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Generating and Calculating One-loop Feynman Diagrams with *FeynArts*, *FormCalc*, and *LoopTools*

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Abstract:
A set of programs is presented for automatically generating and calculating Feynman diagrams. Diagrams are generated with *FeynArts*, then algebraically simplified using a combination of *Mathematica* and *FORM* implemented in the package *FormCalc*, and finally evaluated numerically using the *LoopTools* package. *FormCalc* works either in dimensional regularization or in constrained differential renormalization, the latter of which is equivalent at the one-loop level to regularization by dimensional reduction. *FormCalc* combines the speed of *FORM* with the powerful instruction set of *Mathematica*, and the latter greatly eases further processing of the results (e.g. selecting or modifying terms). The output is in a form well suited for numerical evaluation, which is then straightforward using the implementations of the one-loop integrals in *LoopTools*.

1 Introduction

Explaining the necessity of one-loop calculations in the light of modern-day colliders is like carrying owls to Athens; and as there is no lack of motivation, several program packages have been developed to aid these calculations. However, these programs generally tackle only part of the problem, so there is still considerable work left in making these programs work together.

In this paper three packages, *FeynArts*, *FormCalc*, and *LoopTools*, are presented which work hand in hand. The user has to supply only small driver programs whose main purpose is to specify the necessary input parameters. This makes the whole system very "open" in the sense that the results are returned as *Mathematica* expressions which can easily be manipulated, e.g. to select or modify terms.

Since one-loop calculations can range anywhere from a handful to several hundreds of diagrams (particularly so in models with many particles like the MSSM), speed is an issue, too. *FormCalc*, the program which does the algebraic simplification, therefore uses *FORM* [1] for the time-consuming parts of the calculation. Owing to *FORM*'s speed, *FormCalc* can process, for example, the 1000-odd one-loop diagrams of W–W scattering in the Standard Model [2] in about 5 minutes on an ordinary Pentium PC.

The following table summarizes the steps in a one-loop calculation and the distribution of tasks among the programs *FeynArts*, *FormCalc*, and *LoopTools*:

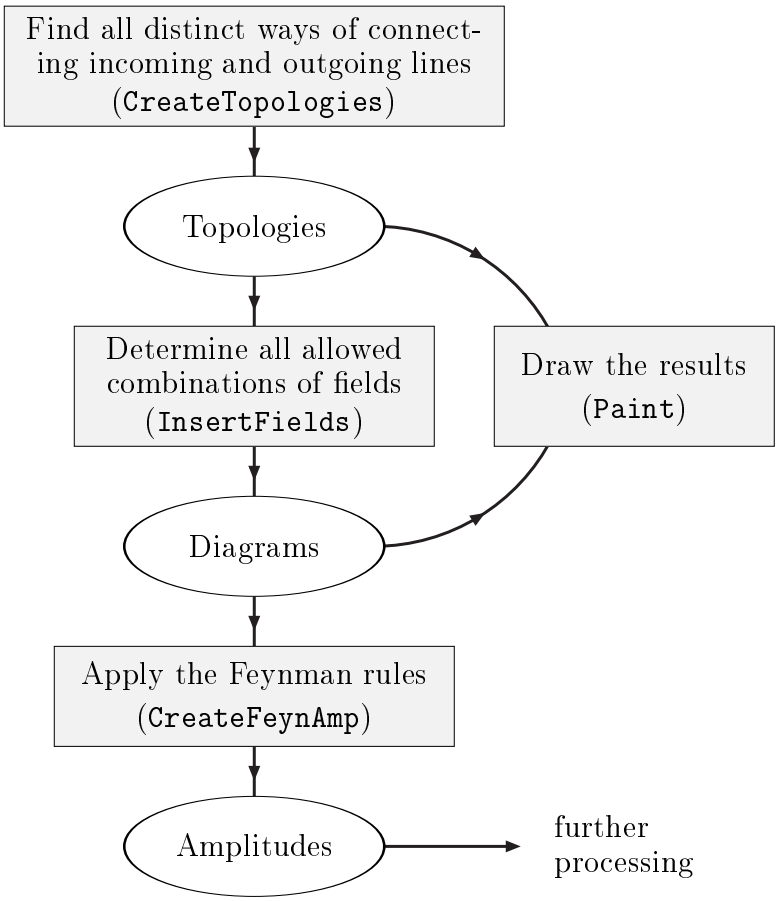
Diagram generation	• Create the topologies • Insert fields • Apply the Feynman rules • Paint the diagrams	*FeynArts*
↓		
Algebraic simplification	• Contract indices • Calculate traces • Reduce tensor integrals • Introduce abbreviations	*FormCalc*
↓		
Numerical evaluation	• Convert *Mathematica* output to Fortran code • Supply a driver program	*FormCalc*
	• Implementation of the integrals	*LoopTools*

The following sections describe the main functions of each program. Also, to demonstrate how the programs are used together, the *FormCalc* package contains two sample calculations in the electroweak Standard Model: $ZZ \to ZZ$ [3] and $e^+e^- \to \bar{t}\,t$ [4].

2 *FeynArts*

FeynArts is a *Mathematica* package for the generation and visualization of Feynman dia-

grams and amplitudes [5]. It works in three basic steps sketched in the following diagram:



The first step is to create all different topologies for a given number of loops and external legs. For example, to create all one-loop topologies for a $2 \to 2$ process except those containing a tadpole, the following call to `CreateTopologies` is used:

```
tops = CreateTopologies[ 1, 2 -> 2, ExcludeTopologies -> {Tadpoles} ]
```

In the second step, the actual particles in the model have to be distributed over the topologies in all allowed ways. E.g. the diagrams for $e^+e^- \to \gamma\gamma$ are produced with

```
inss = InsertFields[ tops, {-F[2, {1}], F[2, {1}]} -> {V[1], V[1]} ]
```

where `F[2, {1}]` is the electron, `-F[2, {1}]` its antiparticle, the positron, and `V[1]` the photon. The fields and their couplings are defined in a special file, the model file, which the user can supply or modify. Model files are currently provided for QED, the electroweak Standard Model, and QCD; a MSSM model file is in preparation.

The diagrams may be painted with `Paint[inss]`. Finally, the analytic expressions for the diagrams are obtained by

```
amps = CreateFeynAmp[ inss ]
```

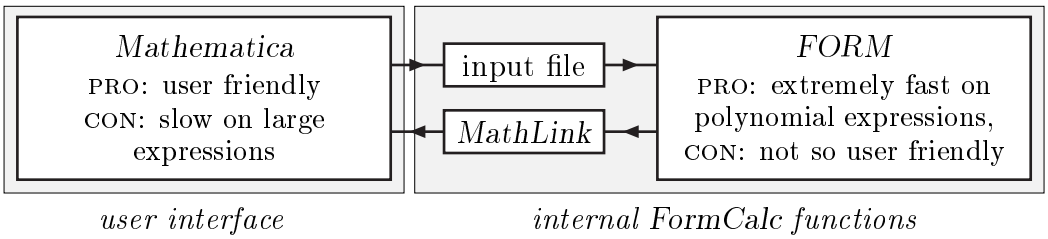
3 *FormCalc*

The evaluation of the *FeynArts* output proceeds in two steps:

1. The symbolic expressions for the diagrams are simplified algebraically with *FormCalc* which returns the results in a form well suited for numerical evaluation.

2. The *Mathematica* expressions then need to be translated into a Fortran program. (The numerical evaluation could, in principle, be done in *Mathematica* directly, but this becomes very slow for large amplitudes.) The translation is done by *NumPrep* which is part of the *FormCalc* package. For compiling the generated code one needs a driver program (`num.F`, also in *FormCalc*), and the numerical implementations of the one-loop integrals in *LoopTools*.

The structure of *FormCalc* is simple: it prepares the symbolic expressions of the diagrams in an input file for *FORM*, runs *FORM*, and retrieves the results. This interaction is transparent to the user. *FormCalc* combines the speed of *FORM* with the powerful instruction set of *Mathematica* and the latter greatly facilitates further processing of the results. The following diagram shows schematically how *FormCalc* interacts with *FORM*:



FormCalc can work either in dimensional regularization or in constrained differential renormalization (CDR) [6], the latter of which is equivalent at the one-loop level to regularization by dimensional reduction [7].

The main function in *FormCalc* is `OneLoop` (the name is not strictly correct since it works also with tree graphs). It is used like this:

```
<< FormCalc.m;
$Dimension = D;
amps = << myamps.m;
alldiags = OneLoop[amps]
```

The file `myamps.m` is assumed here to contain amplitudes generated by *FeynArts*. The dimension—`D` for dimensional regularization or `4` for dimensional reduction / CDR—is set with `$Dimension`. Note that `OneLoop` needs no declarations of the kinematics of the underlying process; it uses the information *FeynArts* hands down.

Even more comprehensive than `OneLoop`, the function `ProcessFile` can process entire files. It collects the diagrams into blocks such that index summations (e.g. over fermion

generations) can later be carried out easily, i.e. only diagrams which are summed over the same indices are put in one block. `ProcessFile` is invoked e.g. as

```
ProcessFile["vertex.amp", "results/vertex"]
```

which reads the *FeynArts* amplitudes from `vertex.amp` and produces files of the form `results/vertex`*id*`.m`, where *id* is an identifier for a particular block.

`OneLoop` and `ProcessFile` return expressions where spinor chains, dot products of vectors, and Levi-Civita tensors contracted with vectors have been collected and abbreviated. A term in such an expression may look like

```
C0i[cc1, MW2, S, MW2, MZ2, MW2, MW2] *
  (P12*S*(-8*a2*MW2 + 4*a2*MW2*S2 - 28*a2*CW^2*MW2*S2 +
          16*a2*CW^2*S*S2 + 4*a2*C2*MW2*SW^2) +
   047*S*(-32*a2*CW^2*MW2*S2 + 8*a2*CW^2*S2*T + 8*a2*CW^2*S2*U) -
   P13*S*(-64*a2*CW^2*MW2*S2 + 16*a2*CW^2*S2*T + 16*a2*CW^2*S2*U))
```

The first line stands for the tensor coefficient function $C_1(M_\mathrm{W}^2, s, M_\mathrm{W}^2, M_\mathrm{Z}^2, M_\mathrm{W}^2, M_\mathrm{W}^2)$ which is multiplied with a linear combination of abbreviations like `047` or `P12` with certain coefficients. These coefficients contain the Mandelstam variables `S`, `T`, and `U` and some shorthands for parameters of the Standard Model, e.g. `a2` $= \alpha^2$.

The abbreviations like `047` or `P12` are introduced automatically and can significantly reduce the size of an amplitude. The definitions of the abbreviations can be retrieved by `Abbreviations[]` which returns a list of rules such that `result //. Abbreviations[]` gives the full, unabbreviated expression.

4 *LoopTools*

LoopTools supplies the actual numerical implementations of the one-loop functions needed for programs made from the *FormCalc* output. It is based on the reliable package *FF* [8] and provides in addition to the scalar integrals of *FF* also the tensor coefficients in the conventions of [9]. *LoopTools* offers three interfaces: Fortran, C++, and *Mathematica*, so most programming tastes should be served.

Using *LoopTools* functions in Fortran and C++ is very similar. In Fortran, it is necessary to include the two files `tools.F` and `tools.h`, the latter one in every function or subroutine. In C++, `ctools.h` must be included once. Before using any *LoopTools* function, `bcaini` must be called and at the end of the calculation `bcaexi` may be called to obtain a summary of errors. It is of course possible to change parameters like the scale μ from dimensional regularization; this is described in detail in the manual [10].

A very simple program would for instance be

Fortran

```
#include "tools.F"

    program simple
#include "tools.h"
    call bcaini
    print *, B0(1000D0, 50D0, 80D0)
    call bcaexi
    end
```

C++

```
#include "ctools.h"

main()
{
  bcaini();
  cout << B0(1000., 50., 80.) << "\n";
  bcaexi();
}
```

The *Mathematica* interface is even simpler to use:

```
In[1]:= Install["bca"]

In[2]:= B0[1000, 50, 80]

Out[2]= -4.40593 + 2.70414 I
```

Acknowledgements

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