Fundamental time asymmetry from nontrivial space topology

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Abstract

It is argued that a fundamental time asymmetry could arise from the global structure of the space manifold. The proposed mechanism relies on the CPT anomaly of certain chiral gauge theories defined over a multiply connected space manifold. The resulting time asymmetry (microscopic arrow of time) is illustrated by a simple thought experiment. The effect could, in principle, play a role in determining the initial conditions of the big bang.

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I. INTRODUCTION

Examining the various time asymmetries present at the macroscopic level, Penrose [1] arrived at the following question: "what special geometric structure did the big bang possess that distinguishes it from the time reverse of the generic singularities of collapse – and why?" He then proposed a particular condition (the vanishing of the Weyl curvature tensor) to hold at any *initial* singularity. Whatever the precise condition may turn out to be, the crucial point is that this condition would *not* hold for *final* singularities. This implies that the unknown physics responsible for the initial singularity necessarily involves T, PT, CT, and CPT violation [2].

But, in that paper, Penrose did not make a concrete proposal for the physical mechanism responsible for this hypothetical T and CPT noninvariance. Subsequently, he has presented some interesting, but speculative, ideas on the possible role of quantum gravity [3].

Here, we suggest the potential relevance of another mechanism that does not involve gravitation directly, but does depend on the global structure (topology) of space. The mechanism is that of the so-called CPT anomaly [4], which occurs for a class of chiral gauge theories that includes the Standard Model of elementary particle physics (modulo a condition on the ultraviolet regularization; see below).

In the remainder of this paper, we first recall the basic features of the CPT anomaly as it applies to Standard Model physics (see also Ref. [5] for a review). We then present a thought experiment (i.e., construct a "clock" C) that would, in principle, be able to distinguish the initial and final singularity. Throughout, we use natural units with $\hbar = c = 1$, except when stated otherwise [6].

II. MODIFIED MAXWELL THEORY

It is our goal to remain as close as possible to known physics. In addition, we prefer to give a single concrete example, rather than to list all possibilities and confuse the reader. We, therefore, proceed in three steps. [In a first reading, it is possible to skip ahead to Eq. (6), which gives the action of the modified Maxwell theory used later on.]

First, consider the $SU(3) \times SU(2) \times U(1)$ Standard Model as embedded in the $SO(10)$ gauge theory with left-handed Weyl fermions in three spinor representations of $SO(10)$. That is, the three families ($N_{\rm fam} = 3$) of known left-handed quarks and leptons, together with three hypothetical left-handed antineutrinos, are grouped into three **16** representations of the $SO(10)$ gauge group [7]. The Higgs fields are not important for our purpose. In short, the chiral gauge theory considered has gauge group G and left-handed fermion representation R_L given by

$$(G, R_L) = (SO(10), \mathbf{16} + \mathbf{16} + \mathbf{16}) \, . \tag{1}$$

Second, take the spacetime manifold M to be

$$\mathrm{M} = \mathbb{R}^3 \times S^1 \,, \tag{2}$$

with Cartesian coordinates

$$x^0 \equiv c\,t,\; x^1,\; x^2 \in \mathbb{R} \quad \text{and} \quad x^3 \in [0, L] \,. \tag{3}$$

The vierbeins (tetrads) are chosen to be trivial and give the Minkowski metric:

$$e^a_\mu(x) = \delta^a_\mu \,, \quad g_{\mu\nu}(x) \equiv e^a_\mu(x)\, e^b_\nu(x)\, \eta_{ab} = \eta_{\mu\nu} \,, \tag{4}$$

with $(\eta_{ab}) \equiv \mathrm{diag}(-1, 1, 1, 1)$. Moreover, the gauge and fermion fields of the $SO(10)$ theory (1) are taken to be periodic in x^3 with period L.

Third, make the gauge-invariant ultraviolet regularization of the matter multiplets in Eq. (1) essentially the same, but with the first and second families (i.e., the electron- and muon-type families) giving cancelling contributions to the CPT anomaly, so that only the contribution of the third (tau-type) family remains. For the regularization used in Ref [4], the odd integer n entering the anomalous term (see below) has then the value

$$n = \sum_{f=1}^{N_{\mathrm{fam}}} \Lambda_0^{(f)} \,/\, |\,\Lambda_0^{(f)}| = +1 - 1 + 1 = +1 \,, \tag{5}$$

with Pauli–Villars cutoffs $\Lambda_0^{(f)}$ for the x^3-independent modes of the fermionic fields contributing to the effective action. For other ultraviolet regularizations, n remains the sum of three odd integers and is therefore nonzero [8], but its value may differ from $+1$. The "correct" value of the odd integer n can perhaps be traced to a more fundamental theory, e.g., quantum gravity. In this paper, we simply assume the value $n = +1$.

The chiral gauge theory defined by Eqs. (1)–(5) turns out to have a Chern–Simons-like term in the effective action for the $SO(10)$ gauge field, which breaks Lorentz invariance and also T and CPT invariance. This term, which is proportional to n/L, has been discussed in great detail in Refs. [4,5]. (Note that the Lorentz and CPT noninvariance have also been observed in a class of exactly solvable models in two spacetime dimensions [9].)

If we now focus on the electromagnetic $U(1)$ gauge field $a_\mu(x)$ embedded in the $SO(10)$ gauge field, we have the following local terms in the effective action at low energies [4,5]:

$$S^{\mathbb{R}^3\times S^1}_{\mathrm{MCS}}[a] = \int_{\mathbb{R}^3} \mathrm{d}x^0 \mathrm{d}x^1 \mathrm{d}x^2 \int_0^L \mathrm{d}x^3 \; \mathcal{L}_{\mathrm{MCS}}[a] \,, \tag{6}$$

$$\mathcal{L}_{\mathrm{MCS}}[a] = -\tfrac{1}{4}\,\eta^{\kappa\mu}\,\eta^{\lambda\nu}\,f_{\kappa\lambda}\,f_{\mu\nu} - \tfrac{1}{4}\,m\,\epsilon^{3\lambda\mu\nu}\,f_{\lambda\mu}\,a_\nu \,, \tag{7}$$

with the Maxwell field strength $f_{\mu\nu} \equiv \partial_\mu a_\nu - \partial_\nu a_\mu$, the completely antisymmetric symbol $\epsilon^{\kappa\lambda\mu\nu}$ normalized by $\epsilon^{0123} = -1$, and the Chern–Simons mass parameter

$$m \sim \alpha/L \,, \tag{8}$$

in terms of the fine-structure constant $\alpha \equiv e^2/(4\pi)$ and the size L of the compact dimension. The precise numerical factor in the definition of m depends on the integer n as given by Eq. (5) and also on the details of the unification and the running of the coupling constant.

The effective action (6) describes the propagation of electromagnetic waves *in vacuo*, taking into account the effects of virtual fermions [i.e., those of the chiral $SO(10)$ theory]. But the reflection of light by a mirror is still described by the usual interactions of quantum electrodynamics, at least to leading order in α.

III. CIRCULARLY POLARIZED LIGHT PULSES

The propagation of light according to the Maxwell–Chern–Simons (MCS) theory (7) has been studied classically in Ref. [10] and quantum mechanically in Ref. [11]. Here, we are primarily interested in the classical propagation of pulses of circularly polarized light.

Specifically, we consider light pulses propagating approximately along the x^2 axis, that is, with wave vector $\vec{k} \equiv (k_1, k_2, k_3)$ obeying

$$k_1 = 0 < m \ll 2\pi/L \ll |k_3| \ll |k_2| \equiv \overline{k} \,, \tag{9}$$

with the fine-structure constant α ($\sim mL$) considered to be parametrically small. The corresponding group velocities for left- and right-handed wave packets have been calculated in Ref. [5], where also the dispersion relation can be found.

For wave vectors (9), the magnitudes of the different group velocities $\vec{v}_{\rm g}(\vec{k})$ of electromagnetic waves *in vacuo* are

$$\begin{aligned} \left|\vec{v}_{\rm g}^{\,R}(0,|k_2|,|k_3|)\right| &= \left|\vec{v}_{\rm g}^{\,L}(0,-|k_2|,-|k_3|)\right| \\ &\approx 1 - \left(m^2/\,k_2^2\right)\left(1 - m/\,|k_3|\right)/\,8 \,, \end{aligned} \tag{10a}$$

$$\begin{aligned} \left|\vec{v}_{\rm g}^{\,L}(0,|k_2|,|k_3|)\right| &= \left|\vec{v}_{\rm g}^{\,R}(0,-|k_2|,-|k_3|)\right| \\ &\approx 1 - \left(m^2/\,k_2^2\right)\left(1 + m/\,|k_3|\right)/\,8 \,, \end{aligned} \tag{10b}$$

up to terms of order m^4 and with the suffixes L and R indicating left- and right-handed circular polarization. For the MCS theory (7), the group velocity is, in general, less or equal to 1. Moreover, the front velocity $v_{\rm f} \equiv \lim_{|\vec{k}|\to\infty} |\vec{v}_{\rm phase}|$ is 1 in all directions and defines c; see Ref. [11].

For future reference, we mention that circularly polarized light pulses traveling along the x^3 axis (which corresponds to the compact dimension of our spacetime manifold M) have equal group velocities:

$$\left|\vec{v}_{\rm g}^{\,L,R}(0,0,k_3)\right| = |k_3|\,/\,\sqrt{k_3^2 + m^2/4}\,. \tag{11}$$

We are now ready to construct our "clocks."

IV. TWO CLOCKS

The type of clock we have in mind is a simple variation of the "light-clock" discussed in Ref. [12], for example. Our first clock C consists of a single pulse of circularly polarized light reflecting between two heavy mirrors, firmly bolted to a common support and placed inside a vacuum chamber. The two mirrors, M_1 and M_2, are parallel to each other and separated by a fixed distance D in the x^2 direction [actually, with a small displacement in the x^3 direction, so as to give the wave vectors (9) from above]; see Fig. 1a.

The source (not shown in Fig. 1a) produces a right-handed light pulse moving towards the right, that is, in the positive x^2 direction. The pulse then oscillates between the mirrors M_1 and M_2. (See, e.g., Ref. [13] for a discussion of the reflection of polarized light.) The "ticks" of the clock now correspond to the light pulse bouncing off the mirror M_1. With each reflection the pulse loses some energy, which is picked up and amplified by an unspecified device. The spacetime diagram corresponding to clock C is shown in Fig. 2a. For the MCS theory (7), the ticks of the clock C are given by ($c \equiv 1$)

$$\Delta t \approx 2\,D\,\left[\,1 + \left(m^2/\,\overline{k}^{\,2}\right)(1 - m/\,|k_3|)\,/\,8\,\right]\,, \tag{12}$$

according to Eq. (10a) for $|k_3|$ and $|k_2| \equiv \overline{k}$ as defined by Eq. (9).

We also construct a time-reversed copy C' of the original clock C, that is, with all motions reversed. (See, e.g., Ref. [14] for a discussion of the time reversal transformation.) Concretely, the source of clock C is turned around (and, if necessary, the aperture modified), so that the initial right-handed pulse starts off to the left. The precise nature of the mirrors in the clock C' is relatively unimportant for the effect we are after and we simply consider them to be the same as those of the clock C [15]. Clock C' is shown in Fig. 1b and the corresponding spacetime diagram in Fig. 2b. According to Eq. (10b) for $|k_2| \equiv \overline{k}$, the light pulse in clock C' travels slower than the one in C, so that the ticks are longer,

$$\Delta t' \approx 2\,D\,\left[\,1 + \left(m^2/\,\overline{k}^{\,2}\right)(1 + m/\,|k_3|)\,/\,8\,\right] > \Delta t\,, \tag{13}$$

provided the Chern–Simons mass parameter m is nonzero and positive; cf. Eqs. (6)–(8).

Note that if both clocks C and C' are turned by 90° around the x^1 axis (so that the initial light pulses travel exactly along the x^3 axis, but in opposite directions), the ticks become equal, according to Eq. (11). The resulting clock $\overline{C}$ has ticks given by

$$\overline{\Delta t} \approx 2\,D\,\left[\,1 + \left(m^2/\,\overline{k}^{\,2}\right)/\,8\,\right]\,, \tag{14}$$

for $|k_3| \equiv \overline{k} \gg m$ and up to terms of order m^4. The behavior of clock $\overline{C}$ does not depend on the direction ($k_3 = \pm\,\overline{k}$) of the initial right-handed pulse and is therefore invariant under time reversal (i.e., motion reversal). But the fact remains that the original two clocks C and C', in the position shown in Fig. 1, would run differently for the MCS theory (7) [16].

V. BIG BANG VS. BIG CRUNCH

The clocks C and C' provide an alternative to the ones discussed implicitly by Aharony and Ne'eman [17], which were based on the behavior of the $K^0-\bar{K}^0$ system with hypothetical CPT violation. As shown by these authors, the $K^0-\bar{K}^0$ system (with nonzero CPT-violating parameter δ) could distinguish between an expanding universe and the time-reversed copy (i.e., a contracting universe), even if the definition of matter-antimatter was left open. The same holds for our clocks C and C' (Figs. 1a and 1b), as long as the matter content of the universe is described by an appropriate chiral gauge field theory like the one of Eq. (1) and the space manifold is multiply connected [18].

Following Ref. [17], consider a hypothetical time-symmetric universe, now with the spacetime topology $\mathbb{R} \times S^2 \times S^1$ (cf. Ref. [18]). Take the time interval $I = [0, \Delta\tau]$ just after the big bang ($t = 0$) and an equal time interval $I' = [\tau - \Delta\tau, \tau]$ just before the big crunch ($t = \tau$), as determined by the use of our reference clock $\overline{C}$ or some other standard clock. Clock C running over the time interval I and the time-reversed copy of clock C (i.e., clock C') running over the time interval I' would then give a different number of ticks [19]. Therefore, the physics near the initial singularity and the physics near the final singularity would be different, even if the final singularity were a time-reversed and time-translated copy of the initial singularity. This fundamental time asymmetry (microscopic arrow of time) is precisely one of the ingredients of the new physics discussed by Penrose [1].

Of course, we do not claim that the CPT anomaly necessarily plays a role in distinguishing the big bang singularity of our own universe. After all, we do not know for sure that the actual spacetime manifold is multiply connected (the topology of the spacetime manifold could very well be $\mathbb{R}^4$ or $\mathbb{R} \times S^3$). But, in principle, the large-scale structure of spacetime could play a role in determining the fundamental time asymmetry of the initial singularity.

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[2] See Sec. 12.4 of Ref. [1]. Recall that C, P, and T stand for the charge conjugation, parity reflection and time reversal transformations, respectively; see, for example, J. J. Sakurai, *Invariance Principles and Elementary Particles* (Princeton University Press, Princeton, NJ, 1964).

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[6] Physically, c corresponds to the front velocity of light, at least for the particular modification of Maxwell theory considered; see below Eq. (10b) in the main text.

[7] See also the discussion on $SO(10)$ grand-unified theories in *Unity of Forces in the Universe*, edited by A. Zee (World Scientific, Singapore, 1982), Chap. 4.

[8] The $SO(10)$ model considered necessarily displays the CPT anomaly, whereas the Standard Model with $N_{\text{fam}} = 3$ may or may not have the anomaly, depending on the type of ultraviolet regularization. The crucial point is the larger gauge group, $SO(10) \supset SU(3) \times SU(2) \times U(1)$, which restricts the allowed regularizations of the fermions (the gauge invariance is to remain exact). See Refs. [4,5] for further details.

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[14] R. G. Sachs, *The Physics of Time Reversal* (Chicago University Press, Chicago, 1987); see also Sakurai in Ref. [2].

[15] For the different spacetime manifold $\mathrm{M} = \mathbb{R} \times S^1 \times S^1 \times S^1$, the mirrors can be dispensed with altogether. Choosing an appropriate ratio $|k_2|/|k_3|$, the light pulse simply returns to its starting point because of the topology of the space manifold.

[16] A different behavior of the clocks C and C' would certainly be a more direct observation of time reversal ("motion reversal") noninvariance than the recent result reported by the CPLEAR Collaboration, A. Angelopoulos et al., Phys. Lett. **B 444**, 43 (1998). But, for the moment, these clocks remain a *Gedankenexperiment*. The main practical difficulty is to keep the two pulses narrow enough to show the effect, which is assumed to be nonvanishing ($m \sim \alpha/L \neq 0$). For light pulses with $|k_3| \sim 2\pi/L \ll |k_2| \equiv 2\pi/\overline{\lambda}$, a rough estimate [using $v_{\mathrm{g}}/c \approx 1 - (1/8)\, m^2/\, k_2^2\,$] gives the condition $c\, T_{\mathrm{exp}} \gg \alpha^{-4}\, L^2/\overline{\lambda}$, where T_{exp} is the total duration of the experiment ($T_{\mathrm{exp}} \approx N\, 2D/c$, for a light pulse making N round trips). The experiment, which consists of clocks C and C' of Fig. 1 pushed against each other and synchronized initially on the mirrors touching, would then have to run for a very long time ($\gg 10^{10}$ years), because the *present* universe is known to be very big ($L \gtrsim 10^{10}$ lightyears). The size of the experiment, $2D \approx c\, T_{\mathrm{exp}}/N$, should also be sufficiently large, in order to reduce the total energy loss of each pulse (the total number of reflections being $2N$). On the other hand, the reflections in the clocks C and C' are essentially the same (by a rotation of 180°), which would, in principle, make for a clean effect $N\, \Delta t' \neq N\, \Delta t$, if it were there. Also, this time difference $N\, (\Delta t' - \Delta t)$ would be much larger than the Planck time $t_{\mathrm{P}} \equiv (\hbar\, G/c^5)^{1/2}$, provided the average wavelength is sufficiently large, $\overline{\lambda} \gg \alpha\, c\, t_{\mathrm{P}}$.

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[18] More precisely, the spacetime manifold should have at least one separable compact spatial dimension (coordinate x^3) with periodic spin structure; see Ref. [4]. An example in the present context would be the spatially homogeneous Kantowski–Sachs model with spacetime topology $\mathbb{R} \times S^2 \times S^1$, which has both closure in time (i.e., recollapse after a period of expansion) and CPT violation (at least, for an appropriate chiral gauge theory). The spatially homogeneous and isotropic "flat" Friedmann–Robertson–Walker (FRW) model with spacetime topology $\mathbb{R}^3 \times S^1$, on the other hand, expands forever (the same holds for the flat FRW model with the topology $\mathbb{R} \times S^1 \times S^1 \times S^1$). For further details on the topology of cosmological models, see, e.g., I. Ciufolini and J. A. Wheeler, *Gravitation and Inertia* (Princeton University Press, Princeton, NJ, 1995), Sec. 4.3.

[19] The ticks from Eqs. (12) and (13) differ because of a crucial sign difference in the $\mathrm{O}(m^3)$ term. This remains so even if the size L of the compact dimension becomes time dependent [cf. Eq. (8)], provided the odd integer n (which traces back to the ultraviolet regularization) is fixed once and for all.

FIGURES

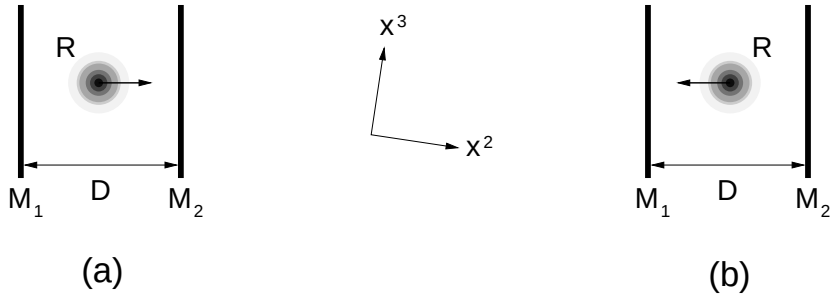

FIG. 1. (a) Sketch of clock C, which consists of a single pulse of circularly polarized light reflecting between two parallel mirrors, M_1 and M_2, separated by a fixed distance D approximately in the x^2 direction. Shown is the time at which the clock is started, with a right-handed (R) light pulse moving towards the right. The nonzero energy density of the pulse is indicated by the shaded area (the contours need not be circular). (b) Sketch of clock C', which has all motions reversed compared to clock C (i.e., clock C' is the time-reversed copy of C; see the main text). Clock C' starts with a right-handed light pulse moving towards the left.

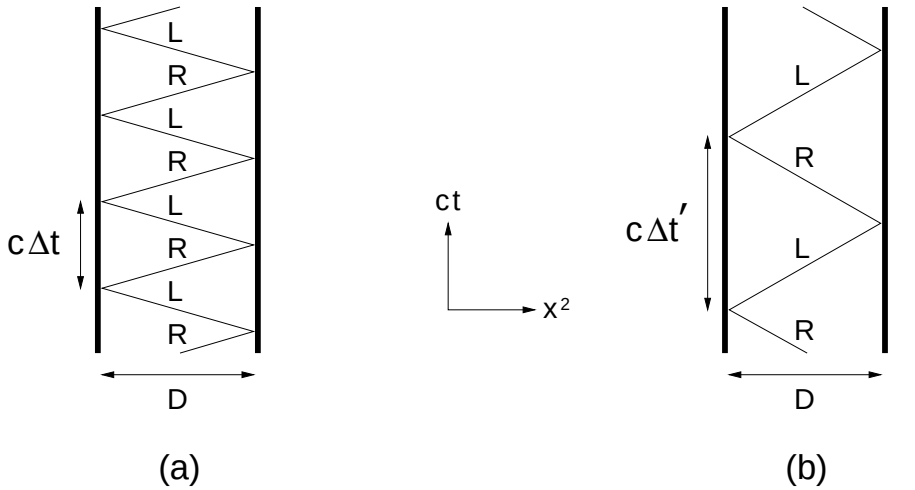

FIG. 2. (a) Schematic spacetime diagram of clock C in the Maxwell–Chern–Simons theory (7), with ticks Δt between the successive reflections of the light pulse and labels L and R indicating left- and right-handed circular polarization. (The slight rotation of the mirrors of clock C in Fig. 1 has been neglected here.) The velocity c is the front velocity of light; see below Eq. (10b) in the main text. (b) Spacetime diagram of clock C', with ticks $\Delta t'$.