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Two-loop electroweak contributions to Δr *

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A review is given on the quantum correction Δr in the W–Z mass correlation at the electroweak two-loop level, as derived from the calculation of the muon lifetime in the Standard Model. Exact results for Δr and the W-mass prediction including $\mathcal{O}(\alpha^2)$ corrections with fermion loops are presented and compared with previous results of a next-to-leading order expansion in the top-quark mass.

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1 Introduction

The interdependence between the W-boson mass, M_W, and the Z-boson mass, M_Z, with the help of the Fermi constant G_F and the fine structure constant α is one of the most important relations for testing the electroweak Standard Model (SM) with high precision. At present, the world-average for the W-boson mass is $M_W^{\rm exp} = 80.434 \pm 0.037$ GeV [1]. The experimental precision on M_W will be further improved with the data taken at LEP2 in their final analysis, at the upgraded Tevatron and at the LHC, where an error of $\delta M_W = 15$ MeV can be expected [2]. At a high-luminosity linear collider running in a low-energy mode at the W^+W^- threshold, a reduction of the experimental error down to $\delta M_W = 6$ MeV may be feasible [3]. This offers the prospect for highly sensitive tests of the electroweak theory [4], provided that the accuracy of the theoretical prediction matches the experimental precision. The basic physical quantity for the M_W–M_Z correlation is the muon lifetime τ_μ, which defines the Fermi constant G_F according to

$$\frac{1}{\tau_\mu} = \frac{G_F^2\, m_\mu^5}{192\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta_{\rm QED})\,, \tag{1}$$

with $F(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4$. By convention, the QED corrections within the Fermi Model, $\Delta_{\rm QED}$, are included in this defining equation for G_F. The one-loop result for $\Delta_{\rm QED}$ [5], which has already been known for several decades, has recently been supplemented by the two-loop correction [6], yielding

$$\Delta_{\rm QED} = 1 - 1.81\,\frac{\alpha(m_\mu)}{\pi} + 6.7\left(\frac{\alpha(m_\mu)}{\pi}\right)^2, \quad \text{with} \quad \alpha(m_\mu) \simeq \frac{1}{135.90}\ . \tag{2}$$

The tree-level W-propagator effect giving rise to the (numerically insignificant) term $3m_\mu^2/(5M_W^2)$ in (1), is conventionally also included in the definition of G_F, although not part of the Fermi Model prediction. From the precisely measured muon-decay width the value [7] $G_F = (1.16637 \pm 0.00001)\, 10^{-5}$ GeV^{-2} for the Fermi constant is derived.

Calculating the muon lifetime within the SM and comparing the SM result with (1) yields the relation

$$M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}\,(1 + \Delta r)\,, \tag{3}$$

where the radiative corrections are summarized in the quantity Δr, first calculated in [8] at one-loop. This relation can be used for deriving the prediction of M_W within the SM (or possible extensions), to be confronted with the experimental result for M_W.

The one-loop result for Δr within the SM can be decomposed as follows (with the notation $s_{\rm W}^2 = 1 - M_W^2/M_Z^2$, $c_{\rm W}^2 = 1 - s_{\rm W}^2$),

$$\Delta r^{(\alpha)} = \Delta\alpha - \frac{c_{\rm W}^2}{s_{\rm W}^2}\Delta\rho + \Delta r_{\rm rem}(M_H), \tag{4}$$

exhibiting the leading fermion-loop contributions $\Delta\alpha$ and $\Delta\rho$, which originate from the charge and mixing-angle renormalization; the remainder part $\Delta r_{\rm rem}$ contains in particular the dependence on the Higgs-boson mass, M_H. The QED-induced shift $\Delta\alpha$ in the fine structure constant contains large logarithms of light-fermion masses. The leading contribution to the ρ parameter from the top/bottom weak-isospin doublet, $\Delta\rho$, gives rise to a term with a quadratic dependence on the top-quark mass, m_t [9].

Beyond the one-loop order, resummations of the leading one-loop contributions $\Delta\alpha$ and $\Delta\rho$ are known [10]. They correctly take into account the terms of the form $(\Delta\alpha)^2$, $(\Delta\rho)^2$, $(\Delta\alpha\Delta\rho)$, and $(\Delta\alpha\Delta r_{\rm rem})$ at the two-loop level and the leading powers in $\Delta\alpha$ to all orders.

QCD corrections to Δr are known at $\mathcal{O}(\alpha\alpha_s)$ [11] and $\mathcal{O}(\alpha\alpha_s^2)$ [12]. Concerning the electroweak two-loop contributions, only partial results are available up to now. Approximative calculations were performed based on expansions for asymptotically large values of M_H [13] and m_t [14–16]. The terms derived by expanding in the top-quark mass of $\mathcal{O}(G_F^2 m_t^4)$ [14] and $\mathcal{O}(G_F^2 m_t^2 M_Z^2)$ [15] were found to be numerically sizeable. The $\mathcal{O}(G_F^2 m_t^2 M_Z^2)$ term, involving three different mass scales, has been obtained by two separate expansions in the regions $M_W, M_Z, M_H \ll m_t$ and $M_W, M_Z \ll m_t, M_H$ and by an interpolation between the two expansions. This formally next-to-leading order term turned out to be of a magnitude similar to that of the formally leading term of $\mathcal{O}(G_F^2 m_t^4)$, entering with the same sign. Its inclusion (both for M_W and the effective weak mixing angle) had important consequences on the indirect constraints on the Higgs-boson mass derived from the SM fit to the precision data.

A more complete calculation of electroweak two-loop effects is hence desirable. As a first step in this direction, exact results were derived for the Higgs-mass dependence of the fermionic two-loop corrections to the precision observables [17]. They have been compared with the results of expanding up to $\mathcal{O}(G_F^2 m_t^2 M_Z^2)$ [15], specifically analysing the effects of the m_t expansion, and good agreement has been found [18]. Beyond the two-loop order, complete results for the pure fermion-loop corrections (*i.e.* contributions containing n fermion loops at n-loop order) have recently been obtained up to four-loop order [19]. These results contain in particular the contributions of the leading powers in $\Delta\alpha$ as well as the ones in $\Delta\rho$ and the mixed terms.

In this talk the complete fermionic electroweak two-loop corrections to Δr are discussed, as calculated exactly without an expansion in the top-quark or the Higgs-boson mass [20]. These are all two-loop diagrams contributing to the muon-decay amplitude and containing at least one closed fermion loop (except the pure QED corrections already contained in the Fermi model result, according to (1)). Fig. 1

displays some typical examples. The considered class of diagrams includes the potentially large corrections both from the top/bottom doublet and from contributions proportional to N_{lf} and N_{lf}^2, where N_{lf} is the number of light fermions (a partial result for the light-fermion contributions is given in [21]). The results presented here improve on the previous results of an expansion in m_t up to next-to-leading order [15] in containing the full dependence on m_t as well as the complete light-fermion contributions at the two-loop order, while in [15] higher-order corrections from light fermions have only been taken into account via a resummation of the one-loop light-fermion contribution.

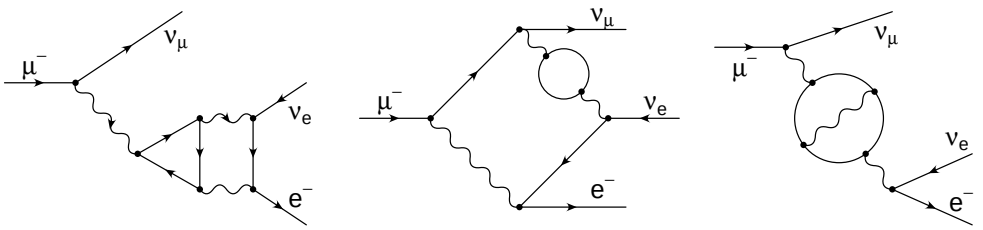

Figure 1: Examples for various types of fermionic two-loop diagrams contributing to muon decay.

2 Outline of the calculation

Since all possibly infrared (IR) divergent photonic corrections are already contained in the definition (1) of the Fermi constant G_F and mass singularities are absorbed in the running of the electromagnetic coupling, M_W represents the scale for the electroweak corrections in Δr. Therefore it is possible to neglect all fermion masses except the top-quark mass and the momenta of the external leptons so that the Feynman diagrams for muon decay reduce to vacuum diagrams.

All QED contributions to the Fermi Model have to be excluded in the computation of Δr since they have already been separated off in the definition of G_F, see eq. (1). Apart from the one-loop contributions, this comprises two-loop QED corrections and mixed contributions of QED and weak corrections of each one-loop order, which have to be removed from $\Delta r^{(\alpha^2)}$. For fermionic two-loop diagrams it is possible to find a one-to-one correspondence between QED graphs in Fermi-Model and SM contributions.

After extracting the IR-divergent QED corrections, the generic diagrams contributing to the muon-decay amplitude can be reduced to vacuum-type diagrams, since the masses of the external particles and the momentum transfer are negligible. The renormalization of is performed in the on-shell scheme. Thus, the mass-

renormalization of the gauge bosons requires the evaluation of two-loop two-point functions with non-zero external momentum, which is more involved from a technical point of view regarding the tensor structure and the evaluation of the scalar integrals. This complication cannot be avoided by performing the calculation within another renormalization scheme, *e.g.* the $\overline{MS}$ scheme, since ultimately one is interested in the relation between the physical parameters M_W, M_Z, α, G_F, where the two-point functions for non-zero momenta enter.

All diagrams and amplitudes for the decay and counterterm contributions have been generated with the program *FeynArts 2.2* [22]. The amplitudes are algebraically reduced by means of a general tensor-integral decomposition for two-loop two-point functions with the program *TwoCalc* [23], leading to a fixed set of standard scalar integrals. Analytic expressions are known for the scalar one-loop [24] and two-loop [25] vacuum integrals, whereas the two-loop self-energy diagrams can be evaluated numerically by means of one-dimensional integral representations [26].

In order to apply an additional check the calculations were performed within a covariant R_ξ gauge, with individual gauge parameters ξ_i for each gauge boson. It has been explicitly checked at the algebraic level that the gauge-parameter dependence of the final result drops out.

At the subloop level, also the Faddeev-Popov ghost sector has to be renormalized. The gauge-fixing part of the Lagrangian, in terms of the gauge fields A^μ, Z^μ, $W^{\pm\mu}$ and the unphysical Higgs scalars χ, $\phi^\pm$ given by

$$\mathcal{L}_{\rm gf} = -\frac{1}{2}\Big[(F^\gamma)^2 + (F^Z)^2 + F^+F^- + F^-F^+\Big], \qquad \text{with}$$

$$F^\gamma = (\xi_1^\gamma)^{-\frac{1}{2}}\,\partial_\mu A^\mu + \frac{\xi^{\gamma Z}}{2}\partial_\mu Z^\mu, \tag{5}$$

$$F^Z = \left(\xi_1^Z\right)^{-\frac{1}{2}}\partial_\mu Z^\mu + \frac{\xi^{Z\gamma}}{2}\partial_\mu A^\mu - \left(\xi_2^Z\right)^{\frac{1}{2}} M_Z\,\chi,$$

$$F^\pm = \left(\xi_1^W\right)^{-\frac{1}{2}}\partial_\mu W^{\pm\mu} \mp i\left(\xi_2^W\right)^{\frac{1}{2}} M_W\,\phi^\pm\,,$$

does not need renormalization. Accordingly, one can either introduce the gauge-fixing term after renormalization or renormalize the gauge parameters in such a way that they compensate the renormalization of the fields and masses. Both ensure that no counterterms arise from the gauge-fixing sector but they differ in the treatment of the ghost Lagrangian, which is given by the variation of the functionals F^a under infinitesimal gauge transformations $\delta\theta_b$,

$$\mathcal{L}_{\rm FP} = \sum_{a,b=\gamma,Z,\pm} \overline{u}^a\,\frac{\delta F^a}{\delta\theta^b}\,u^b. \tag{6}$$

In the latter case, which was applied in our work for simplification of the automatized treatment, additional counterterm contributions for the ghost sector arise from the gauge-parameter renormalization. The parameters ξ_i^a in (5) are renormalized such that their counterterms $\delta\xi_i^a$ exactly cancel the contributions from the renormalization of the fields and masses and that the renormalized gauge parameters comply with the R_ξ gauge.

3 On the γ_5–problem

In four dimensions the algebra of the γ_5–matrix is defined by the two relations

$$\{\gamma_5, \gamma_\alpha\} = 0 \qquad \text{for} \qquad \alpha = 1, \ldots, 4 \tag{7}$$

$$\mathrm{Tr}\{\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\} = 4i\epsilon^{\mu\nu\rho\sigma}. \tag{8}$$

It is impossible to translate both relations simultaneously into $D \neq 4$ dimensions without encountering inconsistencies [27].

A certain treatment of γ_5 might break symmetries, *i.e.* violate Slavnov-Taylor (ST) identities which would have to be restored with extra counterterms. Even after this procedure a residual scheme dependence can persist which is associated with ϵ-tensor expressions originating from the treatment of (8). Such expressions cannot be canceled by counterterms. If they broke ST identities this would give rise to anomalies.

't Hooft and Veltman [27] suggested a consistent scheme, formalized by Breitenlohner and Maison [28], as a separation of the first four and the remaining dimensions of the γ-Matrices (HVBM-scheme). It has been shown [29] that the SM with HVBM regularization is anomaly-free and renormalizable. This shows that ϵ-tensor terms do not get merged with divergences.

The naively anti-commuting scheme, which is widely used for one-loop calculations, extends the rule (7) to D dimensions but abandons (8),

$$\{\gamma_5, \gamma_\alpha\} = 0 \qquad \text{for} \qquad \alpha = 1, \ldots, D \tag{9}$$

$$\mathrm{Tr}\{\gamma_5\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma\} = 0. \tag{10}$$

This scheme is unambiguous but does not reproduce the four-dimensional case.

In the SM, particularly triangle diagrams (like the ones in Figure 2) containing chiral couplings are sensitive to the γ_5–problem. In our context, the one-loop triangle diagrams have been explicitly calculated in both schemes. While the naive scheme immediately respects all ST identities the HVBM scheme requires the introduction of additional finite counterterms. Even after this procedure finite differences remain between the results of the two schemes, showing that the naive scheme is inapplicable in this case.

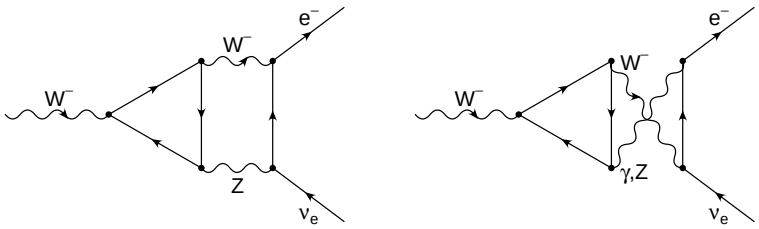

Figure 2: Charged-current vertex diagrams with fermion-triangle subgraphs.

In the calculation of Δr triangle diagrams appear as subloops of two-loop charged current (CC) vertex diagrams (Fig. 2). One finds that for the difference terms between both schemes this loop can be evaluated in four dimensions without further difficulties. This can be explained by the fact that renormalizability forbids divergent contributions to ϵ-tensor terms from higher loops in the HVBM scheme. The ϵ-tensor contributions from the triangle subgraph in the HVBM scheme meet a second ϵ-tensor term from the outer fermion lines in Fig. 2, thereby resulting in a non-zero contribution to Δr.

Computations in the HVBM scheme can in general get very tedious because of the necessity of additional counterterms. For our specific problem, however, it is possible to apply another, simplified, method. One can consider a "mixed" scheme that uses both relations (7) and (8) in D dimensions, despite their mathematical inconsistency, to evaluate the one-loop triangle subgraphs. These results immediately respect all ST identities. As checked explicitly, they differ from the HVBM results (with the appropriate counterterms to restore the ST identities) only by terms of $\mathcal{O}(D-4)$,

$$\Gamma^{\mathrm{HVBM}}_{\Delta(1)} = \Gamma^{\mathrm{mix}}_{\Delta(1)} + \mathcal{O}(D-4). \tag{11}$$

Inserting the one-loop expressions into the the two-loop diagrams one finds that the second loop integration gives a finite result and hence can be performed in four dimensions yielding

$$\Gamma^{\mathrm{HVBM}}_{\mathrm{CC}(2)} = \Gamma^{\mathrm{mix}}_{\mathrm{CC}(2)} + \mathcal{O}(D-4). \tag{12}$$

Thus the mixed scheme can serve in this case as a technically easy prescription for the correct calculation of the CC two-loop contributions. Practical ways of treating γ_5 in higher-order calculations are also discussed in [30].

4 Two-loop renormalization

For the determination of the one-loop counterterms and renormalization constants the conventions of [31] are adopted. Two-loop renormalization constants enter via the

counterterms for the transverse W propagator and the charged current vertex (the counterterms for the transverse Z propagator are analogous):

$$\left[\quad\right]_{\rm T} \;=\; \delta Z^W_{(2)}(k^2 - M_W^2) - \delta M^2_{W(2)} - \delta Z^W_{(1)}\delta M^2_{W(1)}, \tag{13}$$

$$\;=\; i\frac{e}{\sqrt{2}s_{\rm W}}\,\gamma_\mu\omega_-\Big[\delta Z_{e(2)} - \frac{\delta s_{\rm W(2)}}{s_{\rm W}} + \frac{1}{2}\left(\delta Z^{\rm eL}_{(2)} + \delta Z^W_{(2)} + \delta Z^{\nu{\rm L}}_{(2)}\right) + (\text{1-loop renormalization constants})\Big]. \tag{14}$$

$\delta Z^{W,{\rm eL},\nu{\rm L}}$ denote the field-renormalization constants, $\delta M^2_{W,Z}$ the W- and Z-mass counterterms, and δZ_e denotes the charge-renormalization constant. The lower indices in parentheses indicate the loop order. The mixing-angle counterterm $\delta s_{\rm W(2)}$ can be derived from the gauge-boson mass counterterms. The two-loop contributions always include the subloop renormalization.

The on-shell masses are defined as the position of the propagator poles. Starting at the two-loop level, it has to be taken into account that there is a difference between the definition of the mass $\widetilde{M}^2$ as the pole of the real part of the (transverse) propagator,

$$\mathrm{Re}\left\{(D_{\rm T})^{-1}(\widetilde{M}^2)\right\} = 0, \tag{15}$$

and the real part $\overline{M}^2$ of the complex pole,

$$(D_{\rm T})^{-1}(\mathcal{M}^2) = 0, \qquad \mathcal{M}^2 = \overline{M}^2 - i\overline{M}\,\overline{\Gamma}. \tag{16}$$

The imaginary part of the complex pole is associated with the width $\overline{\Gamma}$. The defining condition (16) yields for the W-mass counterterm

$$\delta\overline{M}^2_{W(2)} = \mathrm{Re}\{\Sigma^W_{\rm T(2)}(\overline{M}^2_W)\} - \delta Z^W_{(1)}\delta\overline{M}^2_{W(1)} + \mathrm{Im}\{\Sigma^{W\prime}_{\rm T(1)}(\overline{M}^2_W)\}\,\mathrm{Im}\{\Sigma^W_{\rm T(1)}(\overline{M}^2_W)\}, \tag{17}$$

whereas for the real-pole definition the last term of eq. (17) is missing. $\Sigma^W_{\rm T}$ denotes the transverse W self-energy and $\Sigma^{W\prime}_{\rm T}$ its momentum derivative. Similar expressions hold for the Z boson.

The W and Z mass countertermss determine the two-loop counterterm for the mixing angle, $\delta s_{\rm W(2)}$, which has to be gauge invariant since $s_{\rm W}$ is an observable quantity. With the use of a general R_ξ gauge it has explicitly been checked that $\delta s_{\rm W(2)}$ is gauge-parameter independent for the complex-pole mass definition, whereas the real-pole definition leads to a gauge dependent $\delta s_{\rm W(2)}$. This is in accordance with the expectation from S-matrix theory [32], where the complex pole represents a gauge-invariant mass definition.

It should be noted that the mass definition via the complex pole corresponds to a Breit-Wigner parameterization of the resonance shape with a constant width. For

the experimental determination of the gauge-boson masses, however, a Breit-Wigner ansatz with a running width is used. This has to be accounted for by a shift of the values for the complex pole masses [33],

$$\overline{M} = M - \frac{\Gamma^2}{2M}. \tag{18}$$

which yields the relations

$$\begin{array}{rcl} \overline{M_Z} & = & M_Z - 34.1 \text{ MeV}, \\ \overline{M_W} & = & M_W - 27.4\ (27.0) \text{ MeV} \quad \text{for} \quad M_W = 80.4\ (80.2) \text{ GeV}. \end{array} \tag{19}$$

For M_Z and Γ_Z the experimental numbers are taken. The W mass is a calculated quantity, and therefore also a theoretical value for the W-boson width should be applied here. The results above are obtained from the approximate, but sufficiently accurate expression for the W width,

$$\Gamma_W = 3\frac{G_F M_W^3}{2\sqrt{2}\pi}\left(1 + \frac{2\alpha_s}{3\pi}\right). \tag{20}$$

5 Results

In the previous sections the characteristics of the calculation of electroweak two-loop contributions to Δr have been pointed out. Combining the fermionic $\mathcal{O}(\alpha^2)$ contributions with the one-loop and the QCD corrections yields the total result

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)}. \tag{21}$$

Here N_f, N_f^2 symbolize one and two fermionic loops, respectively. Fig. 3 shows that both the QCD and electroweak two-loop corrections give sizeable contributions of 10–15% with respect to the one-loop result.

In Fig. 4 the prediction for M_W derived from the result (21) and the relation (3) is compared with the experimental value for M_W. Dotted lines indicate one standard deviation bounds. The main uncertainties of the prediction originate from the experimental errors of $m_t = (174.3 \pm 5.1)$ GeV [7] and $\Delta\alpha = 0.05954 \pm 0.00065$ [34]. It is obvious that low Higgs masses are favored; the new results on Δr strengthen the tendency towards a lighter Higgs boson (according to the following comparison).

These results can be compared with the results obtained by expansion of the two-loop contributions up to next-to-leading order in m_t [15]. The predicted values for M_W for several values of M_H are given in Tab. 1. For the input parameters the values of [15] have been chosen, *i.e.* $m_t = 175$ GeV, $M_Z = 91.1863$ GeV, $\Delta\alpha = 0.0594$, $\alpha_s(M_Z) = 0.118$. Agreement is found between the results with maximal deviations of less than 5 MeV in M_W. The deviations in the last column of Tab. 1 can of course not

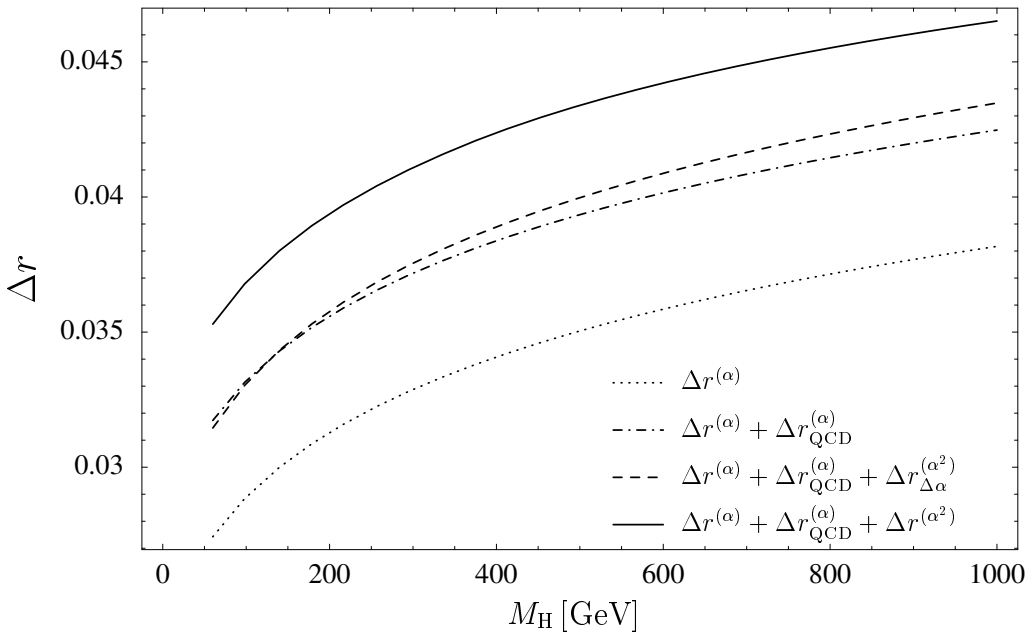

Figure 3: Various stages of Δr, as a function of M_H. The one-loop contribution, $\Delta r^{(\alpha)}$, is supplemented by the two-loop and three-loop QCD corrections, $\Delta r^{(\alpha)}_{\rm QCD} \equiv \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)}$, and the fermionic electroweak two-loop contributions, $\Delta r^{(\alpha^2)} \equiv \Delta r^{(N_{\rm f}\alpha^2)} + \Delta r^{(N_{\rm f}^2\alpha^2)}$. For comparison, the effect of the two-loop corrections induced by a resummation of $\Delta\alpha$, $\Delta r^{(\alpha^2)}_{\Delta\alpha}$, is shown separately.

be attributed exclusively to differences in the two-loop top-quark and light-fermion contributions, because the results also differ by a slightly different treatment of those higher-order terms that are not yet under control, such as purely bosonic two-loop contributions, and effects from scheme dependence.

Similar to [35], a simple numerical parametrization of our result for M_W can be given by the following expression:

$$M_W = M_W^0 - c_1\,\mathrm{dH} - c_5\,\mathrm{dH}^2 + c_6\,\mathrm{dH}^4 - c_2\,\mathrm{d}\alpha + c_3\,\mathrm{dt} - c_7\,\mathrm{dH}\,\mathrm{dt} - c_4\,\mathrm{d}\alpha_s, \quad (22)$$

where

$$\mathrm{dH} = \ln\left(\frac{M_H}{100\,\mathrm{GeV}}\right), \quad \mathrm{dt} = \left(\frac{m_t}{174.3\,\mathrm{GeV}}\right)^2 - 1,$$
$$\mathrm{d}\alpha = \frac{\Delta\alpha}{0.05924} - 1, \qquad \mathrm{d}\alpha_s = \frac{\alpha_s(M_Z)}{0.119} - 1, \quad (23)$$

with the coefficients $M_W^0 = 80.3767$ GeV, $c_1 = 0.05613$, $c_2 = 1.081$, $c_3 = 0.5235$, $c_4 = 0.0763$, $c_5 = 0.00936$, $c_6 = 0.000546$, $c_7 = 0.00573$. and with $M_Z = 91.1785$ GeV. The quality of the approximation (22) to our full result for M_W is within 0.4 MeV, allowing M_H between 65 GeV and 1 TeV.

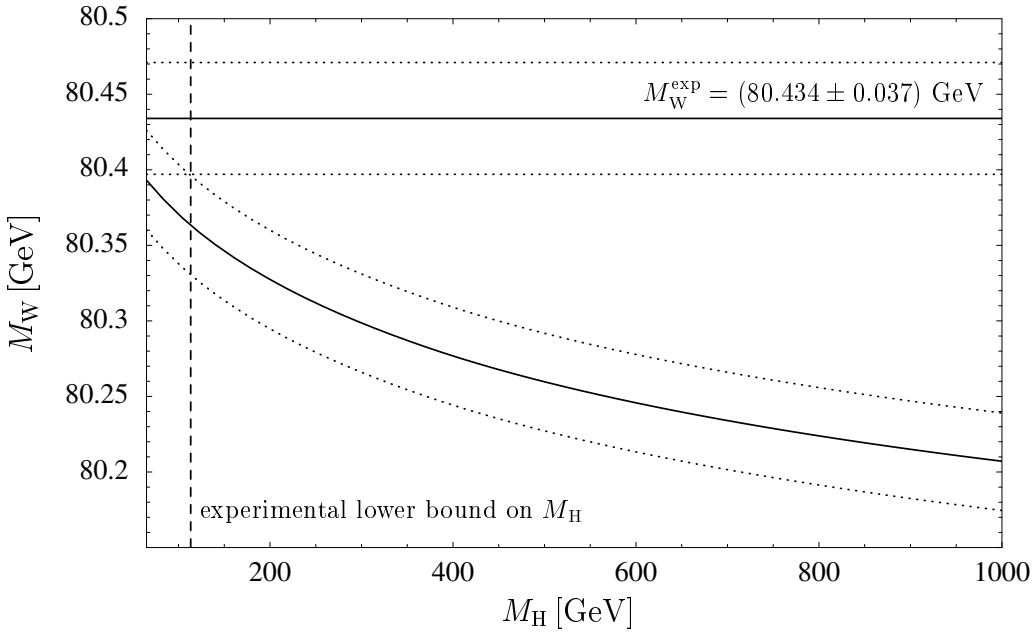

Figure 4: The SM prediction for M_W as a function of M_H for $m_t = 174.3 \pm 5.1$ GeV is compared with the current experimental value, $M_W^{\rm exp} = 80.434 \pm 0.037$ GeV [1].

6 Conclusion

In this talk the realization of an exact two-loop calculation of fermionic contributions in the full electroweak SM and its application to the precise computation of Δr has been reviewed. Numerical illustrations were given for the results, which might serve as ingredients for future SM fits.

Table 1: Comparison between M_W–predictions from an NLO expansion in m_t ($M_W^{\rm expa}$) and the full calculation ($M_W^{\rm full}$). δM_W denotes the difference.

M_H [GeV]	$M_W^{\rm expa}$ [GeV]	$M_W^{\rm full}$ [GeV]	δM_W [MeV]
65	80.4039	80.3997	4.2
100	80.3805	80.3771	3.4
300	80.3061	80.3051	1.0
600	80.2521	80.2521	0.0
1000	80.2129	80.2134	−0.5

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