Accurate Prediction for the Mass of the Lightest Higgs Boson in the MSSM

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Abstract. The mass of the lightest Higgs boson, m_h, in the Minimal Supersymmetric Standard Model (MSSM) is predicted on the basis of explicit Feynman-diagrammatic calculations. The results, containing the complete diagrammatic one-loop corrections, the leading two-loop corrections of $\mathcal{O}(\alpha\alpha_s)$ and further improvements taking into account leading electroweak two-loop and higher-order QCD contributions, are discussed and compared with results obtained by renormalization group calculations. Good agreement is found in the case of vanishing mixing in the scalar top sector, while sizable deviations occur if scalar top mixing is taken into account. By means of a Taylor expansion a compact approximation formula for m_h is derived. The quality of the approximation in comparison with the full result is analyzed.

The search for the lightest Higgs boson provides a direct and very stringent test of Supersymmetry (SUSY) and is one of the main goals at the upgraded Tevatron. A precise prediction of its mass, m_h, is inevitable for determining the discovery and exclusion potential of the Tevatron in this search and for analyzing the accessible MSSM parameter space.

The mass of the lightest Higgs boson is restricted at the tree level to be smaller than the Z-boson mass M_Z. The dominant one-loop corrections arise from the top and scalar-top sector via terms of the form $G_F m_t^4 \ln(m_{\tilde{t}_1} m_{\tilde{t}_2}/m_t^2)$ [1]. These results have been improved by performing a complete one-loop calculation in the on-shell scheme [2–4]. Beyond one-loop order renormalization group (RG) methods have been applied in order to obtain leading logarithmic higher-order contributions [5–9]. Furthermore the leading two-loop QCD corrections have been calculated in the effective potential method [10].

Recently we performed a Feynman-diagrammatic calculation of the leading two-loop corrections of $\mathcal{O}(\alpha\alpha_s)$ to the masses of the neutral $\mathcal{CP}$-even Higgs bosons [11–13]. Compared to the leading one-loop result the two-loop contribution was found to give rise to a considerable reduction of the m_h value. We have combined the full diagrammatic one-loop on-shell result [3] with the leading diagrammatic two-loop result [11] and further corrections concerning the leading two-loop Yukawa corrections of $\mathcal{O}(G_F^2 m_t^6)$ [7,14], and the leading QCD corrections beyond two-loop order. In this way we provide the currently most precise prediction for m_h within the Feynman-diagrammatic approach for arbitrary values of the parameters of the Higgs and scalar top sector of the MSSM [15]. The resulting predictions for m_h are discussed in view of the discovery potential of the upgraded Tevatron and are compared with the results obtained using the RG approach [7–9]. In addition, we derive by means of a Taylor expansion a very compact formula for m_h. It contains in particular a simple expression for the leading logarithmic and non-logarithmic two-loop contributions for general mixing in the $\tilde{t}$-sector. The quality of the approximation formula in comparison to the full result is briefly discussed.

The MSSM Higgs sector can be described with the help of two parameters: $\tan\beta = v_2/v_1$, the ratio of the

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two vacuum expectation values, and M_A, the mass of the $\mathcal{CP}$-odd Higgs boson. The tree-level predictions for the masses m_h and m_H of the neutral $\mathcal{CP}$-even Higgs bosons h and H are derived by diagonalizing the tree-level mass matrix given in terms of the current eigenstates ϕ_1 and ϕ_2 with the help of the rotation

$$\begin{pmatrix} H^0 \\ h^0 \end{pmatrix} = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \phi_1^0 \\ \phi_2^0 \end{pmatrix}. \tag{1}$$

In the Feynman-diagrammatic approach the higher-order corrected Higgs-boson masses are derived by determining the poles of the h, H-propagator matrix whose inverse is given by

$$(\Delta_{\rm Higgs})^{-1} = -i \begin{pmatrix} q^2 - m^2_{H,\rm tree} + \hat{\Sigma}_H(q^2) & \hat{\Sigma}_{hH}(q^2) \\ \hat{\Sigma}_{hH}(q^2) & q^2 - m^2_{h,\rm tree} + \hat{\Sigma}_h(q^2) \end{pmatrix}, \tag{2}$$

where the $\hat{\Sigma}$ denote the renormalized Higgs-boson self-energies, which can be decomposed according to

$$\hat{\Sigma}_s = \hat{\Sigma}_s^{(1)} + \hat{\Sigma}_s^{(2)} + \ldots, \quad s = h, H, hH, \tag{3}$$

into the contributions at one-loop, two-loop order etc.

For the one-loop contribution, $\hat{\Sigma}_s^{(1)}(q^2)$, to these self-energies, we take the result of the complete one-loop on-shell calculation of Ref. [3]. The agreement with the result obtained in Ref. [2] is better than 1 GeV for almost the whole MSSM parameter space.

The leading two-loop corrections, $\hat{\Sigma}_s^{(2)}(0)$, have been obtained in Refs. [11–13] by calculating the $\mathcal{O}(\alpha\alpha_s)$ contribution of the $t, \tilde{t}$-sector to the renormalized Higgs-boson self-energies at zero external momentum from the Yukawa part of the theory.

In the $t, \tilde{t}$-sector the current eigenstates of the scalar quarks, $\tilde{t}_L$ and $\tilde{t}_R$, mix to give the mass eigenstates $\tilde{t}_1$ and $\tilde{t}_2$. The non-diagonal entry in the scalar quark mass matrix is proportional to the mass of the quark and reads for the $\tilde{t}$-mass matrix

$$m_t M_t^{LR} = m_t (A_t - \mu \cot\beta\,), \tag{4}$$

where we have adopted the conventions as in Ref. [13]. Due to the large value of m_t these mixing effects have to be taken into account. Diagonalizing the $\tilde{t}$-mass matrix one obtains the eigenvalues $m_{\tilde{t}_1}$ and $m_{\tilde{t}_2}$ and the $\tilde{t}$-mixing angle $\theta_{\tilde{t}}$.

We have implemented two further corrections beyond $\mathcal{O}(\alpha\alpha_s)$ into the prediction for m_h: The first correction concerns leading QCD corrections beyond two-loop order, taken into account by using the $\overline{\rm MS}$ top-quark mass

$$\overline{m}_t = \overline{m}_t(m_t) \approx \frac{m_t}{1 + \frac{4}{3\pi}\alpha_s(m_t)} \tag{5}$$

for the two-loop contributions instead of the pole mass m_t. The second one is the leading two-loop Yukawa correction of $\mathcal{O}(G_F^2 m_t^6)$, taken over from the result obtained by RG methods [7,14], see eq. (12) below.

For the numerical evaluation we have chosen two values for $\tan\beta$, which are favored by GUT scenarios [16]: $\tan\beta = 1.6$ for the $SU(5)$ scenario and $\tan\beta = 40$ for the $SO(10)$ scenario. Other parameters are $M_Z = 91.187$ GeV, $M_W = 80.39$ GeV, $G_F = 1.16639\,10^{-5}$ GeV^{-2}, $\alpha_s(m_t) = 0.1095$, and $m_t = 175$ GeV, if not otherwise indicated. Further parameters are the mass of the $\mathcal{CP}$-odd Higgs boson M_A, the mass of the gluino $m_{\tilde{g}}$, the Higgs-mixing parameter μ and the $SU(2)$ soft SUSY-breaking parameter $M(\equiv M_2)$. The other gaugino mass parameter, M_1, is fixed via the GUT relation $M_1 = (5\,s_W^2)/(3\,c_W^2)\,M$. The $\tilde{t}$-masses and the mixing angle are derived from the parameters $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$ and M_t^{LR} of the $\tilde{t}$-mass matrix (our conventions are the same as in Ref. [13]). In the figures below we have chosen $m_{\tilde{q}} \equiv M_{\tilde{t}_L} = M_{\tilde{t}_R}$.

Fig. 1 shows the result for m_h obtained from the diagrammatic calculation of the full one-loop and leading two-loop contributions. In the left plot m_h is given as a function of $M_t^{LR}/m_{\tilde{q}}$, where $m_{\tilde{q}}$ is fixed to 500 GeV, in the right plot it is shown as a function of $m_{\tilde{q}}$ for no mixing and maximal mixing in the scenario with $\tan\beta = 1.6$. The two-loop contributions give rise to a large reduction of the one-loop on-shell result by up

to 20 GeV. As can be seen from the left plot, a minimum occurs around $M_t^{LR}/m_{\tilde{q}} = 0$, which corresponds to the case of no mixing in the $\tilde{t}$-sector. A maximum in the two-loop result for m_h is reached for about $|M_t^{LR}/m_{\tilde{q}}| \approx 2$, this case we refer to as 'maximal mixing'. In the two-loop result the maxima are shifted compared to their one-loop values of about $|M_t^{LR}/m_{\tilde{q}}| \approx 2.4$. Varying $\tan\beta$ around the value $\tan\beta = 1.6$ leads to a relatively large effect in m_h (higher values for m_h are obtained for larger $\tan\beta$), while the effect of varying $\tan\beta$ around $\tan\beta = 40$ is marginal. Different values of the gluino mass, $m_{\tilde{g}}$, in the two-loop contribution affect the prediction for m_h by up to ± 2 GeV in the maximal-mixing scenario, while the effect is negligible in the no-mixing scenario. Varying M, which enters via the non-leading one-loop contributions, changes the value of m_h by ± 1.5 GeV. A more detailed analysis of the dependence of our results on the different SUSY parameters has been performed in Ref. [13].

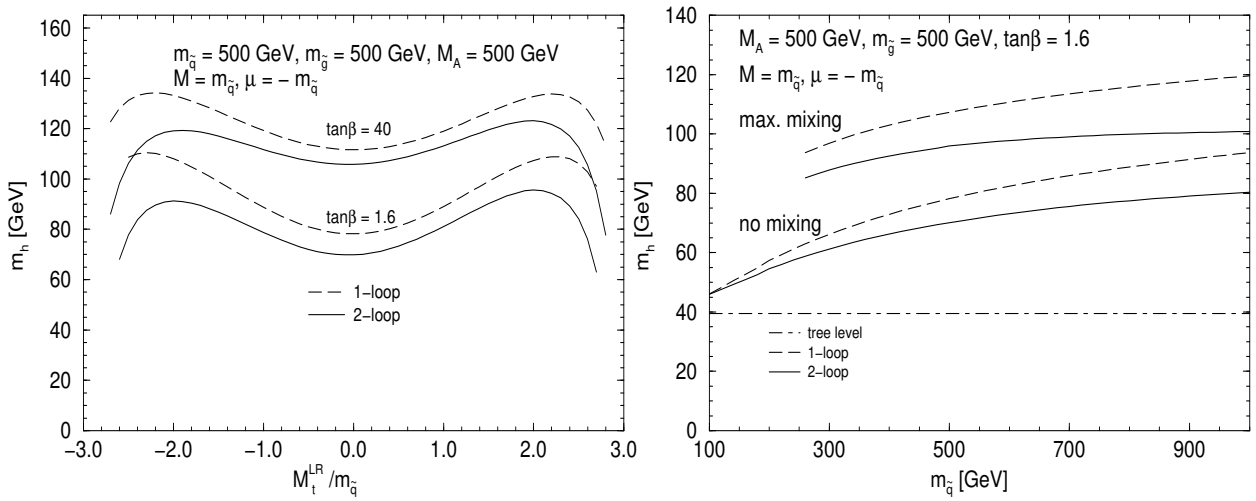

FIGURE 1. One- and two-loop results for m_h as a function of $M_t^{LR}/m_{\tilde{q}}$ for two values of $\tan\beta$ (left plot) and as a function of $m_{\tilde{q}}$ for $\tan\beta = 1.6$ (right plot).

In order to determine the maximally possible value for m_h within the MSSM as a function of $\tan\beta$, we have performed a parameter scan in which $m_{\tilde{g}}, M, \mu, M_A$ and M_t^{LR} have been varied for three values of m_t and fixed values of $m_{\tilde{q}}$ and $\tan\beta$. Fig. 2 shows the maximal Higgs-boson mass value in the range $\tan\beta \leq 5$ for $m_{\tilde{q}} = 1000$ GeV and $m_{\tilde{q}} = 2000$ GeV (in Fig. 2 the choice $M = \mu = 0$ has been made for simplicity; the change in m_h when these parameters are chosen at their experimental lower bounds is negligible). The upper bound is shown for the current experimental value of the top-quark mass, $m_t = 173.8$ GeV, and for values which are higher by one and two standard deviations, respectively. Our results confirm that for the scenario with $\tan\beta = 1.6$ practically the whole parameter space of the MSSM can be covered at LEP2 and the upgraded Tevatron. For slightly larger $\tan\beta$ and maximal mixing, however, some parameter space remains in which the Higgs boson could escape the detection at LEP2. For $\tan\beta = 40$, on the other hand, the prediction for m_h is at the edge of the LEP2 range even in the no-mixing case. The full exploration of the MSSM parameter space for the scenario with large $\tan\beta$ will be a challenge for the upgraded Tevatron as well as for the LHC and a future linear e^+e^--collider.

We now turn to the comparison of our diagrammatic results with the predictions obtained via RG methods. The left plot of Fig. 3 shows the prediction for m_h as a function of $M_t^{LR}/m_{\tilde{q}}$, corresponding to our diagrammatic result and to the result obtained by RG methods [8]. In the no-mixing case (i.e. $M_t^{LR}/m_{\tilde{q}} = 0$) the diagrammatic result agrees well with the RG result. For non-vanishing $\tilde{t}$-mixing sizable deviations between the diagrammatic and the RG results occur, which can reach 5 GeV for moderate mixing and become very large for large values of $|M_t^{LR}/m_{\tilde{q}}|$. As already stressed above, the maximal value for m_h in the diagrammatic approach is reached for $|M_t^{LR}/m_{\tilde{q}}| \approx 2$, whereas the RG results have a maximum at $|M_t^{LR}/m_{\tilde{q}}| \approx 2.4$, i.e. at the one-loop value. Varying the value of $m_{\tilde{g}}$ in our result shifts the diagrammatic result relative to the RG result (which does not contain the gluino mass as a parameter) within ± 2 GeV in the region of large mixing.

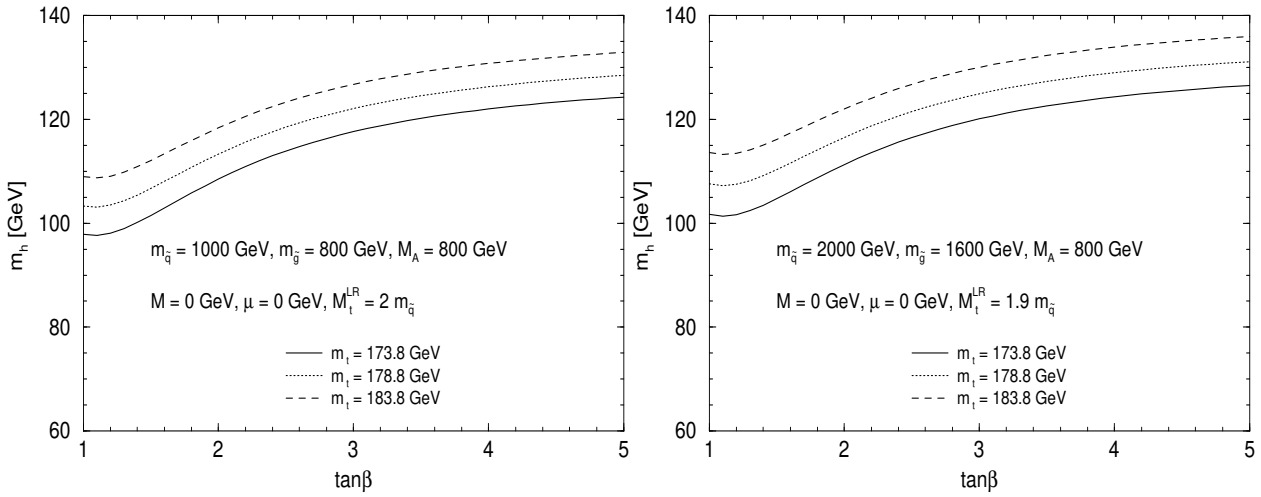

FIGURE 2. The maximally possible value for m_h as a function of $\tan\beta$ for $m_{\tilde{q}} = 1000$ GeV (left plot) and $m_{\tilde{q}} = 2000$ GeV (right plot). The upper bounds are shown for three values of m_t, $m_t = 173.8, 178.8, 183.8$ GeV.

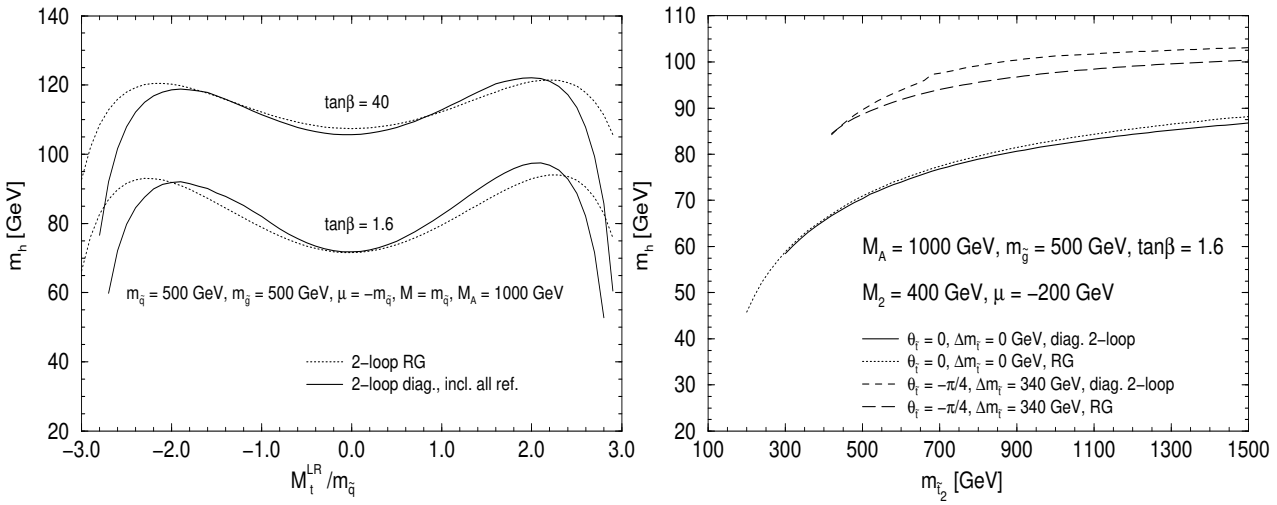

FIGURE 3. Comparison between the Feynman-diagrammatic calculations and the results obtained by renormalization group methods [8]. In the left plot the (unphysical) soft SUSY-breaking parameters of the $\tilde{t}$-mixing matrix are chosen as input, while in the right plot the physical $\tilde{t}$-masses and the mixing angle $\theta_{\tilde{t}}$ are the input parameters. For the curves with $\theta_{\tilde{t}} = 0$ in the right plot a mass difference $\Delta m_{\tilde{t}} = 0$ GeV is taken, whereas for $\theta_{\tilde{t}} = -\pi/4$ we choose $\Delta m_{\tilde{t}} = 340$ GeV, for which the maximal Higgs-boson masses are achieved.

In the left plot of Fig. 3 the results of our diagrammatic on-shell calculation and the RG methods have been compared in terms of the parameters $M_{\tilde{t}_L}$, $M_{\tilde{t}_R}$ and M_t^{LR} of the $\tilde{t}$-mixing matrix. However, since the two approaches rely on different renormalization schemes, the meaning of these (non-observable) parameters is not precisely the same in the two approaches starting from two-loop order. In order to compare results obtained by different approaches making use of different renormalization schemes, we find it preferable to compare predictions for physical observables in terms of other observables (instead of unphysical parameters). As a step into this direction we compare in the right plot of Fig. 3 the diagrammatic results and the RG results as a function of the physical mass $m_{\tilde{t}_2}$ and with the mass difference $\Delta m_{\tilde{t}} = m_{\tilde{t}_2} - m_{\tilde{t}_1}$ and the mixing angle $\theta_{\tilde{t}}$ as parameters. In the context of the RG approach the running $\tilde{t}$-masses, derived from the $\tilde{t}$-mass matrix, are considered as an approximation for the physical masses. As in the comparison performed above in (terms of unphysical parameters), in the right plot of Fig. 3 very good agreement is found between the

results of the two approaches in the case of vanishing $\tilde{t}$-mixing. For the maximal mixing angle $\theta_{\tilde{t}} = -\pi/4$ (and $\Delta m_{\tilde{t}} = 340$ GeV, for which the maximal Higgs-boson masses are achieved), however, the diagrammatic result yields values for m_h which are higher by about 5 GeV.

The upper bound on m_h for a certain value of $\tan\beta$ derived from our diagrammatic results is thus higher in the low $\tan\beta$ region by about 5 GeV than the upper bound derived previously from the RG results. As a result, we find that the $\tan\beta$ region which can fully be covered at the upgraded Tevatron is significantly reduced compared to previous studies.

In order to extract the dominant contributions to m_h from our rather complicated full result and to make a very fast evaluation feasible, we have derived a compact approximation formula for m_h [17]. In deriving the formula the following approximations have been made:

- $M = m_{\tilde{g}} = \sqrt{M_S^2 - \overline{m}_t^2}$, where M_S is given by

$$M_S = \begin{cases} \sqrt{m_{\tilde{q}}^2 + \overline{m}_t^2} & : \quad M_{\tilde{t}_L} = M_{\tilde{t}_R} = m_{\tilde{q}} \\ \left[M_{\tilde{t}_L}^2 M_{\tilde{t}_R}^2 + \overline{m}_t^2 (M_{\tilde{t}_L}^2 + M_{\tilde{t}_R}^2) + \overline{m}_t^4 \right]^{\frac{1}{4}} & : \quad M_{\tilde{t}_L} \neq M_{\tilde{t}_R} \end{cases} \tag{6}$$

- One-loop contribution arising from the $t, \tilde{t}$-sector: we have expanded the Higgs-boson self-energies $\hat{\Sigma}_s^{(1)}(0)$ $(s = h, H, hH)$ up to $\mathcal{O}\Big(\left(M_t^{LR}\right)^8 / M_S^8 \Big)$, $\mathcal{O}(m_t^4/M_S^4)$ and $\mathcal{O}(M_Z^2/M_A^2)$. The momentum depence is small and therefore neglected.

- For the one-loop contributions from the other sectors of the MSSM the leading logarithmic approximation has been used [5].

- The leading two-loop corrections in $\mathcal{O}(\alpha\alpha_s)$: we have expanded the renormalized Higgs-boson self-energies $\hat{\Sigma}_s^{(2)}(0)$ $(s = h, H, hH)$ up to $\mathcal{O}\Big(\left(M_t^{LR}\right)^4 / M_S^4 \Big)$ and $\mathcal{O}(M_Z^2/M_A^2)$.

These approximations yield a very compact formula for m_h^2. In the case $M_A \gg M_Z$ it reads:

$$m_h^2 = m_h^{2,\mathrm{tree}} + \Delta m_h^{2,\alpha,t,\tilde{t}} + \Delta m_h^{2,\alpha,\mathrm{rest}} + \Delta m_h^{2,\alpha\alpha_s} + \Delta m_h^{2,G_F^2}. \tag{7}$$

The tree-level prediction, the one-loop contribution from the $t, \tilde{t}$-sector, and the remaining one-loop contributions are given by

$$m_h^{2,\mathrm{tree}} = \frac{1}{2}\left[M_A^2 + M_Z^2 - \sqrt{(M_A^2 + M_Z^2)^2 - 4 M_Z^2 M_A^2 \cos^2 2\beta}\ , \right] \tag{8}$$

$$\begin{aligned} \Delta m_h^{2,\alpha,t/\tilde{t}} = \frac{\alpha}{\pi} \frac{\overline{m}_t^4}{M_W^2 s_W^2} \Bigg[& \log\left(\frac{\overline{m}_t^2}{M_S^2}\right) \Bigg\{ -\frac{3}{2} - \frac{3}{4}\frac{M_Z^2}{\overline{m}_t^2}\cos 2\beta \\ & -6\frac{M_Z^2}{M_A^2}(1-\sin^2\beta\,)(1-2\sin^2\beta\,)\left(1 + \frac{1}{4}\frac{M_Z^2}{\overline{m}_t^2}(1+4\sin^2\beta\,)\right) \Bigg\} \\ & + \Bigg\{ \frac{\left(M_t^{LR}\right)^2}{M_S^2}\left(\frac{3}{2} - \frac{1}{2}\frac{M_Z^2}{\overline{m}_t^2} - \frac{3}{4}\frac{\overline{m}_t^2}{M_S^2} + \frac{3}{8}\frac{M_Z^2}{M_S^2}\right) + \frac{\left(M_t^{LR}\right)^4}{M_S^4}\left(-\frac{1}{8} + \frac{1}{2}\frac{\overline{m}_t^2}{M_S^2} - \frac{5}{8}\frac{M_Z^2}{M_S^2}\right) \\ & + \frac{\left(M_t^{LR}\right)^6}{M_S^6}\left(-\frac{3}{40}\frac{\overline{m}_t^2}{M_S^2} + \frac{3}{10}\frac{\overline{m}_t^4}{M_S^4}\right) - \frac{\left(M_t^{LR}\right)^8}{M_S^8}\frac{3}{56}\frac{\overline{m}_t^4}{M_S^4} \Bigg\} \\ & \left(1 + 4\frac{M_Z^2}{M_A^2}(1-\sin^2\beta\,)(1-2\sin^2\beta\,)\right) \Bigg], \end{aligned} \tag{9}$$

$$\Delta m_h^{2,\alpha,\mathrm{rest}} = \frac{\alpha}{\pi}\frac{M_Z^4}{24 M_W^2 s_W^2}\left[\log\left(\frac{M_{SUSY}^2}{M_Z^2}\right)\left\{12N_c\frac{m_b^4}{M_Z^4} - 6N_c\cos 2\beta\ \frac{m_b^2}{M_Z^2} + \cos^2 2\beta\ (P_b + P_f)\right.\right.$$
$$\left. + (P_g + P_{2H})(\sin^4\beta\ + \cos^4\beta\ ) - 2\sin^2\beta\ \cos^2\beta\ (P_g' + P_{2H}')\right\}$$
$$+\frac{18 m_b^4}{M_Z^4}\left[2\frac{\left(M_b^{LR}\right)^2}{M_{SUSY}^2}\left(1 - \frac{\left(M_b^{LR}\right)^2}{12\, M_{SUSY}^2}\right) - \frac{M_Z^2}{2\, m_b^2}\cos 2\beta\ \left(\frac{\left(M_b^{LR}\right)^2}{M_{SUSY}^2} + \frac{A_b^2 - \mu^2\tan^2\beta}{3\, M_{SUSY}^2}\right)\right]$$
$$\left. - \log\left(\frac{M_A^2}{M_Z^2}\right)\left\{(\cos^4\beta\ + \sin^4\beta\ )P_{2H} - 2\cos^2\beta\ \sin^2\beta\ P_{2H}' - P_{1H}\right\}\right], \tag{10}$$

with

$$\begin{aligned}
M_b^{LR} &= A_b - \mu\tan\beta \\
P_b &= N_c(1 + 4Q_b s_W^2 + 8Q_b^2 s_W^4) \\
P_f &= Nc(N_g - 1)(2 - 4s_W^2 + 8(Q_t^2 + Q_b^2)s_W^4) + N_g(2 - 4s_W^2 + 8s_W^4) \\
P_g &= -44 + 106 s_W^2 - 62 s_W^4, \quad P_g' \ = \ 10 + 34 s_W^2 - 26 s_W^4 \\
P_{1H} &= -9\cos^4 2\beta\ + (1 - 2s_W^2 + 2s_W^4)\cos^2 2\beta \\
P_{2H} &= -10 + 2s_W^2 - 2s_W^4, \quad P_{2H}' \ = \ 8 - 22 s_W^2 + 10 s_W^4 \\
\text{and} \quad & Q_t = \frac{2}{3},\ Q_b = -\frac{1}{3},\ N_c = 3,\ N_g = 3.
\end{aligned} \tag{11}$$

The leading two-loop Yukawa correction reads [7,14]

$$\Delta m_h^{2,G_F^2} = \frac{9}{16\pi^4} G_F^2 \overline{m}_t^6 \left[\tilde{X}t + t^2\right] \tag{12}$$

$$\text{with} \quad \tilde{X} = \left[\left(\frac{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}{4\overline{m}_t^2}\sin^2 2\theta_{\tilde{t}}\right)^2\left(2 - \frac{m_{\tilde{t}_2}^2 + m_{\tilde{t}_1}^2}{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}\log\left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right)\right)\right.$$
$$\left. + \frac{m_{\tilde{t}_2}^2 - m_{\tilde{t}_1}^2}{2\overline{m}_t^2}\sin^2 2\theta_{\tilde{t}}\log\left(\frac{m_{\tilde{t}_2}^2}{m_{\tilde{t}_1}^2}\right)\right], \qquad t = \frac{1}{2}\log\left(\frac{m_{\tilde{t}_1}^2 m_{\tilde{t}_2}^2}{\overline{m}_t^4}\right). \tag{13}$$

The main ingredient of the approximation formula are the leading logarithmic and non-logarithmic terms of $\mathcal{O}(\alpha\alpha_s)$, which we have derived by expanding the full two-loop result. They are given by the simple expression

$$\Delta m_h^{2,\alpha\alpha_s} = -\frac{\alpha}{\pi}\frac{\alpha_s}{\pi}\frac{\overline{m}_t^4}{M_W^2 s_W^2}\left[4 + 3\log^2\left(\frac{\overline{m}_t^2}{M_S^2}\right) + 2\log\left(\frac{\overline{m}_t^2}{M_S^2}\right) - 6\frac{M_t^{LR}}{M_S} - \frac{\left(M_t^{LR}\right)^2}{M_S^2}\left\{3\log\left(\frac{\overline{m}_t^2}{M_S^2}\right) + 8\right\}\right.$$
$$\left. + \frac{17}{12}\frac{\left(M_t^{LR}\right)^4}{M_S^4}\right]\left(1 + 4\frac{M_Z^2}{M_A^2}(1 - \sin^2\beta\ )(1 - 2\sin^2\beta\ )\right). \tag{14}$$

In Fig. 4 the compact approximation formula is compared with the full result in the low $\tan\beta$ scenario. The compact formula approximates the full result better than about 2 GeV for most parts of the MSSM parameter space. Larger deviations occur for $|M_t^{LR}/m_{\tilde{q}}| > 2$. The quality of the approximation is the same also for the large $\tan\beta$ scenario.

The result of the full calculation has been implemented into the Fortran program *FeynHiggs* [15], which needs about 2 seconds for the evaluation of m_h on a third-generation Alpha 21164 microprocessor (600 MHz processing speed) for one set of parameters. The approximation formula has been implemented into the Fortran program *FeynHiggsFast* [17], which provides a very fast evaluation of m_h with resonably well

accuracy in a large part of the MSSM parameter space. The improvement in the speed of the evaluation with *FeynHiggsFast* compared to *FeynHiggs* is about a factor of 1000. In addition we have implemented the results for the MSSM contributions to $\Delta\rho$ [18] into both programs. Here the corrections arising from $\tilde{t}/\tilde{b}$-loops up to $\mathcal{O}(\alpha\alpha_s)$ have been taken into account. The gluino-exchange contribution which is very lengthy and vanishes for increasing $m_{\tilde{g}}$ has been neglected for *FeynHiggsFast*. The ρ-parameter can be used as an additional constraint (besides the experimental bounds) on the squark masses. A value of $\Delta\rho$ outside the experimentally preferred region of $\Delta\rho^{\rm SUSY} \lesssim 1.3 \cdot 10^{-3}$ [19] indicates experimentally disfavored $\tilde{t}$- and $\tilde{b}$-masses.

Both Fortran programs are available via the WWW page
`http://www-itp.physik.uni-karlsruhe.de/feynhiggs` .

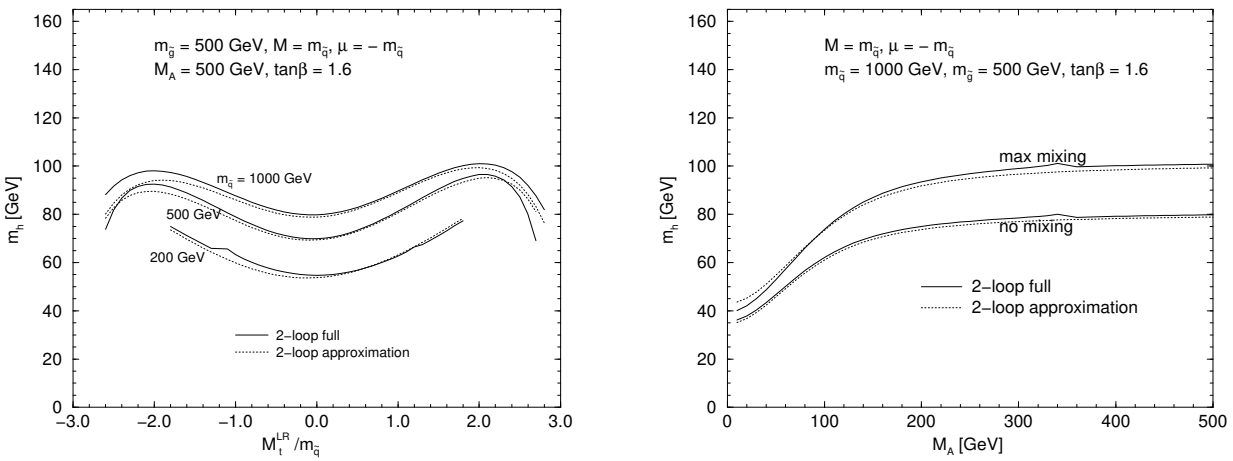

FIGURE 4. Comparison between the compact approximation formula and the full result for m_h.

In conclusion, using the Feynman diagrammatic method we have calculated the leading $\mathcal{O}(\alpha\alpha_s)$ corrections to the masses of the neutral $\mathcal{CP}$-even Higgs bosons in the MSSM. The two-loop result has been combined with the prediction based on the complete diagrammatic one-loop on-shell result. Two further corrections beyond $\mathcal{O}(\alpha\alpha_s)$ have been added in order to incorporate leading electroweak two-loop and higher-order QCD contributions. The full result has been implemented into the Fortran program *FeynHiggs*.

The two-loop corrections lead to a large reduction of the one-loop on-shell result of up to 20 GeV. We have performed a detailed analysis of the dependence of m_h on the various MSSM parameters. A scan over the parameters $\mu, M, m_{\tilde{g}}, M_A$ and M_t^{LR} has been performed in order to determine the maximally possible value for m_h as a function of $\tan\beta$. We find that for the scenario with $\tan\beta = 1.6$ the parameter space of the MSSM can be covered at LEP2 and the upgraded Tevatron, while for slightly larger $\tan\beta$ and large mixing in the scalar top sector the Higgs boson could escape detection at LEP2.

We have compared the results of our Feynman diagrammatic calculation with the results obtained via RG methods. We find good agreement in the case of vanishing mixing in the $\tilde{t}$-sector. In the case of non-vanishing mixing, however, sizable deviations between the results of the two approaches occur, exceeding 5 GeV for large mixing. In the low $\tan\beta$ scenario the maximally possible values for m_h obtained from the diagrammatic calculation are up to 5 GeV larger compared to the RG calculation. We therefore find that the bounds on $\tan\beta$ that can be inferred from the experimental lower bound on m_h are less stringent than in previous analyses based on the RG results.

By means of a Taylor expansion we have derived the leading logarithmic and non-logarithmic contributions in $\mathcal{O}(\alpha\alpha_s)$ to the lightest Higgs-boson mass m_h. This result has been incorporated into a compact formula for m_h^2, which has been implemented into the Fortran program *FeynHiggsFast*. The formula is valid for general mixing in the scalar top sector and arbitrary choices of the parameters in the Higgs sector of the model. The approximation formula has been compared to the full result, and good agreement is found for most parts of the MSSM parameter space.

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