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Renormalization and Symmetry Conditions in Supersymmetric QED

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Abstract

For supersymmetric gauge theories a consistent regularization scheme that preserves supersymmetry and gauge invariance is not known. In this article we tackle this problem for supersymmetric QED within the framework of algebraic renormalization. For practical calculations, a non-invariant regularization scheme may be used together with counterterms from all power-counting renormalizable interactions. From the Slavnov–Taylor identity, expressing gauge invariance, supersymmetry and translational invariance, simple symmetry conditions are derived that are important in a twofold respect: they establish exact relations between physical quantities that are valid to all orders, and they provide a powerful tool for the practical determination of the counterterms. We perform concrete one-loop calculations in dimensional regularization, where supersymmetry is spoiled at the regularized level, and show how the counterterms necessary to restore supersymmetry can be read off easily. In addition, a specific example is given how the supersymmetry transformations in one-loop order are modified by non-local terms.

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1 Introduction

In phenomenological studies of the electroweak standard model (SM) and its extensions it is crucial to take into account radiative corrections. Comparing theoretical predictions with experimental precision data provides tests and comparisons of the models at the level of their quantum structure. In particular, as far as collider energies are too low to produce Higgs or e.g. supersymmetric particles, this is the only way to obtain information about such heavy sectors.

The calculation of these radiative corrections involves a technical problem. The loop integrals are in general divergent and need regularization. But this procedure can break essential symmetries of the underlying theory, such as gauge invariance or supersymmetry. The two most important regularization schemes for the SM and its supersymmetric extensions are dimensional regularization (DReg) [1, 2] and dimensional reduction (DRed) [3], the difference being that in the latter case only the momenta are treated D-dimensional whereas the vector fields and γ^μ matrices are not.

As already noted by the inventor, DRed is inconsistent [4]: it is possible to derive the equation $0 = D(D-1)(D-2)(D-3)(D-4)$ in contradiction to regularization at $D \neq 4$. On the other hand, DReg breaks supersymmetry whereas DRed was designed to preserve supersymmetry[3, 5]. There are many modifications of both schemes; for example, in [6] a version of DRed was suggested which is mathematically consistent but not supersymmetric. In fact, no consistent regularization scheme is known that simultaneously preserves supersymmetry and gauge invariance for supersymmetric gauge theories. A similar problem arises in chiral gauge theories like the standard model.

For practical calculations an invariant scheme is desirable. So in most phenomenological applications requiring supersymmetric calculations schemes such as DRed are used together with arguments that the inconsistencies do not show up in the actual cases [7]. But these arguments have a restricted range of validity, and it is not yet clear if and how they may be applied to calculations beyond one loop in the SM and its supersymmetric extensions [8].

In this article we pursue the opposite way: Instead of searching for an invariant regularization we advocate the use of arbitrary regularization schemes and define the finite (renormalized) Green functions by the basic symmetries, as it is proposed by the abstract approach of algebraic renormalization. (For an introduction to algebraic renormalization see ref. [9].)

From an abstract point of view, the question of the existence of a symmetry-preserving scheme is irrelevant. The theory is defined by symmetry requirements that should be satisfied after renormalization. There are two equivalent ways to achieve that. The first way is to use an invariant scheme keeping the symmetries manifest. In this case, only those counterterms are necessary for renormalization that themselves preserve the symmetries. These are usually just the ones obtained by multiplicative renormalization of the parameters and fields in the Lagrangian of the theory. The second way is to use a non-invariant scheme and to compensate the corresponding symmetry breaking by appropriate non-invariant counterterms. Although less obvious, this possibility was noted in many milestones of renormalization theory, e.g. in [10, 11, 12]. Generally, by using a non-invariant scheme a precise definition of the symmetries one requires from the renormalized theory is mandatory. In order to establish these symmetries one has to allow for all possible counterterms, restricted only by hermiticity, Lorentz invariance and power counting renormalizability, but not by further symmetries.

Of course, if there exists no scheme that keeps the symmetries manifest there could be anomalies making it impossible to restore the symmetries by adjusting the counterterms. But the absence of anomalies, too, may be proven without any recurrence to a particular regularization, only using algebraic properties of the symmetry requirements [12].

The first algebraic analysis of renormalizability of supersymmetric gauge theories was performed in the superspace formalism [13]. For phenomenological applications it is preferable to use the component formulation of supersymmetric gauge theories in the Wess–Zumino gauge, where the unphysical fields are eliminated by the supersymmetric gauge transformation. Finding a well defined identity expressing the symmetry content of supersymmetric gauge theories in the Wess–Zumino gauge is not easy since there the supersymmetry algebra does not close but also includes gauge transformations (see eq. (5)). In particular, a separate treatment of gauge invariance and supersymmetry seems impossible — one would need infinitely many sources and renormalizability could not be proven [14]. The solution of this problem was found in [15, 16] combining ideas of Becchi, Rouet and Stora [12] and Batalin and Vilkovisky [17]. Its essential features are the combination of all symmetries into the BRS transformations, where the algebraic structure is encoded in the nilpotency of the BRS operator. The corresponding Slavnov–Taylor identity includes all symmetries and can be used to prove renormalizability of supersymmetric gauge theories independent of the existence of an invariant regularization scheme [16, 18]. The possible anomalies turn out to be just the supersymmetric extensions of the usual gauge anomalies and are therefore completely characterized by the gauge structure. Furthermore, in [18] it was shown that this setup leads to a theory with the expected physical properties. One can define a set of physical observables, i.e. gauge invariant operators, and generators for supersymmetry transformations and translations, and can prove that the unmodified supersymmetry algebra is realized on the physical observables.

In this article we consider the supersymmetric extension of QED (SQED) as a toy model for general supersymmetric gauge theories and in particular for the supersymmetric extensions of the standard model. From the Slavnov–Taylor identity we derive symmetry conditions, simple identities between renormalized vertex functions. On the one hand, these conditions are exact physical statements expressing symmetry relations, like mass equalities and charge universality, more immediately. On the other hand, they are used to simplify and to streamline the practical determination of counterterms significantly. As examples we apply these identities to various self energies and vertex corrections calculated with DReg. We also examine the effect of "forgetting" a non-invariant but necessary counterterm. It turns out that in this case the numerical error can significantly change the result of the calculation.

The plan of the article is as follows: In section 2 we describe the classical action of SQED and give its symmetries in the form of functional identities, which are the Slavnov–Taylor identity and the gauge Ward identity. In addition, we derive the invariant counterterms and the corresponding normalization conditions. In section 3 the symmetry conditions are derived. In section 4 we demonstrate in several examples, how non-invariant counterterms appearing in DReg are identified and removed by the use of symmetry identities. The Appendix contains the list of the conventions used in this article.

2 Definition of the model

2.1 Classical theory

Supersymmetric QED (SQED) [19] is an abelian gauge theory with the following field content:

1. One vector multiplet $(A^\mu, \lambda^\alpha, \overline{\lambda}^{\dot\alpha})$ consisting of the photon and the photino, described by a vector and a Majorana spinor field.
2. Two chiral multiplets (ψ_L^α, ϕ_L) and (ψ_R^α, ϕ_R) with charges $Q_L = -1$, $Q_R = +1$, each consisting of one Weyl spinor and one scalar field, constituting the left- and right-handed electron and selectron, the matter fields.

The electron Dirac spinor and the photino Majorana spinor are given by

$$\Psi = \begin{pmatrix} \psi_{L\alpha} \\ \overline{\psi}_R^{\dot\alpha} \end{pmatrix}, \quad \tilde\gamma = \begin{pmatrix} -i\lambda_\alpha \\ i\overline{\lambda}^{\dot\alpha} \end{pmatrix}. \tag{1}$$

The SQED Lagrangian contains kinetic, minimal coupling and mass terms and in addition, due to the supersymmetry, coupling terms to the photino and quartic terms in the selectron fields:

$$
\begin{aligned}
\mathcal{L}_{\mathrm{SQED}} = & -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\overline{\tilde{\gamma}}i\gamma^{\mu}\partial_{\mu}\tilde{\gamma} \\
& |D_{\mu}\phi_L|^2 + |D_{\mu}\phi_R^{\dagger}|^2 + \overline{\Psi}i\gamma^{\mu}D_{\mu}\Psi \\
& - \sqrt{2}eQ_L\left(\overline{\Psi}P_R\tilde{\gamma}\phi_L - \overline{\Psi}P_L\tilde{\gamma}\phi_R^{\dagger} + \phi_L^{\dagger}\overline{\tilde{\gamma}}P_L\Psi - \phi_R\overline{\tilde{\gamma}}P_R\Psi\right) \\
& - \frac{1}{2}\left(eQ_L|\phi_L|^2 + eQ_R|\phi_R|^2\right)^2 \\
& - m\overline{\Psi}\Psi - m^2(|\phi_L|^2 + |\phi_R|^2)
\end{aligned} \tag{2}
$$

with the gauge covariant derivative and field strength

$$
D_{\mu} = \partial_{\mu} + ieQA_{\mu} \ , \tag{3}
$$

$$
F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu} \ . \tag{4}
$$

The use of this set of physical fields corresponds to the choice of the Wess–Zumino gauge, where unphysical fields of the vector supermultiplet are eliminated by gauge transformations, and the elimination of further auxiliary fields in the superfield version of SQED. While the former modifies the supersymmetry algebra by gauge transformations, the second contributes terms that vanish only if the equations of motion hold. In fact, the supersymmetry generators $Q_{\alpha}, \overline{Q}_{\dot{\alpha}}$ satisfy

$$
\{Q_{\alpha}, \overline{Q}_{\dot{\alpha}}\} = 2P_{\mu}\sigma^{\mu}_{\alpha\dot{\alpha}} + \delta_{\Lambda} + \text{eqs. of motion}, \tag{5}
$$

where δ_{Λ} is an abelian gauge transformation with the gauge function $\Lambda = -2iA_{\mu}\sigma^{\mu}_{\alpha\dot{\alpha}}$. The equations-of-motion terms appear only when the anticommutator acts on spinor fields.

2.2 Quantization

For quantizing the supersymmetric extension of QED in the Wess–Zumino gauge one has to find symmetries which characterize the classical action and furthermore the one-particle irreducible (1PI) Green functions summarized in their generating functional Γ

$$
\Gamma = \Gamma_{\mathrm{cl}} + \mathcal{O}(\hbar) \ . \tag{6}
$$

The defining symmetries of the gauge invariant action are the abelian gauge invariance and $N = 1$ supersymmetry. As usual, one has to add to the gauge invariant action (2) a gauge fixing term which allows to determine a well-defined photon propagator. The QED gauge fixing, however, breaks the supersymmetry non-linearly in the propagating fields and cannot be used without modifications for a higher order construction. To overcome this difficulty gauge and supersymmetry transformations are included into one BRS transformation with the respective ghosts [15, 16]. It is then possible to extend the gauge fixing by a ghost part in such a way that the complete action is invariant under BRS transformations (cf. (40) and (41)). Moreover, by transforming also the ghosts appropriately the algebra of supersymmetry and gauge transformations is summarized in the nilpotency of the BRS transformations.

For proving renormalizability it has to be shown that the Green functions of SQED satisfy the Slavnov–Taylor identity, which is the functional form of the BRS transformations, to all orders:

$$S(\Gamma) = 0. \tag{7}$$

Renormalizability of N=1 supersymmetric gauge theories in the Wess–Zumino gauge has been proven in [18]. There and in [16] it has been shown in the framework of algebraic renormalization that the only possible anomaly appearing in supersymmetric gauge theories is the supersymmetric extension of the Adler–Bardeen anomaly. If no anomalies are present, as it is in QED and SQED, all breakings are scheme dependent breakings and are removed by adding appropriate counterterms.

It is a basic fact of renormalized perturbation theory [10] that by the requirement of unitarity, causality and Lorentz invariance — leading to the usual Feynman diagram expansion — the higher order contributions to Γ are not uniquely defined: Given Γ renormalized up to the order $\hbar^{n-1}$, the local contributions in the next order $\hbar^n$ are ambiguous. Accordingly, different regularization schemes used to calculate the Feynman diagrams can differ in the results for the local contributions, which in general are divergent; the non-local contributions, however, are unique and finite. That is why the ambiguity inherent in the renormalization procedure is equivalent to the possibility to add local counterterms of order $\hbar^n$ to Γ:

$$\Gamma^{(n)} = \Gamma^{(n)}_{\text{regularized}} + \Gamma^{(n)}_{\text{ct}} \, . \tag{8}$$

The divergent parts of the counterterms must cancel the divergencies of the regularized loop diagrams whereas the finite parts are generally only restricted by hermiticity, Lorentz invariance and power counting renormalizability but otherwise free. All these counterterms may be collected and added to the classical action:

$$\Gamma_{\text{eff}}^{(\leq n)} = \Gamma_{\text{cl}} + \sum_{m=1}^{n} \Gamma_{\text{ct}}^{(m)} \ . \tag{9}$$

Γ_{eff} is the action to be used to derive the Feynman rules of the next order $\hbar^{n+1}$, thus providing an inductive procedure.

All conceivable finite counterterms have to be fixed by the symmetries and by normalization conditions. Proceeding from the lowest order by induction, all scheme-dependent breakings of the Slavnov–Taylor identity $\Delta^{(n)}$ appearing in order n have to be absorbed by adjusting the respective non-invariant counterterms:

$$S(\Gamma^{(\leq n-1)} + \Gamma_{\text{regularized}}^{(n)} + \Gamma_{\text{ct}}^{(n)}) = \Delta^{(n)} + s_{\Gamma_{\text{cl}}}\Gamma_{\text{ct}}^{(n)} = 0 + \mathcal{O}(\hbar^{n+1}). \tag{10}$$

(Here $s_{\Gamma_{\text{cl}}}$ is the linearized Slavnov–Taylor operator defined in (37).) At the same time this equation fixes uniquely all non-invariant counterterms of a specific scheme without referring to invariance properties of the scheme.

Since the construction of supersymmetric gauge theories in the Wess–Zumino gauge by means of the Slavnov–Taylor identity has not been applied yet in phenomenological calculations, we present the construction of the symmetry operators and the ghost action in some detail in the following part of the paper.

2.3 Symmetry requirements

The BRS formalism encodes the complicated structure of (5) in the simple equation

$$s^2 = 0 + \text{eqs. of motion (e.o.m.)}. \tag{11}$$

Here s is the generator of BRS transformations given below. In the BRS transformations the Faddeev–Popov ghost $c(x)$ is used together with space-time independent supersymmetry and translation ghosts $\epsilon^\alpha, \bar{\epsilon}^{\dot{\alpha}}$ and ω^ν as parameters. The transformation rules for the ghosts themselves are given by the structure constants of the symmetry algebra [12]. That yields the following explicit form of the operator s:

$$
\begin{aligned}
sA_\mu &= \partial_\mu c + i\epsilon\sigma_\mu\overline{\lambda} - i\lambda\sigma_\mu\overline{\epsilon} - i\omega^\nu\partial_\nu A_\mu \ , && (12)\\
s\lambda^\alpha &= \frac{i}{2}(\epsilon\sigma^{\rho\sigma})^\alpha F_{\rho\sigma} - i\epsilon^\alpha\, eQ_L(|\phi_L|^2 - |\phi_R|^2) - i\omega^\nu\partial_\nu\lambda^\alpha \ , && (13)\\
s\overline{\lambda}_{\dot\alpha} &= \frac{-i}{2}(\overline{\epsilon}\overline{\sigma}^{\rho\sigma})_{\dot\alpha} F_{\rho\sigma} - i\overline{\epsilon}_{\dot\alpha}\, eQ_L(|\phi_L|^2 - |\phi_R|^2) - i\omega^\nu\partial_\nu\overline{\lambda}_{\dot\alpha} \ , && (14)\\
s\phi_L &= -ieQ_L c\,\phi_L + \sqrt{2}\,\epsilon\psi_L - i\omega^\nu\partial_\nu\phi_L \ , && (15)\\
s\phi_L^\dagger &= +ieQ_L c\,\phi_L^\dagger + \sqrt{2}\,\overline{\psi}_L\overline{\epsilon} - i\omega^\nu\partial_\nu\phi_L^\dagger \ , && (16)\\
s\psi_L^\alpha &= -ieQ_L c\,\psi_L^\alpha - \sqrt{2}\,\epsilon^\alpha\, m\phi_R^\dagger - \sqrt{2}\, i(\overline{\epsilon}\overline{\sigma}^\mu)^\alpha D_\mu\phi_L - i\omega^\nu\partial_\nu\psi_L^\alpha \ , && (17)\\
s\overline{\psi}_{L\dot\alpha} &= +ieQ_L c\,\overline{\psi}_{L\dot\alpha} + \sqrt{2}\,\overline{\epsilon}_{\dot\alpha}\, m\phi_R + \sqrt{2}\, i(\epsilon\sigma^\mu)_{\dot\alpha}(D_\mu\phi_L)^\dagger - i\omega^\nu\partial_\nu\overline{\psi}_{L\dot\alpha} \ , && (18)\\
sc &= 2i\epsilon\sigma^\nu\overline{\epsilon}A_\nu - i\omega^\nu\partial_\nu c \ , && (19)\\
s\epsilon^\alpha &= 0 \ , && (20)\\
s\overline{\epsilon}^{\dot\alpha} &= 0 \ , && (21)\\
s\omega^\nu &= 2\epsilon\sigma^\nu\overline{\epsilon} \ , && (22)\\
s\bar{c} &= B - i\omega^\nu\partial_\nu\bar{c} \ , && (23)\\
sB &= 2i\epsilon\sigma^\nu\overline{\epsilon}\partial_\nu\bar{c} - i\omega^\nu\partial_\nu B && (24)
\end{aligned}
$$

and corresponding transformations for the right-handed fields. Here we have introduced also the antighost $\bar{c}$ and the auxiliary field B appearing in the gauge fixing in later course (see eq. (40)).

The symmetries of the classical Lagrangian are summarized in the equation

$$
s\Gamma_{\mathrm{SQED}} = 0 \qquad (25)
$$

for $\Gamma_{\mathrm{SQED}} = \int d^4x \mathcal{L}_{\mathrm{SQED}}$.

The remaining obstructions are the non-linear BRS transformations and the eqs.-of-motion terms in the nilpotency of s. Both are overcome by using external fields. Each non-linear BRS transformation $s\varphi_i$ is coupled to an external field Y_i:

$$
\begin{aligned}
\Gamma_{\mathrm{ext}} = \int d^4x \Big(& Y_\lambda^\alpha s\lambda_\alpha + Y_{\overline{\lambda}\dot\alpha} s\overline{\lambda}^{\dot\alpha} \\
& + Y_{\phi_L} s\phi_L + Y_{\phi_L^\dagger} s\phi_L^\dagger + Y_{\psi_L}^\alpha s\psi_{L\alpha} + Y_{\overline{\psi}_L\dot\alpha} s\overline{\psi}_L^{\dot\alpha} + (L\to R) \Big) \ . \qquad (26)
\end{aligned}
$$

The statistics, dimension and ghost number of the Y_i is such that Γ_{ext} has the same quantum numbers as Γ_{SQED}. In this way we can use the Y_i as sources for the non-linear BRS transformations and write $s\varphi_i = \delta\Gamma_{\mathrm{ext}}/\delta Y_i$, where the r.h.s. possesses a well-defined extension to higher orders. Moreover, as was realized in [17], it is possible to extend the classical action by terms bilinear in the sources that absorb the eqs.-of-motion terms. Hence, the sum

$$\Gamma_{\rm cl} = \Gamma_{\rm SQED} + \Gamma_{\rm ext} + \Gamma_{\rm bil} \ , \tag{27}$$

$$\Gamma_{\rm bil} = -(Y_\lambda \epsilon)(\overline{\epsilon} Y_{\overline{\lambda}}) - 2(Y_{\psi_L}\epsilon)(\overline{\epsilon} Y_{\overline{\psi}_L}) - 2(Y_{\psi_R}\epsilon)(\overline{\epsilon} Y_{\overline{\psi}_R}) \tag{28}$$

satisfies the Slavnov–Taylor identity

$$S(\Gamma_{\rm cl}) = 0 \ . \tag{29}$$

The Slavnov–Taylor operator acting on a general functional $\mathcal{F}$ is defined as

$$\begin{aligned} S(\mathcal{F}) = \int d^4x \Big(& sA^\mu \frac{\delta \mathcal{F}}{\delta A^\mu} + sc\frac{\delta \mathcal{F}}{\delta c} + s\bar{c}\frac{\delta \mathcal{F}}{\delta \bar{c}} + sB\frac{\delta \mathcal{F}}{\delta B} \\ & + \frac{\delta \mathcal{F}}{\delta Y_{\lambda\alpha}}\frac{\delta \mathcal{F}}{\delta \lambda^\alpha} + \frac{\delta \mathcal{F}}{\delta Y_{\overline{\lambda}}^{\dot\alpha}}\frac{\delta \mathcal{F}}{\delta \overline{\lambda}_{\dot\alpha}} \\ & + \frac{\delta \mathcal{F}}{\delta Y_{\phi_L}}\frac{\delta \mathcal{F}}{\delta \phi_L} + \frac{\delta \mathcal{F}}{\delta Y_{\phi_L^\dagger}}\frac{\delta \mathcal{F}}{\delta \phi_L^\dagger} + \frac{\delta \mathcal{F}}{\delta Y_{\psi_L\alpha}}\frac{\delta \mathcal{F}}{\delta \psi_L^\alpha} + \frac{\delta \mathcal{F}}{\delta Y_{\overline{\psi}_L}^{\dot\alpha}}\frac{\delta \mathcal{F}}{\delta \overline{\psi}_{L\dot\alpha}} + (L\to R)\Big) \\ & + s\epsilon^\alpha \frac{\partial \mathcal{F}}{\partial \epsilon^\alpha} + s\overline{\epsilon}_{\dot\alpha}\frac{\partial \mathcal{F}}{\partial \overline{\epsilon}_{\dot\alpha}} + s\omega^\nu \frac{\partial \mathcal{F}}{\partial \omega^\nu} \\ \equiv \int \Big(& s\varphi'_i \frac{\delta \mathcal{F}}{\delta \varphi'_i} + \frac{\delta \mathcal{F}}{\delta Y_i}\frac{\delta \mathcal{F}}{\delta \varphi_i}\Big) \ . \end{aligned} \tag{30}$$

In the last line a symbolic abbreviation has been introduced in which φ'_i runs over all linearly transforming fields and the global ghosts. The electron contributions to $\Gamma_{\rm ext}$ and $S(\mathcal{F})$ can be written in terms of 4-spinors as

$$\Gamma_{\rm ext}|_\Psi = \int d^4x \left(Y_\Psi s\Psi + Y_{\overline{\Psi}} s\overline{\Psi}\right) \ , \tag{31}$$

$$S(\mathcal{F})|_\Psi = \int d^4x \Big(\frac{\delta \mathcal{F}}{\delta Y_\Psi}\frac{\delta \mathcal{F}}{\delta \Psi} + \frac{\delta \mathcal{F}}{\delta Y_{\overline{\Psi}}}\frac{\delta \mathcal{F}}{\delta \overline{\Psi}}\Big) \tag{32}$$

with the 4-spinors from eq. (1) and

$$Y_\Psi = \left(Y_{\psi_L}{}^\alpha, Y_{\overline{\psi}_R\dot\alpha}\right) \ , \quad Y_{\overline{\Psi}} = \begin{pmatrix} -Y_{\psi_R\alpha} \\ -Y_{\overline{\psi}_L}{}^{\dot\alpha}\end{pmatrix} \ , \tag{33}$$

$$\frac{\delta}{\delta Y_\Psi} = \begin{pmatrix} \frac{\delta}{\delta Y_{\psi_L}{}^\alpha} \\ \frac{\delta}{\delta Y_{\overline{\psi}_R\dot\alpha}} \end{pmatrix} \ , \quad \frac{\delta}{\delta Y_{\overline{\Psi}}} = \left(\frac{\delta}{\delta Y_{\psi_R\alpha}}, \frac{\delta}{\delta Y_{\overline{\psi}_L}{}^{\dot\alpha}}\right) \ . \tag{34}$$

The Slavnov–Taylor identity is the key for solving the above mentioned problems since it may be extended to higher orders and it contains both the invariance (25) and the nilpotency (11): The invariance of $\Gamma_{\rm SQED}$ is expressed in the terms without Y_j, and the terms linear in the Y_j express the symmetry algebra acting on the corresponding fields φ_j:

$$(Y_j)^0: \int\Big(s\varphi'_i\frac{\delta\Gamma_{\rm SQED}}{\delta\varphi'_i}+\frac{\delta\Gamma_{\rm ext}}{\delta Y_i}\frac{\delta\Gamma_{\rm SQED}}{\delta\varphi_i}\Big) \qquad \to s\Gamma_{\rm SQED}=0\ , \tag{35}$$

$$(Y_j): \int\Big(s\varphi'_i\frac{\delta\Gamma_{\rm ext}}{\delta\varphi'_i}+\frac{\delta\Gamma_{\rm ext}}{\delta Y_i}\frac{\delta\Gamma_{\rm ext}}{\delta\varphi_i}+\frac{\delta\Gamma_{\rm bil}}{\delta Y_i}\frac{\delta\Gamma_{\rm SQED}}{\delta\varphi_i}\Big)\to s^2\varphi_j = \text{ e.o.m.} \tag{36}$$

The linearized Slavnov–Taylor operator, defined for bosonic functionals $\mathcal{F}$, is given by

$$s_{\mathcal{F}}=\int\Big(s\varphi'_i\frac{\delta}{\delta\varphi'_i}+\frac{\delta\mathcal{F}}{\delta Y_i}\frac{\delta}{\delta\varphi_i}+\frac{\delta\mathcal{F}}{\delta\varphi_i}\frac{\delta}{\delta Y_i}\Big)\ . \tag{37}$$

The full Slavnov–Taylor operator and its linearized version have the nilpotency property

$$s_{\mathcal{F}}S(\mathcal{F})=0 \tag{38}$$

if the functional $\mathcal{F}$ satisfies the linear identity

$$i\epsilon\sigma^\mu\frac{\delta\mathcal{F}}{\delta Y_{\overline{\lambda}}}-i\frac{\delta\mathcal{F}}{\delta Y_\lambda}\sigma^\mu\overline{\epsilon}+i\omega^\nu\partial_\nu(i\epsilon\sigma^\mu\overline{\lambda}-i\lambda\sigma^\mu\overline{\epsilon})-2i\epsilon\sigma_\nu\overline{\epsilon}F^{\nu\mu}=0\ , \tag{39}$$

which is equivalent to nilpotency on A^μ: $s^2_{\mathcal{F}}A^\mu=0$. Eq. (39) is satisfied in particular by $\Gamma_{\rm cl}$.

The gauge fixing term has to be chosen in such a way that renormalizability by power-counting is ensured. We define

$$\begin{aligned}\Gamma_{\rm fix}&=\int d^4x\ s_{\Gamma_{\rm cl}}[\bar{c}(\partial^\mu A_\mu+\frac{\xi}{2}B)]\\&=\int d^4x\Big(B\partial^\mu A_\mu+\frac{\xi}{2}B^2-\bar{c}\Box c\\&\qquad-\bar{c}\partial^\mu(i\epsilon\sigma_\mu\overline{\lambda}-i\lambda\sigma_\mu\overline{\epsilon})+\xi i\epsilon\sigma^\nu\overline{\epsilon}(\partial_\nu\bar{c})\bar{c}\Big)\end{aligned} \tag{40}$$

with a real gauge parameter ξ. This gauge fixing term is added to the classical action:

$$\Gamma_{\rm cl} \to \Gamma_{\rm cl} + \Gamma_{\rm fix} \ . \tag{41}$$

Introducing the gauge fixing in this way the Slavnov–Taylor identity remains valid. Indeed we see that in addition to the usual QED gauge fixing and ghost terms, which break supersymmetry, there arise compensating terms dependent on the constant ghost fields $\epsilon, \bar{\epsilon}$.

Symmetry requirements on Γ: The symmetry properties of $\Gamma_{\rm cl}$ are now imposed as constraints on Γ. In addition to the Slavnov–Taylor identity several linear equations and manifest symmetries are imposed. To summarize:

- Slavnov–Taylor identity and nilpotency of s_Γ:

  $$S(\Gamma) = 0 \ , \tag{42}$$
  $$s_\Gamma^2 A^\mu = 0 \ . \tag{43}$$

  The latter condition is equivalent to eq. (39) for $\mathcal{F} = \Gamma$, and according to eq. (38) it is already sufficient for the nilpotency relation $s_\Gamma S(\Gamma) = 0$.

- Gauge fixing condition, ghost equations:

  $$\frac{\delta \Gamma}{\delta B} = \frac{\delta \Gamma_{\rm cl}}{\delta B}, \quad \frac{\delta \Gamma}{\delta c} = \frac{\delta \Gamma_{\rm cl}}{\delta c}, \quad \frac{\delta \Gamma}{\delta \omega^\mu} = \frac{\delta \Gamma_{\rm cl}}{\delta \omega^\mu}, \quad \frac{\delta \Gamma}{\delta \bar{c}} = \frac{\delta \Gamma_{\rm cl}}{\delta \bar{c}} \ . \tag{44}$$

  It is possible to require that these derivatives do not receive quantum corrections since they are linear in the dynamical fields at the classical level. These equations serve as normalization conditions; their physical consequences are explained in the next subsection.

- Manifest symmetries: We require Γ to be invariant under the discrete symmetries R, C, CP and to be electrically and ghost charge neutral, Lorentz invariant and bosonic. The quantum numbers of the fields are determined by the corresponding symmetries of $\Gamma_{\rm cl}$ and are listed in tab. 1, 2. Note that the usual R-parity is the same as our R^2 and thus less restrictive than our R. Contrary to the preceding symmetries, we assume these ones to be manifestly preserved, which is true for all common regularization schemes.

χ	x^μ	A^μ	$-i\lambda^\alpha$	ϕ_L	ϕ_R	ψ_L^α	ψ_R^α	c	ϵ^α	ω^ν	$\bar{c}$	B
$R\chi$	x^μ	A^μ	$-\lambda^\alpha$	$-i\phi_L$	$-i\phi_R$	ψ_L^α	ψ_R^α	c	$-i\epsilon^\alpha$	ω^ν	$\bar{c}$	B
$C\chi$	x^μ	$-A^\mu$	$i\lambda^\alpha$	ϕ_R	ϕ_L	ψ_R^α	ψ_L^α	$-c$	ϵ^α	ω^ν	$-\bar{c}$	$-B$
$CP\chi$	$(\mathcal{P}x)^\mu$	$-(\mathcal{P}A)^\mu$	$-\overline{\lambda}_{\dot\alpha}$	$\phi_L^\dagger$	$\phi_R^\dagger$	$i\overline{\psi}_{L\dot\alpha}$	$i\overline{\psi}_{R\dot\alpha}$	$-c$	$-i\overline{\epsilon}_{\dot\alpha}$	$(\mathcal{P}\omega)^\nu$	$-\bar{c}$	$-B$

Table 1: Discrete symmetries. The transformation rules for the sources Y_i can be deduced from the requirement that $\Gamma_{\rm ext}$ is invariant and the transformation rules for the complex conjugate fields are obvious except for the CP conjugation of the spinors. We define for $\chi \in \{\lambda, \psi_L, \psi_R, \epsilon\}$:

$$\chi^\alpha \stackrel{CP}{\to} a\overline{\chi}_{\dot\alpha} \Rightarrow \overline{\chi}_{\dot\alpha} \stackrel{CP}{\to} -a^*\chi^\alpha \ , \ \chi_\alpha \stackrel{CP}{\to} -a\overline{\chi}^{\dot\alpha} \ , \ \overline{\chi}^{\dot\alpha} \stackrel{CP}{\to} a^*\chi_\alpha \ .$$

χ	x^μ	A^μ	$-i\lambda^\alpha$	ϕ_L	ϕ_R	ψ_L^α	ψ_R^α	c	ϵ^α	ω^ν	$\bar{c}$	B
Q	0	0	0	-1	$+1$	-1	$+1$	0	0	0	0	0
Q_c	0	0	0	0	0	0	0	$+1$	$+1$	$+1$	-1	0
GP	0	0	1	0	0	1	1	1	0	1	1	0
dim	-1	1	3/2	1	1	3/2	3/2	0	$-1/2$	-1	2	2

Table 2: Quantum numbers. Q, Q_c, GP, dim denote electrical and ghost charge, Grassmann parity and the mass dimension, respectively. The quantum numbers of the sources Y_i can be obtained from the requirement that $\Gamma_{\rm ext}$ is neutral, bosonic and has $dim = 4$. The commutation rule for two general fields is $\chi_1\chi_2 = (-1)^{GP_1 GP_2}\chi_2\chi_1$.

2.4 Immediate consequences

The conditions for $\bar{c}$ and B in eq. (44) forbid any quantum corrections to $\Gamma_{\rm fix}$ and thus play the role of gauge fixing conditions. The ghost equations in eq. (44) for c, ω^μ have a direct physical consequence: They imply, in connection with the Slavnov–Taylor identity, Ward identities for electrical current conservation and translational invariance. This can be seen from the following consistency equations for general bosonic functionals $\mathcal{F}$:

$$\frac{\delta}{\delta c}S(\mathcal{F}) + s_{\mathcal{F}}\frac{\delta\mathcal{F}}{\delta c} = -\partial^\mu\frac{\delta\mathcal{F}}{\delta A^\mu} - i\omega^\nu\partial_\nu\frac{\delta\mathcal{F}}{\delta c} \ , \tag{45}$$

$$\frac{\delta}{\delta\omega^\mu}S(\mathcal{F}) + s_{\mathcal{F}}\frac{\delta\mathcal{F}}{\delta\omega^\mu} = -i\int(\partial_\mu\varphi_i')\frac{\delta\mathcal{F}}{\delta\varphi_i'} \ , \tag{46}$$

$$\frac{\delta}{\delta\bar{c}}S(\mathcal{F}) + s_{\mathcal{F}}\frac{\delta\mathcal{F}}{\delta\bar{c}} = -2i\epsilon\sigma^\nu\bar{\epsilon}\partial_\nu\frac{\delta\mathcal{F}}{\delta B} - i\omega^\nu\partial_\nu\frac{\delta\mathcal{F}}{\delta\bar{c}} \ , \tag{47}$$

$$\frac{\delta}{\delta B}S(\mathcal{F}) - s_{\mathcal{F}}\frac{\delta\mathcal{F}}{\delta B} = \frac{\delta\mathcal{F}}{\delta\bar{c}} + i\omega^\nu\partial_\nu\frac{\delta\mathcal{F}}{\delta B} \ . \tag{48}$$

For $\mathcal{F} = \Gamma$ and $S(\Gamma) = 0$ the first two equations lead to the announced Ward identities:

$$\partial^\mu\frac{\delta\Gamma}{\delta A^\mu} = -iew_{\mathrm{em}}\Gamma - \Box B + \mathcal{O}(\omega) \ , \tag{49}$$

$$\begin{aligned} w_{\mathrm{em}} = Q_L\Big(&\phi_L\frac{\delta}{\delta\phi_L} - Y_{\phi_L}\frac{\delta}{\delta Y_{\phi_L}} + \psi_L\frac{\delta}{\delta\psi_L} - Y_{\psi_L}\frac{\delta}{\delta Y_{\psi_L}} \\ &- \phi_L^\dagger\frac{\delta}{\delta\phi_L^\dagger} + Y_{\phi_L^\dagger}\frac{\delta}{\delta Y_{\phi_L^\dagger}} - \overline{\psi}_L\frac{\delta}{\delta\overline{\psi}_L} - Y_{\overline{\psi}_L}\frac{\delta}{\delta Y_{\overline{\psi}_L}}\Big) \\ &+ (\scriptstyle{L\to R}) \end{aligned} \tag{50}$$

and

$$0 = \int d^4x\Big(\partial_\mu\varphi_i'\frac{\delta\Gamma}{\delta\varphi_i'} + \partial_\mu\varphi_i\frac{\delta\Gamma}{\delta\varphi_i} + \partial_\mu Y_i\frac{\delta\Gamma}{\delta Y_i}\Big) \ . \tag{51}$$

The ω-dependent terms in the electromagnetic Ward identity (49) arise because translations do not commute with local gauge transformations.

Conversely, if the Ward identities and the linear eqs. (43), (44) hold, the consistency equations yield

$$\frac{\delta}{\delta c}S(\Gamma) = \frac{\delta}{\delta\omega^\nu}S(\Gamma) = \frac{\delta}{\delta\bar{c}}S(\Gamma) = \frac{\delta}{\delta B}S(\Gamma) = 0 \ . \tag{52}$$

In this case, therefore, the Slavnov–Taylor identity can not be broken by terms depending on $c, \omega^\nu, \bar{c}, B$.

2.5 Most general symmetric counterterms

The symmetry requirements fix Γ up to additive symmetric counterterms in each order. To find the symmetric counterterms we take two solutions Γ and $\tilde{\Gamma} = \Gamma + \zeta\Gamma_{\rm sym}$ of the symmetry requirements at first order in the infinitesimal parameter ζ and calculate the most general counterterms $\Gamma_{\rm sym}$. The requirements that the Slavnov–Taylor identity eq. (42) is satisfied at first order in ζ can be cast into the form

$$s_{\Gamma_{\rm cl}}\Gamma_{\rm sym} = \int \left(s\varphi_i' \frac{\delta\Gamma_{\rm sym}}{\delta\varphi_i'} + \frac{\delta\Gamma_{\rm cl}}{\delta Y_i}\frac{\delta\Gamma_{\rm sym}}{\delta\varphi_i} + \frac{\delta\Gamma_{\rm sym}}{\delta Y_i}\frac{\delta\Gamma_{\rm cl}}{\delta\varphi_i} \right) = 0 \; , \tag{53}$$

and eq. (44) prevents a dependence of $\Gamma_{\rm sym}$ on $B, c, \omega^\nu, \bar{c}$. The solution reads

$$\begin{aligned}
\Gamma_{\rm sym} = \Big[& \delta Z_m m \frac{\partial}{\partial m} \\
& + \frac{1}{2}\delta Z_\gamma \Big(-e\frac{\partial}{\partial e} + 2\xi\frac{\partial}{\partial\xi} \\
& + \int d^4x \Big(A^\mu \frac{\delta}{\delta A^\mu} + \lambda^\alpha \frac{\delta}{\delta\lambda^\alpha} - Y_\lambda{}^\alpha \frac{\delta}{\delta Y_\lambda{}^\alpha} + \overline{\lambda}_{\dot\alpha}\frac{\delta}{\delta\overline{\lambda}_{\dot\alpha}} - Y_{\overline{\lambda}\dot\alpha}\frac{\delta}{\delta Y_{\overline{\lambda}\dot\alpha}} \\
& + c\frac{\delta}{\delta c} - \bar{c}\frac{\delta}{\delta\bar{c}} - B\frac{\delta}{\delta B} \Big) \Big) \\
& + \frac{1}{2}\delta Z_\phi \int d^4x \Big(\phi_L \frac{\delta}{\delta\phi_L} - Y_{\phi_L}\frac{\delta}{\delta Y_{\phi_L}} + \phi_R\frac{\delta}{\delta\phi_R} - Y_{\phi_R}\frac{\delta}{\delta Y_{\phi_R}} \\
& + \phi_L^\dagger \frac{\delta}{\delta\phi_L^\dagger} - Y_{\phi_L^\dagger}\frac{\delta}{\delta Y_{\phi_L^\dagger}} + \phi_R^\dagger\frac{\delta}{\delta\phi_R^\dagger} - Y_{\phi_R^\dagger}\frac{\delta}{\delta Y_{\phi_R^\dagger}} \Big) \\
& + \frac{1}{2}\delta Z_\Psi \int d^4x \Big(\Psi\frac{\delta}{\delta\Psi} - Y_\Psi\frac{\delta}{\delta Y_\Psi} + \overline{\Psi}\frac{\delta}{\delta\overline{\Psi}} - Y_{\overline{\Psi}}\frac{\delta}{\delta Y_{\overline{\Psi}}} \Big) \Big] \; \Gamma_{\rm cl}
\end{aligned} \tag{54}$$

with four free constants δZ_m, δZ_γ, δZ_ϕ, δZ_Ψ. The condition for $\frac{\delta\Gamma}{\delta c}$ in (44) and the Ward identity (49) result in e being the effective charge in the Thomson limit (see section 3.3) and thus prevent an independent charge renormalization. The action of this differential operator on the classical action just corresponds to a multiplicative renormalization of the parameters and fields appearing therein. That means that after restoring the symmetries all divergencies from the loop diagrams may be absorbed by redefinitions of the parameters and fields appearing in $\Gamma_{\rm cl}$, which is the usual understanding of multiplicative renormalizability.

2.6 Normalization conditions

To fix the remaining ambiguity of the symmetric counterterms we impose the usual normalization conditions[2] for QED. These are on-shell normalization conditions for the mass parameter and the photon self energy, and conditions at an arbitrary scale κ for the normalization of the matter self energies:[3]

$$\Gamma_{\phi_L \phi_L^\dagger}(-p,p) = 0 \text{ for } p^2 = m^2 \ , \tag{55}$$

$$\lim_{p^2\to 0} \frac{1}{p^2}\Gamma_{A^\mu A^\nu}(-p,p)|_{g_{\mu\nu}-\text{part}} = -g_{\mu\nu} \ , \tag{56}$$

$$\frac{\partial}{\partial p^2}\Gamma_{\phi_L \phi_L^\dagger}(-p,p) = 1 \text{ for } p^2 = \kappa^2 \ , \tag{57}$$

$$\Gamma_V(p^2) + 2m^2(\Gamma_V'(p^2) - \Gamma_S'(p^2)) = 1 \text{ for } p^2 = \kappa^2 \ . \tag{58}$$

Here we have used a covariant decomposition for the electron self energy:

$$\Gamma_{\Psi\overline{\Psi}}(p,-p) = \not{p}\Gamma_V(p^2) - m\Gamma_S(p^2) \tag{59}$$

with scalar functions $\Gamma_{V,S}$. Since these normalization conditions have a unique classical (i.e. tree level) solution, they fix Γ uniquely to all orders.

We did not require the residua of the matter propagators to be unity on-shell. It is useful to define the functions

$$Z_\phi(p^2) = \left(\partial_{p^2}\Gamma_{\phi_L\phi_L^\dagger}(-p,p)\right)^{-1} \ , \tag{60}$$

$$Z_\Psi(p^2) = \left(\Gamma_V(p^2) + 2m^2(\Gamma_V'(p^2) - \Gamma_S'(p^2))\right)^{-1} \ . \tag{61}$$

They approach the usual (infrared divergent) Z factors in the limit $p^2 \to m^2$ and appear in the LSZ reduction formula as wave function renormalization factors:

$$S_{fi} = \lim_{p^2\to m^2} \left(i Z_\phi{}^{-1/2}(p^2)(-p^2+m^2)\ldots\langle 0|T\phi\ldots|0\rangle(p,\ldots)\right) \ . \tag{62}$$

For the present paper they play a role in the symmetry conditions derived in the next section.

[2]In the literature also labeled as "renormalization conditions".

[3]$\kappa^2 = m^2$ would lead to infrared divergences in the normalization conditions.

3 Symmetry conditions

The Slavnov–Taylor identity (42) is a complicated non-linear equation for the effective action with an enormous information content. In this section we will show that it is possible to obtain much simpler symmetry conditions as a consequence of the Slavnov–Taylor identity and the normalization conditions. One virtue of these symmetry conditions is that they are well suited for practical applications. Together with the normalization conditions and the conditions in eqs. (43), (44) they form a complete set of simple identities that determine the counterterms of all power-counting renormalizable interactions. A similar strategy was applied by [20] in the context of the abelian Higgs-Kibble model.

We begin this section with a particularly simple symmetry condition, to illustrate our general method. This example also shows that is useful to divide the symmetry conditions into two parts: the ones for vertex functions containing external sources, expressing the higher order modifications to the symmetry transformations, and the ones for the vertex functions for physical fields.

Let us make some remarks on our notation and conventions. The manifest symmetries are always implicitly used, in particular R-parity violating vertex functions are not mentioned and the conditions involving one selectron field are only given for one of the fields ϕ_L, ϕ_R, $\phi_L^\dagger$, $\phi_R^\dagger$. Since it is easier to work with fields of a definite R-parity the 2-spinors $\lambda, \overline{\lambda}, \epsilon, \overline{\epsilon}$ and the 4-spinors $\Psi, \overline{\Psi}$ are used during the derivations and only for the final results either a pure 2-spinor or a pure 4-spinor notation is chosen. Most of the following identities stem from some derivative of the Slavnov–Taylor identity $\delta S(\Gamma)/\delta\chi_1 \dots \delta\chi_n = 0$, leading to products of the form

$$\Gamma_{\chi_1 \dots \chi_m Y_i}(p_1, \dots, p_m, -p)\Gamma_{\chi_{m+1} \dots \chi_n \varphi_i}(p_{m+1}, \dots, p_n, p) \tag{63}$$

with $p = p_1 + \dots + p_m = -p_{m+1} - \dots - p_n$ due to momentum conservation. The definition of the vertex functions is given in app. A.2. Because this structure is general, the momenta in the arguments are not always written down explicitly.

3.1 Electron–selectron mass identity

The normalization condition

$$\Gamma_{\phi_L \phi_L^\dagger}(-p, p) = 0 \text{ for } p^2 = m^2 \tag{64}$$

defines m to be the physical selectron mass. Using the Slavnov–Taylor identity we will now prove the following symmetry condition:

$$\Gamma_{\Psi\overline{\Psi}}(p,-p)u(p) = 0 \text{ for } p^2 = m^2 \ , \tag{65}$$

where $u(p)$ is a spinor satisfying the Dirac equation $(\not{p} - m)u(p) = 0$. Physically this means that m is equal to the physical electron mass, and thus the electron and selectron masses are equal.

The strategy for the proofs of the symmetry conditions is first to obtain identities between vertex functions in the usual way taking suitable derivatives of the Slavnov–Taylor identity and setting all fields to zero afterwards. These non-linear identities can then be solved for particular vertex functions and further simplified if one evaluates them at the special momenta of the normalization conditions.

Since the condition we want to prove is due to supersymmetry, we use one derivative with respect to ϵ:

$$\frac{\partial}{\partial\epsilon}\frac{\delta^2}{\delta\phi_L^\dagger(-p)\delta\Psi(p)}S(\Gamma)|_{\varphi_i=Y_i=0} = 0 \ . \tag{66}$$

After setting all fields to zero most of the terms vanish due to charge non-conservation, and only two terms contribute:

$$\Gamma_{\Psi\epsilon Y_{\phi_L}}(p,-p)\Gamma_{\phi_L^\dagger\phi_L}(-p,p) + \Gamma_{\phi_L^\dagger\epsilon Y_{\overline{\Psi}}}(-p,p)\Gamma_{\Psi\overline{\Psi}}(p,-p) = 0 \ . \tag{67}$$

For $p^2 = m^2$ the normalization condition (64) and $\Gamma_{\phi_L^\dagger\epsilon Y_{\overline{\Psi}}} \neq 0$ show that the spinor matrix $\Gamma_{\Psi\overline{\Psi}}(p,-p)$ has the eigenvalue zero and thus cannot be invertible. Since it must be built out of the covariants 1 and $\not{p}$ it can only be proportional to $(\not{p} - m)$ or $(\not{p} + m)$. Taking into account the lowest order result the second possibility is excluded and the announced result (65) follows.

3.2 Higher order supersymmetry

Eq. (67) exhibits a general feature of the equations derived below, namely the appearance of prefactors that are themselves vertex functions with external sources and ghost fields, reflecting the non-linearity of the Slavnov–Taylor identity. Their physical meaning is to represent renormalized higher order corrections to the symmetry transformations coupled to the sources in Γ_{ext}, which will be explained in more detail in sec. 4.5. It is necessary to derive symmetry conditions for such vertex functions before we are able to derive further identities for vertex functions involving only physical fields.

In fact, all vertex functions involving external c or ω^μ ghosts — expressing the exact gauge transformations and translations — are already fixed to all orders by the requirements in eq. (44). The vertex functions involving external ϵ ghosts and Y fields express the supersymmetry transformations. They may acquire higher order corrections, but it is still possible to derive symmetry conditions constraining these modifications because the symmetry algebra is fixed to all orders.

First we derive the supersymmetry transformations of the photino, i.e. the vertex functions with external ϵ and Y_λ. There are only three terms of dimension ≤ 4 possible: $Y_\lambda\epsilon A^\mu$, $Y_\lambda\epsilon|\phi_{L,R}|^2$, $Y_\lambda\epsilon Y_{\overline{\lambda}}\overline{\epsilon}$ and their CP-conjugates. To constrain the first one we use the nilpotency on A^μ, which expresses the supersymmetry algebra:

$$0 = \frac{\partial^2}{\partial\overline{\epsilon}\partial\epsilon}\frac{\delta}{\delta A^\rho}s_\Gamma^2 A^\mu \tag{68}$$

$$\begin{aligned}\Rightarrow 0 = {}& i\Gamma_{A^\rho\epsilon^\beta Y_{\lambda\alpha}}\sigma^\mu_{\alpha\dot{\beta}} + i\sigma^\mu_{\beta\dot{\alpha}}\Gamma_{A^\rho\overline{\epsilon}^{\dot{\beta}}Y_{\overline{\lambda}\dot{\alpha}}} \\ & + 2\not{p}_{\beta\dot{\beta}}g_\rho{}^\mu - 2p^\mu\sigma_{\rho\beta\dot{\beta}}\ .\end{aligned} \tag{69}$$

The first line contains products of the transformation of the photon into a photino and vice versa, the second line a sum of a translation and a gauge transformation. The explicit σ matrices originate from $\frac{\partial^2}{\partial\epsilon\partial\overline{\lambda}}sA^\mu$ and $h.c.$, and the terms in the second line from the BRS transformations of the ω and c ghosts in sA^μ. All terms are fixed except for the ones containing $Y_\lambda, Y_{\overline{\lambda}}$. Taking into account the Ward identity (49), leading to $p^\rho\Gamma_{A^\rho\epsilon Y_\lambda} = 0$, implies in connection with CP invariance:

$$\Gamma_{A^\mu\epsilon^\beta Y_{\lambda\alpha}}(p,-p) = p^\rho(\sigma_{\rho\mu})_\beta{}^\alpha\ . \tag{70}$$

Next we use the supersymmetry algebra acting on λ, which is expressed in the Slavnov–Taylor identity by the terms proportional to $\overline{\epsilon}\epsilon Y_\lambda$:

$$0 = \frac{\partial^2}{\partial\overline{\epsilon}\partial\epsilon}\frac{\delta^2}{\delta\lambda\delta Y_\lambda}S(\Gamma) \tag{71}$$

$$\Rightarrow 0 = \tfrac{\partial^2(sA^\mu)}{\partial\lambda\partial\overline{\epsilon}}\Gamma_{\epsilon Y_\lambda A^\mu} + \Gamma_{\overline{\epsilon}\epsilon Y_\lambda Y_{\overline{\lambda}}}\Gamma_{\lambda\overline{\lambda}} + \tfrac{\partial^2(s\omega^\mu)}{\partial\overline{\epsilon}\partial\epsilon}\Gamma_{\lambda Y_\lambda\omega^\mu}\ . \tag{72}$$

The only unknown here is the vertex function with two external sources corresponding to an eqs.-of-motion term in the algebra (5). Solving (72) yields

$$\Gamma_{\overline{\epsilon}_{\dot{\beta}}\epsilon^\beta Y_{\lambda\gamma}Y_{\overline{\lambda}}{}^{\dot{\gamma}}}(p,-p)\Gamma_{\lambda_\alpha\overline{\lambda}_{\dot{\gamma}}}(-p,p) = \delta_\beta{}^\gamma\not{p}^{\dot{\beta}\alpha}\ . \tag{73}$$

For the supersymmetry transformation of the photino into $|\phi_{L,R}|^2$ we derive the equation

$$0 = \frac{\partial}{\partial\epsilon} \frac{\delta^3}{\delta\phi_L^\dagger \delta\phi_L \delta\overline{\lambda}} S(\Gamma) \tag{74}$$

$$\Rightarrow 0 = \tfrac{\partial^2(sA^\mu)}{\partial\overline{\lambda}\partial\epsilon} \Gamma_{\phi_L^\dagger \phi_L A^\mu} + \Gamma_{\phi_L^\dagger \phi_L \epsilon Y_\lambda} \Gamma_{\overline{\lambda}\lambda} + \Gamma_{\phi_L^\dagger \epsilon Y_{\overline{\Psi}}} \Gamma_{\phi_L \overline{\lambda}\overline{\Psi}} \ . \tag{75}$$

For $p_{\overline{\lambda}} = 0$ this equation may be used to determine $\Gamma_{\phi_L \overline{\lambda}\overline{\Psi}}$ (see eq. (102)), for $p_{\overline{\lambda}} \neq 0$ it may be used as a symmetry condition for $\Gamma_{\phi_L^\dagger \phi_L \epsilon Y_\lambda}$.

Now we proceed with symmetry conditions for the supersymmetry transformations of the matter fields. While the mass identity (67) fixes the ratio of the supersymmetry transformations $\phi \leftrightarrow \Psi$, the supersymmetry algebra

$$0 = \frac{\partial^2}{\partial\overline{\epsilon}\partial\epsilon} \frac{\delta^2}{\delta\phi_L \delta Y_{\phi_L}} S(\Gamma) \tag{76}$$

$$\Rightarrow 0 = \Gamma_{Y_{\phi_L} \overline{\epsilon}\epsilon Y_{\phi_L^\dagger}} \Gamma_{\phi_L \phi_L^\dagger} + \Gamma_{\phi_L \overline{\epsilon} Y_\Psi} \Gamma_{\epsilon Y_{\phi_L} \Psi} + \tfrac{\partial^2(s\omega^\mu)}{\partial\overline{\epsilon}\partial\epsilon} \Gamma_{\phi_L Y_{\phi_L} \omega^\mu} \tag{77}$$

fixes the product. For on-shell momentum the eqs.-of-motion term vanishes and (77) reduces to

$$2\not{p}_{\beta\dot{\beta}} = \Gamma_{\phi_L \overline{\epsilon}^{\dot{\beta}} Y_\Psi}(p,-p) \Gamma_{\epsilon^\beta Y_{\phi_L} \Psi}(-p,p) \text{ for } p^2 = m^2 \ . \tag{78}$$

Solving for the individual vertex functions is best done using the covariant decompositions

$$\Gamma_{\phi_L \overline{\epsilon}^{\dot{\beta}} Y_{\overline{\psi}_R \dot{\alpha}}}(p,-p) = -\sqrt{2}\Theta_1(p^2) m \delta^{\dot{\alpha}}{}_{\dot{\beta}} \ , \tag{79}$$

$$\Gamma_{\phi_L \overline{\epsilon}^{\dot{\beta}} Y_{\psi_L}{}^\alpha}(p,-p) = -\sqrt{2}\Theta_2(p^2) \not{p}_{\alpha\dot{\beta}} \tag{80}$$

with $\Theta_1(m^2) = \Theta_2(m^2)$ due to (67), (65). The results are the following symmetry conditions:

$$\text{for } p^2 = m^2 \ :$$

$$\Gamma_{\phi_L \overline{\epsilon}^{\dot{\beta}} Y_{\psi_L}{}^\alpha}(p,-p) = -\sqrt{2}\not{p}_{\alpha\dot{\beta}} \ \Theta \ , \tag{81}$$

$$\Gamma_{\phi_L \overline{\epsilon}^{\dot{\beta}} Y_{\overline{\psi}_R \dot{\alpha}}}(p,-p) = -\sqrt{2} m \delta^{\dot{\alpha}}{}_{\dot{\beta}} \ \Theta \ , \tag{82}$$

$$\not{p}_{\alpha\dot{\beta}} \Gamma_{\psi_{L\alpha} \epsilon^\beta Y_{\phi_L}}(p,-p) = -m \Gamma_{\overline{\psi}_R{}^{\dot{\beta}} \epsilon^\beta Y_{\phi_L}}(p,-p) - \sqrt{2}\not{p}_{\beta\dot{\beta}} \frac{1}{\Theta} \ , \tag{83}$$

$$\Theta = \lim_{p^2 \to m^2} \sqrt{Z_\psi(p^2)/Z_\phi(p^2)} \ . \tag{84}$$

Using these results together with the gauge covariance of the supersymmetry transformation of ψ_L

$$0 = \frac{\partial}{\partial \bar{\epsilon}^{\dot{\beta}}} \frac{\delta^3}{\delta c \delta \phi_L \delta Y^\alpha_{\psi_L}} S(\Gamma) \tag{85}$$

then yields

$$q^\mu \Gamma_{A^\mu \phi_L \bar{\epsilon}^{\dot{\beta}} Y^\alpha_{\psi_L}}(q, p, p') = \sqrt{2} e Q_L \not{q}_{\alpha\dot{\beta}}\ \Theta \text{ for } p^2 = p'^2 = m^2 \ . \tag{86}$$

Finally we determine the coefficient of the eqs.-of-motion term in the supersymmetry algebra acting on ψ_L, given by $\Gamma_{\epsilon\bar{\epsilon}Y_{\psi_L} Y_{\overline{\psi}_L}}$:

$$0 = \frac{\partial^2}{\partial \epsilon \partial \bar{\epsilon}} \frac{\delta^2}{\delta \psi_L \delta Y_{\psi_L}} S(\Gamma) \tag{87}$$

$$\begin{aligned} \Rightarrow 0 = {} & \Gamma_{\epsilon^\beta \bar{\epsilon}^{\dot{\beta}} Y^\gamma_{\psi_L} Y^{\dot{\delta}}_{\overline{\psi}_L}} \Gamma_{\psi_{L\alpha} \overline{\psi}_{L\dot{\delta}}} + \Gamma_{\epsilon^\beta \bar{\epsilon}^{\dot{\beta}} Y^\gamma_{\psi_L} Y_{\psi_R \delta}} \Gamma_{\psi_{L\alpha} \psi^\delta_R} \\ & \Gamma_{\psi_{L\alpha} \epsilon^\beta Y_{\phi_L}} \Gamma_{Y^\gamma_{\psi_L} \bar{\epsilon}^{\dot{\beta}} \phi_L} + \Gamma_{\psi_{L\alpha} \bar{\epsilon}^{\dot{\beta}} Y_{\phi^\dagger_R}} \Gamma_{Y^\gamma_{\psi_L} \epsilon^\beta \phi^\dagger_R} \\ & - 2 \not{p}_{\beta\dot{\beta}} \delta^\gamma_\alpha \ . \end{aligned} \tag{88}$$

Since all other vertex functions of dimension ≤ 4 have already been fixed, this identity can be viewed as a symmetry condition for $\Gamma_{\epsilon\bar{\epsilon}Y_{\psi_L} Y_{\overline{\psi}_L}}$.

3.3 Physical conditions

In addition to the mass equality from subsection 3.1 here we derive further symmetry conditions for physical vertex functions. Thereby we make use of the conditions derived in sec. 3.2, expressing the higher order modifications to the supersymmetry transformations, and of the requirements (44), (49) that there are no higher order corrections to gauge transformations.

Due to supersymmetry, the photon and photino self energies are related:

$$0 = \frac{\partial}{\partial \epsilon} \frac{\delta^2}{\delta A^\rho \delta \overline{\lambda}} S(\Gamma) \tag{89}$$

$$\Rightarrow 0 = \tfrac{\partial^2 (s A^\mu)}{\partial \overline{\lambda} \partial \epsilon} \Gamma_{A^\rho A^\mu} + \Gamma_{A^\rho \epsilon Y_\lambda} \Gamma_{\overline{\lambda}\lambda} \ . \tag{90}$$

The prefactor $\Gamma_{A^\rho \epsilon Y_\lambda}$, expressing the supersymmetry transformation of the photino, is determined to all orders by (70) and thus

$$\sigma^\mu_{\beta\dot\alpha}\Gamma_{A^\rho A^\mu}(p,-p) = -ip^\nu(\sigma_{\nu\rho})_\beta{}^\alpha \Gamma_{\overline{\lambda}^{\dot\alpha}\lambda^\alpha}(-p,p) \ . \tag{91}$$

We can use this identity together with the normalization condition (56) and the symmetry condition (73) to get simpler conditions:

$$\Gamma_{\overline{\lambda}^{\dot\alpha}\lambda^\alpha}(-p,p) = \not{p}_{\alpha\dot\alpha} \text{ for } p^2 = 0 \ , \tag{92}$$

$$\Gamma_{\overline{\epsilon}_{\dot\beta}\epsilon^\beta Y_{\lambda\gamma} Y_{\overline{\lambda}}{}^{\dot\gamma}}(p,-p) = \delta^{\dot\beta}{}_{\dot\gamma}\delta_\beta{}^\gamma \text{ for } p^2 = 0 \ . \tag{93}$$

Using suitable derivatives of the Ward identity (49) we find that gauge invariance restricts the remaining power-counting renormalizable photon and photino interactions:

$$0 = p^\mu \Gamma_{A^\rho A^\mu}(-p,p) \ , \tag{94}$$

$$0 = p^\mu \Gamma_{A^\rho A^\sigma A^\mu}(p',-p-p',p) \ , \tag{95}$$

$$0 = p^\mu \Gamma_{A^\rho A^\sigma A^\nu A^\mu}(p',p'',-p-p'-p'',p) \ , \tag{96}$$

$$0 = p^\mu \Gamma_{\lambda\overline{\lambda}A^\mu}(p',-p-p',p) \ . \tag{97}$$

Similarly, gauge invariance (49) yields symmetry conditions for the photon–matter interactions, in particular

$$q^\mu \Gamma_{\Psi\overline{\Psi}A^\mu}(p,p',q) = -eQ_L\left(\Gamma_{\Psi\overline{\Psi}}(-p',p') - \Gamma_{\Psi\overline{\Psi}}(p,-p)\right) \ . \tag{98}$$

Taking the derivative with respect to q^μ at $q = 0$ and the limit $p^2 \to m^2$ and multiplying with spinors satisfying the Dirac equation $(\not{p} - m)u(p) = 0$, yields the Thomson-limit condition

$$\bar u(p) Z_\Psi \Gamma_{\Psi\overline{\Psi}A^\mu}(p,-p,0)u(p) = \bar u(p)\left(-eQ_L\gamma_\mu\right)u(p) \qquad \text{for } p^2 = m^2 \ . \tag{99}$$

Thomson-limit conditions for the photon–selectron interactions may be obtained in the same way:

$$Z_\phi \Gamma_{\phi_L \phi_L^\dagger A^\mu}(p,-p,0) = -2eQ_L p_\mu \quad \text{for } p^2 = m^2 \ , \tag{100}$$

$$Z_\phi \Gamma_{\phi_L \phi_L^\dagger A^\nu A^\mu}(p,-p,0,0) = 2(eQ_L)^2 g_{\mu\nu} \quad \text{for } p^2 = m^2 \ . \tag{101}$$

The functions $Z_\Psi(p^2), Z_\phi(p^2)$ have been defined in eqs. (60), (61). For brevity the momentum arguments have been suppressed. Instead of gauge invariance, supersymmetry is responsible for a Thomson-limit condition for the photino–matter interaction. Using (75) for $p_{\overline{\lambda}} = 0$ together with (100) and (81), (82) it can be derived either in terms of 2-spinors:

$$\begin{aligned}\sqrt{Z_\phi Z_\Psi}\Big(&\Gamma_{\phi_L^\dagger \psi_{L\alpha}\lambda^\beta}(-p,p,0)\not{p}_{\alpha\dot\beta} \\ &+ \Gamma_{\phi_L^\dagger \overline{\psi}_R^{\dot\alpha}\lambda^\beta}(-p,p,0)m\delta^{\dot\alpha}_{\dot\beta}\Big) = -i\sqrt{2}eQ_L\not{p}_{\beta\dot\beta} \quad \text{for } p^2 = m^2 \ , \end{aligned} \tag{102}$$

or of 4-spinors:

$$\sqrt{Z_\phi Z_\Psi}\Gamma_{\phi_L^\dagger \Psi\overline{\tilde\gamma}}(-p,p,0)u(p) = -\sqrt{2}eQ_L P_L u(p) \quad \text{for } p^2 = m^2 \ . \tag{103}$$

The remaining power-counting renormalizable interactions are the four-scalar interactions. Supersymmetry relates them to the photino–matter interaction and thus to the gauge coupling in the following way:

$$0 = \frac{\partial}{\partial\epsilon}\frac{\delta^4}{\delta\phi_L^\dagger\delta\phi_L\delta\phi_L^\dagger\delta\psi_L}S(\Gamma) \tag{104}$$

$$\begin{aligned}\Rightarrow 0 = {} & 2\Gamma_{\phi_L^\dagger\phi_L\epsilon Y_\lambda}\Gamma_{\phi_L^\dagger\psi_L\lambda} + 2\Gamma_{\phi_L\phi_L^\dagger\psi_L\epsilon Y_{\phi_L}}\Gamma_{\phi_L^\dagger\phi_L} \\ &+ \Gamma_{\phi_L^\dagger\phi_L^\dagger\psi_L\epsilon Y_{\phi_L^\dagger}}\Gamma_{\phi_L\phi_L^\dagger} + \Gamma_{\psi_L\epsilon Y_{\phi_L}}\Gamma_{\phi_L^\dagger\phi_L\phi_L^\dagger\phi_L} \\ &+ \Gamma_{\phi_L^\dagger\phi_L\phi_L^\dagger\epsilon Y_{\overline{\Psi}}}\Gamma_{\psi_L\overline{\Psi}} + 2\Gamma_{\phi_L^\dagger\epsilon Y_{\overline{\Psi}}}\Gamma_{\phi_L^\dagger\phi_L\psi_L\overline{\Psi}} \ ,\end{aligned} \tag{105}$$

$$0 = \frac{\partial}{\partial\epsilon}\frac{\delta^4}{\delta\phi_R^\dagger\delta\phi_R\delta\phi_L^\dagger\delta\psi_L}S(\Gamma) \tag{106}$$

$$\begin{aligned}\Rightarrow 0 = {} & \Gamma_{\phi_R^\dagger\phi_R\epsilon Y_\lambda}\Gamma_{\phi_L^\dagger\psi_L\lambda} + \Gamma_{\phi_R\phi_R^\dagger\psi_L\epsilon Y_{\phi_L}}\Gamma_{\phi_L^\dagger\phi_L} + \Gamma_{\phi_R\phi_L^\dagger\psi_L\epsilon Y_{\phi_R}}\Gamma_{\phi_R^\dagger\phi_R} \\ &+ \Gamma_{\phi_L^\dagger\phi_R^\dagger\psi_L\epsilon Y_{\phi_R^\dagger}}\Gamma_{\phi_R\phi_R^\dagger} + \Gamma_{\psi_L\epsilon Y_{\phi_L}}\Gamma_{\phi_R^\dagger\phi_R\phi_L^\dagger\phi_L} \\ &+ \Gamma_{\phi_R^\dagger\phi_R\phi_L^\dagger\epsilon Y_{\overline{\Psi}}}\Gamma_{\psi_L\overline{\Psi}} + \Gamma_{\phi_L^\dagger\epsilon Y_{\overline{\Psi}}}\Gamma_{\phi_R^\dagger\phi_R\psi_L\overline{\Psi}} + \Gamma_{\phi_R^\dagger\epsilon Y_\Psi}\Gamma_{\phi_L^\dagger\phi_R\psi_L\Psi} \ ,\end{aligned} \tag{107}$$

$$0 = \frac{\partial}{\partial\epsilon}\frac{\delta^4}{\delta\phi_L\delta\phi_R\delta\phi_R\delta\psi_L}S(\Gamma) \tag{108}$$

$$\begin{aligned}\Rightarrow 0 = {} & 2\Gamma_{\phi_L\phi_R\epsilon Y_{\overline{\lambda}}}\Gamma_{\phi_R\psi_L\overline{\lambda}} + 2\Gamma_{\phi_R\phi_L\psi_L\epsilon Y_{\phi_R^\dagger}}\Gamma_{\phi_R\phi_R^\dagger} + \Gamma_{\phi_R\phi_R\psi_L\epsilon Y_{\phi_L^\dagger}}\Gamma_{\phi_L\phi_L^\dagger} \\ &+ \Gamma_{\psi_L\epsilon Y_{\phi_L}}\Gamma_{\phi_L\phi_R\phi_R\phi_L} + \Gamma_{\phi_L\phi_R\phi_R\epsilon Y_{\overline{\Psi}}}\Gamma_{\psi_L\overline{\Psi}} \ .\end{aligned} \tag{109}$$

The momentum arguments in these terms are dropped (see explanation at the beginning of this section). The factors 2 in front of several terms imply symmetrization with respect to the momenta of the two $\phi_L^\dagger$ and ϕ_R fields, respectively. These equations constitute symmetry conditions for $\Gamma_{\phi_L^\dagger\phi_L\phi_L^\dagger\phi_L}$, $\Gamma_{\phi_R^\dagger\phi_R\phi_L^\dagger\phi_L}$ and $\Gamma_{\phi_R\phi_R\phi_L\phi_L}$, since these are the only power-counting renormalizable vertex functions not yet determined.

3.4 Collection of all symmetry and normalization conditions

We now list all symmetry and normalization conditions for an easy reference and to make transparent the similarity in their mathematical structure. Taking into account also eqs. (43), (44) and the manifest symmetries there is a condition for each vertex function corresponding to a power-counting renormalizable interaction.

Photon and photino only:

$$\lim_{p^2\to 0}\frac{1}{p^2}\Gamma_{A^\mu A^\nu}(-p,p)|_{g_{\mu\nu}-\text{part}} = -g_{\mu\nu}\ , \tag{110}$$

$$\Gamma_{\overline{\lambda}^{\dot\alpha}\lambda^\alpha}(-p,p) = \not{p}_{\alpha\dot\alpha}\ \text{for}\ p^2=0\ , \tag{111}$$

$$p^\mu\Gamma_{A^\rho A^\mu}(-p,p) = 0\ , \tag{112}$$

$$p^\mu\Gamma_{A^\rho A^\sigma A^\mu}(p',-p-p',p) = 0\ , \tag{113}$$

$$p^\mu\Gamma_{A^\rho A^\sigma A^\nu A^\mu}(p',p'',-p-p'-p'',p) = 0\ , \tag{114}$$

$$p^\mu\Gamma_{\lambda\overline{\lambda}A^\mu}(p',-p-p',p) = 0\ , \tag{115}$$

Interactions involving matter fields:

$$\Gamma_{\phi_L\phi_L^\dagger}(-p,p) = 0 \quad \text{for}\ p^2=m^2\ , \tag{116}$$

$$\Gamma_{\Psi\overline{\Psi}}(p,-p)u(p) = 0 \quad \text{for}\ p^2=m^2\ , \tag{117}$$

$$\frac{\partial}{\partial p^2}\Gamma_{\phi_L\phi_L^\dagger}(-p,p) = 1 \quad \text{for}\ p^2=\kappa^2\ , \tag{118}$$

$$\Gamma_V(p^2)+2m^2(\Gamma_V'(p^2)+\Gamma_S'(p^2)) = 1 \quad \text{for}\ p^2=\kappa^2\ , \tag{119}$$

$$
\begin{aligned}
&Z_\phi \Gamma_{\phi_L \phi_L^\dagger A^\mu}(p,-p,0) && = -2eQ_L p_\mu \quad \text{for } p^2 = m^2 \ , && (120)\\
&Z_\phi \Gamma_{\phi_L \phi_L^\dagger A^\nu A^\mu}(p,-p,0,0) && = 2(eQ_L)^2 g_{\mu\nu} \quad \text{for } p^2 = m^2 \ , && (121)\\
&\bar{u}(p) Z_\Psi \Gamma_{\Psi \overline{\Psi} A^\mu}(p,-p,0) u(p) && = \bar{u}(p) \left(-eQ_L \gamma_\mu\right) u(p) && \\
& && \qquad\qquad\qquad \text{for } p^2 = m^2 \ , && (122)\\
&\sqrt{Z_\phi Z_\Psi} \Gamma_{\phi_L^\dagger \Psi \overline{\gamma}}(-p,p,0) u(p) && = -\sqrt{2} eQ_L P_L u(p) && \\
& && \qquad\qquad\qquad \text{for } p^2 = m^2 \ , && (123)
\end{aligned}
$$

$$
\begin{aligned}
0 = {} & 2\Gamma_{\phi_L^\dagger \phi_L \epsilon Y_\lambda} \Gamma_{\phi_L^\dagger \psi_L \lambda} + 2\Gamma_{\phi_L \phi_L^\dagger \psi_L \epsilon Y_{\phi_L}} \Gamma_{\phi_L^\dagger \phi_L} \\
& + \Gamma_{\phi_L^\dagger \phi_L^\dagger \psi_L \epsilon Y_{\phi_L^\dagger}} \Gamma_{\phi_L \phi_L^\dagger} + \Gamma_{\psi_L \epsilon Y_{\phi_L}} \Gamma_{\phi_L^\dagger \phi_L \phi_L^\dagger \phi_L} \\
& + \Gamma_{\phi_L^\dagger \phi_L \phi_L^\dagger \epsilon Y_{\overline{\Psi}}} \Gamma_{\psi_L \overline{\Psi}} + 2\Gamma_{\phi_L^\dagger \epsilon Y_{\overline{\Psi}}} \Gamma_{\phi_L^\dagger \phi_L \psi_L \overline{\Psi}} \ , && (124)\\
0 = {} & \Gamma_{\phi_R^\dagger \phi_R \epsilon Y_\lambda} \Gamma_{\phi_L^\dagger \psi_L \lambda} + \Gamma_{\phi_R \phi_R^\dagger \psi_L \epsilon Y_{\phi_L}} \Gamma_{\phi_L^\dagger \phi_L} + \Gamma_{\phi_R \phi_L^\dagger \psi_L \epsilon Y_{\phi_R}} \Gamma_{\phi_R^\dagger \phi_R} \\
& + \Gamma_{\phi_L^\dagger \phi_R^\dagger \psi_L \epsilon Y_{\phi_R^\dagger}} \Gamma_{\phi_R \phi_R^\dagger} + \Gamma_{\psi_L \epsilon Y_{\phi_L}} \Gamma_{\phi_R^\dagger \phi_R \phi_L^\dagger \phi_L} \\
& + \Gamma_{\phi_R^\dagger \phi_R \phi_L^\dagger \epsilon Y_{\overline{\Psi}}} \Gamma_{\psi_L \overline{\Psi}} + \Gamma_{\phi_L^\dagger \epsilon Y_{\overline{\Psi}}} \Gamma_{\phi_R^\dagger \phi_R \psi_L \overline{\Psi}} + \Gamma_{\phi_R^\dagger \epsilon Y_\Psi} \Gamma_{\phi_L^\dagger \phi_R \psi_L \Psi} \ , && (125)\\
0 = {} & 2\Gamma_{\phi_L \phi_R \epsilon Y_{\overline{\lambda}}} \Gamma_{\phi_R \psi_L \overline{\lambda}} + 2\Gamma_{\phi_R \phi_L \psi_L \epsilon Y_{\phi_R^\dagger}} \Gamma_{\phi_R \phi_R^\dagger} + \Gamma_{\phi_R \phi_R \psi_L \epsilon Y_{\phi_L^\dagger}} \Gamma_{\phi_L \phi_L^\dagger} \\
& + \Gamma_{\psi_L \epsilon Y_{\phi_L}} \Gamma_{\phi_L \phi_R \phi_R \phi_L} + \Gamma_{\phi_L \phi_R \phi_R \epsilon Y_{\overline{\Psi}}} \Gamma_{\psi_L \overline{\Psi}} \ , && (126)
\end{aligned}
$$

Interactions involving ghost fields:

$$
\begin{aligned}
&\Gamma_{A^\mu \epsilon^\beta Y_{\lambda\alpha}}(p,-p) && = p^\rho (\sigma_{\rho\mu})_\beta{}^\alpha \ , && (127)\\
&\Gamma_{\overline{\epsilon}_{\dot\beta} \epsilon^\beta Y_{\lambda\gamma} Y_{\overline{\lambda}}{}^{\dot\gamma}}(p,-p) && = \delta^{\dot\beta}{}_{\dot\gamma} \delta_\beta{}^\gamma \qquad \text{for } p^2 = 0 \ , && (128)\\
&-\Gamma_{\phi_L^\dagger \phi_L \epsilon Y_\lambda} \Gamma_{\overline{\lambda}\lambda} && = \Gamma_{\overline{\lambda} \epsilon Y_{A^\mu}} \Gamma_{\phi_L^\dagger \phi_L A^\mu} + \Gamma_{\phi_L^\dagger \epsilon Y_{\overline{\Psi}}} \Gamma_{\phi_L \overline{\lambda \Psi}} \ , && (129)
\end{aligned}
$$

$$
\begin{aligned}
&\Gamma_{\phi_L \overline{\epsilon}^{\dot\beta} Y_{\psi_L}{}^\alpha}(p,-p) && = -\sqrt{2} \not{p}_{\alpha\dot\beta}\, \Theta \qquad \text{for } p^2 = m^2 \ , && (130)\\
&\Gamma_{\phi_L \overline{\epsilon}^{\dot\beta} Y_{\overline{\psi}_R \dot\alpha}}(p,-p) && = -\sqrt{2} m \delta^{\dot\alpha}{}_{\dot\beta}\, \Theta \qquad \text{for } p^2 = m^2 \ , && (131)\\
&\not{p}_{\alpha\dot\beta} \Gamma_{\psi_{L\alpha} \epsilon^\beta Y_{\phi_L}}(p,-p) && = -m \Gamma_{\overline{\psi}_R{}^{\dot\beta} \epsilon^\beta Y_{\phi_L}}(p,-p) - \sqrt{2} \not{p}_{\beta\dot\beta} \frac{1}{\Theta} && \\
& && \qquad\qquad \text{for } p^2 = m^2 \ , && (132)\\
&q^\mu \Gamma_{A^\mu \phi_L \overline{\epsilon}^{\dot\beta} Y^\alpha_{\psi_L}}(q,p,p') && = \sqrt{2} eQ_L \not{q}_{\alpha\dot\beta}\, \Theta \qquad \text{for } p^2 = p'^2 = m^2 \ , && (133)\\
&\qquad\qquad 0 && = \Gamma_{\epsilon^\beta \overline{\epsilon}^{\dot\beta} Y^\gamma_{\psi_L} Y^{\dot\delta}_{\overline{\psi}_L}} \Gamma_{\psi_{L\alpha} \overline{\psi}_{L\dot\delta}} + \Gamma_{\epsilon^\beta \overline{\epsilon}^{\dot\beta} Y^\gamma_{\psi_L} Y_{\psi_R \delta}} \Gamma_{\psi_{L\alpha} \psi_R^\delta} && \\
& && \quad \Gamma_{\psi_{L\alpha} \epsilon^\beta Y_{\phi_L}} \Gamma_{Y^\gamma_{\psi_L} \overline{\epsilon}^{\dot\beta} \phi_L} + \Gamma_{\psi_{L\alpha} \overline{\epsilon}^{\dot\beta} Y_{\phi_R^\dagger}} \Gamma_{Y^\gamma_{\psi_L} \epsilon^\beta \phi_R^\dagger} && \\
& && \quad - 2\not{p}_{\beta\dot\beta} \delta^\gamma_\alpha \ . && (134)
\end{aligned}
$$

4 Applications

The general prescription for higher order calculations not relying on an invariant regularization is:

- Calculate the necessary loop diagrams using some arbitrary (preferably consistent) regularization.
- To every power-counting renormalizable interaction there is an independent counterterm.
- For each counterterm the proper coefficient can be read off from one of the conditions collected in section 3.4.
- From the considerations in section 2 we know that this leads uniquely to a renormalized theory respecting all defining symmetries.

In this section we show some sample calculations of renormalized higher order corrections using dimensional regularization as defined in [2]. In particular we use $\{\gamma^\mu, \gamma^5\} = 2\hat{g}^{\mu\nu}\gamma_\nu\gamma^5$ with $\hat{g}^\mu{}_\mu = D - 4$ and set $\hat{g}^{\mu\nu} = 0$ only in the final results. This regularization scheme is known to break supersymmetry. In establishing the symmetries of the renormalized theory, the symmetry conditions we have derived will prove to be an efficient tool, due to the common structure of most of them:

$$\Gamma_{ABC}|_{\text{on shell}} = \Gamma^{\text{regularized}}_{ABC} + \Gamma^{\text{ct}}_{ABC} = \text{definite value}. \tag{135}$$

Non-supersymmetric counterterms in dimensional regularization have already been calculated in the literature [21]. The equality of the effective couplings to gauge bosons and gauginos we have proven in sec. 3 as a consequence of the defining symmetry requirements was anticipated there as a symmetry condition and used for the determination of the counterterms.

4.1 Elimination of B

Although for theoretical purposes the auxiliary B field is useful, it complicates practical calculations whenever we are not interested in Green functions involving external B fields. Therefore it is convenient to eliminate B by its equation of motion. Due to the gauge condition in eq. (44) we can write

$$\Gamma(B, A_\mu, \ldots) = \Gamma_{\text{no } B}(A_\mu, \ldots) + \Gamma_{\text{with } B}(B, A_\mu)\ , \tag{136}$$

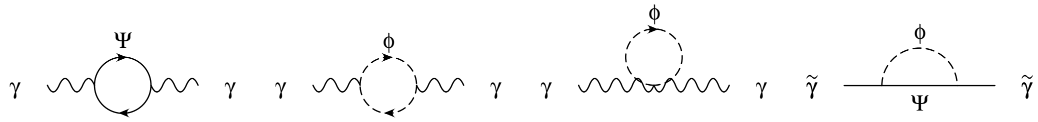

Figure 1: One-loop diagrams contributing to the photon and photino self energies.

where the first term does not depend on B and

$$\Gamma_{\text{with }B}(B, A_\mu) = \int d^4x \Big(B\partial^\mu A_\mu + \frac{\xi}{2} B^2 \Big) \ . \tag{137}$$

The solution of the equation of motion is $B = -\frac{1}{\xi}(\partial A)$ to all orders, and one can show that the effective action

$$\begin{aligned} \tilde{\Gamma}(A_\mu, \dots) &= \Gamma_{\text{no }B}(A_\mu, \dots) + \Gamma_{\text{with }B}(B = -\tfrac{1}{\xi}(\partial A), A_\mu) \\ &= \Gamma_{\text{no }B}(A_\mu, \dots) - \frac{1}{2\xi} \int d^4x (\partial^\mu A_\mu)^2 \ , \end{aligned} \tag{138}$$

where $\tilde{\Gamma}$ does not depend on B, generates the same connected Green functions as $\Gamma(B, A_\mu, \dots)$. In the passage from Γ to $\tilde{\Gamma}$, the only vertex function that changes is $\Gamma_{A^\mu A^\rho}$, which receives a longitudinal part. In the rest of this section we always work with $\tilde{\Gamma}$, so we drop the $\tilde{}$ and denote by Γ the effective action without B. This yields

$$p^\mu \Gamma_{A^\rho A^\mu}(-p, p) = -\frac{1}{\xi} p^2 p_\rho \tag{139}$$

instead of eq. (112), while all other conditions in section 3.4 are unchanged.

4.2 Photon and photino self energies

The one-loop diagrams contributing to the photon and photino self energies are depicted in fig. 1. In terms of the one-loop integrals defined in app. A.3, the results are ($\alpha = \frac{e^2}{4\pi}$)

$$\Gamma^{\text{regularized}}_{A^\mu A^\rho}(-p, p) = \left(-g_{\mu\rho} p^2 + p_\mu p_\rho \right) \left(1 + \Pi^\gamma(p^2) \right) - \frac{1}{\xi} p_\mu p_\rho \ , \tag{140}$$

$$\Gamma^{\text{regularized}}_{\overline{\lambda}^{\dot\alpha} \lambda^\alpha}(-p, p) = \not{p}_{\alpha\dot\alpha} (1 + \Pi^{\tilde\gamma}(p^2)) \ , \tag{141}$$

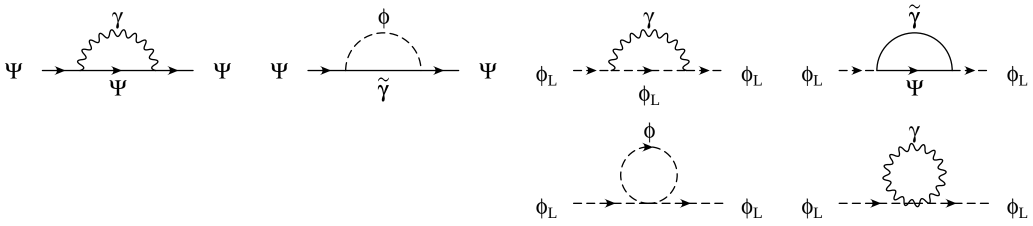

Figure 2: One-loop diagrams contributing to the electron and selectron self energies.

where the one-loop corrections

$$\Pi^{\gamma}(p^2) = \Pi^{\tilde{\gamma}}(p^2) = \frac{\alpha}{4\pi} 2B_0(m^2, m^2, p^2) \tag{142}$$

turn out to be equal, so the identity (91) is already satisfied at the regularized level (up to the new longitudinal part of $\Gamma_{A^\mu A^\rho}$). To renormalize we have to define counterterms such that the conditions (110), (111) are satisfied. The correct choice is

$$\mathcal{L}_{\rm ct} = \delta Z_\gamma \left(-\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} \overline{\tilde{\gamma}} i \gamma^\mu \partial_\mu \tilde{\gamma}\right) \tag{143}$$

with

$$\delta Z_\gamma = -\Pi^\gamma(0) \; , \tag{144}$$

yielding to $\mathcal{O}(\alpha)$

$$\Gamma_{A^\mu A^\rho}(-p,p) = \left(-g_{\mu\rho} p^2 + p_\mu p_\rho\right) \left(1 + \Pi^\gamma(p^2) + \delta Z_\gamma\right) - \frac{1}{\xi} p_\mu p_\rho \; , \tag{145}$$

$$\Gamma_{\overline{\lambda}^{\dot{\alpha}} \lambda^\alpha}(-p,p) = \not{p}_{\alpha\dot{\alpha}} (1 + \Pi^\gamma + \delta Z_\gamma(p^2)) \; . \tag{146}$$

Note that mass and gauge fixing counterterms are not ruled out a priori but they turn out to vanish because of the concrete form of the regularized self energies.

4.3 Electron and selectron self energies

The one-loop contributions to the matter self energies can be written as follows:

$$\Gamma^{\text{regularized}}_{\phi_L\phi_L^\dagger}(p,-p) = p^2 - m^2 + \Sigma_\phi(p^2)\ , \tag{147}$$
$$\Gamma^{\text{regularized}}_{\Psi\overline{\Psi}}(p,-p) = \not{p} - m + \not{p}\Sigma_V(p^2) - m\Sigma_S(p^2)\ . \tag{148}$$

For later purposes we also introduce the abbreviation

$$\Sigma'_\Psi(p^2) = \Sigma_V(p^2) + 2m^2(\Sigma'_V(p^2) - \Sigma'_S(p^2))\ . \tag{149}$$

The contributing Feynman diagrams are shown in fig. 2 and yield[4]

$$\Sigma_\phi(p^2) = \frac{\alpha}{4\pi}[-4m^2B_0(0,m^2,p^2) + 4(D-4)B_{22}(0,m^2,p^2)]\ , \tag{150}$$
$$\Sigma_V(p^2) = \frac{\alpha}{4\pi}[(D-2)B_0(0,m^2,p^2) + (D-4)B_1(0,m^2,p^2)]\ , \tag{151}$$
$$\Sigma_S(p^2) = \frac{\alpha}{4\pi}[DB_0(0,m^2,p^2)]\ . \tag{152}$$

The most general counterterms contributing to these self energies are

$$\begin{aligned}\mathcal{L}_{\text{ct}} = {}& \delta Z_\phi(|\partial_\mu\phi_L|^2 - m^2|\phi_L|^2 + (_{L\to R})) - 2m\delta m_\phi(|\phi_L|^2 + |\phi_R|^2) \\ & + \delta Z_\Psi\overline{\Psi}(i\gamma^\mu\partial_\mu - m)\Psi - \delta m_\Psi\overline{\Psi}\Psi\ .\end{aligned} \tag{153}$$

For each counterterm one of the conditions (116–119) applies. Expressed in terms of the quantities in $\mathcal{L}_{\text{ct}}$ they read:

$$\Sigma_\phi(m^2) - 2m\delta m_\phi = 0\ , \tag{154}$$
$$m\Sigma_V(m^2) - m\Sigma_S(m^2) - \delta m_\Psi = 0\ , \tag{155}$$
$$\Sigma'_\phi(\kappa^2) + \delta Z_\phi = 0\ , \tag{156}$$
$$\Sigma'_\Psi(\kappa^2) + \delta Z_\Psi = 0\ , \tag{157}$$

from which the coefficients of the counterterms follow immediately:

$$\delta m_\phi = \frac{\alpha}{4\pi}m\left[-2B_0(0,m^2,m^2) - \frac{2}{3}\right]\ , \tag{158}$$
$$\delta m_\Psi = \frac{\alpha}{4\pi}m[-2B_0(0,m^2,m^2) + 1]\ , \tag{159}$$
$$\delta Z_\phi = \frac{\alpha}{4\pi}\left[4m^2B'_0(0,m^2,\kappa^2) - \frac{2}{3}\right]\ , \tag{160}$$
$$\delta Z_\Psi = \frac{\alpha}{4\pi}[-2B_0(0,m^2,\kappa^2) + 4m^2B'_0(0,m^2,\kappa^2) + 1]\ , \tag{161}$$

[4] For the rest of this section we use the gauge parameter $\xi = 1$.

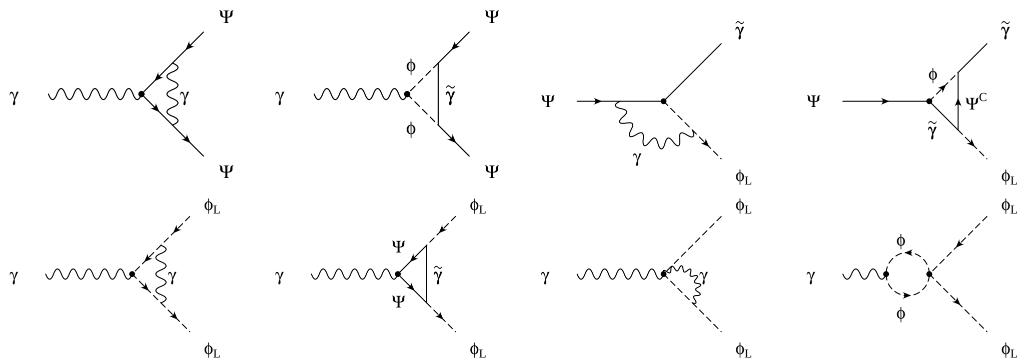

Figure 3: One-loop vertex corrections.

where in the finite terms the limit $D \to 4$ has been taken.

This non-vanishing difference $\delta m_\Psi - \delta m_\phi$ is our first encounter of a supersymmetry-violating counterterm, necessary because dimensional regularization itself breaks supersymmetry. It is precisely this choice for the counterterms that restores (116–117) and thus the equality of the renormalized masses, a necessary consequence of supersymmetry.

The different δZ counterterms do not correspond to a symmetry breaking, as shown in section 2.5.

4.4 Photon and photino interactions with electron and selectron

We define scalar functions containing the regularized one-loop contributions to the photon–/photino–matter interactions in the following way:

$$\Gamma^{\text{regularized}}_{\phi_L \phi_L^\dagger A^\mu}(p,-p,0) = \Lambda_{\phi\phi A}(p^2)\ (-2eQ_L p_\mu)\ , \tag{162}$$

$$\bar{u}(p)\Gamma^{\text{regularized}}_{\Psi\overline{\Psi}A^\mu}(p,-p,0)u(p) = \Lambda_{\Psi\Psi A}(p^2)\ \bar{u}(p)(-eQ_L\gamma_\mu)u(p)\ , \tag{163}$$

$$\Gamma^{\text{regularized}}_{\phi_L^\dagger \Psi \overline{\tilde{\gamma}}}(-p,p,0)u(p) = \Lambda_{\phi\Psi\tilde{\gamma}}(p^2)\ (-\sqrt{2}eQ_L P_L)u(p)\ . \tag{164}$$

For each of these vertex functions there is one independent counterterm. To make the comparison with the case of symmetric counterterms transparent we denote them by

$$\mathcal{L}_{\rm ct} = (\delta Z_\phi + \frac{1}{2}\delta Z_\gamma + \delta Z_{\phi\phi A}) ieQ_L A^\mu (\phi_L^\dagger \partial_\mu \phi_L - \phi_L \partial_\mu \phi_L^\dagger) + ({\scriptstyle L\to R})$$
$$(\delta Z_\Psi + \frac{1}{2}\delta Z_\gamma + \delta Z_{\Psi\Psi A}) \overline{\Psi}(-eQ_L A^\mu \gamma_\mu)\Psi$$
$$(\frac{\delta Z_\phi + \delta Z_\Psi + \delta Z_\gamma}{2} + \delta Z_{\phi\Psi\tilde{\gamma}})(-\sqrt{2}eQ_L)(\phi_L^\dagger \overline{\tilde{\gamma}} P_L \Psi - \phi_R \overline{\tilde{\gamma}} P_R \Psi + h.c.) \tag{165}$$

According to section 2.5 these counterterms are symmetric if $\delta Z_{\phi\phi A} = \delta Z_{\Psi\Psi A} = \delta Z_{\phi\Psi\tilde{\gamma}}$. Their values are determined by the conditions (120–123). The functions Z_ϕ, Z_Ψ are given in one-loop order by

$$Z_\phi(p^2) = 1 - \Sigma'_\phi(p^2) - \delta Z_\phi \ , \tag{166}$$
$$Z_\Psi(p^2) = 1 - \Sigma'_\Psi(p^2) - \delta Z_\Psi \ ; \tag{167}$$

therefore in (120–123) the matter field renormalization factors $\delta Z_\phi, \delta Z_\Psi$ drop out and the remaining conditions are

$$\Lambda_{\phi\phi A}(p^2) - \Sigma'_\phi(p^2) + \frac{1}{2}\delta Z_\gamma + \delta Z_{\phi\phi A} = 0 \quad \text{for } p^2 = m^2, \tag{168}$$
$$\Lambda_{\Psi\Psi A}(p^2) - \Sigma'_\Psi(p^2) + \frac{1}{2}\delta Z_\gamma + \delta Z_{\Psi\Psi A} = 0 \quad \text{for } p^2 = m^2, \tag{169}$$
$$\Lambda_{\phi\Psi\tilde{\gamma}}(p^2) - \frac{1}{2}(\Sigma'_\phi(p^2) + \Sigma'_\Psi(p^2)) + \frac{1}{2}\delta Z_\gamma + \delta Z_{\phi\Psi\tilde{\gamma}} = 0 \quad \text{for } p^2 = m^2. \tag{170}$$

Again, the counterterms can be read off easily from the corresponding conditions once the loop diagrams shown in fig. 3 have been calculated. Inspection of the Feynman integrands shows that both conditions for the photon interactions already hold at the regularized level, so we have to choose

$$\delta Z_{\phi\phi A} = \delta Z_{\Psi\Psi A} = -\frac{1}{2}\delta Z_\gamma \ . \tag{171}$$

Physically these conditions express the gauge invariance of the renormalized theory, and the structure of these counterterms shows that gauge invariance is preserved by dimensional regularization.

The one-loop correction to the photino interaction is given by

$$\Lambda_{\phi\Psi\tilde{\gamma}}(p^2) = \frac{\alpha}{4\pi}[B_0(0, m^2, p^2) + 4m^2(C_0 + C_{11}) + \mathcal{O}(p^2 - m^2)] \tag{172}$$

with $C_{ij} = C_{ij}(0, m^2, m^2, p^2, 0, p^2)$, and the derivatives of the matter self energies are

$$\Sigma'_\phi(p^2) = \frac{\alpha}{4\pi}\left[-4m^2 B'_0(0, m^2, p^2) + \frac{2}{3}\right] , \tag{173}$$
$$\Sigma'_\Psi(p^2) = \frac{\alpha}{4\pi}[2B_0(0, m^2, p^2) - 4m^2 B'_0(0, m^2, p^2) - 1] . \tag{174}$$

Using $B'_0 = -C_0 - C_{11}$ shows that the correct choice for the counterterm is

$$\delta Z_{\phi\Psi\tilde{\gamma}} = -\frac{1}{2}\delta Z_\gamma - \frac{1}{6}\frac{\alpha}{4\pi} . \tag{175}$$

This result exhibits three important aspects. First, in eq. (170) the non-local terms cancel. This is a regularization-independent fact due to the supersymmetry. Second, on the dimensionally regularized level there is a local violation of eq. (170). This supersymmetry breaking has to be cancelled choosing the charge counterterm $\delta Z_{\phi\Psi\tilde{\gamma}}$ different from the charge counterterms for the photon interactions. Physically these non-supersymmetric counterterms lead uniquely to charge universality in the renormalized theory as required by eqs. (120–123). Third, obviously the determination of this counterterm $\delta Z_{\phi\Psi\tilde{\gamma}}$ is just as straightforward as the determination of the charge counterterms for the photon interactions before, in spite of the supersymmetry breaking. The reason is that the main work has already been done in the derivation of the corresponding symmetry condition.

The photino–matter interaction also constitutes an example where a naive one-loop calculation can lead to a large numerical error. Naively one might think that the required symmetries restrict the counterterms to those of section 2.5 corresponding to field and parameter renormalization. According to this line of reasoning one would ignore the effects of the regularization and choose $\delta Z_{\phi\Psi\tilde{\gamma}} = \delta Z_{\phi\phi A} = \delta Z_{\Psi\Psi A}$. In this section we have shown that for dimensional regularization this amounts to forgetting the necessary term $(-\frac{1}{6}\frac{\alpha}{4\pi})$ and spoiling charge universality and thus supersymmetry of the renormalized theory. Since all contributions to $\Lambda_{\phi\Psi\tilde{\gamma}}(p^2)$ are basically of the order $\frac{\alpha}{4\pi}$, the numerical error in the renormalized one-loop correction to the photino–electron–selectron interaction is in general quite sizeable.

4.5 Supersymmetry transformations at one loop

The Slavnov–Taylor identity may be rewritten in the form of an invariance relation (φ'_i runs over the linearly transforming fields including the global ghosts,

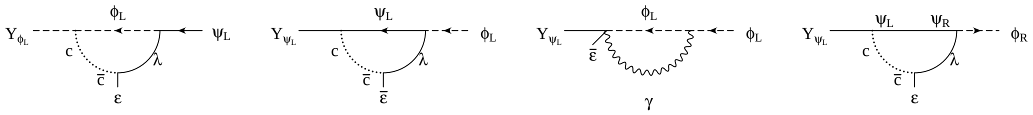

Figure 4: One-loop contributions to the supersymmetry transformations of ϕ_L and ψ_L.

φ_i, Y_i over the non-linearly transforming fields and the corresponding external fields):

$$\Gamma(\varphi_i' + \theta s_\Gamma \varphi_i', \varphi_i + \theta s_\Gamma \varphi_i, Y_i) = \Gamma(\varphi_i', \varphi_i, Y_i) \ , \tag{176}$$

where θ is an infinitesimal fermionic parameter and s_Γ is the quantum analogue to the classical BRS operator:

$$s_\Gamma \varphi_i' = s\varphi_i' \ , \tag{177}$$

$$s_\Gamma \varphi_i = \frac{\delta \Gamma}{\delta Y_i} = \langle s_{\Gamma_{\rm cl}} \varphi_i \rangle_J \ , \tag{178}$$

$$s_{\Gamma_{\rm cl}} \varphi_i = s\varphi_i + \frac{\delta \Gamma_{\rm bil}}{\delta Y_i} \ . \tag{179}$$

$s_\Gamma \varphi_i$ is equal to the expectation value of the composite operator $s_{\Gamma_{\rm cl}} \varphi_i$ in the presence of sources $J = -\frac{\delta \Gamma}{\delta \varphi}$. Thus $s_\Gamma \varphi_i$ — and equivalently the vertex functions involving an external Y_i — contain quantum corrections to the BRS transformations. These quantum corrections can be non-trivial but are constrained by eqs. (43), (44).

We focus now on the transformation of the electron and selectron fields as particular examples:

$$\begin{aligned} s_\Gamma \phi_L(x) &= -ieQ_L c(x)\, \phi_L(x) - i\omega^\nu \partial_\nu \phi_L(x) \\ &\quad - \int d^4y\ \epsilon^\beta\, \psi_{L\alpha}(y)\ \Gamma_{\psi_{L\alpha}\epsilon^\beta Y_{\phi_L}}(y,x) \\ &\quad - \int d^4y\ \epsilon^\beta\, \overline{\psi}_R{}^{\dot\alpha}(y)\ \Gamma_{\overline{\psi}_R{}^{\dot\alpha}\epsilon^\beta Y_{\phi_L}}(y,x) + \dots\ , \end{aligned} \tag{180}$$

$$\begin{aligned} s_\Gamma \psi_L^\alpha(x) &= -ieQ_L c(x)\, \psi_L^\alpha(x) - i\omega^\nu \partial_\nu \psi_L^\alpha(x) \\ &\quad + \int d^4y\ \overline{\epsilon}_{\dot\beta}\, \phi_L(y)\ \Gamma_{\phi_L \overline{\epsilon}_{\dot\beta} Y_{\psi_L \alpha}}(y,x) \\ &\quad + \int d^4y\ \epsilon^\beta\, \phi_R^\dagger(y)\ \Gamma_{\phi_R^\dagger \epsilon^\beta Y_{\psi_{L\alpha}}}(y,x) + \dots\ , \end{aligned} \tag{181}$$

where the dots denote terms involving higher powers of the fields. So the renormalized supersymmetry transformations $\phi \leftrightarrow \psi$ are governed by vertex functions of the type $\Gamma_{\psi\epsilon Y_\phi}$ and $\Gamma_{\phi\epsilon Y_\psi}$.

At one-loop order these vertex functions are given by the Feynman diagrams displayed in fig. 4 and by the counterterms determined through eqs. (130–134) with $\Theta = 1 + \frac{\alpha}{4\pi}(B_0(0,m^2,\kappa^2) - B_0(0,m^2,m^2))$. In momentum space the results are ($B_0 = B_0(0,m^2,p^2)$)

$$\Gamma_{\psi_{L\alpha}\epsilon^\beta Y_{\phi_L}} = -\sqrt{2}\delta_\beta{}^\alpha \left[1 + \frac{\alpha}{4\pi}B_0 + \frac{1}{2}\left(\delta Z_\psi - \delta Z_\phi - \frac{5}{3}\frac{\alpha}{4\pi}\right)\right] , \tag{182}$$

$$\Gamma_{\overline{\psi}_R{}^{\dot\alpha}\epsilon^\beta Y_{\phi_L}} = 0 , \tag{183}$$

$$\Gamma_{\phi_L\bar\epsilon_{\dot\beta} Y_{\psi_{L\alpha}}} = -\sqrt{2}\not{p}_{\phi_L}^{\dot\beta\alpha} \left[1 - \frac{\alpha}{4\pi}B_0 - \frac{1}{2}\left(\delta Z_\psi - \delta Z_\phi - \frac{5}{3}\frac{\alpha}{4\pi}\right)\right] , \tag{184}$$

$$\begin{aligned}\Gamma_{\phi_R^\dagger\epsilon^\beta Y_{\psi_{L\alpha}}} &= -\sqrt{2}m\delta_\beta{}^\alpha \\ &\times \left[1 + \frac{\alpha}{4\pi}B_0 - \frac{1}{2}\left(\delta Z_\psi - \delta Z_\phi - \frac{5}{3}\frac{\alpha}{4\pi}\right) + \frac{\delta m_\phi}{m} + \frac{2}{3}\frac{\alpha}{4\pi}\right] .\end{aligned} \tag{185}$$

Again, non-invariant counterterms are necessary.

These results show that in one-loop order the supersymmetry transformations are modified by non-local terms. One reason for this modification is the non-linearity of the BRS transformations permitting all the vertices involving Y fields in fig. 4. Another reason can be traced back to the gauge fixing fermion $F = \bar c(\partial^\mu A_\mu + \frac{\xi}{2}B)$. Since F breaks supersymmetry, there are terms in $s_{\Gamma_{\rm cl}}F$ involving the ϵ ghosts, in particular the $\bar c\epsilon\lambda$ vertices appearing in three of the graphs in fig. 4. These supersymmetry transformations are related to physical vertex functions by identities such as eq. (67), (75) expressing non-trivial relations among self energies and vertex corrections.

4.6 Summary of counterterms

We had to use non-invariant counterterms in many of the vertex functions we calculated. However, one should note that the separation $\Gamma_{\rm ct} = \Gamma_{\rm sym} + \Gamma_{\rm non-inv}$ is not unique. The simplest expression for $\Gamma_{\rm non-inv}$ is obtained using special renormalization constants in $\Gamma_{\rm sym}$ as given by eq. (54). If one uses $(\delta Z_\phi + \frac{2}{3}\frac{\alpha}{4\pi})$, $(\delta Z_\Psi - \frac{\alpha}{4\pi})$ as field renormalization constants instead of δZ_ϕ, δZ_Ψ, and the mass counterterm $m\delta Z_m = (\delta m_\phi + \frac{2}{3}\frac{\alpha}{4\pi})$, then the non-invariant counterterms are confined to the matter self energies and the photon interactions:

$$\Gamma_{\rm n.i.} = \int d^4x\, \frac{\alpha}{4\pi}\left(\overline{\Psi}(i\not{D} - 2m)\Psi - \frac{2}{3}|D_\mu\phi_L|^2 + 2m^2|\phi_L|^2 + ({\scriptstyle L\to R})\right) . \tag{186}$$

5 Conclusions

In this article we have constructed the Green functions of SQED in the Wess–Zumino gauge from the Slavnov–Taylor identity without referring to the existence of an invariant scheme. The Slavnov–Taylor identity expresses gauge invariance, supersymmetry and translational invariance in a single symmetry identity. For its formulation one has to introduce several unphysical fields, namely the Faddeev–Popov ghost c, global ghosts $\epsilon, \overline{\epsilon}, \omega^\mu$ and sources Y_i for all non-linear BRS transformations. The Slavnov–Taylor identity is a complicated non-linear equation involving Green functions with physical and unphysical fields.

We have evaluated this identity and have derived simple symmetry conditions that resemble the normalization conditions in their mathematical structure. These symmetry conditions constitute exact physical statements that are valid to all orders and express lucidly the various aspects of the symmetries. Two important examples are the equality of the electron and selectron masses and the charge universality in the photon and photino interactions with electron and selectron. These are thus proven exclusively in the Wess–Zumino gauge without using superspace methods or referring to the realization of the supersymmetry algebra in the Hilbert space of physical states.

We have seen that in the renormalization of the one-loop self energies and vertex corrections using DReg several non-invariant counterterms are necessary. Still the calculation has been just as straightforward as if we would have relied on an invariant regularization and used only invariant counterterms. The reason is that the symmetry conditions may be used as an efficient tool for the practical determination of counterterms. This is particularly important for calculations beyond one-loop order since there the behaviour of invariant but inconsistent schemes such as DRed is not really under control. One should note, however, that using DRed in the 1-loop examples of this article invariant counterterms are sufficient to renormalize correctly not only the self energies and vertex corrections, as is well known [5], but also the vertex functions expressing the higher order corrections to supersymmetry transformations.

Higher order corrections to the non-linear supersymmetry transformations are determined in terms of vertex functions involving external Y fields and ϵ ghosts and are in general non-local. The corresponding counterterms may be read off from appropriate symmetry conditions. As an example we have calculated the one-loop corrections to the supersymmetry transformations of the electron and selectron. Via the Slavnov–Taylor identity they appear in the relations between physical vertex functions and may thus have also phenomenological implications.

The whole study can be generalized to supersymmetric models with soft breakings and eventually to the supersymmetric extensions of the standard model.

For the standard model the algebraic renormalization has been worked out in [22], soft breakings have been introduced in [23]. Although the corresponding Slavnov–Taylor identities are more involved since they have to express not only the symmetries but also the spontaneous or soft breaking, their structure is the same as in SQED. So it is possible also for these models to derive symmetry conditions which may be exploited in practical calculations if the existence of consistent invariant regularization schemes is questionable.

A Conventions

A.1 Spinors

2-Spinor indices and scalar products:

$$\epsilon_{\alpha\beta} = -\epsilon_{\beta\alpha}, \quad \epsilon_{12} = 1, \quad \epsilon^{\alpha\beta}\epsilon_{\beta\gamma} = \delta^{\alpha}{}_{\gamma}, \tag{187}$$
$$\epsilon_{\dot\alpha\dot\beta} = -\epsilon_{\dot\beta\dot\alpha}, \quad \epsilon_{\dot1\dot2} = 1, \quad \epsilon^{\dot\alpha\dot\beta}\epsilon_{\dot\beta\dot\gamma} = \delta^{\dot\alpha}{}_{\dot\gamma}, \tag{188}$$
$$\psi\chi = \psi^\alpha\chi_\alpha \ , \quad \psi^\alpha = \epsilon^{\alpha\beta}\psi_\beta \ , \tag{189}$$
$$\overline\psi\overline\chi = \overline\psi_{\dot\alpha}\overline\chi^{\dot\alpha} \ , \quad \overline\psi_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\overline\psi^{\dot\beta} \ . \tag{190}$$

σ matrices:

$$\sigma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{191}$$
$$\sigma^\mu_{\alpha\dot\alpha} = (1, \sigma^k)_{\alpha\dot\alpha} \ , \quad \overline\sigma^{\mu\dot\alpha\alpha} = (1, -\sigma^k)^{\dot\alpha\alpha} \ , \tag{192}$$
$$(\sigma^{\mu\nu})_\alpha{}^\beta = \frac{i}{2}(\sigma^\mu\overline\sigma^\nu - \sigma^\nu\overline\sigma^\mu)_\alpha{}^\beta \ , \quad (\overline\sigma^{\mu\nu})^{\dot\alpha}{}_{\dot\beta} = \frac{i}{2}(\overline\sigma^\mu\sigma^\nu - \overline\sigma^\nu\sigma^\mu)^{\dot\alpha}{}_{\dot\beta} \ . \tag{193}$$

Complex conjugation:

$$(\psi\theta)^\dagger = \overline{\theta\psi} \ , \tag{194}$$
$$(\psi\sigma^\mu\overline\theta)^\dagger = \theta\sigma^\mu\overline\psi \ , \tag{195}$$
$$(\psi\sigma^{\mu\nu}\theta)^\dagger = \overline\theta\overline\sigma^{\mu\nu}\overline\psi \ . \tag{196}$$

Derivatives:

$$\frac{\partial}{\partial\theta^\alpha}\theta^\beta = \delta_\alpha{}^\beta \ , \quad \frac{\partial}{\partial\theta_\alpha}\theta_\beta = \epsilon^{\alpha\gamma}\epsilon_{\beta\delta}\delta_\gamma{}^\delta = -\delta_\beta{}^\alpha \ , \tag{197}$$

$$\frac{\partial}{\partial\overline{\theta}_{\dot\alpha}}\overline{\theta}_{\dot\beta} = \delta^{\dot\alpha}{}_{\dot\beta} \ , \quad \frac{\partial}{\partial\overline{\theta}^{\dot\alpha}}\overline{\theta}^{\dot\beta} = \epsilon_{\dot\alpha\dot\gamma}\epsilon^{\dot\beta\dot\delta}\delta^{\dot\gamma}{}_{\dot\delta} = -\delta^{\dot\beta}{}_{\dot\alpha} \ . \tag{198}$$

4-Spinors: The general relations between a 4-spinor and derivatives with respect to it are defined in such a way that $\frac{\delta}{\delta\Psi}\Psi = 1$, $\frac{\delta}{\delta\overline{\Psi}}\overline{\Psi} = 1$:

$$\Psi = \begin{pmatrix}\psi_\alpha \\ \overline{\chi}^{\dot\alpha}\end{pmatrix} , \quad \overline{\Psi} = \begin{pmatrix}\chi^\alpha & \overline{\psi}_{\dot\alpha}\end{pmatrix} \ , \tag{199}$$

$$\frac{\delta}{\delta\Psi} = \left(-\frac{\delta}{\delta\psi_\alpha}, -\frac{\delta}{\delta\overline{\chi}^{\dot\alpha}}\right) \ , \quad \frac{\delta}{\delta\overline{\Psi}} = \begin{pmatrix}\frac{\delta}{\delta\chi^\alpha} \\ \frac{\delta}{\delta\overline{\psi}_{\dot\alpha}}\end{pmatrix} . \tag{200}$$

γ **matrices:**

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \overline{\sigma}^\mu & 0\end{pmatrix}, \quad \gamma^5 = \begin{pmatrix}-1 & 0 \\ 0 & 1\end{pmatrix}, \quad P_{L,R} = \frac{1\mp\gamma^5}{2}. \tag{201}$$

A.2 Vertex functions

Vertex functions with external $\chi_1, \chi_2, \dots$ are defined as

$$\Gamma_{\chi_1\chi_2\dots}(x_1, x_2, \dots) = \frac{\delta\Gamma(\varphi_i' = \varphi_i = Y_i = 0)}{\delta\chi_1(x_1)\delta\chi_2(x_2)\dots} \ . \tag{202}$$

The χ_i may be any of the physical fields, ghosts, or Y fields. For χ_i being one of the global ghosts it is understood that there is no corresponding x_i argument, and that the functional derivative reduces to a partial derivative.

The sign of the momenta in Fourier transforms is defined in such a way that momenta are always diagrammatically incoming. The Fourier transform of vertex functions thus involves the opposite sign for the momenta, as compared to the fields:

$$\chi(x) = \int \frac{d^4p}{(2\pi)^4} e^{-ipx}\chi(p) \ , \tag{203}$$

$$(2\pi\delta)^4(p_1 + \dots)\Gamma_{\chi_1\dots}(p_1, \dots) = \int d^4x_1 \dots e^{-i(p_1x_1+\dots)}\Gamma_{\chi_1\dots}(x_1, \dots) \ . \tag{204}$$

A.3 One-loop integrals

We use the following one-loop two- and three-point functions [24]:

$$B_{\{0,\mu,\mu\nu\}} := \int \frac{\{1, k_\mu, k_\mu k_\nu\}}{[k^2 - m_0^2][(k+p_1)^2 - m_1^2]} \ , \tag{205}$$

$$C_{\{0,\mu\}} := \int \frac{\{1, k_\mu\}}{[k^2 - m_0^2][(k+p_1)^2 - m_1^2][(k+p_1+p_2)^2 - m_2^2]} \tag{206}$$

with

$$\int \to \mu^{4-D} \frac{16\pi^2}{i} \int \frac{d^D k}{(2\pi)^D} \tag{207}$$

and the tensor decomposition

$$B_\mu = p_{1\mu} B_1 \ , \tag{208}$$

$$B_{\mu\nu} = p_{1\mu} p_{1\nu} B_{21} + g_{\mu\nu} B_{22} \ , \tag{209}$$

$$C_\mu = p_{1\mu} C_{11} + p_{2\mu} C_{12} \ , \tag{210}$$

$$B_{ij} = B_{ij}(m_0^2, m_1^2, p_1^2) \ , \tag{211}$$

$$C_{ij} = C_{ij}(m_0^2, m_1^2, m_2^2, p_1^2, p_2^2, (p_1+p_2)^2) \tag{212}$$

in the conventions of [25].

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