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The Implementation of the Minimal Supersymmetric Standard Model in *FeynArts* and *FormCalc*

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Abstract

We describe the implementation of the MSSM in the diagram generator *FeynArts* and the calculational tool *FormCalc*. This extension allows to perform loop calculations of MSSM processes almost fully automatically. The actual implementation has two aspects: The MSSM Feynman rules are specified in a new model file for *FeynArts*. The computation of the parameters in the MSSM Lagrangian from the input parameters is realized as a Fortran subroutine in the framework of *FormCalc*. The model file does not depend on the latter, however, and can be used even if one does not want to continue the calculation with *FormCalc*. The Feynman rules have been entered in a very generic way to allow e.g. scenarios with complex parameters, and have been tested extensively by reproducing known results for several non-trivial scattering processes.

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1 Introduction

One of the main problems of Feynman-diagrammatic computations is the enormous growth of the number of Feynman diagrams, not only with the loop order, but also with the number of particles in a model. While many precision calculations in the Standard Model (SM) could still be performed by hand exactly, the same is very difficult in models like the Minimal Supersymmetric Standard Model (MSSM). Yet it is highly desirable to perform unabridged calculations in the MSSM, too, since also the MSSM allows to make precise predictions in terms of a set of input parameters.

With the availability of powerful software packages, the basic problem of bookkeeping and calculation of the diagrams has been solved for many common cases. Still, it is not entirely trivial to code a model of the complexity of the MSSM in such a system, since this has to be done in a reasonably general way (i.e. not only for special cases of the parameters) and many checks have to be performed to test all sectors of the model.

The present paper documents the implementation of the MSSM in the *FeynArts* [1] and *FormCalc* [2] packages. Other programs for which an MSSM model file exists are GRACE [3, 4] and CompHEP [5, 6]. Conceptually, the *FeynArts*–*FormCalc* system works in three stages, as sketched in the following diagram.

Diagram generation	→	**Algebraic simplification**	→	**Numerical evaluation**
FeynArts		*FormCalc*		*FormCalc*/*LoopTools*
Mathematica		*Mathematica*/FORM		Fortran

A more detailed discussion of the interplay between *FeynArts* and *FormCalc* can be found in [7].

This paper is organized as follows. Corresponding to the three stages, Diagram generation, Algebraic simplification, and Numerical evaluation, there are three sections which describe the modifications to each stage that are necessary for calculations in the MSSM.

Sect. 2 describes the changes in *FeynArts*, namely the new MSSM model file. The model file declares the properties of the fields, their propagators, and their couplings. It contains the parameters of the MSSM Lagrangian. The Feynman rules have been entered in a very generic way to allow e.g. scenarios with complex parameters.

Sects. 3 and 4 describe the changes in *FormCalc*. Sect. 3 outlines the changes in the algebraic simplification and Sect. 4 explains the calculation of the parameters that are used in the model file. The model-file parameters are derived from a (reasonably small) set of input parameters. This is realized as a model-initializing Fortran subroutine in the framework of *FormCalc*. The model file does not depend on the latter, however, and can be used even if one does not want to continue the calculation with *FormCalc*.

2 Diagram generation

2.1 Algorithms

Supersymmetric theories contain fermion-number-violating couplings, e.g. quark–squark–gluino, so that the usual method of ordering the Dirac matrices oppositely to their occurrence along the arrows on fermionic lines breaks down since one cannot define a fermion-number flow.

FeynArts uses the "flipping-rule" algorithm [8]. This algorithm was invented precisely to solve the problem of fermion-number-violating couplings and works as follows: Instead of traversing the fermion lines along the fermion-number flow imposed from the outside, *FeynArts* chooses a direction for each fermion chain. If it turns out later that, for a Dirac fermion, the chosen direction is opposite to the actual fermion flow, *FeynArts* "flips" the coupling, i.e. it derives the coupling appropriate for the reversed fermion flow from the known coupling. The flipping of a coupling is in fact nothing but a charge (as opposed to hermitian) conjugation.

2.2 The MSSM model file

The model file is the source of all physics information in *FeynArts*. It declares the properties of the fields, their propagators, and their couplings. In the model file the parameters of the Lagrangian are used, not a restricted set of input parameters.

There are two versions of the MSSM model file in *FeynArts*, both of which follow the conventions of [9, 10, 11]. The file `MSSMQCD.mod` defines the complete (electroweak and strong) MSSM, whereas `MSSM.mod` contains only the electroweak subset, defined as everything except the gluon, its ghost, and the gluino. The four-sfermion couplings appear in `MSSM.mod` although they have both electroweak and strong parts. Counter-terms are not yet entered in the model files.

Table 1 gives the names for the fields and corresponding masses defined in `MSSM.mod` and `MSSMQCD.mod`. The symbols used for the MSSM parameters are specified in Table 2. The complete list of couplings is too long to be included here, but is contained in the *FeynArts* distribution as a PostScript file.

The MSSM model files further declare a number of restrictions with which certain groups of particles can be excluded in the diagram generation. These restrictions are listed in Table 3.

3 Algebraic simplification

3.1 Algorithms

The algebraic simplification has to address two problems that arise in one-loop calculations in supersymmetric theories in general and in the MSSM in particular. The first is the conceptual

leptons	$f = f^\dagger$	field	mass	sleptons	$f = f^\dagger$	field	mass
ν_g		`F[1,` $\{g\}$`]`	`0`	$\tilde{\nu}_g$		`S[11,` $\{g\}$`]`	`MSf`
ℓ_g		`F[2,` $\{g\}$`]`	`MLE`	$\tilde{\ell}^s_g$		`S[12,` $\{s, g\}$`]`	`MSf`

quarks				squarks			
u_g		`F[3,` $\{g, o\}$`]`	`MQU`	$\tilde{u}^s_g$		`S[13,` $\{s, g, o\}$`]`	`MSf`
d_g		`F[4,` $\{g, o\}$`]`	`MQD`	$\tilde{d}^s_g$		`S[14,` $\{s, g, o\}$`]`	`MSf`

gauge bosons				neutralinos, charginos			
γ	yes	`V[1]`	`0`	$\tilde{\chi}^0_n$	yes	`F[11,` $\{n\}$`]`	`MNeu`
Z	yes	`V[2]`	`MZ`	$\tilde{\chi}^-_c$		`F[12,` $\{c\}$`]`	`MCha`
W^-		`V[3]`	`MW`				

Higgs bosons				ghosts			
h^0	yes	`S[1]`	`Mh0`	u_γ		`U[1]`	`0`
H^0	yes	`S[2]`	`MHH`	u_Z		`U[2]`	`MZ`
A^0	yes	`S[3]`	`MA0`	u_+		`U[3]`	`MW`
G^0	yes	`S[4]`	`MG0`	u_-		`U[4]`	`MW`
H^-		`S[5]`	`MHp`	u_g		`U[5,` $\{u\}$`]`	`0`
G^-		`S[6]`	`MGp`				

gluon				gluino			
g	yes	`V[5,` $\{u\}$`]`	`0`	$\tilde{g}$	yes	`F[15,` $\{u\}$`]`	`MGl`

where the following indices are used:

$$
\begin{aligned}
g &= \texttt{Index[Generation]} = 1\ldots 3\,, & s &= \texttt{Index[Sfermion]} = 1\ldots 2\,,\\
o &= \texttt{Index[Colour]} = 1\ldots 3\,, & n &= \texttt{Index[Neutralino]} = 1\ldots 4\,,\\
u &= \texttt{Index[Gluon]} = 1\ldots 8\,, & c &= \texttt{Index[Chargino]} = 1\ldots 2\,.
\end{aligned}
$$

Table 1: The particle set-up in `MSSM.mod` and `MSSMQCD.mod`. The gluon, its ghost, and the gluino, which are defined only in the latter, are written in grey.

MhO, MHH, MAO, MGO	neutral Higgs masses
MHp, MGp	charged Higgs masses
CB, SB, TB	$\cos\beta$, $\sin\beta$, $\tan\beta$
CA, SA	$\cos\alpha$, $\sin\alpha$
C2A, S2A, C2B, S2B	$\cos 2\alpha$, $\sin 2\alpha$, $\cos 2\beta$, $\sin 2\beta$
CAB, SAB, CBA, SBA	$\cos(\alpha+\beta)$, $\sin(\alpha+\beta)$, $\cos(\beta-\alpha)$, $\sin(\beta-\alpha)$
MUE	Higgs-doublet mixing parameter μ
MGl	gluino mass
MNeu[n]	neutralino masses
ZNeu[n, n']	neutralino mixing matrix
MCha[c]	chargino masses
UCha[c, c'], VCha[c, c']	chargino mixing matrices
MSf[s, t, g]	sfermion masses
USf[t, g][s, s']	sfermion mixing matrix
Af[t, g]	(scalar) soft-breaking A-parameters

The indices enumerate the following properties:

sfermion generation: $g = 1 \ldots 3\,,$

$$\text{sfermion type:}\quad t = \begin{cases} 1 & \text{sneutrinos}\,, \\ 2 & \text{sleptons}\,, \\ 3 & \text{up-type squarks}\,, \\ 4 & \text{down-type squarks}\,, \end{cases}$$

eigenstates:	mass	gauge
sfermion	$s = 1 \ldots 2\,,$	$s' = 1 \ldots 2\,,$
neutralino	$n = 1 \ldots 4\,,$	$n' = 1 \ldots 4\,,$
chargino	$c = 1 \ldots 2\,,$	$c' = 1 \ldots 2\,.$

Table 2: Symbols representing the MSSM parameters introduced by `MSSM.mod` and `MSSMQCD.mod`.

NoGeneration1	exclude generation-1 fermions (ν_e, e, u, d)
NoGeneration2	exclude generation-2 fermions (ν_μ, μ, c, s)
NoGeneration3	exclude generation-3 fermions (ν_τ, τ, t, b)
NoElectronHCoupling	exclude all couplings involving electrons and any Higgs field
NoLightFHCoupling	exclude all couplings between light fermions (all fermions except the top) and any Higgs field
NoSUSYParticles	exclude the particles not present in the SM: sfermions, charginos, neutralinos, and the Higgs fields H^0, A^0, $H^\pm$
THDMParticles	exclude the particles not present in the two-Higgs-doublet model: the sfermions, charginos, and neutralinos

Table 3: Pre-defined restrictions in `MSSM.mod` and `MSSMQCD.mod`.

problem of implementing a supersymmetry-preserving regularization scheme, the second is the technical problem of dealing with more diagrams and larger expressions.

The default regularization scheme employed by *FormCalc* is dimensional regularization. Unfortunately, this scheme is known to break supersymmetry [12]. *FormCalc* has therefore been equipped with an alternative scheme suited for calculations in supersymmetric theories. It is the constrained differential renormalization scheme (CDR) [13] which is equivalent to regularization by dimensional reduction at the one-loop level [2]. The schemes are chosen with the `Dimension` option of the `CalcFeynAmp` function: `Dimension -> D` selects dimensional regularization (the default), `Dimension -> 4` switches to CDR.

The second problem is a technical one. In the SM, both the number of diagrams and the comparatively simple coupling structure allow to perform calculations with "brute force." To wit, the diagrams are generated with all indices explicitly written out, and then calculated.

The situation is less favourable in the MSSM, however: not only is the number of diagrams considerably higher in most cases, but also the coupling structure is more involved. This is because particles like sfermions, or gauginos and higgsinos in general mix to form mass eigenstates and hence their couplings contain plenty of mixing matrices and can become rather lengthy. This means that the algebraic simplification has to be performed more carefully in order to maintain a decent performance and keep the size of the results as small as possible.

The way in which *FormCalc* proceeds is already suggested by the level structure of the *FeynArts* amplitudes. The lowest (generic) level completely determines the kinematical structure of a diagram, so *FormCalc* first performs the time-consuming simplifications involving kinematical quantities, like the tensor reduction, on the far fewer generic diagrams only. For each diagram at the next higher (classes) level, it then substitutes the generic coupling constants and masses by their actual values. The diagrams are mostly specified by then, except that index summations, e.g. over fermion generations, are not yet carried out. These index summations are performed only in the Fortran code, which means that *FormCalc* has to keep track of all indices and write out the corresponding loop instructions in Fortran. For a model like the MSSM, the savings incurred with this method of simplification may easily amount to an order of magnitude in both CPU time and size of the Fortran code.

4 Numerical evaluation

In the *FormCalc* framework, the *Mathematica* expressions resulting from the algebraic simplification are translated to Fortran code for numerical evaluation. The generated Fortran code of course has to be provided with the proper numerical values for the parameters appearing in the model, i.e. the variables in Table 2. This is solved by a subroutine which is called at the beginning of the calculation to initialize all model parameters.

Technically, the input parameters are specified in common blocks defined in `model.h`. Some less commonly changed inputs, notably some breaking parameters in the sfermion sector, are realized as preprocessor variables which take default values if not defined by the user. Preprocessor variables are also used for switches, such as whether squark mixing should be turned on or off.

4.1 Parameters of the MSSM

Supersymmetry (SUSY) completely determines the supersymmetric part of the MSSM Lagrangian once the SM parameters are known. The complete MSSM, however, with softly broken SUSY in its general form, necessarily introduces a large number of masses, phases, and mixing angles to parametrize the SUSY breaking. At a final count, 105 of these degrees of freedom cannot be absorbed in some other quantity or rotated away [14]. In order to get a handle on so many parameters, several assumptions have been made in `mssm_ini.F` to arrive at a moderate number of input parameters while retaining reasonable generality.

The first assumption is that the SUSY-GUT relation holds, which relates the U(1), SU(2), and

SU(3) gaugino mass parameters M_1, M_2, and M_3 according to

$$M_1 = \frac{5}{3}\frac{s_W^2}{c_W^2} M_2 \quad \text{and} \quad M_3 = \frac{\overline{\alpha}_s(s)}{\overline{\alpha}(s)} s_W^2 M_2 \quad \text{with} \quad m_{\tilde{g}} \equiv |M_3| \,, \tag{1}$$

where $\sqrt{s}$ is the CMS energy, $c_W = M_W/M_Z$, and $s_W = \sqrt{1-c_W^2}$. The running couplings $\overline{\alpha}$ and $\overline{\alpha}_s$ are $\overline{\text{MS}}$ values; we use the CERNlib function `ALPHAS2` [15] for $\overline{\alpha}_s(s)$ and

$$\frac{1}{\overline{\alpha}(s)} = \frac{1}{\overline{\alpha}(M_Z^2)} - \frac{1}{3\pi}\sum_{f\neq t} Q_f^2 N_c \ln\frac{s}{M_Z^2} = \frac{1}{\overline{\alpha}(M_Z^2)} - \frac{20}{9\pi}\ln\frac{s}{M_Z^2} \tag{2}$$

with $1/\overline{\alpha}(M_Z^2) = 127.934$ [16].

Secondly, we assume the SUSY-breaking parameters in the sfermion sector to be flavour-blind and distinguish only two A-parameters for different isospin values, hence the sfermion sector is governed by seven scalar parameters:

$$\begin{aligned}
\mathbf{M}_{\tilde{Q}}^2 &= M_{\tilde{Q}}^2 \mathbb{1}\,, & \mathbf{M}_{\tilde{U}}^2 &= M_{\tilde{U}}^2 \mathbb{1}\,, & \mathbf{A}_U &= A_u \mathbb{1}\,, \\
& & \mathbf{M}_{\tilde{D}}^2 &= M_{\tilde{D}}^2 \mathbb{1}\,, & \mathbf{A}_D &= A_d \mathbb{1}\,, \\
\mathbf{M}_{\tilde{L}}^2 &= M_{\tilde{L}}^2 \mathbb{1}\,, & \mathbf{M}_{\tilde{E}}^2 &= M_{\tilde{E}}^2 \mathbb{1}\,, & \mathbf{A}_L &= A_d \mathbb{1}\,,
\end{aligned} \tag{3}$$

where in addition the various $M_{\tilde{X}}$ get the default value M_{SUSY}, a common SUSY-breaking mass, i.e.

$$M_{\tilde{Q}} = M_{\tilde{L}} = M_{\tilde{U}} = M_{\tilde{D}} = M_{\tilde{E}} = M_{\text{SUSY}} \quad \text{(by default)}\,. \tag{4}$$

The remaining MSSM input parameters thus have 10 degrees of freedom:

$$\begin{array}{lll}
1 & \text{quotient of VEV's:} & \tan\beta \text{ (real)}, \\
1 & \text{Higgs mass:} & M_A \text{ (real)}, \\
6 & \text{breaking parameters:} & A_u, A_d \text{ (complex)},\ M_{\text{SUSY}}, M_2 \text{ (real)}, \\
2 & \text{mixing parameter:} & \mu \text{ (complex)}.
\end{array} \tag{5}$$

The inputs to `mssm_ini.F` are summarized in Table 4. All other variables in the MSSM Lagrangian (see Table 2) are determined once this reduced set of MSSM input parameters and the SM inputs are specified and are calculated by `mssm_ini.F`. This is the reason why `mssm_ini.F` is much more involved than its companion file for the SM, `sm_ini.F`. The calculation of the parameters can be directed by several switches, which are also listed in Table 4.

When the particle masses are calculated from these input parameters, `mssm_ini.F` checks whether they are consistent with the current exclusion limits and automatically omits already-excluded points with a warning. The following bounds are used:

$$\begin{aligned}
m_{\tilde{t}} &\geqslant 80 \text{ GeV [17]}\,, & M_h &\geqslant \begin{cases} 91 \text{ GeV for real input parameters [19]}\,, \\ 85 \text{ GeV for complex input parameters [20]}\,, \end{cases} \\
m_{\tilde{b}} &\geqslant 70 \text{ GeV [17]}\,, & & \\
m_{\tilde{q}\neq\tilde{b},\tilde{t}} &\geqslant 150 \text{ GeV [17]}\,, & m_{\tilde{\chi}} &\geqslant 90 \text{ GeV [21]}\,, \\
m_{\tilde{\ell}} &\geqslant 70 \text{ GeV [18]}\,, & m_{\tilde{\chi}^0} &\geqslant 30 \text{ GeV [21]}\,, \\
\Delta\rho_{\tilde{t},\tilde{b}} &\leqslant 3\times 10^{-3} \text{ [16]}\,, & m_{\tilde{g}} &\geqslant 175 \text{ GeV [22]}\,.
\end{aligned} \tag{6}$$

The detailed formulas built into `mssm_ini.F` for computing the various parameters are presented in the following sections, organized according to the various sectors of the MSSM.

4.2 The Higgs sector

The neutral Higgs sector is fixed by choosing a value for $\tan\beta = v_2/v_1$ (the ratio of the vacuum expectation values of the two Higgs doublets) and for the mass M_A of the CP-odd neutral Higgs boson A^0. For the CP-even Higgs masses, which receive sizable radiative corrections, we use the approximation formula of [23] which agrees with the full two-loop calculation [24] to within less than 2 GeV.

parameter	Fortran name	type	default value
$\tan\beta$	TB	double precision	
M_A	MA0	double precision	
A_u	Au	double complex	
A_d	Ad	double complex	
M_{SUSY}	MSusy	double precision	
M_2	M_2	double precision	
μ	MUE	double complex	
$M_{\tilde{Q}}$	MSQ	preprocessor variable	MSusy
$M_{\tilde{L}}$	MSL	preprocessor variable	MSusy
$M_{\tilde{U}}$	MSU	preprocessor variable	MSusy
$M_{\tilde{D}}$	MSD	preprocessor variable	MSusy
$M_{\tilde{E}}$	MSE	preprocessor variable	MSusy

switch	action
NO_SQUARK_MIXING	Sets $A_u = \mu^* \cot\beta$ and $A_d = \mu^* \tan\beta$, so that the off-diagonal entries of the sfermion mass matrices vanish (cf. Eq. (37)), i.e. makes sfermion mass eigenstates = sfermion gauge eigenstates.
COMPLEX_PARAMETERS	Uses a simpler (one-loop) approximation for the Higgs masses which is valid for all parameters. This switch must be set if complex input parameters are used because otherwise the more precise (two-loop) approximation for the Higgs masses is taken, which is valid only for real parameters.
SM_ONLY	Calculates M_h as usual, but then reverse-engineers the mixing in the Higgs sector (α and β) such that the MSSM Higgs sector looks like a SM Higgs sector (see [11], p. 356), with the light CP-even Higgs boson h figuring as the SM Higgs boson.
NO_EXCLUSION_LIMITS	Ignores the experimental bounds, i.e. does not exclude points in parameter space if the bounds in Eq. (6) are violated.

Table 4: The inputs and switches for `mssm_ini.F`. Note: A preprocessor variable is defined with "`#define` *var value*" and a switch is set with "`#define` *switch*". The `#define` must stand at the beginning of the line.

4.2.1 The neutral CP-even Higgs bosons h and H

In this section we discuss the case of real MSSM parameters, for which a more sophisticated computation of the Higgs masses is implemented. The case of complex parameters is treated in Sect. 4.2.2.

The gauge eigenstates ϕ_1 and ϕ_2 of the neutral CP-even Higgs bosons mix via the mass matrix

$$M^2_{\text{Higgs}} = \begin{pmatrix} M_A^2 \sin^2\beta + M_Z^2 \cos^2\beta - \hat{\Sigma}_{\phi_1} & -(M_A^2 + M_Z^2)\sin\beta\cos\beta - \hat{\Sigma}_{\phi_1\phi_2} \\ -(M_A^2 + M_Z^2)\sin\beta\cos\beta - \hat{\Sigma}_{\phi_1\phi_2} & M_A^2 \cos^2\beta + M_Z^2 \sin^2\beta - \hat{\Sigma}_{\phi_2} \end{pmatrix}. \tag{7}$$

Diagonalizing this matrix yields the Higgs masses

$$\begin{aligned} M^2_{H,h} = {} & \frac{M_A^2 + M_Z^2 - \hat{\Sigma}_{\phi_1} - \hat{\Sigma}_{\phi_2}}{2} \pm \Bigg[\frac{(M_A^2 + M_Z^2)^2 + (\hat{\Sigma}_{\phi_1} - \hat{\Sigma}_{\phi_2})^2}{4} - M_A^2 M_Z^2 \cos^2 2\beta \\ & + \frac{1}{2}(M_A^2 - M_Z^2)\cos 2\beta\,(\hat{\Sigma}_{\phi_1} - \hat{\Sigma}_{\phi_2}) + (M_A^2 + M_Z^2)\sin 2\beta\,\hat{\Sigma}_{\phi_1\phi_2} + \hat{\Sigma}^2_{\phi_1\phi_2}\Bigg]^{1/2}. \end{aligned} \tag{8}$$

The mixing angle α of the CP-even Higgs doublet is hence

$$\alpha = \arctan\left(\frac{-(M_A^2 + M_Z^2)\sin\beta\cos\beta - \hat{\Sigma}_{\phi_1\phi_2}}{M_Z^2\cos^2\beta + M_A^2\sin^2\beta - \hat{\Sigma}_{\phi_1} - M_h^2}\right). \tag{9}$$

The presence of renormalized self-energies $\hat{\Sigma}$ indicates that the mass matrix contains significant radiative corrections which must be taken into account to make quantitatively correct predictions. The self-energies of course receive contributions from all sectors of the MSSM, but not all are numerically of equal relevance. We take into account only the following terms:

- $\hat{\Sigma}^{(1,t/\tilde{t})}(0)$: the one-loop $t/\tilde{t}$-contributions at zero momentum transfer up to m_t^4 [25],
- $\hat{\Sigma}^{(2,t/\tilde{t})}(0)$: the dominant two-loop $t/\tilde{t}$-contributions of $\mathcal{O}(\alpha\alpha_s)$ at zero momentum transfer [23] and the leading two-loop Yukawa correction of $\mathcal{O}(\alpha^2)$ [26],
- $\hat{\Sigma}^{(1,\text{rest})}$: the one-loop leading-log contributions from all other sectors [27].

In the following, we give the explicit expressions for these contributions as implemented in `mssm_ini.F`.

- The one-loop $t/\tilde{t}$-contributions at zero momentum transfer up to m_t^4:

$$\hat{\Sigma}^{(1,t/\tilde{t})}_{\phi_1}(0) = \frac{G_F\sqrt{2}}{\pi^2} M_Z^4 \Lambda \cos^2\beta \ln\frac{m_t^2}{M_S^2}\,, \tag{10}$$

$$\hat{\Sigma}^{(1,t/\tilde{t})}_{\phi_1\phi_2}(0) = -\frac{G_F\sqrt{2}}{\pi^2} M_Z^2 \cot\beta \left(-\frac{3}{8}m_t^2 + M_Z^2\Lambda\sin^2\beta\right)\ln\frac{m_t^2}{M_S^2}\,, \tag{11}$$

$$\begin{aligned} \hat{\Sigma}^{(1,t/\tilde{t})}_{\phi_2}(0) = {} & \frac{G_F\sqrt{2}}{\pi^2}\frac{m_t^4}{8\sin^2\beta}\Bigg[-2\frac{M_Z^2}{m_t^2} + \frac{11}{10}\frac{M_Z^4}{m_t^4} \\ & + \left(12 - 6\frac{M_Z^2}{m_t^2}\sin^2\beta + 8\frac{M_Z^4}{m_t^4}\Lambda\sin^4\beta\right)\ln\frac{m_t^2}{M_S^2} \\ & + \left(-12 + 4\frac{M_Z^2}{m_t^2} + 6\frac{m_t^2}{M_S^2}\right)\frac{(M_t^{LR})^2}{M_S^2} + \left(1 - 4\frac{m_t^2}{M_S^2} + 3\frac{m_t^4}{M_S^4}\right)\frac{(M_t^{LR})^4}{M_S^4} \\ & + \left(\frac{3}{5}\frac{m_t^2}{M_S^2} - \frac{12}{5}\frac{m_t^4}{M_S^4} + 2\frac{m_t^6}{M_S^6}\right)\frac{(M_t^{LR})^6}{M_S^6} \\ & + \left(\frac{3}{7}\frac{m_t^4}{M_S^4} - \frac{12}{7}\frac{m_t^6}{M_S^6} + \frac{3}{2}\frac{m_t^8}{M_S^8}\right)\frac{(M_t^{LR})^8}{M_S^8}\Bigg] \end{aligned} \tag{12}$$

with

$$M_t^{LR} = A_u - \mu^* \tan\beta\,, \tag{13}$$

$$M_S = \left[M_{\tilde Q}^2 M_{\tilde U}^2 + m_t^2(M_{\tilde Q}^2 + M_{\tilde U}^2) + m_t^4\right]^{1/4}, \tag{14}$$

$$\Lambda = \frac{1}{8} - \frac{1}{3}s_W^2 + \frac{4}{9}s_W^4\,. \tag{15}$$

- The dominant two-loop $t/\tilde t$-contributions of $\mathcal{O}(\alpha\alpha_s)$ at zero momentum transfer and the leading two-loop Yukawa correction of $\mathcal{O}(\alpha^2)$:

$$\hat\Sigma_{\phi_1}^{(2,t/\tilde t)}(0) = \hat\Sigma_{\phi_1\phi_2}^{(2,t/\tilde t)}(0) = 0\,, \tag{16}$$

$$\begin{aligned}\hat\Sigma_{\phi_2}^{(2,t/\tilde t)}(0) = {} & \frac{G_F\sqrt{2}}{\pi^2}\frac{\overline\alpha_s(m_t^2)}{\pi}\frac{\overline m_t^4}{\sin^2\beta}\left[3\ln^2\frac{\overline m_t^2}{M_S^2} - 6\ln\frac{\overline m_t^2}{M_S^2} - 6\frac{M_t^{LR}}{M_S}\right.\\ & \left. - 3\frac{(M_t^{LR})^2}{M_S^2}\ln\frac{\overline m_t^2}{M_S^2} + \frac{3}{4}\frac{(M_t^{LR})^4}{M_S^4}\right]\\ & - \frac{9G_F^2}{16\pi^4}\frac{\overline m_t^6}{\sin^2\beta}\left[\tilde X_t \ln\frac{m_{\tilde t_1}m_{\tilde t_2}}{\overline m_t^2} + \ln^2\frac{m_{\tilde t_1}m_{\tilde t_2}}{\overline m_t^2}\right],\end{aligned} \tag{17}$$

where

$$\tilde X_t = \left(\frac{m_{\tilde t_2}^2 - m_{\tilde t_1}^2}{\overline m_t^2}(U_{11}^t U_{12}^t)^2\right)^2\left(2 - \frac{m_{\tilde t_2}^2 + m_{\tilde t_1}^2}{m_{\tilde t_2}^2 - m_{\tilde t_1}^2}\ln\frac{m_{\tilde t_2}^2}{m_{\tilde t_1}^2}\right) + 2\frac{m_{\tilde t_2}^2 - m_{\tilde t_1}^2}{\overline m_t^2}(U_{11}^t U_{12}^t)^2 \ln\frac{m_{\tilde t_2}^2}{m_{\tilde t_1}^2}\,, \tag{18}$$

and U^t is the stop mixing matrix defined in Eq. (35). We use the $\overline{\rm MS}$ top mass

$$\overline m_t = \overline m_t(m_t) \approx \frac{m_t}{1 + \frac{4}{3\pi}\overline\alpha_s(m_t^2)} \tag{19}$$

instead of the pole mass to include also the leading $t/\tilde t$-contributions beyond $\mathcal{O}(\alpha\alpha_s)$.

- The one-loop leading-log contributions from all other sectors:

$$\begin{aligned}\hat\Sigma_{\phi_1}^{(1,\rm rest)} = {} & -\frac{G_F M_Z^4}{12\sqrt{2}\pi^2}\left[\left(\frac{12N_c m_b^4}{M_Z^4\cos^4\beta} - \frac{6N_c m_b^2}{M_Z^2\cos^2\beta} + P_b + P_f + P_g + P_{2H}\right)\ln\frac{M_{\rm SUSY}^2}{M_Z^2}\right.\\ & \left. + \theta(M_A - M_Z)(P_{1H} - P_{2H})\ln\frac{M_A^2}{M_Z^2}\right]\cos^2\beta\\ & - \frac{G_F N_c m_b^2}{4\sqrt{2}\pi^2 M_{\rm SUSY}^2}\left[\frac{4m_b^2 A_b M_b^{LR}}{\cos^2\beta}\left(1 - \frac{A_b M_b^{LR}}{12M_{\rm SUSY}^2}\right) - M_Z^2 A_b\left(M_b^{LR} + \frac{1}{3}A_b\right)\right],\end{aligned} \tag{20}$$

$$\begin{aligned}\hat\Sigma_{\phi_1\phi_2}^{(1,\rm rest)} = {} & -\frac{G_F M_Z^4}{12\sqrt{2}\pi^2}\left[\left(\frac{3N_c m_b^2}{M_Z^2\cos^2\beta} - P_b - P_f - P_g' - P_{2H}'\right)\ln\frac{M_{\rm SUSY}^2}{M_Z^2}\right.\\ & \left. + \theta(M_A - M_Z)(P_{1H} + P_{2H}')\ln\frac{M_A^2}{M_Z^2}\right]\sin\beta\cos\beta\\ & + \frac{G_F N_c m_b^2}{8\sqrt{2}\pi^2 M_{\rm SUSY}^2}\left[\frac{4m_b^2 \mu M_b^{LR}}{\cos^2\beta}\left(1 - \frac{A_b M_b^{LR}}{6M_{\rm SUSY}^2}\right)\right.\\ & \left. - M_Z^2\tan\beta\left(M_b^{LR}(A_b + \mu\cot\beta) + \frac{1}{3}(\mu^2 + A_b^2)\right)\right],\end{aligned} \tag{21}$$

$$\hat\Sigma_{\phi_2}^{(1,\rm rest)} = -\frac{G_F M_Z^4}{12\sqrt{2}\pi^2}\left[(P_b + P_f + P_g + P_{2H})\ln\frac{M_{\rm SUSY}^2}{M_Z^2}\right.$$

$$
\left. + \theta(M_A - M_Z)(P_{1H} - P_{2H}) \ln \frac{M_A^2}{M_Z^2} \right] \sin^2\beta
$$

$$
+ \frac{G_F N_c m_b^2}{4\sqrt{2}\pi^2 M_{\rm SUSY}^2} \left[\frac{m_b^2 \mu^2 (M_b^{LR})^2}{3 M_{\rm SUSY}^2 \cos^2\beta} + M_Z^2 \mu \tan\beta \left(M_b^{LR} + \frac{1}{3}\mu \tan\beta \right) \right] \tag{22}
$$

with

$$
\begin{aligned}
M_b^{LR} &= A_u - \mu^* \cot\beta \,, \\
P_b &= N_c (1 + 4 Q_b s_W^2 + 8 Q_b^2 s_W^4) \,, \\
P_f &= N_c (N_g - 1) \big(2 - 4 s_W^2 + 8 (Q_t^2 + Q_b^2) s_W^4 \big) + N_g (2 - 4 s_W^2 + 8 s_W^4) \,, \\
P_g &= -44 + 106 s_W^2 - 62 s_W^4 \,, \\
P_g' &= 10 + 34 s_W^2 - 26 s_W^4 \,, \\
P_{1H} &= -9 \cos^4 2\beta + (1 - 2 s_W^2 + 2 s_W^4) \cos^2 2\beta \,, \\
P_{2H} &= -10 + 2 s_W^2 - 2 s_W^4 \,, \\
P_{2H}' &= 8 - 22 s_W^2 + 10 s_W^4 \,, \\
Q_t &= \frac{2}{3} \,, \qquad Q_b = -\frac{1}{3} \,, \qquad N_c = 3 \,, \qquad N_g = 3 \,.
\end{aligned} \tag{23}
$$

4.2.2 The case of complex parameters

The sophisticated calculation of the Higgs masses as in the preceding section is valid only for real parameters. When complex parameters are used, the preprocessor variable `COMPLEX_PARAMETERS` must be defined (see Table 4), which makes `mssm_ini.F` use a simple (one-loop) approximation for the Higgs masses that is valid also for complex parameters.

For the Higgs masses the dominant one-loop contributions are the m_t^4 terms [25]. Neglecting the splitting of the $\tilde{t}$ masses, the ϕ_2 self-energy of Eq. (12) becomes

$$
\hat{\Sigma}_{\phi_2}^{(1,t/\tilde{t})}(0) = \frac{3 G_F}{2\sqrt{2}\pi^2} \frac{\overline{m}_t^4}{\sin^2\beta} \ln \left(1 + \frac{M_{\tilde{Q}}^2 \, M_{\tilde{U}}^2}{\overline{m}_t^4} + \frac{M_{\tilde{Q}}^2 + M_{\tilde{U}}^2}{\overline{m}_t^2} \right) \tag{24}
$$

with the running top mass $\overline{m}_t$ as defined in Eq. (19). All other radiative corrections in the Higgs mass matrix are set to zero.

4.2.3 The charged Higgs bosons

There are only small radiative corrections for the charged Higgs masses. We use the following relation, valid for $M_A \sim \mathcal{O}(M_Z)$ [27], up to M_A-values of 250 GeV:

$$
\begin{aligned}
M_{H^\pm}^2 = M_A^2 + M_W^2 &+ \frac{\alpha M_W^2}{12\pi s_W^2} \left[N_c (N_g - 1) + N_g - 9 + 15 \frac{s_W^2}{c_W^2} \right] \ln \frac{M_{\rm SUSY}^2}{M_W^2} \\
&+ \frac{N_c \alpha}{8\pi s_W^2 M_W^2} \left[\frac{2 \overline{m}_t^2 m_b^2}{\sin^2\beta \cos^2\beta} - M_W^2 \left(\frac{\overline{m}_t^2}{\sin^2\beta} + \frac{m_b^2}{\cos^2\beta} \right) + \frac{2}{3} M_W^4 \right] \ln \frac{M_{\rm SUSY}^2}{\overline{m}_t^2} \,,
\end{aligned} \tag{25}
$$

with $N_c = N_g = 3$ and the running top mass $\overline{m}_t$ as defined in Eq. (19). Above 250 GeV, this formula reduces to $M_{H^\pm}^2 = M_A^2 + M_W^2$.

4.3 The chargino sector

The charged gauginos and higgsinos mix via the mass matrix [9, 10]

$$
X = \begin{pmatrix} M_2 & \sqrt{2} M_W \sin\beta \\ \sqrt{2} M_W \cos\beta & \mu \end{pmatrix} . \tag{26}
$$

X is diagonalized with two unitary matrices U and V according to

$$U^* X V^\dagger = \mathrm{diag}(m_{\tilde\chi_1}, m_{\tilde\chi_2})\,, \tag{27}$$

which yields the chargino masses

$$m^2_{\tilde\chi_{1,2}} = \frac{M_2^2 + |\mu|^2 + 2M_W^2}{2} \mp \sqrt{\frac{(M_2^2 + |\mu|^2 + 2M_W^2)^2}{4} - |M_W^2 \sin 2\beta - \mu M_2|^2}\,. \tag{28}$$

For the numerical diagonalization, the LAPACK subroutine ZGESVD [28] is used.

4.4 The neutralino sector

The neutral gauginos and higgsinos mix via the mass matrix [9, 10]

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_W \cos\beta & M_Z s_W \sin\beta \\ 0 & M_2 & M_Z c_W \cos\beta & -M_Z c_W \sin\beta \\ -M_Z s_W \cos\beta & M_Z c_W \cos\beta & 0 & -\mu \\ M_Z s_W \sin\beta & -M_Z c_W \sin\beta & -\mu & 0 \end{pmatrix}. \tag{29}$$

Y is diagonalized with a unitary matrix N according to

$$N^* Y N^\dagger = \mathrm{diag}(m_{\tilde\chi_1^0}, m_{\tilde\chi_2^0}, m_{\tilde\chi_3^0}, m_{\tilde\chi_4^0})\,, \tag{30}$$

which yields the four neutralino mass eigenstates. For the numerical diagonalization, the LAPACK subroutine ZGESVD [28] is used. Note that we work with a general complex matrix N and also check that the computed mass values are nonnegative.

4.5 The sfermion sector

The sfermion gauge eigenstates are connected via the mass matrix [9, 10]

$$Z = \begin{pmatrix} M_f^{LL} + m_f^2 & m_f (M_f^{LR})^* \\ m_f M_f^{LR} & M_f^{RR} + m_f^2 \end{pmatrix} \tag{31}$$

where

$$M_f^{LL} = M_Z^2 (I_3^f - Q_f s_W^2)\cos 2\beta + \begin{cases} M^2_{\tilde Q} & \text{for left-handed squarks,} \\ M^2_{\tilde L} & \text{for left-handed sleptons,} \end{cases} \tag{32}$$

$$M_f^{LR} = A_f - \mu^* \begin{cases} \cot\beta & \text{for } u\text{-type sfermions } (I_3^f = +1/2), \\ \tan\beta & \text{for } d\text{-type sfermions } (I_3^f = -1/2), \end{cases} \tag{33}$$

$$M_f^{RR} = M_Z^2 Q_f s_W^2 \cos 2\beta + \begin{cases} M^2_{\tilde U} & \text{for right-handed, } u\text{-type squarks,} \\ M^2_{\tilde D} & \text{for right-handed, } d\text{-type squarks,} \\ M^2_{\tilde E} & \text{for right-handed sleptons.} \end{cases} \tag{34}$$

The mass eigenstates are obtained by diagonalizing Z with a unitary matrix U^f, viz.

$$U^f Z U^{f\dagger} = \mathrm{diag}(m^2_{\tilde f_1}, m^2_{\tilde f_2})\,, \tag{35}$$

which results in the sfermion masses

$$m^2_{\tilde f_{1,2}} = m_f^2 + \frac{1}{2}\left(M_f^{LL} + M_f^{RR} \mp \sqrt{\left(M_f^{LL} - M_f^{RR}\right)^2 + 4m_f^2 \left|M_f^{LR}\right|^2}\right). \tag{36}$$

For the numerical diagonalization the LAPACK subroutine ZHEEV [28] is used.

Sfermion mixing can be switched off by defining the preprocessor variable `NO_SQUARK_MIXING` (see Table 4). This causes the breaking parameters A_u and A_d to be set to the values

$$A_u = \mu^* \cot\beta\,, \qquad A_d = \mu^* \tan\beta \tag{37}$$

so that the off-diagonal terms in the mass matrix Z vanish.

4.5.1 Supersymmetric contributions to $\Delta\rho$

In addition to the experimental bounds on the squark masses, `mssm_ini.F` also checks whether the MSSM contributions to the ρ-parameter are consistent with the current exclusion limits (see Eq. (6)). This bound becomes relevant mainly when parameters are chosen to achieve a large mass splitting between $\tilde{t}_1$ and $\tilde{t}_2$ or $\tilde{b}_1$ and $\tilde{b}_2$.

We have implemented the calculation of the MSSM contributions to $\Delta\rho$ according to [29]. The sfermion contributions are significant only when the masses of the isospin partners are very different, and since this is governed by the quark masses (see Eq. (36)), only corrections from $\tilde{t}/\tilde{b}$-loops have been included. Contributions from gluino exchange, which are very lengthy and vanish for large $m_{\tilde{g}}$, have similarly been neglected.

These contributions to $\Delta\rho$ are given by

$$\begin{aligned}\Delta\rho_{\tilde{t},\tilde{b}} &= U^t_{11}U^t_{22}U^{t*}_{12}U^{t*}_{21}F(m^2_{\tilde{t}_1},m^2_{\tilde{t}_2}) + U^b_{11}U^b_{22}U^{b*}_{12}U^{b*}_{21}F(m^2_{\tilde{b}_1},m^2_{\tilde{b}_2}) \\ &\quad + |U^t_{11}|^2|U^b_{11}|^2F(m^2_{\tilde{t}_1},m^2_{\tilde{b}_1}) + |U^t_{11}|^2|U^b_{21}|^2F(m^2_{\tilde{t}_1},m^2_{\tilde{b}_2}) \\ &\quad + |U^t_{21}|^2|U^b_{11}|^2F(m^2_{\tilde{t}_2},m^2_{\tilde{b}_1}) + |U^t_{21}|^2|U^b_{21}|^2F(m^2_{\tilde{t}_2},m^2_{\tilde{b}_2})\,, \end{aligned} \tag{38}$$

where the U^f are the sfermion mixing matrices defined in Eq. (35) and the function

$$F(x,y) = \frac{3G_F}{8\sqrt{2}\pi^2}F_1(x,y) + \frac{G_F}{4\sqrt{2}\pi^2}\frac{\overline{\alpha}_s(m_t^2)}{\pi}F_2(x,y) \tag{39}$$

includes the one- and two-loop contributions

$$F_1(x,y) = x + y - \frac{2xy}{x-y}\ln\frac{x}{y}\,, \tag{40}$$

$$F_2(x,y) = x + y - \frac{2xy}{x-y}\left(2 + \frac{x}{y}\ln\frac{x}{y}\right)\ln\frac{x}{y} + \frac{x^2(x+y)}{(x-y)^2}\ln^2\frac{x}{y} - 2(x-y)\,\mathrm{Li}_2\left(1 - \frac{x}{y}\right). \tag{41}$$

The two-loop contribution is of the order of 10 to 15% of the one-loop result.

5 Tests

The model files `MSSM.mod` and `MSSMQCD.mod` have been checked against results known from the literature for a variety of scattering processes. In cases where the Fortran programs of the original authors were available, we found perfect agreement for all differential cross-sections. In the other cases we could reproduce qualitatively the figures. The list of processes we checked together with the agreement we achieved is given in Table 5.

6 Availability, Requirements

The *FeynArts* package including the `MSSM.mod` and `MSSMQCD.mod` model files can be downloaded from `http://www.feynarts.de`. The program itself requires *Mathematica* 3 or above. For the topology editor, which is not necessarily invoked, a Java VM and the J/Link package are needed, both of which can be obtained free of charge (see the instructions on the web site).

The *FormCalc* package is available from `http://www.feynarts.de/formcalc` and includes the initialization file `mssm_ini.F`. It runs on Unix-like platforms and requires *Mathematica* 3, *FORM* 3, and the GNU C compiler (gcc). To link the Fortran code generated by *FormCalc*, one needs in addition the *LoopTools* library (`http://www.feynarts.de/looptools`) and the CERNlib (`http://wwwinfo.cern.ch/asd/index.html`).

FeynArts and *FormCalc* each include a comprehensive manual which explains installation and usage. Both are open-source programs and stand under the GNU library general public license.

process	agreement	reference	scope
$q\bar{q} \to t\bar{t}$	11 digits	[30]	SUSY-QCD only
$q\bar{q} \to t\bar{t}$	10 digits	[31]	full MSSM
$gg \to t\bar{t}$	qualitative	[32]	SUSY-QCD only
$gg \to t\bar{t}$	10 digits	[31]	full MSSM
$gg \to H^+H^-$	10 digits	[33]	full MSSM
$gg \to W^-H^+$	qualitative	[34]	full MSSM
$e^+e^- \to t\bar{t}$	11 digits	[35]	full MSSM
$e^+e^- \to W^+W^-$	qualitative	[36]	sfermion contributions
$e^+e^- \to H^+H^-$	7 digits	[37]	full MSSM
dipole moments	11 digits	[38]	full MSSM

Table 5: Checks performed with *FeynArts* and *FormCalc* to test the implementation of the MSSM.

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