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Non-renormalization theorems in softly broken SQED and the soft β-functions

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Abstract

The renormalization of softly broken SQED is related to the one of supersymmetric QED by using the construction with a local gauge supercoupling and by taking into account softly broken anomalous axial $U(1)$ symmetry. From this extended model one obtains the non-renormalization theorems of SQED and the counterterms of the soft breaking parameters as functions of the supersymmetric counterterms. Due to the Adler–Bardeen anomaly of the axial current an invariant regularization scheme does not exist, and therefore the β-functions of soft breaking parameters are derived from an algebraic construction of the Callan–Symanzik equation and of the renormalization group equation. We obtain the soft β-functions in terms of the gauge β-function and of the anomalous dimension of the supersymmetric matter mass. In particular, we find that the X-term of the scalar mass β-function as well as the gauge β-function in $l \geq 2$ are due to the Adler–Bardeen anomaly of the axial symmetry.

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1 Introduction

Starting with the investigations of Yamada [1] there has been considerable progress in relating the β-functions of soft supersymmetry breaking parameters [2] to the β-functions and anomalous dimensions of the supersymmetric model [3, 4, 5, 6].

There are two different approaches in extracting β-functions of the softly broken model from the respective supersymmetric model: The first approach [1, 3, 4] is based on the superfield formalism [7, 8, 9, 10] and exploits supergraph techniques to relate the UV counterterms of softly broken and supersymmetric gauge theories. In the second approach [11, 5, 12, 6] renormalization group invariant quantities are used for deriving all-order expressions for the β-functions of soft breaking parameters. The latter computation is mainly based on the component formalism using dimensional reduction (DRED) as a supersymmetric regularization scheme.

From the beginning it has been seen that the calculation of the scalar mass β-function is somewhat involved: In dimensional reduction a mass term for ϵ scalars is introduced, which enters the β-function of the scalar mass in two-loop order [13]. By a redefinition from the DRED to a DRED$'$ scheme the scalar mass β-function can be defined in terms of physical parameters [14, 15], and one gets then an extra contribution, called X-term [6]. It is the X-term, in which the results gained from a simple application of supergraph techniques differ from the explicit expressions from 2-loop order onwards.

It is obvious that such ambiguities can be only resolved by a scheme-independent construction of softly broken supersymmetric models. In the present paper we introduce such a scheme-independent definition of softly broken SQED, which describes at the same time the non-renormalization theorems and the relation of soft breaking to the supersymmetric parameters. In particular it is shown that the X-term is in effect generated by the supersymmetric extension of the Adler–Bardeen anomaly.

For the construction we use an extended model of SQED which has been considered in a recent paper [16] for the derivation of non-renormalization theorems in the Wess–Zumino gauge. In addition to the physical fields it contains a local gauge coupling with its superpartners, a chiral multiplet coupled to the mass term of matter and an axial vector multiplet. In the extended model the photino mass is generated by the gauge supercoupling and the left-right mixing scalar mass is generated by the chiral field of the matter mass term as proposed in the literature [3, 11]. Differently from their approaches, the scalar mass of matter is generated by the D-component of the axial vector multiplet. The interaction of the axial vector multiplet with the matter fields is governed by softly broken axial symmetry. Axial symmetry, however, is broken by the Adler-Bardeen anomaly. In presence of the local gauge coupling the Slavnov–Taylor identity can be modified in such a way that it includes the anomaly. Then the renormalization of softly broken SQED is defined by the anomalous Slavnov–Taylor identity. From this identity we obtain the non-renormalization theorems in the same way as in the supersymmetric model, and the relations between softly broken SQED and supersymmetric QED are derived from a scheme independent algebraic construction.

In this respect the present construction is superior to the constructions [17, 18] of softly broken supersymmetric theories in the framework of algebraic renormalization. Irrespective of the fact whether the soft breakings are introduced by BRS doublets or by spurion fields, their divergences do not appear to be related to the divergences of the supersymmetric theories. Like the non-renormalization theorems, the specific renormalization properties of softly broken supersymmetry are not direct consequence of supersymmetry, but are a consequence of the multiplet structure of supersymmetric Lagrangians [19, 16]. Indeed, the soft mass breakings are the lowest components of the Lagrangian multiplets and this property is only exploited with the use of a local supercoupling and of axial symmetry as proposed in the present paper.

The plan of the paper is as follows: In section 2 we introduce the classical action of the extended model of SQED including the soft supersymmetry breaking. In section 3 we give the defining symmetries for the construction of higher-order Green functions. In particular, we show that the Green functions satisfy an anomalous Slavnov-Taylor identity, which includes the Adler Bardeen anomaly, to all orders. In section 4 we derive the invariant counterterms and comment on the non-renormalization theorems. In section 5 we discuss the normalization conditions. Section 6 and section 7 is devoted to the derivation of soft mass β-functions: We first derive the Callan-Symanzik equation and prove then, that in mass independent schemes their coefficient functions are related to the ones of the renormalization group equation. The all-order expressions of soft mass β-functions follow from symmetries of the extended model in the same way as the closed form of the gauge β function. In the appendices we summarize the BRS transformations and the symmetric operators in their general form. The conventions are the ones we have given in appendix A of [16].

2 The classical action

In a recent paper [16] the non-renormalization theorems of SQED have been derived in the Wess-Zumino gauge [20] by extending the gauge coupling to an external real superfield and by taking into account softly broken axial symmetry with its anomaly. The extended model of SQED can be immediately used also for a consistent description of the soft supersymmetry breaking. The soft breaking terms of the Giradello–Grisaru class [2] are implicitly included in the extended model since they couple to the highest components of external field multiplets. They are generated when we introduce a constant shift in these external field components. Explicitely we introduce the following external field multiplets with associated shifts:

- The local gauge coupling $e(x)$ is the lowest component of a real superfield $E(x,\theta,\overline{\theta})$, which itself is composed of a chiral and an antichiral field multiplet $\boldsymbol{\eta}(x,\theta)$ and $\overline{\boldsymbol{\eta}}(x,\overline{\theta})$:

$$E(x,\theta,\overline{\theta}) = (\boldsymbol{\eta}(x,\theta,\overline{\theta}) + \overline{\boldsymbol{\eta}}(x,\theta,\overline{\theta}))^{-\frac{1}{2}} \equiv e(x) + \mathcal{O}(\theta,\overline{\theta}) \qquad (1)$$

  with

$$\boldsymbol{\eta}(x,\theta) = \eta + \theta^\alpha \chi_\alpha + \theta^2 f \ , \qquad \overline{\boldsymbol{\eta}}(x,\overline{\theta}) = \overline{\eta} + \theta_{\dot\alpha}\overline{\chi}^{\dot\alpha} + \overline{\theta}^2 \overline{f} \qquad (2)$$

  in the chiral and antichiral representation, respectively. Shifting the highest components of the chiral and antichiral field multiplet

$$f \to f + \frac{M_\lambda}{e^2} \ , \qquad \bar{f} \to \bar{f} + \frac{M_\lambda}{e^2} \ , \qquad (3)$$

  we will obtain a mass term for the photino in the classical action.

- The axial current is coupled to an external axial vector multiplet. When we shift the D-component of the axial vector multiplet

$$V^i = (V^\mu, \tilde{\lambda}^\alpha, \overline{\tilde{\lambda}}^{\dot\alpha}, \tilde{D}) \qquad (4)$$

  with

$$\tilde{D} \to \tilde{D} - 2M^2 \ , \qquad (5)$$

  a mass term for the scalar superpartners of the electron is present in the classical action.

- The chiral and antichiral field multiplets

$$q^i = (q, q^\alpha, q_F) \quad \text{and} \quad \overline{q}^i = (\overline{q}, \overline{q}^\alpha, \overline{q}_F) \tag{6}$$

  with dimension one couple to the supersymmetric mass term. By a shift in their F-components one generates the left-right-mixing mass term, usually called the b-parameter, for the scalar superpartners of the electron in the classical action:

$$q_F \to q_F - b\ , \qquad \overline{q}_F \to \overline{q}_F - b\ . \tag{7}$$

  The b-parameter has mass dimension 2.

The shifts appear not only in the classical action, but modify also the axial transformations and the supersymmetry transformations of the corresponding fields. However, their presence in the symmetry transformations does not change the algebraic characterization and the algebraic structure of symmetry transformations and the model with soft breaking is described by the same symmetries as the supersymmetric model. Hence, the renormalization properties remain unchanged in softly broken SQED. It is the purpose of the present paper to work out explicitly the symmetric counterterms and the β-functions in presence of soft supersymmetry breaking. All expressions are immediately obtained from the expressions in the symmetric model [16], but for a clear presentation we summarize the construction in a condensed form.

In the Wess–Zumino gauge the algebra of supersymmetry transformations closes on field dependent gauge transformations and the gauge fixing of the photon cannot be given in a supersymmetric form. To overcome these difficulties one uses a BRS formalism and combines the symmetries of the model in the BRS-transformations [21, 22]. On fields with ghost charge zero the BRS operator acts as a combination of gauge symmetry, axial symmetry, supersymmetry and translations:

$$\mathbf{s}\phi = (\delta^{\rm gauge}_{c(x)} + \delta^{\rm axial}_{\tilde{c}(x)} + \epsilon^\alpha \delta_\alpha + \bar{\delta}_{\dot\alpha}\overline{\epsilon}^{\dot\alpha} - i\omega^\mu \partial_\mu)\phi\ . \tag{8}$$

The ghost fields $c(x), \tilde{c}(x)$ replace the local transformation parameters of gauge transformations and axial transformations, and the constant ghosts ϵ^α, $\overline{\epsilon}^{\dot\alpha}$ and ω^μ are the constant supersymmetry and translational ghosts, respectively. BRS-transformations of the ghosts are determined by the structure constants of the algebra and the algebra of symmetry transformations is expressed in on-shell nilpotency of the BRS operator. The BRS transformations of the fields are summarized in appendix A. They differ from the BRS transformations in the symmetric model by shifts in the scalar components of external fields.

The complete classical action is decomposed into the physical part $\Gamma_{\rm susy}$, the gauge fixing and ghost part $\Gamma_{\rm g.f}$ and an external field part $\Gamma_{\rm ext.f}$, which makes possible to describe the BRS invariance of the action by the Slavnov–Taylor identity:

$$\Gamma_{\rm cl} = \Gamma_{\rm susy} + \Gamma_{\rm g.f.} + \Gamma_{\rm ext.f.} \ . \tag{9}$$

$\Gamma_{\rm susy}$ is invariant under gauge transformations with the local gauge coupling, supersymmetry transformations and axial transformations. Up to normalization constants it is determined by these symmetries and the construction results in the following form:

$$\begin{aligned}
\Gamma_{\rm susy} = \int d^4x \, \Big(& -\frac{1}{4e^2} F^{\mu\nu}(eA) F_{\mu\nu}(eA) + \frac{i}{2}(\lambda\sigma\partial\overline{\lambda} - \partial\lambda\sigma\overline{\lambda}) \\
& + \frac{i}{2}(\eta - \overline{\eta})\partial_\mu(e^2\lambda\sigma^\mu\overline{\lambda}) - \frac{i}{8}(\eta - \overline{\eta})\epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}(eA)F_{\rho\sigma}(eA) \\
& + \frac{i}{4}e(\chi\sigma^{\mu\nu}\lambda - \overline{\lambda}\bar{\sigma}^{\mu\nu}\overline{\chi})F_{\mu\nu}(eA) - \frac{1}{2}e^2 f\lambda\lambda - \frac{1}{2}e^2\overline{f\lambda\lambda} \\
& - \frac{1}{2}M_\lambda(\lambda\lambda + \overline{\lambda\lambda}) \\
+ \Big(& D^\mu\overline{\varphi}_L D_\mu\varphi_L + i\psi_L^\alpha\sigma^\mu_{\alpha\dot\alpha}D_\mu\overline{\psi}_L^{\dot\alpha} + ieQ_L\sqrt{2}(\lambda\psi_L\overline{\varphi}_L - \overline{\lambda\psi}_L\varphi_L) \\
& + i\sqrt{2}(\tilde{\lambda}\psi_L\overline{\varphi}_L - \overline{\tilde{\lambda}}\overline{\psi}_L\varphi_L) + \frac{1}{2}\tilde{D}\overline{\varphi}_L\varphi_L + (L\to R)\Big) \\
& - M^2(\overline{\varphi}_L\varphi_L + \overline{\varphi}_R\varphi_R) \\
- \frac{1}{8}\big(& ie^2(\chi\lambda - \overline{\chi}\overline{\lambda}) + 2eQ_L(\varphi_L\overline{\varphi}_L - \varphi_R\overline{\varphi}_R)\big)^2 \\
- (q + & m)(\overline{q} + m)(\varphi_R\overline{\varphi}_R + \varphi_L\overline{\varphi}_L) - \big((q+m)\psi_L^\alpha\psi_{R\alpha} + \text{c.c.}\big) \\
- \frac{1}{\sqrt{2}}\big(& q^\alpha(\psi_{L\alpha}\varphi_R + \psi_{R\alpha}\varphi_L) + \overline{q}_{\dot\alpha}(\overline{\psi}_L^{\dot\alpha}\overline{\varphi}_R + \overline{\psi}_R^{\dot\alpha}\overline{\varphi}_L)\big) \\
+ q_F & \varphi_L\varphi_R + \bar{q}_F\overline{\varphi}_L\overline{\varphi}_R - b(\varphi_L\varphi_R + \overline{\varphi}_L\overline{\varphi}_R)\Big) \ .
\end{aligned} \tag{10}$$

The fields A^μ and $\lambda^\alpha, \overline{\lambda}^{\dot\alpha}$ are the photon and photino and $\varphi_A, \psi_A, A = L, R$ and their complex conjugate the left- and right-handed matter fields with charge $Q_L = -1$ and $Q_R = 1$. The covariant derivatives are covariant with respect to gauge transformations and axial transformations:

$$D_\mu\phi_A = (\partial_\mu + iQ_A A_\mu + iV_\mu)\phi_A \ , \qquad D_\mu\bar{\phi}_A = (D_\mu\phi_A)^\dagger \ , \quad \phi = \psi, \varphi \ . \tag{11}$$

The gauge fixing and ghost part of the action, $\Gamma_{\rm g.f.}$, as well as the external field part $\Gamma_{\rm ext.f}$ only depend on the local gauge coupling $e(x)$ and have the same form as in the symmetric model [16]. Using the auxiliary field B for describing the gauge fixing term, the gauge fixing and ghost part can be written as a BRS-variation:

$$\Gamma_{\rm g.f.} = \mathbf{s} \int d^4x \,\big(\frac{1}{2}\xi \bar{c} B + \bar{c}\frac{1}{e}\partial(eA)\big) = \int d^4x \,\big(\frac{1}{2}\xi B^2 + B\frac{1}{e}\partial(eA)\big) + \Gamma_{\rm ghost} \; . \tag{12}$$

The explicit form of $\Gamma_{\rm ghost}$ is not relevant for the further construction.

The classical action satisfies the Slavnov-Taylor identity

$$\mathcal{S}(\Gamma_{\rm cl}) = 0 \; , \tag{13}$$

with the usual Slavnov-Taylor operator (see (80)). It describes BRS invariance of the classical action as well as the algebraic structure of symmetry transformations.

When we take the limit to constant coupling and set all further external fields to zero, the classical action becomes the classical action of SQED with soft supersymmetry breaking:

$$\lim_{E\to e} \Gamma_{\rm cl}\Big|_{\substack{V^i=0\\ q^i=0}} = \Gamma_{\rm cl}^{\rm SQED} + \Gamma_{\rm soft} \equiv \Gamma_{\rm cl}^{\rm sSQED} \tag{14}$$

with

$$\begin{aligned}\Gamma_{\rm soft} = \int d^4x \,\Big(&-\frac{1}{2}M_\lambda(\lambda\lambda + \overline{\lambda\lambda}) \\ &- M^2(\varphi_L\overline{\varphi}_L + \varphi_R\overline{\varphi}_R) - b(\varphi_L\varphi_R + \overline{\varphi}_L\overline{\varphi}_R)\Big) \; .\end{aligned} \tag{15}$$

The classical action (9) is the starting point for the perturbative calculations. It turns out that it is complete in the sense of multiplicative renormalization.

3 Symmetries and renormalization

In the perturbative construction the local coupling and its superpartners are considered as external fields which appear in the same way as ordinary external fields in the generating functional of 1PI Green functions Γ. However the chiral and antichiral fields which compose the local coupling are distinguished from the dimensionless spurion fields by the property that the local gauge coupling is the perturbative expansion parameter. This is the content of the topological formula

$$N_{e(x)} = N_{\rm amp.legs} + N_Y + 2N_f + 2N_\chi + 2N_{\eta-\overline{\eta}} + 2(l-1) \; , \tag{16}$$

which determines the number of local couplings in a specific diagram in dependence of the loop order l. Here $N_{\text{amp.legs}}$ counts the number of external amputated legs with propagating fields ($A^\mu, \lambda, \varphi_A, \psi_A, c, \overline{c}$ and the respective complex conjugate fields), N_Y gives the number of BRS insertions, counted by the number of differentiations with respect the the external fields Y_ϕ. N_f, N_χ and $N_{\eta-\overline{\eta}}$ gives the number of insertions corresponding to the respective external fields. As for the classical action the validity of the topological formula ensures that the limit to constant coupling results in the 1PI Green functions of ordinary SQED with soft breaking.

The topological formula is not the only restriction on the appearance of the $\boldsymbol{\eta}$ and $\overline{\boldsymbol{\eta}}$-multiplets, but it is seen that the classical action depends on the parity odd scalar field $\eta - \overline{\eta}$ only via a total derivative. The corresponding Ward identity

$$\int d^4x \, (\frac{\delta}{\delta\eta} - \frac{\delta}{\delta\overline{\eta}})\Gamma = 0 \tag{17}$$

can be maintained in the course of renormalization. It was shown that this identity in combination with supersymmetry is the basis for the non-renormalization theorems [16].

Supersymmetry, abelian gauge symmetry and axial symmetry are included in the Slavnov–Taylor identity. However, at the quantum level axial symmetry is broken by the Adler–Bardeen anomaly [24, 25]. In the model with local gauge coupling the Adler-Bardeen anomaly can be absorbed into a modified Slavnov-Taylor identity and due to the non-renormalization of the anomaly it can be proven that the generating functional of 1PI Green functions satisfies the following anomalous Slavnov–Taylor identity to all orders [16]:

$$\mathcal{S}(\Gamma) + r^{(1)}\delta\mathcal{S}\Gamma = 0 \ , \tag{18}$$

where $\mathcal{S}(\Gamma)$ is defined in (80) and

$$\begin{aligned} \delta\mathcal{S}\Gamma = -4i \int d^4x \, \Big(\tilde{c}\Big(\frac{\delta}{\delta\eta} - \frac{\delta}{\delta\overline{\eta}}\Big) &+ 2i(\epsilon\sigma^\mu)^{\dot\alpha} V_\mu \frac{\delta}{\delta\overline{\chi}^{\dot\alpha}} - 2i(\sigma^\mu\overline{\epsilon})^\alpha V_\mu \frac{\delta}{\delta\chi^\alpha} \\ &+ 2\overline{\epsilon}_{\dot\alpha}\overline{\tilde{\lambda}}^{\dot\alpha}\frac{\delta}{\delta f} - 2\tilde{\lambda}^\alpha\epsilon_\alpha\frac{\delta}{\delta\overline{f}}\Big)\Gamma \ . \end{aligned} \tag{19}$$

Here $r^{(1)}$ is the coefficient of the anomaly determined from the usual triangle diagrams:

$$r^{(1)} = -\frac{1}{16\pi^2} \ . \tag{20}$$

The operator (19) describes the supersymmetric extension of the Adler–Bardeen anomaly. Due to the appearance of the supersymmetry ghosts ϵ^α and $\bar\epsilon^{\dot\alpha}$ it modifies the supersymmetry transformations of the axial vector multiplet. It has been shown in ref. [16] that these modifications are the Wess-Zumino-gauge analogue of the Konishi anomaly in superspace [26, 27].

The anomalous Slavnov-Taylor identity (18) is the defining symmetry of higher-order Green functions. The anomaly part turns out to have indeed important implications for the β-functions of the gauge coupling and as we will show here, for the β-function of the scalar mass M.

From the ghost equations

$$\frac{\delta\Gamma}{\delta c} = \frac{\delta\Gamma_{\rm cl}}{\delta c}\,, \qquad \frac{\delta\Gamma}{\delta\tilde c} = \frac{\delta\Gamma_{\rm cl}}{\delta\tilde c}\,, \tag{21}$$

one derives the gauge Ward identity and the anomalous Ward identity of axial symmetry [16].

In addition to these symmetries the Green functions are invariant under charge conjugation and parity. R-parity as defined in [16] is broken by the mass term of the photino and the left-right mass term of scalars. However, R-parity is defined by a global $U(1)$-symmetry with a discrete transformation angle. The corresponding global Ward identity can be derived also in the case of soft breaking and includes the mass shifts expressing soft breaking of the global $U(1)$ symmetry:

$$\mathcal{W}^R\Gamma = 0 \tag{22}$$

with

$$\begin{aligned}\mathcal{W}^R = i\int d^4x\,\Big(&\sum_{A=L,R}(\varphi_A\frac{\delta}{\delta\varphi_A} - Y_{\varphi_A}\frac{\delta}{\delta Y_{\varphi_A}}) + \lambda^\alpha\frac{\delta}{\delta\lambda^\alpha} - Y^\alpha_\lambda\frac{\delta}{\delta Y^\alpha_\lambda}\\ &+\tilde\lambda^\alpha\frac{\delta}{\delta\tilde\lambda^\alpha} - q^\alpha\frac{\delta}{\delta q^\alpha} - \chi^\alpha\frac{\delta}{\delta\chi^\alpha}\\ &-2(q_F-b)\frac{\delta}{\delta q_F} - 2(f+\frac{M_\lambda}{e^2})\frac{\delta}{\delta f} - \text{c.c.}\Big)\\ +\,i\Big(&\epsilon^\alpha\frac{\partial}{\partial\epsilon^\alpha} - \bar\epsilon^{\dot\alpha}\frac{\partial}{\partial\epsilon^{\dot\alpha}}\Big)\,.\end{aligned} \tag{23}$$

R-parity in the usage of transforming all susy fields to their negative is maintained but its consequences are already included in the global Ward identity (22).

Together with the topological formula (16), the anomalous Slavnov–Taylor identity (18), the identity (17) and the global Ward identity (22) uniquely define the 1PI Green functions of the extended model up to invariant counterterms. Taking the fields of the axial vector multiplet and the fields of the q multiplet to zero we find in limit to constant coupling the 1PI Green functions of softly broken SQED, which we denote with $\Gamma^{\rm sSQED}$:

$$\lim_{E\to e} \Gamma\Big|_{\substack{V^i=0\\ q^i=0}} = \Gamma^{\rm sSQED} \ . \tag{24}$$

4 Symmetric counterterms

The appearance of symmetric counterterms gives a hint on the divergences of the model. In particular, absence of symmetric counterterms means that the corresponding Green functions are determined by non-local expressions, which cannot appear with independent divergences. In [16] we have found that counterterms to the chiral vertices and counterterms to the photon self energy in $l \geq 2$ are excluded as a consequence of supersymmetry and the identity (17). The same arguments apply here in the model with soft supersymmetry breaking and we find the same types of symmetric counterterms as in the supersymmetric model. Due to the shifts in the external fields, the symmetric counterterms include now the soft breaking parameters as functions of the supersymmetric parameters.

Symmetric counterterms are invariants with respect to the symmetries of the model. However, a general classical solution, i.e. a local action satisfying the anomalous Slavnov–Taylor identity, does not exist. Thus, an invariant regularization scheme for the extended model cannot be constructed and invariant counterterms can only be given order by order in the perturbative expansion. As such, they are restricted by the symmetries of the classical action:

$$s_{\Gamma_{\rm cl}} \Gamma^{(l)}_{\rm ct,inv} = 0 \ , \qquad \int d^4x \left(\frac{\delta}{\delta\eta} - \frac{\delta}{\delta\overline{\eta}}\right)\Gamma^{(l)}_{\rm ct,inv} = 0 \ , \qquad \mathcal{W}^R \Gamma^{(l)}_{\rm ct,inv} = 0 \ , \tag{25}$$

and $\Gamma_{\rm ct,inv}$ is invariant under the discrete symmetries C and P. A further constraint on the counterterms is the topological formula (16), which determines the order in the local coupling.

Owing to these restrictions we have five types of invariant counterterms: a one-loop counterterm to the kinetic term of the photon multiplet, a counterterm to the matter term of the action and three gauge dependent field redefinitions for the matter fields. The counterterms are best expressed in form of $\mathbf{s}_{\Gamma_{\rm cl}}$-invariant operators acting on the classical action. The five invariant operators corresponding to the invariant counterterms are given in their general form in appendix B. Here we discuss the limit to constant coupling.

- The one-loop counterterm to the kinetic term of the photon multiplet $\Gamma^{(1)}_{\rm ct,kin}$ is determined by the symmetric operator $\mathcal{D}_{\rm kin}$ (83). Using the identity

$$\lim_{E\to e}\int d^4x\ M_\lambda\Big(\frac{\delta}{\delta f}+\frac{\delta}{\delta \bar f}\Big)\Gamma_{\rm cl}=e^2M_\lambda\partial_{M_\lambda}\Gamma^{\rm sSQED}_{\rm cl}\ , \tag{26}$$

it reads in the limit to constant coupling

$$\begin{aligned}\lim_{E\to e}\Gamma^{(1)}_{\rm ct,kin}=-\frac{1}{2}e^2\Big(&e\partial_e+2M_\lambda\partial_{M_\lambda}-N_A-N_\lambda\\&-N_c+N_{Y_\lambda}+N_B+N_{\bar c}-2\xi\partial_\xi\Big)\Gamma^{\rm sSQED}_{\rm cl}\end{aligned} \tag{27}$$

with

$$\begin{aligned}N_\phi&=\int d^4x\ \frac{\delta}{\delta\phi}\ ,\quad \text{if } \phi \text{ is a real field,}\\ N_\phi&=\int d^4x\ (\frac{\delta}{\delta\phi}+\frac{\delta}{\delta\bar\phi})\ ,\quad \text{if } \phi \text{ is a complex field.}\end{aligned} \tag{28}$$

Hence, the symmetric counterterm $\Gamma^{(1)}_{\rm ct,kin}$ describes the renormalization of the gauge coupling, field redefinitions of the photon and photino and the renormalization of the photino mass.

- The counterterm to the matter part of the action is decomposed into a field redefinition of matter fields and additional invariant counterterm Γ_{Vv}. In its general form it contains redefinitions of the axial vector multiplet into components of the supercoupling E^{2l} and the q-field renormalization. It is determined by the operator $\mathcal{D}_{Vv}$ (85) and results in the limit to constant coupling in a counterterm for the matter mass terms and the q-vertices:

$$\begin{aligned}\lim_{E\to e}\Gamma^{(l)}_{{\rm ct},Vv}=-2e^{2l}\Big(&N_{q^i}+m\partial_m\frac{1}{2}l(l+1)\frac{M_\lambda^2}{M^2}M\partial_M\\&+(2l\frac{M_\lambda m}{b}+1\Big)b\partial_b)\Gamma^{\rm sSQED}_{\rm cl}\ .\end{aligned} \tag{29}$$

There we have used the following relations:

$$\begin{aligned}m\partial_m\Gamma_{\rm cl}&=m\int d^4x\ \Big(\frac{\delta}{\delta q}+\frac{\delta}{\delta\overline{q}}\Big)\Gamma_{\rm cl}\ ,\\ M\partial_M\Gamma_{\rm cl}&=-4M^2\int d^4x\ \frac{\delta}{\delta D}\Gamma_{\rm cl}\ ,\\ b\partial_b\Gamma_{\rm cl}&=-b\int d^4x\ \Big(\frac{\delta}{\delta q_F}+\frac{\delta}{\delta\overline{q}_F}\Big)\Gamma_{\rm cl}\ ,\end{aligned} \tag{30}$$

which make possible to eliminate for constant coupling the field differentiation appearing in the symmetric operator $\mathcal{D}_{Vv}$ in favour of a mass derivative.

- There are three types of gauge dependent field redefinitions (see (91),((92) and (93)). Two of them correspond to the individual field redefinitions of electrons and selectrons:

$$\lim_{E\to e}\Gamma^{(l)}_{\mathrm{ct},\varphi} = e^{2l} f^{(l)}_{\varphi}(\xi)(N_{\varphi_L}+N_{\varphi_R}-N_{Y_{\varphi_L}}-N_{Y_{\varphi_R}})\Gamma^{\mathrm{sSQED}}_{\mathrm{cl}} \ ,$$
$$\lim_{E\to e}\Gamma^{(l)}_{\mathrm{ct},\psi} = e^{2l} f^{(l)}_{\psi}(\xi)(N_{\psi_L}+N_{\psi_R}-N_{Y_{\psi_L}}-N_{Y_{\psi_R}})\Gamma^{\mathrm{sSQED}}_{\mathrm{cl}} \ . \tag{31}$$

  The third one redefines an electron into a selectron and the spinor component of the local coupling. For constant coupling there remains only a contribution in the external field part:

$$\lim_{E\to e}\Gamma^{(l)}_{\mathrm{ct},\psi\varphi} = -2le^{2l} f^{(l)}_{\psi\varphi}(\xi)\sqrt{2}M_\lambda\Big(\epsilon^\alpha Y_{\psi_L\alpha}\varphi_L - \overline{\epsilon}_{\dot\alpha} Y^{\dot\alpha}_{\overline{\psi}_L}\overline{\varphi}_L + (L\to R)\Big) \tag{32}$$

  Its appearance shows that we have to expect an independent divergence in the external field part describing the supersymmetry transformation of the electron. Since the counterterm is linear in propagating fields, it cannot be inserted into loop diagrams and it is not relevant for the definition of physical Green functions.

We want to note that there is no symmetric counterterm corresponding to a field redefinition of selectron field φ_L into the right-handed selectron field $\overline{\varphi}_R$, because it is excluded by the global Ward identity $\mathcal{W}^R\Gamma_{\mathrm{ct,inv}}=0$.

The action of invariant counterterms in loop order l is a linear combination of the symmetric counterterms (27), (29), (31) and (32):

$$\Gamma^{(l)}_{\mathrm{ct,inv}} = z^{(1)}_{\mathrm{kin}}\Gamma^{(1)}_{\mathrm{ct,kin}}\delta_{l1} + z^{(l)}_{Vv}\Gamma^{(l)}_{\mathrm{ct},Vv}$$
$$+ z^{(l)}_{\varphi}\Gamma^{(l)}_{\mathrm{ct},\varphi}(\xi) + z^{(l)}_{\psi}\Gamma^{(l)}_{\mathrm{ct},\psi}(\xi) + z^{(l)}_{\psi\varphi}\Gamma^{(l)}_{\mathrm{ct},\psi\varphi}(\xi) \ . \tag{33}$$

It is illuminating to rewrite these counterterms as parameter and field renormalizations in the classical action:

$$\Gamma_{\mathrm{cl}}((1+z^{(l)}_e)e,(1+z^{(l)}_\phi)\phi,(1+z^{(l)}_{M_i})M_i) = \Gamma_{\mathrm{cl}}(e,\phi,M_i) + \Gamma^{(l)}_{\mathrm{ct,inv}} + \mathcal{O}(\hbar^{2l}) \tag{34}$$

Comparing the z-factors in (34) with the ones in (33) yields:

$$z^{(1)}_e = -\frac{1}{2}e^2 z^{(1)}_{\mathrm{kin}} \quad \text{and} \quad z^{(l)}_m = -2e^{2l} z^{(l)}_{Vv} \ , \tag{35}$$

and we obtain for the z-factors of soft mass parameters

$$
\begin{aligned}
z^{(1)}_{M_\lambda} &= 2z^{(1)}_e \ , \\
z^{(l)}_b &= (2l\frac{M_\lambda m}{b} + 1)z^{(l)}_m \ , \quad z^{(l)}_M = \frac{1}{2}l(l+1)\frac{M_\lambda^2}{M^2}z^{(l)}_m \ .
\end{aligned} \tag{36}
$$

Thus, the z-factors of the soft mass parameters are entirely expressed in terms of z_e and z_m. The same relations have to hold for the independent symmetric divergences of the corresponding loop diagrams when using a regularization scheme with an UV regulator.

These results are now exemplified at the one-loop level using dimensional regularization. Although this scheme breaks supersymmetry, the divergent one-loop contributions preserve all symmetry constraints.

In dimensional regularization, the one-loop divergences for the self energies read ($\alpha = \frac{e^2}{4\pi}$):

$$\Sigma^{T\,\mathrm{div}}_A = \frac{\alpha}{4\pi}2p^2\Delta \ , \tag{37}$$

$$\Sigma^{\mathrm{div}}_\lambda = \frac{\alpha}{4\pi}2p_\mu\gamma^\mu\Delta \ , \tag{38}$$

$$\Sigma^{\mathrm{div}}_{\varphi_L\overline{\varphi}_L} = \frac{\alpha}{4\pi}(-4M_\lambda^2 - 4m^2)\Delta \ , \tag{39}$$

$$\Sigma^{\mathrm{div}}_{\varphi_L\overline{\varphi}_R} = \frac{\alpha}{4\pi}(4mM_\lambda - 2b)\Delta \ , \tag{40}$$

$$\Sigma^{\mathrm{div}}_{\Psi\bar{\Psi}} = \frac{\alpha}{4\pi}(2p_\mu\gamma^\mu - 4m)\Delta \ , \tag{41}$$

where Ψ is the electron Dirac spinor composed of the left and right-handed Weyl spinors $\psi_{L\alpha}$ and $\overline{\psi}^{\dot{\alpha}}_R$. The divergences for $D \to 4$ appear in the combination $\Delta = \frac{2}{4-D} - \gamma_E + \log 4\pi$. To absorb them, the divergent parts of the symmetric counterterms have to be chosen as follows:

$$z^{(1)}_e = \frac{\alpha}{4\pi}\Delta \qquad \text{and} \qquad z^{(1)}_m = \frac{\alpha}{4\pi}2\Delta \ , \tag{42}$$

$$z^{(1)}_{M_\lambda} = \frac{\alpha}{4\pi}2\Delta = 2z^{(1)}_e \ , \tag{43}$$

$$z^{(1)}_M = \frac{\alpha}{4\pi}2\Delta\frac{M_{\lambda^2}}{M^2} = z^{(1)}_m\frac{M_{\lambda^2}}{M^2} \ , \tag{44}$$

$$z^{(1)}_b = \frac{\alpha}{4\pi}\Delta\left(2 + \frac{4mM_\lambda}{b}\right) = z^{(1)}_m\left(1 + \frac{2mM_\lambda}{b}\right) \ . \tag{45}$$

Indeed, these expressions are in agreement with the general results (36).

Like in the supersymmetric model we find in addition that independent symmetric counterterms to the chiral q-vertex and to the photon self energy in $l \geq 2$ are absent expressing the non-renormalization theorem of chiral vertices and the generalized non-renormalization theorem of the photon self energy.

With the same techniques as in [16] we are able to relate the photon self energy in $l \geq 2$ and the chiral vertices to non-local expressions. The explicit expressions are modified by soft contributions but the content and the analysis of non-renormalization theorems is the same as in the supersymmetric case: Chiral Green functions are up to the gauge dependent wave function renormalization related to superficially convergent Green functions, whereas the photon self energy is related to linearly divergent Green functions, whose divergent part is determined from non-local expressions via gauge invariance.

Hence, non-renormalization theorems and the relations of soft parameter renormalization to the supersymmetry parameters are implied by the same symmetries, namely by the multiplet structure of supersymmetric Lagrangians and by the identity (17) which identifies the coupling as the lowest component of a constrained real superfield.

5 Normalization conditions

Renormalization of softly broken theories is only complete, when the coefficients of symmetric counterterms are fixed by suitable normalization conditions. Then symmetries and normalization conditions together define the Green functions independently from properties of a specific scheme used for the subtraction of divergences.

The z-factors $z_{\rm Vv}$, z_ψ and z_φ in (33) appear in the same way as in the supersymmetric model and can be fixed for constant coupling by a normalization condition on the electron mass and on the residua of matter fields (see [23]). Similarly, the 1-loop parameter $z^{(1)}_{\rm kin}$ is determined by a normalization condition on the photon residuum in one-loop order. With local gauge coupling the photon self energy in $l \geq 2$ is determined by non-local Green functions, and it is not possible to dispose of it by a normalization condition without a modification of the defining symmetries.

In softly broken SQED there remains in addition a contribution from the gauge dependent parameter $z_{\psi\varphi}$ (32). As a normalization condition one can require that the respective vertex function vanishes at the normalization point κ^2:

$$\Gamma_{\epsilon^\alpha Y^\beta_{\psi_L}\varphi_L}(-p,p)\Big|_{p^2=\kappa^2} = 0 \ . \tag{46}$$

To complete the definition of softly broken SQED we have to prove that the theory as constructed here has a physical meaning in the sense that all fields can be interpreted as particles, i.e. it has to be shown that the two-point Green functions have a pole in perturbation theory. For this purpose we impose pole conditions on the 2-point functions and prove that these conditions are in agreement with the defining symmetries of the model.

Apparently, there are not enough symmetric counterterms for setting normalization conditions of soft parameters, but these can be imposed when we include finite redefinitions of the mass parameters. In the extended model with local coupling such redefinitions induce higher order corrections to the shifts in the Slavnov-Taylor identity and in the Ward-identity of R-symmetry (22):

$$
\begin{aligned}
f(x) &\to f(x) + \frac{1}{e^2}(M_\lambda + \sum_{l=1}^{\infty} v_\lambda^{(l)} e^{2l}) \, , \\
q_F &\to q_F(x) - (b + \sum_{l=1}^{\infty} v_b^{(l)} e^{2l}) \, , \\
\tilde{D} &\to \tilde{D} - 2(M + \sum_{l=1}^{\infty} v_M^{(l)} e^{2l})^2 \, ,
\end{aligned}
\tag{47}
$$

but do not change the structure of symmetry transformations.

In this context we want to note that due to parity conservation mass eigenstates of the selectron fields are constructed to all orders by an orthogonal transformation on the left- and right-handed selectron fields

$$
\begin{aligned}
\varphi_1 &= \frac{1}{2}\sqrt{2}(\varphi_L - \overline{\varphi}_R) \, , \qquad & \overline{\varphi}_1 &= \frac{1}{2}\sqrt{2}(\overline{\varphi}_L - \varphi_R) \, , \\
\varphi_2 &= \frac{1}{2}\sqrt{2}(\varphi_L + \overline{\varphi}_R) \, , \qquad & \overline{\varphi}_2 &= \frac{1}{2}\sqrt{2}(\overline{\varphi}_L + \varphi_R) \, .
\end{aligned}
\tag{48}
$$

These fields are also eigenstates with respect to charge conjugation and parity transformation:

$$
\varphi_1^C = -\overline{\varphi}_1 \, , \quad \varphi_2^C = \overline{\varphi}_2 \quad \text{and} \quad \varphi_1^P = -\overline{\varphi}_1 \, , \quad \varphi_2^P = \overline{\varphi}_2 \, , \tag{49}
$$

and the vertex function $\Gamma_{\varphi_2 \overline{\varphi}_1}$ vanishes due to parity conservation. With the finite redefinitions (47) pole conditions for the photino, and the scalar fields φ_1 and φ_2 can be established by adjusting the parameter v_λ, v_b and v_M as functions of the mass parameters of softly broken SQED.

The appearance of the higher order shifts (47) does not change the analysis of symmetric counterterms and the content and analysis of non-renormalization theorems. However, the Callan–Symanzik and renormalization group functions which we determine in the subsequent sections are scheme-dependent and depend on the specific form of the normalization conditions and symmetries. It is possible to include the higher-order shifts induced by pole conditions of soft parameters into the construction without difficulty, but their appearance obscures the algebraic structure of the Callan-Symanzik and renormalization group coefficients. For this reason we will restrict to the classical shifts for the remaining part of the paper and note that these conditions match the ones used in the MS-scheme of dimensional reduction. For generalized normalization conditions, the respective expressions can be obtained by carrying out a redefinition of the form (47) in the Callan–Symanzik equation (54) and the related mass equations (67).

6 The Callan–Symanzik equation

The relations of soft breaking parameters to the supersymmetric parameters as well as the non-renormalization theorems have immediate implications for the Callan–Symanzik coefficients and renormalization group coefficients of softly broken SQED. Indeed, from the symmetries of the model with local gauge coupling and gauged axial symmetry we obtain the gauge β-function in its closed form and the all-order expressions for the anomalous mass dimensions and β-functions of soft parameters. We start the construction with the Callan–Symanzik equation and continue the analysis to the renormalization group equation in the next section.

The Callan–Symanzik (CS) equation is the partial differential equation connected with the breaking of dilatations. The dilatations act on the Green functions in the same way as a scaling of all mass parameters of the theory including the normalization point κ according to their mass dimension:

$$\mathcal{W}^D\Gamma = -(m\partial_m + M_\lambda\partial_{M_\lambda} + M\partial_M + 2b\partial_b + \kappa\partial_\kappa)\Gamma \equiv -\mu\partial_\mu \; . \tag{50}$$

At the tree level, dilatations are broken by the vertex functions with non-vanishing mass dimension. By means of the external fields we obtain the following expression in the classical approximation:

$$\begin{aligned}\mu\partial_\mu\Gamma_{\rm cl} = {} & m\int d^4x\, \Big(\frac{\delta}{\delta q} + \frac{\delta}{\delta \overline{q}}\Big)\Gamma_{\rm cl} + M_\lambda \int d^4x\, \frac{1}{e^2}\Big(\frac{\delta}{\delta f} + \frac{\delta}{\delta \overline{f}}\Big)\Gamma_{\rm cl} \\ & -4M^2\int d^4x\, \frac{\delta}{\delta \tilde{D}}\Gamma_{\rm cl} - 2b\int d^4x\, \Big(\frac{\delta}{\delta q_F} + \frac{\delta}{\delta \overline{q}_F}\Big)\Gamma_{\rm cl} \; .\end{aligned} \tag{51}$$

We rewrite the classical equation (51) in the form

$$\mu D_\mu \Gamma_{\rm cl} = 0 \ , \tag{52}$$

and note that μD_μ is a symmetric operator with respect to the anomalous Slavnov–Taylor operator (18) and with respect to the Ward operator of softly broken R-symmetry (22).

In higher orders the CS equation of softly broken SQED is constructed in the same way as for SQED. For this reason we skip the construction and refer for details to [16]. There it was shown that the CS operator has to be constructed as a symmetric operator with respect to the anomalous Slavnov–Taylor identity. We find five independent $\mathbf{s}_{\Gamma_{\rm cl}}$-invariant operators with the correct quantum numbers – they are just the ones which we have already used for the construction of invariant counterterms. These five operators can be extended to $\mathbf{s}_\Gamma + r^{(1)}\delta\mathcal{S}$-symmetric operators $\mathcal{D}_{\rm kin}, \mathcal{D}^{\rm sym}_{Vv}, \mathcal{N}_\varphi, \mathcal{N}_\psi$ and $\mathcal{N}_{\psi\varphi}$. The CS operator is composed as a linear combination of these five operators:

$$\mathcal{C} = \mu_i D_{\mu_i} + \hat\beta^{(1)}_e \mathcal{D}_{\rm kin} - \sum_l \big(\hat\gamma^{(l)}\mathcal{D}^{{\rm sym}(l)}_{Vv} + \hat\gamma^{(l)}_\varphi \mathcal{N}^{(l)}_\varphi + \hat\gamma^{(l)}_\psi \mathcal{N}^{(l)}_\psi + \hat\gamma^{(l)}_{\psi\varphi}\mathcal{N}^{(l)}_{\psi\varphi}\big). \tag{53}$$

The algebraic construction yields the CS equation

$$\mathcal{C}\Gamma = \Delta_Y \tag{54}$$

for the extended model of SQED with soft breakings. In (54) Δ_Y is a field monomial linear in propagating fields and contains those parts of invariants, which are not expressed in form of operators. It is also uniquely defined in the algebraic construction and depends on the CS coefficients of the left-hand side.

The invariant operators appearing in the CS operator (53) are given in their general form in appendix B, here we want to point out the property that the symmetric operator $\mathcal{D}^{{\rm sym}(l)}_{Vv}$ includes not only an l-loop operator, but also operators of order $l+1$:

$$\mathcal{D}^{{\rm sym}(l)}_{Vv} \equiv \mathcal{D}^{(l)}_{Vv} - r^{(1)}\big(4\mathcal{D}^{(l+1)}_e + 8l(\mathcal{N}^{(l+1)}_V - 8(l+1)r^{(1)}\delta\mathcal{N}^{(l+2)}_V)\big). \tag{55}$$

In this expression the operators of order $l+1$ are determined by the anomaly absorbing part $r^{(1)}\delta\mathcal{S}$ of the anomalous Slavnov–Taylor identity. The operator $\mathcal{D}_e$ contains a differential operator with respect to the gauge coupling and determines the gauge β-function in its closed form [26, 28]. The operator $\mathcal{N}_V$ describes an anomalous dimension of the axial vector multiplet and generates the X-term to the scalar mass β-function [6].

In its structure the CS equation (54) coincides with the one of the supersymmetric model. It contains the two gauge independent coefficients $\beta^{(1)}$ and γ and the gauge dependent anomalous dimensions of matter fields, which are specific for the Wess-Zumino gauge. Furthermore, the one-loop coefficients are mass independent, and are therefore the same functions as in SQED. In particular one has

$$\beta_e^{(1)} = e^2 \frac{1}{8\pi^2} \quad \text{and} \quad \gamma^{(1)} = -\frac{e^2}{2\pi^2} \, . \tag{56}$$

For $l \geq 2$ the anomalous dimension γ is in general mass dependent and depends on the specific normalization conditions for defining the coefficient of the invariant counterterm z_{Vv}.

From the expression (54) we can extract the anomalous mass dimensions of soft parameters in their usual form. For this purpose we set the axial vector field, its superpartners and the fields of the q-multiplet to zero. Then we find the CS equation for the Green function of SQED with soft supersymmetry breaking and constant coupling:

$$\begin{aligned}
&\Big(\mu_i\partial_{\mu_i} + e^2(\hat\beta_e^{(1)} + 4r^{(1)}\gamma)(e\partial_e - N_A - N_\lambda + N_{Y_\lambda} - N_c + N_B + N_{\overline{c}} - 2\xi\partial_\xi) \\
&-\gamma_\varphi(N_{\varphi_L} + N_{\varphi_R} - N_{Y_{\varphi_L}} - N_{Y_{\varphi_R}}) - \gamma_\psi(N_{\psi_L} + N_{\psi_R} - N_{Y_{\psi_L}} - N_{Y_{\psi_R}})\Big)\,\Gamma^{\mathrm{sSQED}} \\
&= \Big(m(1-2\gamma)\int d^4x \,(\frac{\delta}{\delta q} + \frac{\delta}{\delta \overline{q}}) + \frac{M_\lambda}{e^2}(1-\gamma_{M_\lambda})\int d^4x\,\Big(\frac{\delta}{\delta f} + \frac{\delta}{\delta \overline{f}}\Big) \\
&-4M^2(1-\gamma_M)\int d^4x\,\frac{\delta}{\delta \tilde{D}} - 2b(1-\gamma_b)\int d^4x\,\Big(\frac{\delta}{\delta q_F} + \frac{\delta}{\delta \overline{q}_F}\Big)\Big)\Gamma\Big|_{\substack{V^i,q^i=0\\ E\to e}} \\
&-e\partial_e\gamma_{\psi\varphi}\sqrt{2}M_\lambda\Big(Y^\alpha_{\psi_L}\epsilon_\alpha\varphi_L - Y_{\overline{\psi}_L\dot\alpha}\overline{\epsilon}^{\dot\alpha}\overline{\varphi}_L + (L\to R)\Big)\, .
\end{aligned} \tag{57}$$

As usually it is an inhomogeneous partial differential equation containing on the right-hand-side the soft mass insertions of the breaking of dilatations.

In the CS equation (57) the anomalous dimensions of soft parameter, γ_{M_λ}, γ_M and γ_b, are entirely determined by the anomalous dimensions of the supersymmetric mass parameter γ and the one-loop β-function $\beta^{(1)}$.

Like the higher-order contributions to the gauge β-function the anomalous dimension of the photino mass is determined by the operator $\mathcal{D}_{\mathrm{kin}}$ and the operator $\mathcal{D}_e^{(l)}$:

$$\begin{aligned}
&\lim_{E\to e}(\hat\beta_e^{(1)}\mathcal{D}_{\mathrm{kin}} + 4r^{(1)}\sum_l \hat\gamma^{(l)}\mathcal{D}_e^{(l+1)})\Gamma \\
&= \beta_e(e\partial_e + \cdots)\Gamma_{E\to e} + \frac{M_\lambda}{e^2}\gamma_{M_\lambda}\int d^4x\,\Big(\frac{\delta}{\delta f} + \frac{\delta}{\delta \overline{f}}\Big)\Gamma\Big|_{E\to e}\, .
\end{aligned} \tag{58}$$

From this expression we obtain the gauge β-function β_e in its closed form,

$$\beta_e = e^2(\hat{\beta}_e^{(1)} + 4r^{(1)}\gamma) \quad \text{with} \quad \gamma = \sum_l \hat{\gamma}^{(l)} e^{2l} \ , \tag{59}$$

and the anomalous dimension γ_{M_λ},

$$\gamma_{M_\lambda} = \left(2e^2(\hat{\beta}_e^{(1)} + 4r^{(1)}\gamma) + 8r^{(1)} \sum_l \hat{\gamma}^{(l)} l e^{2(l+1)}\right) \ , \tag{60}$$

in terms of $\hat{\beta}_e^{(1)}$, $\hat{\gamma}^{(l)}$ and the anomaly coefficient $r^{(1)}$.

Contributions to the anomalous mass dimension of the scalar mass M arise from the operator $\mathcal{D}_{Vv}$ and from $\mathcal{N}_V$. In addition, from $\mathcal{D}_{Vv}$ one obtains the anomalous dimension of the b parameter. Evaluating the operators $\mathcal{D}_{Vv}$ (85) and $\mathcal{N}_V$ (86) for constant coupling we find:

$$\hat{\gamma}_M^{(l)} = \hat{\gamma}^{(l)}(l(l+1)\frac{M_\lambda^2}{M^2} + 4e^2 r^{(1)} l) \ , \tag{61}$$

$$\hat{\gamma}_b^{(l)} = \hat{\gamma}^{(l)}(1 + 2l\frac{M_\lambda m}{b}) \ . \tag{62}$$

Finally it is possible to rearrange the expressions for γ_{M_λ}, γ_b and γ_M into a more familiar form by applying the differential operator $e\partial_e$ on the gauge β-function β_e and the anomalous dimension γ (59). Defining the anomalous dimensions of soft mass parameters as a power series in the coupling

$$\gamma_M = \sum_l e^{2l}\hat{\gamma}_M^{(l)} \quad \text{and} \quad \gamma_b = \sum_l e^{2l}\hat{\gamma}_b^{(l)} \ , \tag{63}$$

one obtains from (60), (61) and (62) the following expressions:

$$\gamma_{M_\lambda} = e\partial_e(\beta_e^{(1)} + 4r^{(1)}e^2\gamma) \ , \tag{64}$$

$$\gamma_M = \frac{M_\lambda^2}{4M^2} e\partial_e(e\partial_e\gamma) + \frac{M_\lambda^2}{2M^2} e\partial_e\gamma + 2r^{(1)}e^3\partial_e\gamma \ , \tag{65}$$

$$\gamma_b = \gamma + \frac{M_\lambda m}{b} e\partial_e\gamma \ . \tag{66}$$

These expressions coincide with similar expressions given for the β-functions of soft mass parameters in refs. [5, 3] including the X-term for the scalar mass β-function [6]. The relations of the anomalous mass dimensions in the CS equation to the renormalization group β-functions are discussed finally in section 7.

In summary we find that the CS equation of softly broken SQED contains the same gauge independent coefficient as the CS equation of SQED. These are the one-loop β-function and the anomalous mass dimension γ appearing with the renormalization of the supersymmetric mass parameter m. Hence, the gauge β-function is expressed in its closed form, and one obtains a common anomalous dimension for the chiral supersymmetric vertices. Both are implications of the non-renormalization theorems for chiral vertices and for the photon self energy in $l \geq 2$. Moreover, and this is the remarkable point of the construction, the anomalous contributions to the soft breakings are completely expressed in terms of the two gauge independent CS coefficients $\beta_e^{(1)}$, γ and the anomaly coefficient $r^{(1)}$.

7 The renormalization group equation and the limit to supersymmetric QED

The CS equation is not the only partial differential equation for scaling of mass parameters in softly broken SQED, but we can derive similar equations as the CS equation for the differentiation with respect to all mass parameters of the theory.

The corresponding mass equations take the general form ($M_i = M_\lambda, M, m, b$):

$$\begin{aligned}&\big(M_i D_{M_i} + \hat\beta_e^{i(1)} \mathcal{D}_{\rm kin}\\ &- \sum_l (\hat\gamma^{i(l)} \mathcal{D}^{{\rm sym}(l)}_{Vv} + \hat\gamma^{i(l)}_\varphi \mathcal{N}^{(l)}_\varphi + \hat\gamma^{i(l)}_\psi \mathcal{N}^{(l)}_\psi + \hat\gamma^{i(l)}_{\psi\varphi} \mathcal{N}^{(l)}_{\psi\varphi})\big)\Gamma = \Delta^i_Y \ .\end{aligned} \tag{67}$$

Here $M_i D_{M_i}$ are the symmetric operators corresponding to the mass differentiation $M_i \partial_{M_i}$. Explicitly one has:

$$\begin{aligned} mD_m &\equiv m\partial_m - m\int d^4x \,\Big(\frac{\delta}{\delta q} + \frac{\delta}{\delta \overline{q}}\Big) \ , \\ M_\lambda D_{M_\lambda} &\equiv M_\lambda \partial_{M_\lambda} - \int d^4x \,\frac{M_\lambda}{e^2}\Big(\frac{\delta}{\delta f} + \frac{\delta}{\delta \overline{f}}\Big)\Gamma \ , \\ M_\lambda D_{M_\lambda} &\equiv M\partial_M \Gamma + 4M^2 \int d^4x \,\frac{\delta}{\delta \tilde D}\Gamma \ , \\ bD_b &\equiv b\partial_b + b\int d^4x \,\Big(\frac{\delta}{\delta q_F} + \frac{\delta}{\delta \overline{q}_F}\Big)\Gamma \ . \end{aligned} \tag{68}$$

Starting from the classical equations it is obvious, that the breaking of higher orders in eq. (67) is a linear combination of the five $\mathbf{s}_\Gamma + r^{(1)}\delta\mathcal{S}$-invariant operators, which we had used in the construction of the CS equation. Hence, in the limit to constant coupling we find among the coefficient functions of the individual mass equations the same relations as for the coefficient functions of the CS equation (see (59) and (64) – (66)). The β function $\hat{\beta}_e^{i(1)}$ and the anomalous dimension $\hat{\gamma}^{i(l)}$ appearing in (67) are scheme dependent and depend on the specific normalization conditions already in one-loop order.

By subtraction of the single mass equations from the CS equation we obtain the renormalization group (RG) equation for the variation of the normalization point κ. The β-functions of the RG equation can be compared with the β-functions of soft mass parameters as defined in refs. [5, 6, 3]. The latter are determined in specific mass independent schemes. The intrinsic normalization conditions of such schemes are normalization conditions at an asymptotic normalization point κ^2_∞ which is much larger than all mass parameters of the theory [29]:

$$|\kappa^2_\infty| \gg m^2, M^2_\lambda, b, M^2 \; . \tag{69}$$

We take the usual normalization conditions for the residua of matter fields and of the photon field in one-loop at the asymptotic normalization point κ^2_∞ [23]. For a mass independent definition of the invariant counterterm $\Gamma_{\mathrm{ct},V v}$ (29) we choose a normalization condition on a dimensionless vertex function at the symmetric point as for example:

$$\Gamma_{q\psi_{R\alpha}\psi^\alpha_L}\Big|_{\substack{p_i^2=\kappa^2_\infty \\ 2p_ip_j=-\kappa^2_\infty, i\neq j}} = 2 \; . \tag{70}$$

Evaluating the equations (67) for the normalization vertices at the asymptotic normalization point we find that the mass equations are in their tree form to all orders of perturbation theory, i.e.

$$M_i D_{M_i}\Gamma = 0 \; , \qquad M_i = m, M, M_\lambda, b \; . \tag{71}$$

(We omit the contributions $\gamma_{\psi\varphi}$ of the trivial insertion in the subsequent discussion.)

By means of these mass equations the field differentiation on the right-hand-side of the CS equation (57) can be eliminated for constant coupling in favour of a mass differentiation. The resulting equation is the RG equation of softly broken SQED for asymptotic normalization conditions. It is a homogeneous partial differential equation for the vertex functional of softly broken SQED Γ^{sSQED}:

$$\begin{aligned}\Big(&\kappa_\infty\partial_{\kappa_\infty} + \beta_e(e\partial_e - N_A - N_\lambda + N_{Y_\lambda} - N_c + N_B + N_{\bar{c}} - 2\xi\partial_\xi) \\ &+\gamma_{M_\lambda} M_\lambda\partial_{M_\lambda} + \gamma_M M\partial_M + 2\gamma_b b\partial_b + 2\gamma m\partial_m - \gamma_\varphi(N_{\varphi_L} + N_{\varphi_R} \\ &\quad -N_{Y_{\varphi_L}} - N_{Y_{\varphi_R}}) - \gamma_\psi(N_{\psi_L} + N_{\psi_R} - N_{Y_{\psi_L}} - N_{Y_{\psi_R}})\Big)\,\Gamma^{\mathrm{sSQED}} = 0\end{aligned} \tag{72}$$

Here β_e and $\gamma_M, \gamma_{M_\lambda}$ and γ_b are functions of $\hat\beta_e^{(1)}$ and γ as given in (59) and (64) – (66). Interpreting the coefficients of mass differentiation as β-functions for soft parameters one gets:

$$\beta_{M_\lambda} = M_\lambda e \partial_e \beta_e \ , \tag{73}$$

$$\beta_M = M\Big(\frac{M_\lambda^2}{4M^2} e\partial_e\big(e\partial_e\gamma\big) + \frac{M_\lambda^2}{2M^2} e\partial_e\gamma + \frac{1}{2}e^3\partial_e\frac{\beta_e}{e^2}\Big) \ , \tag{74}$$

$$\beta_b = 2b(\gamma + \frac{M_\lambda m}{b} e\partial_e\gamma) \ . \tag{75}$$

These expressions can be immediately compared with previous results on the β-functions, finding correspondence for the β-functions of the photino and the b-parameter [3, 5], and the correct all order expression for the β-function of the scalar mass as suggested in [6].

Finally using consistency conditions of the mass equations (71) with the RG equation (72) we find that the coefficients γ, γ_φ and γ_ψ are mass independent [29]. For this reason the CS coefficients of the supersymmetric theory and the CS coefficients as well as the RG coefficients of softly broken SQED coincide for asymptotic normalization conditions. In this sense the RG functions of the softly broken theory are indeed determined by the ones of supersymmetric QED.

8 Conclusions

In the present paper the specific renormalization properties of softly broken SQED have been worked out on the basis of algebraic renormalization. The construction is based on the extended model of SQED with a local gauge coupling and gauged axial symmetry. In the extended model softly broken supersymmetry is a natural generalization of unbroken supersymmetry. By a constant shift in the highest components of external field multiplets the soft mass terms are generated without changing the structure of defining symmetry transformations.

The non-renormalization theorems of chiral vertices and the generalized non-renormalization theorem of the photon self energy in $l \geq 2$ are deduced in the same way as in the supersymmetric model: Chiral vertices are superficially convergent up to gauge dependent field redefinitions, and the photon self energy can be related to non-local expressions via gauge invariance. Furthermore, it is seen that the symmetric counterterms of soft mass parameters are related to the counterterms of the supersymmetric mass and to the one-loop counterterm of the photon self energy.

These relations imply also restrictions on the β-functions of soft mass parameters inferring the closed form of the gauge β function and the exact all-order formulas for the soft β-functions from the algebraic construction.

In comparison to related constructions (see [3, 30] and [31]) the relevant difference concerns the inclusion of anomalous axial symmetry with the axial vector multiplet in the present construction. In ref. [16] we have shown, that the axial vector multiplet has to be introduced in order to complete SQED with the local gauge coupling to a multiplicatively renormalizable theory. Including axial symmetry we have to care about the Adler–Bardeen anomaly with its supersymmetric extension. Then, of course, a consistent higher-order construction cannot be performed with arguments on invariant schemes and, in particular, it is not possible to infer the RG β-functions from symmetric counterterms in general.

It is the remarkable point in the construction that the local coupling makes possible to absorb the anomaly into a modified anomalous Slavnov–Taylor identity, and in this case algebraic renormalization can be performed in presence of the anomaly. The closed form of the gauge β-function and the all-order expressions of soft β-functions are the result of the algebraic construction of the CS and RG equation in presence of the anomaly in the Slavnov–Taylor identity.

Finally we want to remark that the present construction is not restricted to the Wess-Zumino gauge, but can be performed in the same way with linear supersymmetry transformations and a supersymmetric gauge fixing using the superspace formalism.

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A The BRS transformations and the Slavnov-Taylor identity

In this appendix we list the BRS transformations of external fields including the shifts. The BRS transformations of the remaining fields take the conventional form with local gauge coupling and are not modified by the shifts. Their explicit form can be found in ref. [16].

- BRS transformations of the axial vector multiplet and of axial ghost

$$
\begin{aligned}
sV_\mu &= \partial_\mu \tilde{c} + i\epsilon\sigma_\mu \bar{\tilde{\lambda}} - i\tilde{\lambda}\sigma_\mu\overline{\epsilon} - i\omega^\nu\partial_\nu V_\mu \ , \\
s\tilde{\lambda}^\alpha &= \frac{i}{2}(\epsilon\sigma^{\rho\sigma})^\alpha F_{\rho\sigma}(V) + \frac{i}{2}\epsilon^\alpha\,(\tilde{D} - 2M^2) - i\omega^\nu\partial_\nu\tilde{\lambda}^\alpha \ , \\
s\bar{\tilde{\lambda}}_{\dot\alpha} &= \frac{-i}{2}(\overline{\epsilon}\overline{\sigma}^{\rho\sigma})_{\dot\alpha} F_{\rho\sigma}(V) + \frac{i}{2}\overline{\epsilon}_{\dot\alpha}(\tilde{D} - 2M^2) - i\omega^\nu\partial_\nu\bar{\tilde{\lambda}}_{\dot\alpha} \ , \\
s\tilde{D} &= 2\epsilon\sigma^\mu\partial_\mu\overline{\tilde{\lambda}} + 2\partial_\mu\tilde{\lambda}\sigma^\mu\overline{\epsilon} - i\omega^\nu\partial_\nu\tilde{D} \ , \\
s\tilde{c} &= 2i\epsilon\sigma^\nu\overline{\epsilon}V_\nu - i\omega^\nu\partial_\nu\tilde{c}
\end{aligned}
\tag{76}
$$

- BRS transformations of the local coupling and its superpartners (1)

$$
\begin{aligned}
s\eta &= \epsilon^\alpha\chi_\alpha - i\omega^\nu\partial_\nu\eta \ , \\
s\overline{\eta} &= \overline{\chi}_{\dot\alpha}\overline{\epsilon}^{\dot\alpha} - i\omega^\nu\partial_\nu\overline{\eta} \ , \\
s\chi_\alpha &= 2i(\sigma^\mu\overline{\epsilon})_\alpha\partial_\mu\eta + 2\epsilon_\alpha(f + \frac{M_\lambda}{e^2}) - i\omega^\mu\partial_\mu\chi_\alpha \\
s\overline{\chi}_{\dot\alpha} &= 2i(\epsilon\sigma^\mu)_{\dot\alpha}\partial_\mu\overline{\eta} - 2\overline{\epsilon}_{\dot\alpha}(\bar{f} + \frac{M_\lambda}{e^2}) - i\omega^\mu\partial_\mu\overline{\chi}_{\dot\alpha} \\
sf &= -M(\epsilon\chi + \overline{\chi\epsilon}) + i\partial_\mu\chi\sigma^\mu\overline{\epsilon} - i\omega^\mu\partial_\mu f \\
s\bar{f} &= -M(\epsilon\chi + \overline{\chi\epsilon}) - i\epsilon\sigma^\mu\partial_\mu\overline{\chi} - i\omega^\mu\partial_\mu\bar{f}
\end{aligned}
\tag{77}
$$

- BRS transformations of q-multiplets (6)

$$
\begin{aligned}
sq &= +2i\tilde{c}(q+m) + \epsilon^\alpha q_\alpha - i\omega^\nu\partial_\nu q \ , \\
s\overline{q} &= -2i\tilde{c}(\overline{q}+m) + \overline{q}_{\dot\alpha}\overline{\epsilon}^{\dot\alpha} - i\omega^\nu\partial_\nu\overline{q} \ , \\
sq_\alpha &= +2i\tilde{c}q_\alpha + 2i(\sigma^\mu\overline{\epsilon})_\alpha D_\mu q + 2\epsilon_\alpha(q_F - b) - i\omega^\mu\partial_\mu q_\alpha \ , \\
s\overline{q}_{\dot\alpha} &= -2i\tilde{c}\overline{q}_{\dot\alpha} + 2i(\epsilon\sigma^\mu)_{\dot\alpha} D_\mu\overline{q} - 2\overline{\epsilon}_{\dot\alpha}(\overline{q}_F - b) - i\omega^\mu\partial_\mu\overline{q}_{\dot\alpha} \ , \\
sq_F &= +2i\tilde{c}(q_F - b) + iD_\mu q^\alpha\sigma^\mu_{\alpha\dot\alpha}\overline{\epsilon}^{\dot\alpha} - 4i\overline{\tilde{\lambda}}_{\dot\alpha}\overline{\epsilon}^{\dot\alpha}(q+m) - i\omega^\mu\partial_\mu q_F \ , \\
s\overline{q}_F &= -2i\tilde{c}(\overline{q}_F - b) - i\epsilon^\alpha\sigma^\mu_{\alpha\dot\alpha}D_\mu\overline{q}^{\dot\alpha} + 4i\epsilon^\alpha\tilde{\lambda}_\alpha(\overline{q}+m) - i\omega^\mu\partial_\mu\overline{q}_F \ .
\end{aligned}
\tag{78}
$$

The covariant derivative is defined by

$$
D_\mu q^i = (\partial_\mu - 2iV_\mu)(q^i + (m, 0, -b))
\tag{79}
$$

The classical Slavnov–Taylor identity (13) expresses in functional form BRS invariance of the classical action and on-shell nilpotency of BRS transformations. The Slavnov–Taylor operator acting on a general functional $\mathcal{F}$ is defined as

$$\begin{aligned}\mathcal{S}(\mathcal{F}) = \int d^4x \, \Big(& sA^\mu \frac{\delta\mathcal{F}}{\delta A^\mu} + \frac{\delta\mathcal{F}}{\delta Y_{\lambda\alpha}}\frac{\delta\mathcal{F}}{\delta\lambda^\alpha} + \frac{\delta\mathcal{F}}{\delta Y_{\overline{\lambda}}^{\dot\alpha}}\frac{\delta\mathcal{F}}{\delta\overline{\lambda}_{\dot\alpha}} \\ & + sc\frac{\delta\mathcal{F}}{\delta c} + sB\frac{\delta\mathcal{F}}{\delta B} + s\bar{c}\frac{\delta\mathcal{F}}{\delta\bar{c}} + s\xi\frac{\delta\mathcal{F}}{\delta\xi} + s\chi_\xi\frac{\delta\mathcal{F}}{\delta\chi_\xi} \\ & + \frac{\delta\mathcal{F}}{\delta Y_{\varphi_L}}\frac{\delta\mathcal{F}}{\delta\varphi_L} + \frac{\delta\mathcal{F}}{\delta Y_{\overline{\varphi}_L}}\frac{\delta\mathcal{F}}{\delta\overline{\varphi}_L} + \frac{\delta\mathcal{F}}{\delta Y_{\psi_L\alpha}}\frac{\delta\mathcal{F}}{\delta\psi_L^\alpha} + \frac{\delta\mathcal{F}}{\delta Y_{\overline{\psi}_L}^{\dot\alpha}}\frac{\delta\mathcal{F}}{\delta\overline{\psi}_{L\dot\alpha}} + (L\to R) \\ & + s\eta^i\frac{\delta\mathcal{F}}{\delta\eta^i} + s\overline{\eta}^i\frac{\delta\mathcal{F}}{\delta\overline{\eta}^i} + sq^i\frac{\delta\mathcal{F}}{\delta q^i} + s\overline{q}^i\frac{\delta\mathcal{F}}{\delta\overline{q}^i} \\ & + sV^i\frac{\delta\mathcal{F}}{\delta V^i} + s\tilde{c}\frac{\delta\mathcal{F}}{\delta\tilde{c}}\Big) + s\omega^\nu\frac{\partial\mathcal{F}}{\partial\omega^\nu} \ . \end{aligned} \tag{80}$$

B Invariant operators

The invariant operators $\mathcal{O}^{\rm sym}$ are defined as being symmetric with respect to the symmetries of functional of 1PI Green functions: They are invariant with respect to the anomalous Slavnov-Taylor identity:

$$\big(s_\Gamma + r^{(1)}\delta\mathcal{S}\big)\mathcal{O}^{\rm sym}\Gamma = \mathcal{O}^{\rm sym}\big(\mathcal{S} + r^{(1)}\delta\mathcal{S}\big)\Gamma + \big(s_\Gamma + r^{(1)}\delta\mathcal{S}\big)\Delta_Y \ . \tag{81}$$

The expression Δ_Y is defined to be a collection of field monomials which are linear in propagating fields. And they commute with the Ward identity of R-symmetry:

$$\Big[\mathcal{O}^{\rm sym}, \mathcal{W}^R\Big] = 0 \ . \tag{82}$$

According to the identity (17) they depend on $\eta - \overline{\eta}$ only by a derivative. The numbers of local couplings contributing in the operator of loop order l is restricted by the topological formula (16).

The five invariant operators are defined by the following expressions [16]:

- The gauge independent operator $\mathcal{D}_{kin}$ is strictly one-loop and expresses the renormalization of the local coupling:

$$\begin{aligned}\mathcal{D}^{(1)}_{\rm kin} = \int d^4x \; e^2\Big(& e\frac{\delta}{\delta e} - A^\mu\frac{\delta}{\delta A^\mu} - \lambda^\alpha\frac{\delta}{\delta\lambda^\alpha} - \overline{\lambda}^{\dot\alpha}\frac{\delta}{\delta\overline{\lambda}^{\dot\alpha}} \\ & + 2\frac{M_\lambda}{e^2}\big(\frac{\delta}{\delta f} + \frac{\delta}{\delta\overline{f}}\big) \\ & + Y_\lambda^\alpha\frac{\delta}{\delta Y_\lambda^\alpha} + Y_{\overline{\lambda}}^{\dot\alpha}\frac{\delta}{\delta Y_{\overline{\lambda}}^{\dot\alpha}} - c\frac{\delta}{\delta c} \\ & + B\frac{\delta}{\delta B} + \overline{c}\frac{\delta}{\delta\overline{c}} - 2(\xi(x) + \xi)\frac{\delta}{\delta\xi} - 2\chi_\xi\frac{\delta}{\delta\chi_\xi}\Big) \end{aligned} \tag{83}$$

- The gauge independent operator $\mathcal{D}_{vV}^{\mathrm{sym}}$ extends the $\mathbf{s}_\Gamma$ invariant operator $\mathcal{D}_{vV}$ to an $\mathbf{s}_\Gamma + r^{(1)}\delta\mathcal{S}$-symmetric operator:

$$\mathcal{D}_{Vv}^{\mathrm{sym}} \equiv \mathcal{D}_{Vv}^{(l)} - r^{(1)}\big(4\mathcal{D}_e^{(l+1)} + 8l(\mathcal{N}_V^{(l+1)} - 8(l+1)r^{(1)}\delta\mathcal{N}_V^{(l+2)})\big). \quad (84)$$

The $\mathbf{s}_\Gamma$ symmetric operator is defined by:

$$\begin{aligned}\mathcal{D}_{Vv}^{(l)} \equiv \int d^4x \, \Bigg(& v^{(E^{2l})\mu}\frac{\delta}{\delta V^\mu} + \lambda^{(E^{2l})\alpha}\frac{\delta}{\delta\tilde\lambda^\alpha} + \overline{\tilde\lambda}^{(E^{2l})\dot\alpha}\frac{\delta}{\delta\overline{\tilde\lambda}^{\dot\alpha}} \\ & + d^{(E^{2l})}\frac{\delta}{\delta\tilde D} - i(\epsilon^\alpha\chi_\alpha^{(E^{2l})} - \overline{\chi}_{\dot\alpha}^{(E^{2l})}\overline{\epsilon}^{\dot\alpha})\frac{\delta}{\delta\tilde c} \\ & - 2e^{2l}(q+m)\frac{\delta}{\delta q} - 2\big(e^{2l}q^\alpha + 2\chi^{(E^{2l})\alpha}(q+m)\big)\frac{\delta}{\delta q^\alpha} \\ & - 2\big(e^{2l}(q_F - b) + 2f^{(E^{2l})}(q+m) - \chi^{(E^{2l})\alpha}q_\alpha\big)\frac{\delta}{\delta q_F} \\ & - 2e^{2l}(\overline{q}+m)\frac{\delta}{\delta\overline{q}} - 2\big(e^{2l}\overline{q}^{\dot\alpha} + 2\overline{\chi}^{(E^{2l})\dot\alpha}(\overline{q}+m)\big)\frac{\delta}{\delta\overline{q}^{\dot\alpha}} \\ & - 2\big(e^{2l}(\overline{q}_F - b) + 2\overline{f}^{(E^{2l})}(\overline{q}+m) - \overline{\chi}_{\dot\alpha}^{(E^{2l})}\overline{q}^{\dot\alpha}\big)\frac{\delta}{\delta\overline{q}_F}\Bigg) \end{aligned} \quad (85)$$

The operator $\mathcal{N}_V^{(l+1)}$ contributes to an anomalous dimension of the axial vector field and its superpartners:

$$\begin{aligned}\mathcal{N}_V^{(l)} = \int d^4x \, \Bigg(& e^{2l}\Big(V^\mu\frac{\delta}{\delta V^\mu} + \tilde\lambda^\alpha\frac{\delta}{\delta\tilde\lambda^\alpha} + \overline{\tilde\lambda}^{\dot\alpha}\frac{\delta}{\delta\overline{\tilde\lambda}^{\dot\alpha}} + (\tilde D - 2M^2)\frac{\delta}{\delta\tilde D}\Big) \\ & -\frac{i}{2}V_\mu(\sigma^\mu\overline{\chi}^{(E^{2l})})^\alpha\frac{\delta}{\delta\tilde\lambda^\alpha} + \frac{i}{2}V_\mu(\chi^{(E^{2l})}\sigma^\mu)^{\dot\alpha}\frac{\delta}{\delta\overline{\tilde\lambda}^{\dot\alpha}} \\ & +2\Big(V^\mu v_\mu^{(E^{2l})} + i\tilde\lambda^\alpha\chi_\alpha^{(E^{2l})} - i\overline{\chi}_{\dot\alpha}^{(E^{2l})}\overline{\tilde\lambda}^{\dot\alpha}\Big)\frac{\delta}{\delta\tilde D}\Bigg), \end{aligned} \quad (86)$$

and

$$\delta\mathcal{N}_V^{(l)} = \int d^4x \; e^{2l}V^\mu V_\mu\frac{\delta}{\delta\tilde D} \; ; \quad (87)$$

the operator $\mathcal{D}_e$ describes a redefinition of the coupling $e(x)$ and its superpartners:

$$
\begin{aligned}
\mathcal{D}_e^{(l+1)} = \int d^4x \, \Big(& e^{2l+3}\frac{\delta}{\delta e} - 2\big(\chi^{(E^{2l})\alpha}\frac{\delta}{\delta\chi^\alpha} + \overline{\chi}^{(E^{2l})\dot\alpha}\frac{\delta}{\delta\overline{\chi}^{\dot\alpha}}\big) \\
& - 2\big((f^{(E^{2l})} - M_\lambda l e^{2l})\frac{\delta}{\delta f} + (\overline{f}^{(E^{2l})} - M_\lambda l e^{2l})\frac{\delta}{\delta\overline{f}}\big)\Big) \\
& - e^{2(l+1)}\Big(A^\mu\frac{\delta}{\delta A^\mu} + \lambda^\alpha\frac{\delta}{\delta\lambda^\alpha} + \overline{\lambda}^{\dot\alpha}\frac{\delta}{\delta\overline{\lambda}^{\dot\alpha}} - B\frac{\delta}{\delta B} \\
& \qquad - Y_\lambda^\alpha\frac{\delta}{\delta Y_\lambda^\alpha} - Y_{\overline{\lambda}}^{\dot\alpha}\frac{\delta}{\delta Y_{\overline{\lambda}}^{\dot\alpha}} + c\frac{\delta}{\delta c} - \overline{c}\frac{\delta}{\delta\overline{c}} \\
& \qquad + 2(\xi(x) + \xi)\frac{\delta}{\delta\xi} + 2\chi_\xi\frac{\delta}{\delta\chi_\xi}\Big) \; . \qquad (88)
\end{aligned}
$$

In eqs. (85),(86) and (88) the components of the multiplet E^{2l} are defined by the following expansion:

$$
\begin{aligned}
E^{2l}(x,\theta,\overline{\theta}) &= \big(\boldsymbol{\eta}(x,\theta,\overline{\theta}) + \overline{\boldsymbol{\eta}}(x,\theta,\overline{\theta}) + \frac{M_\lambda}{e^2}(\theta^2 + \overline{\theta}^2)\big)^{-l} \\
&\equiv e^{2l}(x) + \theta^\alpha\chi_\alpha^{(E^{2l})} + \overline{\chi}_{\dot\alpha}^{(E^{2l})}\overline{\theta}^{\dot\alpha} + \theta^2 f^{(E^{2l})} + \overline{\theta}^2\overline{f}^{(E^{2l})} \\
&\quad + \theta\sigma^\mu\overline{\theta}v_\mu^{(E^{2l})} + i\theta^2(\overline{\lambda}^{(E^{2l})} + \frac{1}{2}\partial_\mu\chi\sigma^\mu)\overline{\theta} \\
&\quad - i\overline{\theta}^2\theta(\lambda^{(E^{2l})} + \frac{1}{2}\sigma^\mu\partial_\mu\overline{\chi}^{(E^{2l})}) + \frac{1}{4}\theta^2\overline{\theta}^2(d^{(E^{2l})} - \Box e^{2l}) \qquad (89)
\end{aligned}
$$

For constant coupling one gets:

$$
\begin{aligned}
&\lim_{E\to e} f^{(E^{2l})} = \lim_{E\to e}\overline{f}^{(E^{2l})} = -lM_\lambda e^{2l} \; , \\
&\lim_{E\to e} d^{(E^{2l})} = 4l(l+1)e^{2l}M_\lambda^2 \; , \qquad (90)
\end{aligned}
$$

all other θ-components are zero.

- The three operators corresponding to the field redefinitions are BRS variations and as such they are gauge dependent:

$$
\mathcal{N}_\varphi^{(l)}\Gamma + \Delta_{Y,\varphi}^{(l)} \equiv s_\Gamma \int d^4x \; e^{2l} f_\varphi^{(l)}(\tilde\xi)\big(Y_{\varphi_L}\varphi_L + Y_{\overline{\varphi}_L}\overline{\varphi}_L + (L\to R)\big) \qquad (91)
$$

$$
\mathcal{N}_\psi^{(l)}\Gamma + \Delta_{Y,\psi}^{(l)} \equiv s_\Gamma \int d^4x \; e^{2l} f_\psi^{(l)}(\tilde\xi)\big(\psi_L Y_{\psi_L} + \overline{\psi}_L Y_{\overline{\psi}_L} + (L\to R)\big) \qquad (92)
$$

$$
\begin{aligned}
\mathcal{N}_{\psi\varphi}^{(l)}\Gamma + \Delta_{Y,\psi\varphi}^{(l)} \equiv (s_\Gamma + r^{(1)}\delta\mathcal{S}) \int d^4x \; \sqrt{2} f_{\psi\varphi}^{(l)}(\tilde\xi)\big(&\chi^{E^{(2l)}}\varphi_L Y_{\psi_L} \\
&+ \overline{\chi}^{E^{(2l)}}\overline{\varphi}_L Y_{\overline{\psi}_L} + (L\to R)\big) \qquad (93)
\end{aligned}
$$

Explicit expressions are immediately obtained by evaluating the $\mathbf{s}_\Gamma + r^{(1)}\delta\mathcal{S}$-variation. They can be also found in [16].

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