeynArts*
↓		
Algebraic simplification	• Contract indices • Calculate traces • Reduce tensor integrals • Introduce abbreviations	*FormCalc*
↓		
Numerical evaluation	• Convert *Mathematica* output to Fortran code • Supply a driver program	*FormCalc*
	• Implementation of the integrals	*LoopTools*

The following sections describe the main functions of each program. Furthermore, the *FormCalc* package contains two sample calculations in the electroweak Standard Model, $ZZ \to ZZ$ [7] and $e^+e^- \to \bar{t}\,t$ [8], which demonstrate how the programs are used together.

2. *FeynArts*

FeynArts is a *Mathematica* package for the generation and visualization of Feynman diagrams and amplitudes [9]. It works in the three basic steps sketched in Fig. 1.

The first step is to create all different topologies for a given number of loops and external legs. For example, to create all one-loop topologies for a $1 \to 2$ process, the following call to `CreateTopologies` is used:

```
top = CreateTopologies[1, 1 -> 2]
```

In the second step, the actual particles in the model have to be distributed over the topologies in all allowed ways. E.g. the diagrams for $Z \to b\bar{b}$ are produced with

```
ins = InsertFields[top, V[2] -> {F[4,{3}], -F[4,{3}]}]
```

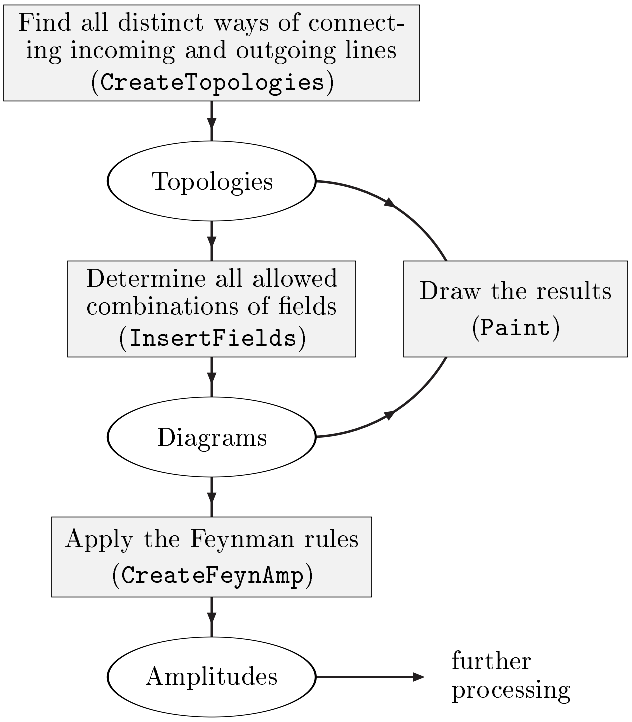


Fig. 1. Flowchart for the generation of Feynman amplitudes with *FeynArts*.

where `F[4,{3}]` is the b-quark, `-F[4,{3}]` its antiparticle, and `V[2]` the Z boson. The fields and their couplings are defined in a special file, the model file, which the user can supply or modify. Model files are currently provided for QED, the electroweak Standard Model, and QCD; a MSSM model file is in preparation.

The diagrams can be drawn with `Paint[ins]`, depending on the options either on screen, or in a PostScript or LaTeX file. Finally, the analytic expressions for the diagrams are obtained by

```
amp = CreateFeynAmp[ins]
```

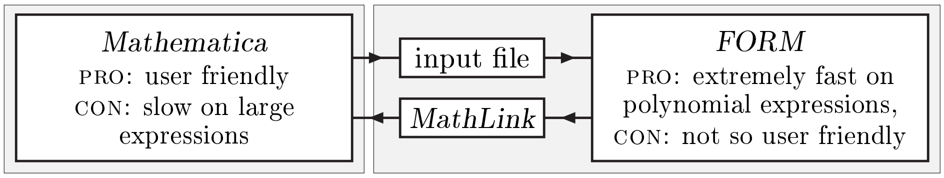
3. *FormCalc*

The evaluation of the *FeynArts* output proceeds in two steps:

1. The symbolic expressions for the diagrams are simplified algebraically with *FormCalc* which returns the results in a form well suited for numerical evaluation.

2. The *Mathematica* expressions then need to be translated into a Fortran program. (The numerical evaluation could, in principle, be done in *Mathematica* directly, but this becomes very slow for large amplitudes.) The translation is done by the program *NumPrep* which is part of the *FormCalc* package. For compiling the generated code one needs a driver program (also in *FormCalc*), and the numerical implementations of the one-loop integrals in *LoopTools*.

The structure of *FormCalc* is simple: it prepares the symbolic expressions of the diagrams in an input file for *FORM*, runs *FORM*, and retrieves the results. This interaction is transparent to the user. *FormCalc* combines the speed of *FORM* with the powerful instruction set of *Mathematica* and the latter greatly facilitates further processing of the results. The following diagram shows schematically how *FormCalc* interacts with *FORM*:



user interface *internal FormCalc functions*

The main function in *FormCalc* is `OneLoop` (the name is not strictly correct since it works also with tree graphs). It is used like this:

```
<< FormCalc`
amps = << myamps.m     (* load some amplitudes *)
result = OneLoop[amps]
```

where it is assumed that the file `myamps.m` contains amplitudes generated by *FeynArts*. `OneLoop` uses dimensional regularization by default. To calculate in constrained differential renormalization ($\equiv$ dimensional reduction at the one-loop level), one has to put `$Dimension = 4` before invoking `OneLoop`. Note that `OneLoop` needs no declarations of the kinematics of the underlying process; it uses the information *FeynArts* hands down.

Even more comprehensive than `OneLoop`, the function `ProcessFile` can process entire files. It collects the diagrams into blocks such that index summations (*e.g.* over fermion generations) can later be carried out easily, *i.e.* only diagrams which are summed over the same indices are put in one block. `ProcessFile` is invoked *e.g.* as

```
ProcessFile["vertex.amp", "results/vertex"]
```

which reads the *FeynArts* amplitudes from `vertex.amp` and produces files of the form `results/vertex`*id*`.m`, where *id* is an identifier for a particular block.

`OneLoop` and `ProcessFile` return expressions where spinor chains, dot products of vectors, and Levi-Civita tensors contracted with vectors have been collected and abbreviated. A term in such an expression may look like

```
C0i[cc1, MW2, S, MW2, MZ2, MW2, MW2] *
  ( AbbSum12*(-8*a2*MW2 + 4*a2*MW2*S2 - 2*a2*CW^2*MW2*S2 +
      16*a2*CW^2*S*S2 + 4*a2*C2*MW2*SW^2) +
    Abb47*(-32*a2*CW^2*MW2*S2 + 8*a2*CW^2*S2*T +
      8*a2*CW^2*S2*U) -
    AbbSum13*(-64*a2*CW^2*MW2*S2 + 16*a2*CW^2*S2*T +
      16*a2*CW^2*S2*U) )
```

Here, the tensor coefficient function $C_1(M_{\mathrm{W}}^2, s, M_{\mathrm{W}}^2, M_{\mathrm{Z}}^2, M_{\mathrm{W}}^2, M_{\mathrm{W}}^2)$ is multiplied with a linear combination of abbreviations like `Abb47` or `AbbSum12` with certain coefficients. These coefficients contain the Mandelstam variables `S`, `T`, and `U` and some short-hands for parameters of the Standard Model, *e.g.* `a2` $= \alpha^2$.

The abbreviations like `Abb47` or `AbbSum12` are introduced automatically and can significantly reduce the size of an amplitude. The definitions of the abbreviations can be retrieved by `Abbreviations[]` which returns a list of rules such that `result //. Abbreviations[]` gives the full, unabbreviated expression.

4. *LoopTools*

LoopTools supplies the actual numerical implementations of the one-loop functions needed for programs made from the *FormCalc* output. It is based on the reliable package *FF* [10] and provides in addition to the scalar integrals of *FF* also the tensor coefficients in the conventions of [11]. *LoopTools* offers three interfaces: Fortran, C++, and *Mathematica*, so most programming tastes should be served.

Using *LoopTools* functions in Fortran and C++ is very similar. In Fortran it is necessary to include the two files `tools.F` and `tools.h`, the latter one in every function or subroutine. In C++, `ctools.h` must be included once. Before using any *LoopTools* function, `bcaini` must be called and at the end of the calculation `bcaexi` may be called to obtain a summary of errors. It is of course possible to change parameters like the scale μ from dimensional regularization; this is described in detail in the manual [12].

A very simple program would for instance be

Fortran
```
#include "tools.F"

    program simple
#include "tools.h"
    call bcaini
    print *, B0(1000D0,50D0,80D0)
    call bcaexi
    end
```

C++
```
#include "ctools.h"

main()
{
 bcaini();
 cout << B0(1000.,50.,80.) << "\n";
 bcaexi();
}
```

The *Mathematica* interface is even simpler to use:

```
In[1]:= Install["bca"]

In[2]:= B0[1000, 50, 80]

Out[2]= -4.40593 + 2.70414 I
```

5. Requirements and Availability

All three packages require *Mathematica* 2.2 or above; *FormCalc* needs in addition *FORM*, preferably version 2 or above; *LoopTools* needs a Fortran compiler, `gcc/g++`, and GNU make.

The packages should compile and run without change on any Unix platform. They are specifically known to work under DEC Unix, HP-UX, Linux, Solaris, and AIX. All three packages are open source and stand under the GNU library general public license. They are available from

```
http://www-itp.physik.uni-karlsruhe.de/feynarts
http://www-itp.physik.uni-karlsruhe.de/formcalc
http://www-itp.physik.uni-karlsruhe.de/looptools
```

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