Twenty Years of Δr: The W–Z Mass Correlation *

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Abstract

A review is given on the quantum correction Δr in the W–Z mass correlation, as derived from the calculation of the muon lifetime in the Standard Model. Δr was calculated for the first time in 1980 at the one-loop level. The one-loop result has been improved over the last two decades by numerically important QCD and electroweak higher-order terms, establishing thus a powerful relation that allows accurate predictions of the W-boson mass M_W for high-precision tests of the Standard Model.

1 Introduction

The interdependence between the W-boson mass, M_W, and the Z-boson mass, M_Z, with the help of the Fermi constant G_F and the fine structure constant α is one of the most important relations for testing the electroweak Standard Model (SM) with high precision. At present, the world-average for the W-boson mass is $M_W^{\rm exp} = 80.434 \pm 0.037$ GeV [1]. The experimental precision on M_W will be further improved with the data taken at LEP2 in their final analysis, at the upgraded Tevatron and at the LHC, where an error of $\delta M_W = 15$ MeV can be expected [2]. At a high-luminosity linear collider running in a low-energy mode at the W^+W^- threshold, a reduction of the experimental error down to $\delta M_W = 6$ MeV may be feasible [3]. This offers the prospect for highly sensitive tests of the electroweak theory [4], provided that the accuracy of the theoretical prediction matches the experimental precision. The basic physical quantity for the M_W–M_Z correlation is the muon lifetime τ_μ, which defines the Fermi constant G_F according to

$$\frac{1}{\tau_\mu} = \frac{G_F^2\, m_\mu^5}{192\pi^3}\, F\left(\frac{m_e^2}{m_\mu^2}\right)\left(1 + \frac{3}{5}\frac{m_\mu^2}{M_W^2}\right)(1 + \Delta_{\rm QED})\,, \tag{1}$$

with $F(x) = 1 - 8x - 12x^2 \ln x + 8x^3 - x^4$. By convention, the QED corrections within the Fermi Model, $\Delta_{\rm QED}$, are included in this defining equation for G_F. The one-loop

*Presented at the Symposium "50 Years of Precision Electroweak Physics" in honor of Alberto Sirlin's 70-th Birthday, New York University, October 27-28, 2000 result for $\Delta_{\rm QED}$ [5], has already been known for several decades; it has recently been supplemented by the two-loop correction [6], yielding

$$\Delta_{\rm QED} = 1 - 1.81\,\frac{\alpha(m_\mu)}{\pi} + 6.7\left(\frac{\alpha(m_\mu)}{\pi}\right)^2, \quad \text{with} \quad \alpha(m_\mu) \simeq \frac{1}{135.90}\ . \tag{2}$$

The tree-level W-propagator effect giving rise to the (numerically insignificant) term $3m_\mu^2/(5M_W^2)$ in (1), is conventionally also included in the definition of G_F, although not part of the Fermi Model prediction. From the precisely measured muon-decay width the value [7] $G_F = (1.16637 \pm 0.00001)\,10^{-5}\ \mathrm{GeV}^{-2}$ for the Fermi constant is derived.

Calculating the muon lifetime within the SM and comparing the SM result with (1) yields the relation

$$M_W^2\left(1 - \frac{M_W^2}{M_Z^2}\right) = \frac{\pi\alpha}{\sqrt{2}G_F}\,(1+\Delta r)\,, \tag{3}$$

where the radiative corrections are summarized in the quantity Δr, which was calculated for the first time by Alberto Sirlin [8] at the one-loop level. The one-loop result has been improved over the last two decades by numerically important QCD and electroweak higher-order terms, establishing thus a powerful relation that can be used to predict M_W within the SM (or possible extensions), to be confronted with the experimental result for M_W.

2 One-loop calculation of Δr

The quantum correction Δr can be obtained from the SM calculation of τ_μ at one-loop and the comparison with (1). Thereby, a set of infra-red divergent QED-correction graphs has to be removed. These omitted diagrams, together with the real bremsstrahlung contributions, reproduce the one-loop part of the QED-correction factor of the Fermi-model result in Eq. (1). They have no influence on the relation between G_F and the Standard Model parameters. In the on-shell renormalization scheme, with the notations

$$c_{\rm w}^2 = \frac{M_W^2}{M_Z^2}, \quad s_{\rm w}^2 = 1 - c_{\rm w}^2,$$

one finds the expression

$$\begin{aligned} \Delta r \;&=\; \Pi^\gamma(0) - \frac{c_{\rm w}^2}{s_{\rm w}^2}\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) + \frac{\Sigma^W(0) - \delta M_W^2}{M_W^2} \\ &\quad + 2\,\frac{c_{\rm w}}{s_{\rm w}}\,\frac{\Sigma^{\gamma Z}(0)}{M_Z^2} + \frac{\alpha}{4\pi s_{\rm w}^2}\left(6 + \frac{7-4s_{\rm w}^2}{2s_{\rm w}^2}\log c_{\rm w}^2\right)\,. \end{aligned} \tag{4}$$

The quantity $\Delta r = \Delta r(e, M_W, M_Z, M_H, m_t)$ depends on the entire set of the standard-model parameters. It is composed from the on-shell mass counterterms and the photon vacuum polarization from charge renormalization in the classical limit.

The photon vacuum polarization is a basic entry in the predictions for electroweak precision observables. The difference

$$\mathrm{Re}\,\hat{\Pi}^{\gamma}(M_Z^2) = \mathrm{Re}\,\Pi^{\gamma}(M_Z^2) - \Pi^{\gamma}(0) \tag{5}$$

is a finite quantity. The purely fermionic part corresponds to standard QED and does not depend on the details of the electroweak theory. It can be split into a leptonic and a hadronic contribution, yielding the quantity

$$\Delta\alpha = \Delta\alpha_{\rm lept} + \Delta\alpha_{\rm had} = -\,\mathrm{Re}\,\hat{\Pi}^{\gamma}_{\rm lept}(M_Z^2) - \mathrm{Re}\,\hat{\Pi}^{\gamma}_{\rm had}(M_Z^2)\,, \tag{6}$$

which represents a QED-induced shift in the electromagnetic fine structure constant

$$\alpha \to \alpha(1+\Delta\alpha)\,. \tag{7}$$

It can be resummed according to the renormalization group, accommodating all the leading logarithms of the type $\alpha^n \log^n(M_Z/m_f)$, to give an effective fine-structure constant at the Z mass scale,

$$\alpha(M_Z^2) = \frac{\alpha}{1-\Delta\alpha}\,. \tag{8}$$

The leptonic content of $\Delta\alpha$ can be directly evaluated in terms of the known lepton masses, yielding at three-loop order [9]

$$\Delta\alpha_{\rm lept} = 314.97687\cdot 10^{-4}\,. \tag{9}$$

For the light hadronic part, perturbative QCD is not applicable and quark masses are no reasonable input parameters. Instead, the 5-flavour contribution to $\hat{\Pi}^{\gamma}_{\rm had}$ can be derived from experimental data with the help of a dispersion relation

$$\Delta\alpha_{\rm had} = -\frac{\alpha}{3\pi}\, M_Z^2\,\mathrm{Re}\int_{4m_\pi^2}^{\infty} \mathrm{d}s'\,\frac{R^{\gamma}(s')}{s'(s'-M_Z^2-i\varepsilon)} \tag{10}$$

where

$$R^{\gamma}(s) = \frac{\sigma(e^+e^- \to \gamma^* \to \mathrm{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)}$$

is an experimental input quantity for the low energy range, yielding the value [10] $\Delta\alpha = 0.02804\pm 0.00065$. Including new data points vom BES [11] leads to an update [1] $\Delta\alpha = 0.02755 \pm 0.00046$ [1].

The heavy quark doublet (t,b) contributes predominatly via the ρ parameter [13]

$$\Delta\rho = \frac{3G_F m_t^2}{8\pi^2\sqrt{2}} \tag{11}$$

to Δr, which thus has a simple form in the light- and heavy-fermion terms:

$$\Delta r = \Delta\alpha - \frac{c_{\rm W}^2}{s_{\rm W}^2}\,\Delta\rho + \Delta r_{\rm rem}\,. \tag{12}$$

$\Delta\alpha$ contains the large logarithmic corrections from the light fermions and $\Delta\rho$ the quadratic correction from m_t. All other terms are collected in the remainder $\Delta r_{\rm rem}$, which has a typical size of the order ~ 0.01.

[1] for further error reduction based on more theory input see [12].

3 Higher-order contributions

Beyond the one-loop order, resummations of the leading one-loop contributions $\Delta\alpha$ and $\Delta\rho$ are known [14], which result in the replacement

$$1 + \Delta r \;\to\; \frac{1}{(1-\Delta\alpha)\cdot\left(1+\frac{c_{\mathrm{W}}^2}{s_{\mathrm{W}}^2}\Delta\rho\right) - \Delta r_{\mathrm{rem}}}\,. \tag{13}$$

A similar form was obtained in an $\overline{MS}$-scheme calculation [15]. The resummations correctly take into account the leading powers in $\Delta\alpha$ to all orders and the terms of the form $(\Delta\alpha\Delta r_{\mathrm{rem}})$, $(\Delta\rho)^2$, $(\Delta\alpha\Delta\rho)$ at the two-loop level when the 1-particle irreducible 2-loop contributions to the ρ parameter are included, *i.e.* replacing $\Delta\rho$ from (12) by the following higher-order expression with electroweak [2] [16] and QCD terms [17, 18],

$$\Delta\rho = 3x_t\left[1 + x_t\,\rho^{(2)} + \delta\rho_{\mathrm{QCD}}\right], \quad x_t = \frac{G_F m_t^2}{8\pi^2\sqrt{2}}\,, \tag{14}$$

with

$$\delta\rho_{\mathrm{QCD}} = -\,\frac{\alpha_s(\mu)}{\pi}\,c_1 + \left(\frac{\alpha_s(\mu)}{\pi}\right)^2 c_2(\mu), \quad c_1 = \frac{2}{3}\left(\frac{\pi^2}{3}+1\right), \tag{15}$$

and the three-loop coefficent [18] for the on-shell top mass m_t,

$$c_2 = -14.59 \quad (\mu = m_t \text{ and 6 flavors}). \tag{16}$$

The three-loop term reduces the scale dependence of $\Delta\rho$ significantly and hence is an important entry to decrease the theoretical uncertainty in the W mass prediction.

QCD corrections Δr beyond the contribution via $\Delta\rho$ are known at $\mathcal{O}(\alpha\alpha_s)$ [19] and $\mathcal{O}(\alpha\alpha_s^2)$ [20]. Concerning the electroweak two-loop contributions, only partial results are available up to now. Approximative calculations were performed based on expansions for asymptotically large values of M_H [21] and m_t [22]. The term derived by expanding in the top-quark mass beyond the leading m_t^4-order at the two-loop level [22] was found to be numerically sizeable. This $\mathcal{O}(G_F^2 m_t^2 M_Z^2)$ term, involving three different mass scales, has been obtained by two separate expansions in the regions $M_W, M_Z, M_H \ll m_t$ and $M_W, M_Z \ll m_t, M_H$ and by an interpolation between the two expansions. This formally next-to-leading order term turned out to be of a magnitude similar to that of the formally leading term of $\mathcal{O}(G_F^2 m_t^4)$, entering with the same sign. Its inclusion (both for M_W and the effective weak mixing angle) had important consequences on the indirect constraints on the Higgs-boson mass derived from the SM fit to the precision data.

A more complete calculation of electroweak two-loop effects is hence desirable. As a first step in this direction, exact results were derived for the Higgs-mass dependence of the fermionic two-loop corrections to the precision observables [24]. They have been compared with the results of expanding up to $\mathcal{O}(G_F^2 m_t^2 M_Z^2)$ [22], specifically analysing the effects of the m_t expansion, and good agreement has been found [25]. Beyond the

[2] recently also the 3-loop contribution to the ρ parameter was calculated [23].

two-loop order, complete results for the pure fermion-loop corrections (*i.e.* contributions containing n fermion loops at n-loop order) have recently been obtained up to four-loop order [26]. These results contain in particular the contributions of the leading powers in $\Delta\alpha$ as well as the ones in $\Delta\rho$ and the mixed terms.

In the following I want to discuss the complete fermionic electroweak two-loop corrections to Δr, as calculated exactly without an expansion in the top-quark or the Higgs-boson mass [27]. These are all two-loop diagrams contributing to the muon-decay amplitude and containing at least one closed fermion loop (except the pure QED corrections already contained in the Fermi model result, according to (1)). Fig. 1 displays some typical examples. The considered class of diagrams includes the potentially large corrections both from the top/bottom doublet and from contributions proportional to N_{lf} and N_{lf}^2, where N_{lf} is the number of light fermions (a partial result for the light-fermion contributions is given in [28]). These results improve on the previous results of an expansion in m_t up to next-to-leading order [22] in containing the full dependence on m_t as well as the complete light-fermion contributions at the two-loop order, while in [22] higher-order corrections from light fermions have only been taken into account via a resummation of the one-loop light-fermion contribution.

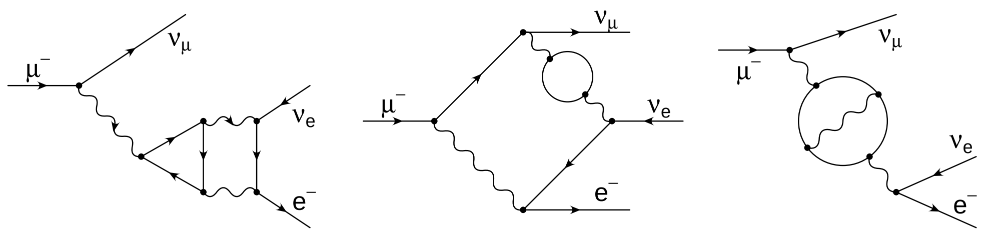


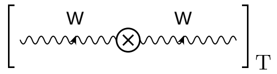
Figure 1: Examples for various types of fermionic two-loop diagrams contributing to muon decay.

All QED contributions to the Fermi Model have to be excluded in the computation of Δr since they have already been separated off in the definition of G_F, see eq. (1). Apart from the one-loop contributions, this comprises two-loop QED corrections and mixed contributions of QED and weak corrections of each one-loop order, which have to be removed from $\Delta r^{(\alpha^2)}$. For fermionic two-loop diagrams it is possible to find a one-to-one correspondence between QED graphs in Fermi-Model and SM contributions.

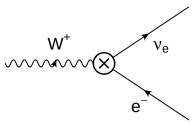
After extracting the IR-divergent QED corrections, the generic diagrams contributing to the muon-decay amplitude can be reduced to vacuum-type diagrams, since the masses of the external particles and the momentum transfer are negligible. The renormalization of is performed in the on-shell scheme. Thus, the mass-renormalization of the gauge bosons requires the evaluation of two-loop two-point functions with non-zero external momentum, which is more involved from a technical point of view regarding the tensor structure and the evaluation of the scalar integrals. This complication cannot be avoided by performing the calculation within another renormalization scheme, *e.g.* the $\overline{MS}$ scheme, since ultimately one is interested in the relation between the physical parameters M_W, M_Z, α, G_F, where the two-point functions for non-zero momenta enter.

All diagrams and amplitudes for the decay and counterterm contributions have been generated with the program *FeynArts 2.2* [29]. The amplitudes are algebraically reduced by means of a general tensor-integral decomposition for two-loop two-point functions with the program *TwoCalc* [30], leading to a fixed set of standard scalar integrals. Analytic expressions are known for the scalar one-loop [31] and two-loop [32] vacuum integrals, whereas the two-loop self-energy diagrams can be evaluated numerically by means of one-dimensional integral representations [33].

For the determination of the one-loop counterterms and renormalization constants the conventions of [34] are adopted. Two-loop renormalization constants enter via the counterterms for the transverse W propagator and the charged current vertex (the counterterms for the transverse Z propagator are analogous):



$$= \delta Z^W_{(2)}(k^2 - M_W^2) - \delta M^2_{W(2)} - \delta Z^W_{(1)} \delta M^2_{W(1)}, \tag{17}$$



$$= i \frac{e}{\sqrt{2} s_\mathrm{W}} \gamma_\mu \omega_- \Big[\delta Z_{e(2)} - \frac{\delta s_{\mathrm{W}(2)}}{s_\mathrm{W}} + \frac{1}{2} \left(\delta Z^{e\mathrm{L}}_{(2)} + \delta Z^W_{(2)} + \delta Z^{\nu\mathrm{L}}_{(2)} \right) + (\text{1-loop renormalization constants}) \Big]. \tag{18}$$

$\delta Z^{W, e\mathrm{L}, \nu\mathrm{L}}$ denote the field-renormalization constants, $\delta M^2_{W,Z}$ the W- and Z-mass counterterms, and δZ_e denotes the charge-renormalization constant. The lower indices in parentheses indicate the loop order. The mixing-angle counterterm $\delta s_{\mathrm{W}(2)}$ can be derived from the gauge-boson mass counterterms. The two-loop contributions always include the subloop renormalization.

The on-shell masses are defined as the position of the propagator poles. Starting at the two-loop level, it has to be taken into account that there is a difference between the definition of the mass $\widetilde{M}^2$ as the pole of the real part of the (transverse) propagator,

$$\mathrm{Re}\left\{ (D_\mathrm{T})^{-1}(\widetilde{M}^2) \right\} = 0, \tag{19}$$

and the real part $\overline{M}^2$ of the complex pole,

$$(D_\mathrm{T})^{-1}(\mathcal{M}^2) = 0, \qquad \mathcal{M}^2 = \overline{M}^2 - i \overline{M}\, \overline{\Gamma}. \tag{20}$$

The imaginary part of the complex pole is associated with the width $\overline{\Gamma}$. The defining condition (20) yields for the W-mass counterterm

$$\delta \overline{M}^2_{W(2)} = \mathrm{Re}\{\Sigma^W_{\mathrm{T}(2)}(\overline{M}^2_W)\} - \delta Z^W_{(1)} \delta \overline{M}^2_{W(1)} + \mathrm{Im}\{\Sigma^{W\prime}_{\mathrm{T}(1)}(\overline{M}^2_W)\}\, \mathrm{Im}\{\Sigma^W_{\mathrm{T}(1)}(\overline{M}^2_W)\}, \tag{21}$$

whereas for the real-pole definition the last term of eq. (21) is missing. Σ^W_T denotes the transverse W self-energy and $\Sigma^{W\prime}_\mathrm{T}$ its momentum derivative. Similar expressions hold for the Z boson.

The W and Z mass countertermss determine the two-loop counterterm for the mixing angle, $\delta s_{\mathrm{W}(2)}$, which has to be gauge invariant since s_W is an observable quantity. With the use of a general R_ξ gauge it has explicitly been checked that $\delta s_{\mathrm{W}(2)}$ is gauge-parameter independent for the complex-pole mass definition, whereas the real-pole definition leads to a gauge dependent $\delta s_{\mathrm{W}(2)}$. This is in accordance with the expectation from S-matrix theory [35], where the complex pole represents a gauge-invariant mass definition.

It should be noted that the mass definition via the complex pole corresponds to a Breit-Wigner parameterization of the resonance shape with a constant width. For the experimental determination of the gauge-boson masses, however, a Breit-Wigner ansatz with a running width is used. This has to be accounted for by a shift of the values for the complex pole masses [36],

$$\overline{M} = M - \frac{\Gamma^2}{2M}. \tag{22}$$

which yields the relations

$$\begin{array}{rcl} \overline{M}_Z &=& M_Z - 34.1 \text{ MeV}, \\ \overline{M}_W &=& M_W - 27.4\ (27.0) \text{ MeV} \quad \text{for} \quad M_W = 80.4\ (80.2) \text{ GeV}. \end{array} \tag{23}$$

For M_Z and Γ_Z the experimental numbers are taken. The W mass is a calculated quantity, and therefore also a theoretical value for the W-boson width should be applied here. The results above are obtained from the approximate, but sufficiently accurate expression for the W width,

$$\Gamma_W = 3\frac{G_F M_W^3}{2\sqrt{2}\pi}\left(1 + \frac{2\alpha_s}{3\pi}\right). \tag{24}$$

4 Precise predictions of the W mass

In the previous sections the characteristics of the calculation of electroweak two-loop contributions to Δr have been pointed out. Combining the fermionic $\mathcal{O}(\alpha^2)$ contributions with the one-loop and the QCD corrections yields the total result

$$\Delta r = \Delta r^{(\alpha)} + \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)} + \Delta r^{(N_f\alpha^2)} + \Delta r^{(N_f^2\alpha^2)}. \tag{25}$$

Here N_f, N_f^2 symbolize one and two fermionic loops, respectively. Fig. 2 shows that both the QCD and electroweak two-loop corrections give sizeable contributions of 10–15% with respect to the one-loop result.

In Fig. 3 the prediction for M_W derived from the result (25) and the relation (3) is compared with the experimental value for M_W. Dotted lines indicate one standard deviation bounds. The main uncertainties of the prediction originate from the experimental errors of $m_t = (174.3 \pm 5.1)$ GeV [7] and $\Delta\alpha = 0.05954 \pm 0.00065$ [10]. It is obvious that low Higgs masses are favored; the new results on Δr strengthen the tendency towards a lighter Higgs boson (according to the following comparison).

These results can be compared with the results obtained by expansion of the two-loop contributions up to next-to-leading order in m_t [22]. The predicted values for M_W for several values of M_H are given in Tab. 1. For the input parameters the values of [22] have been chosen, *i.e.* $m_t = 175$ GeV, $M_Z = 91.1863$ GeV, $\Delta\alpha = 0.0594$, $\alpha_s(M_Z) = 0.118$. Agreement is found between the results with maximal deviations of less than 5 MeV in M_W. The deviations in the last column of Tab. 1 can of course

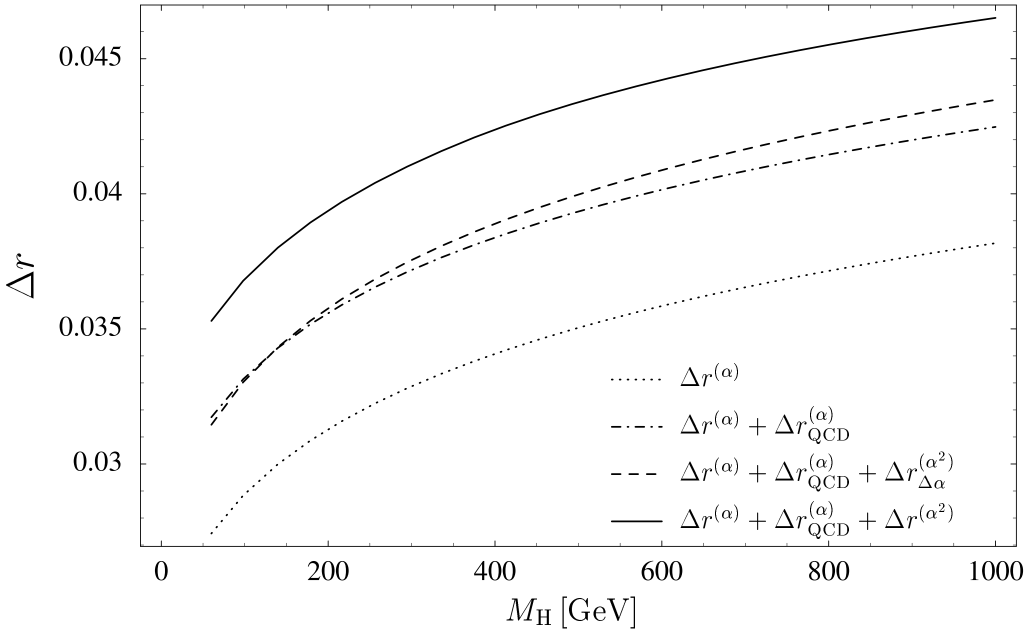


Figure 2: Various stages of Δr, as a function of M_H. The one-loop contribution, $\Delta r^{(\alpha)}$, is supplemented by the two-loop and three-loop QCD corrections, $\Delta r^{(\alpha)}_{\rm QCD} \equiv \Delta r^{(\alpha\alpha_s)} + \Delta r^{(\alpha\alpha_s^2)}$, and the fermionic electroweak two-loop contributions, $\Delta r^{(\alpha^2)} \equiv \Delta r^{(N_{\rm f}\alpha^2)} + \Delta r^{(N_{\rm f}^2\alpha^2)}$. For comparison, the effect of the two-loop corrections induced by a resummation of $\Delta\alpha$, $\Delta r^{(\alpha^2)}_{\Delta\alpha}$, is shown separately.

not be attributed exclusively to differences in the two-loop top-quark and light-fermion contributions, because the results also differ by a slightly different treatment of those higher-order terms that are not yet under control, such as purely bosonic two-loop contributions, and effects from scheme dependence.

Similar to [37], a simple numerical parametrization of our result for M_W can be given by the following expression:

$$M_W = M_W^0 - c_1\,\mathrm{dH} - c_5\,\mathrm{dH}^2 + c_6\,\mathrm{dH}^4 - c_2\,\mathrm{d}\alpha + c_3\,\mathrm{dt} - c_7\,\mathrm{dH}\,\mathrm{dt} - c_4\,\mathrm{d}\alpha_s, \qquad (26)$$

Table 1: Comparison between M_W–predictions from an NLO expansion in m_t ($M_W^{\rm expa}$) and the full calculation ($M_W^{\rm full}$). δM_W denotes the difference.

M_H [GeV]	$M_W^{\rm expa}$ [GeV]	$M_W^{\rm full}$ [GeV]	δM_W [MeV]
65	80.4039	80.3997	4.2
100	80.3805	80.3771	3.4
300	80.3061	80.3051	1.0
600	80.2521	80.2521	0.0
1000	80.2129	80.2134	−0.5

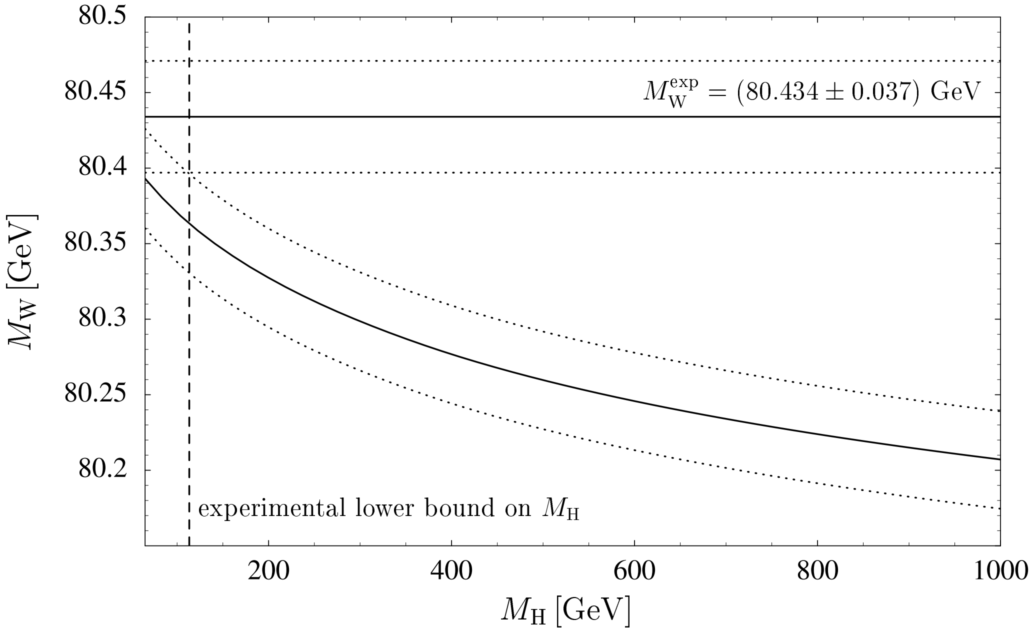


Figure 3: The SM prediction for M_W as a function of M_H for $m_t = 174.3 \pm 5.1$ GeV is compared with the current experimental value, $M_W^{\mathrm{exp}} = 80.434 \pm 0.037$ GeV [1].

where

$$\mathrm{dH} = \ln\left(\frac{M_H}{100\,\mathrm{GeV}}\right), \quad \mathrm{dt} = \left(\frac{m_t}{174.3\,\mathrm{GeV}}\right)^2 - 1,$$
$$\mathrm{d}\alpha = \frac{\Delta\alpha}{0.05924} - 1, \qquad \mathrm{d}\alpha_s = \frac{\alpha_s(M_Z)}{0.119} - 1, \tag{27}$$

with the coefficients $M_W^0 = 80.3767$ GeV, $c_1 = 0.05613$, $c_2 = 1.081$, $c_3 = 0.5235$, $c_4 = 0.0763$, $c_5 = 0.00936$, $c_6 = 0.000546$, $c_7 = 0.00573$. and with $M_Z = 91.1785$ GeV. The quality of the approximation (26) to our full result for M_W is within 0.4 MeV, allowing M_H between 65 GeV and 1 TeV.

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