# Higgs pair production in the Composite Higgs model 

Higgs Paarproduktion im Composite Higgs Modell

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Februar 2011

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Ich versichere, dass ich diese Arbeit selbstständig verfasst und keine anderen als die angegebenen Hilfsmittel benutzt habe.

Ramona Gröber<br>Karlsruhe, den 1. Februar 2011

Als Diplomarbeit anerkannt.

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## CHAPTER 1

## Introduction

Ever since the moment that the Large Hadron Collider (LHC) provided its first data a new era has begun for particle physics. The data so far confirms the Standard Model (SM) with remarkable precision. The SM is based on an $S U(3)_{C} \times S U(2)_{L} \times U(1)_{Y}$ gauge symmetry. The electroweak $S U(2)_{L} \times U(1)_{Y}$ symmetry is spontaneously broken down to the electromagnetic symmetry $U(1)_{e m}$. In the SM, electroweak symmetry breaking is realized in the most economic way: A single scalar field obtains a vacuum expectation value [1] 7]. The scalar field through its vacuum expectation value generates masses for the gauge bosons as well as for the fermions. Three of the degrees of freedom of the scalar field are thereby "eaten" by the $W^{ \pm}$and $Z$ bosons. The remaining degree of freedom provides a physical particle - the Higgs boson. The Higgs boson has another very important feature: It unitarizes the amplitudes for the scattering of longitudinally polarized gauge bosons.
As soon as the Higgs boson is found and its mass - the only free parameter left - is measured, the SM will be completely verified. However, since the SM suffers from unsolved problems such as the hierarchy problem the LHC data might soon suggest extensions of the SM. Since the SM cannot unify all four fundamental forces - it does not include the gravity - it seems to be only an effective, low energy description. Therefore, physics beyond the SM is expected to be found at the TeV scale. However, new physics has to fit into electroweak precision data from e.g. Tevatron or LEP, which until now have confirmed the SM with high precision. This highly constrains new physics extensions. An alternative idea to the elementary Higgs boson incorporated in the SM is that the Higgs boson might be a bound state of a strongly interacting sector [8-14]. The Higgs boson is realized as a pseudo Nambu-Goldstone boson [15] and is therefore naturally lighter than the other usual resonances of the strongly-interacting sector. The Higgs mass is generated via quantum effects and protected by an enlarged global symmetry. This keeps the Higgs mass naturally small. Originally, these Composite Higgs models could not fulfill electroweak precision tests, but has recently been revived by constructions in higher dimensional curved space-times [16, 17].

In Composite Higgs models there is an additional parameter, $\xi$, which lies between 0 and 1 . The value $\xi=0$ corresponds to the SM whereas $\xi=1$ corresponds the technicolor limit. In order to fulfill electroweak precision tests only slight deviations from the SM are desirable. This implies small values for $\xi$. The particle content of these models is the same as in the SM for low energies. Only the couplings change and now depend on $\xi$. Such a change in the couplings can lead to significant changes in the Higgs pair production cross sections and branching ratios.
The subject of this diploma thesis is to investigate Higgs pair production in the context of Composite Higgs models. In Higgs pair production, the trilinear Higgs self-coupling is accessible, which provides an insight into the Higgs potential and the mechanism of electroweak symmetry breaking. In the first part of chapter 2 a short review of the Higgs mechanism of the Standard model is given. In order to explain the difficulties of the SM, the hierarchy problem will be described. In the second part of chapter 2 the general concepts of Composite Higgs models will be introduced, both in the context of a model-independent Lagrangian and for two explicit models. In the last part of chapter 2 a modification of one of the models is discussed, where the fermions of the new strongly interacting sector are not integrated out but appear explicitly. In chapter 3, Higgs pair production processes at the LHC are investigated. The cross sections for gluon fusion, vector boson fusion and double Higgs-strahlung are examined. For the gluon fusion process, parameter regions where the trilinear Higgs self couplings might be accessible are constructed. Higgs pair production at $e^{+} e^{-}$-colliders is discussed in chapter 4. In this context, Higgs pair production in double Higgs-strahlung off $Z$ bosons and $W$ boson fusion is investigated. Cross sections as a function of the Higgs mass and the collider energy are shown. Also the prospects of measuring the trilinear Higgs coupling in these processes is discussed. In chapter 5, Higgs production in a model with a partially composite top quark, where the new fermions of the strongly interacting sector are explicitly taken into account, is presented. The first part of that chapter is devoted to the diagonalization of the mass matrix. Then Higgs production via gluon fusion is compared to a model where the new fermions are integrated out. Afterwards, the way in which the new fermions affect Higgs pair production via gluon fusion is investigated. In chapter 6, a summary and an outlook is given.

## CHAPTER 2

## Theoretical background

### 2.1. The Standard Model

The Standard Model (SM) of particle physics describes the electromagnetic, weak and strong interactions of particles. The SM proves to be a theory of high predictive power for many physical processes. The free parameters, however, have to be determined by experiments. Furthermore, there is one particle which has not yet been found experimentally - the Higgs boson. In this section the properties of the SM Higgs boson will be described. In the next section an alternative to the Higgs sector of the SM will be discussed.

### 2.1.1. The Higgs Mechanism

The Standard Model of particle physics is based on an $S U(3)_{C} \times S U(2)_{L} \times U(1)_{Y}$ gauge symmetry. Gauge boson masses cannot be introduced ad hoc in the SM Lagrangian, because these mass terms would violate gauge invariance. However, gauge boson masses can be implemented through spontaneous breaking of the electroweak $S U(2)_{L} \times U(1)_{Y}$ symmetry down to the electromagnetic symmetry $U(1)_{e m}$. Therefore a (Lorentz) scalar field $\Phi$, which transforms as a doublet under the $S U(2)$ and has got a $U(1)$ charge of $1 / 2$, has to be introduced [18]. So the SM Lagrangian $\mathcal{L}$ is extended by

$$
\begin{equation*}
\mathcal{L}_{s}=\left(D^{\mu} \Phi\right)^{\dagger}\left(D_{\mu} \Phi\right)-V(\Phi)=\left(D^{\mu} \Phi\right)^{\dagger}\left(D_{\mu} \Phi\right)-\mu^{2} \Phi^{\dagger} \Phi-\lambda\left(\Phi^{\dagger} \Phi\right)^{2} . \tag{2.1}
\end{equation*}
$$

The first part is the kinetic term and the second part $V(\Phi)$ is the potential for the field $\Phi$. The potential is the key ingredient for electroweak symmetry breaking. Since for $\mu^{2} \leq 0$ the field has a non-zero minimum, electroweak symmetry breaking can be triggered. The scalar field acquires a vacuum expectation value (VEV)

$$
\begin{equation*}
v=\sqrt{\frac{-\mu^{2}}{\lambda}}=246 \mathrm{GeV} \tag{2.2}
\end{equation*}
$$

The vacuum field $\langle\Phi\rangle$ reads

$$
\begin{equation*}
\langle\Phi\rangle=\frac{1}{\sqrt{2}}\binom{0}{v} . \tag{2.3}
\end{equation*}
$$

The electroweak covariant derivative is given by

$$
\begin{equation*}
D_{\mu} \Phi=\left(\partial_{\mu}-i g \frac{\sigma^{a}}{4} W_{\mu}^{a}-i g^{\prime} \frac{1}{2} B_{\mu}\right) \Phi \tag{2.4}
\end{equation*}
$$

Combining Eqs.(2.3) and (2.4) yields

$$
\begin{array}{r}
\left(D^{\mu}\langle\Phi\rangle\right)^{\dagger}\left(D_{\mu}\langle\Phi\rangle\right)=\frac{1}{2} \frac{v^{2}}{4}\left(g^{2}\left(W_{\mu}^{1}\right)^{2}+g^{2}\left(W_{\mu}^{2}\right)^{2}+\left(-g W_{\mu}^{3}+g^{\prime} B_{\mu}\right)^{2}\right) \\
=\frac{g^{2} v^{2}}{4}\left(W_{\mu}^{1}+i W_{\mu}^{2}\right)\left(W^{1 \mu}-i W^{2 \mu}\right)+\frac{1}{2} \sqrt{g^{2}+g^{\prime 2}} \frac{v}{2} \frac{1}{\sqrt{g^{2}+g^{\prime 2}}}\left(g W_{\mu}^{3}-g^{\prime} B_{\mu}\right)^{2} . \tag{2.5}
\end{array}
$$

As it turns out, there are three massive gauge bosons

$$
\begin{align*}
& W_{\mu}^{ \pm}=\frac{1}{\sqrt{2}}\left(W_{\mu}^{1} \pm i W_{\mu}^{2}\right) \quad \text { with masses } \quad M_{W}=\frac{g v}{2},  \tag{2.6}\\
& Z_{\mu}^{0}=\frac{1}{\sqrt{g^{2}+g^{\prime 2}}}\left(g W_{\mu}^{3}-g^{\prime} B_{\mu}\right) \quad \text { with mass } \quad M_{Z}=\sqrt{g^{2}+g^{\prime 2}} \frac{v}{2} . \tag{2.7}
\end{align*}
$$

The non-vanishing VEV of the scalar field hence gives masses to the gauge bosons. If $\mu^{2}$ is not smaller than zero, $\Phi$ is just a massive gauge-coupled scalar field with self-interactions but cannot give masses to the gauge bosons.
There is a fourth vector field, which remains massless and can thus be identified with the photon field

$$
A_{\mu}=\frac{1}{\sqrt{g^{2}+g^{\prime 2}}}\left(g W_{\mu}^{3}+g^{\prime} B_{\mu}\right) .
$$

Expressed in mass eigenstates, the covariant derivative (2.4) becomes

$$
\begin{equation*}
D_{\mu}=\partial_{\mu}-i \frac{g}{\sqrt{2}}\left(W_{\mu}^{+} T^{+}+W_{\mu}^{-} T^{-}\right)-i \frac{1}{\sqrt{g^{2}+g^{\prime 2}}} Z_{\mu}\left(g^{2} T^{3}-g^{\prime 2} Y\right)-i e A_{\mu}\left(T^{3}+Y\right) \tag{2.8}
\end{equation*}
$$

with $T^{ \pm}=\frac{1}{2}\left(\sigma^{1} \pm i \sigma^{2}\right), T^{3}=\frac{1}{2} \sigma^{3}$ and $e=\frac{g g^{\prime}}{\sqrt{g^{2}+g^{\prime 2}}}$. The combination

$$
\begin{equation*}
Q=T^{3}+Y \tag{2.9}
\end{equation*}
$$

can be identified with the electric charge quantum number in units of the elementary charge $e$. We have e.g. $\mathrm{Q}=-1$ for the electron.
Expanding the Lagrangian around the vacuum expectation value

$$
\begin{equation*}
\Phi(x)=\binom{\theta_{2}(x)+i \theta_{1}(x)}{\frac{1}{\sqrt{2}}(v+H(x))-i \theta_{3}(x)}=e^{i \theta_{a} \frac{\sigma_{a}}{2 v}}\binom{0}{\frac{1}{\sqrt{2}}(v+H(x))} \tag{2.10}
\end{equation*}
$$

and using the unitary gauge

$$
\begin{equation*}
\Phi \rightarrow \Phi^{\prime}=e^{-i \theta_{a} \frac{\sigma_{a}}{2 v}} \Phi=\binom{0}{\frac{1}{\sqrt{2}}(v+H(x))} \tag{2.11}
\end{equation*}
$$

leaves a real field $H$ - the Higgs field. Linear combinations of the other three would-be Goldstone bosons are "eaten" by the massive gauge bosons. As this three Goldstone fields could be rotated away, they only appear in the spectrum as the longitudinal polarizations of the $W$ and $Z$ bosons [19]. By inserting Eq. (2.11) into Eq. (2.1), the covariant derivative not only gives masses to the gauge bosons but also provides couplings of the Higgs to the gauge bosons and the kinetic term for the Higgs particle. The mass term of the Higgs boson and its self-couplings are included in the potential term $V(\Phi)$ of (2.1)

$$
\begin{equation*}
V(H)=-\frac{\mu^{2}}{2}(v+H)^{2}+\frac{\lambda}{4}(v+H)^{4}=\frac{\lambda}{4} H^{4}+\lambda v H^{3}+\frac{1}{2} \underbrace{2 \lambda v^{2}}_{M_{H}^{2}} H^{2}+\text { const. . } \tag{2.12}
\end{equation*}
$$

As the Higgs self-couplings are part of the Higgs potential, the measurement of Higgs self-couplings gives an insight into the mechanism of electroweak symmetry breaking and is therefore very important for the experimental verification of the Higgs mechanism.

### 2.1.2. The Yukawa couplings

As the $W$ bosons only couple to the left-handed fermions, left- and right-handed fermions belong to different representations (fundamental and trivial respectively) of the $S U(2)_{L}$ gauge group. In the first generation the fermionic fields are given by the left-handed doublets and the right-handed singlets

$$
\begin{equation*}
L=\binom{\nu_{e}}{e}_{L}, \quad e_{R}, \quad Q_{L}=\binom{u}{d}_{L}, \quad u_{R}, d_{R} . \tag{2.13}
\end{equation*}
$$

It is not possible to just write down a mass term $\mathcal{L}=-m_{e}\left(\bar{e}_{L} e_{R}+\bar{e}_{R} e_{L}\right)$, because the left- and right-handed fermionic fields are in different representations (e.g. $e_{L}$ is part of an $S U(2)_{L}$ doublet and $e_{R}$ is an $S U(2)_{L}$ singlet). Such a term would violate the $S U(2)_{L}$ gauge invariance. There are, however, gauge-invariant Yukawa couplings to the $\Phi$ field. For the electrons they read

$$
\begin{equation*}
\mathcal{L}_{f}=-\lambda_{e} \bar{L} \cdot \Phi e_{R}+h . c . \tag{2.14}
\end{equation*}
$$

When $\Phi$ acquires a vacuum expectation value, the fermions obtain their masses. For example, for the electron we have

$$
\begin{equation*}
\mathcal{L}_{f}=-\frac{1}{\sqrt{2}} \lambda_{e} v \bar{e}_{L} e_{R}+h . c . \quad m_{e}=\frac{1}{\sqrt{2}} \lambda_{e} v . \tag{2.15}
\end{equation*}
$$

The couplings of the Higgs particles to fermions can be obtained by inserting (2.11) into (2.14).

### 2.1.3. Custodial symmetry

The SM has an additional symmetry: An approximate global $S U(2)$ symmetry. This symmetry fixes the ratio [20]

$$
\begin{equation*}
\rho=\frac{M_{W}}{M_{Z} \cos \theta_{W}} \equiv 1 . \tag{2.16}
\end{equation*}
$$

This can be seen if the scalar field $\Phi$ is extended to a bidoublet under $S U(2)_{L} \times S U(2)_{R}$.

$$
\begin{equation*}
\tilde{\Phi}=\frac{1}{\sqrt{2}}\left(\Phi^{C}, \Phi\right) \tag{2.17}
\end{equation*}
$$

where $\Phi^{C}=i \sigma_{2} \Phi^{*}$ denotes as usual the charged conjugate of $\Phi$. This means that $\Phi$ transforms under $S U(2)_{R} \times S U(2)_{L}$ as a $(2,2)$. A general global $S U(2)_{L} \times S U(2)_{R}$ transformation is given by

$$
\begin{equation*}
\tilde{\Phi} \mapsto U_{L} \tilde{\Phi} U_{R}^{\dagger}, \tag{2.18}
\end{equation*}
$$

with $U_{R} \in S U(2)_{R}$ and $U_{L} \in S U(2)_{L}$. The Lagrangian (2.1) can be rewritten as

$$
\begin{equation*}
\mathcal{L}_{s}=\operatorname{Tr}\left[\left(D^{\mu} \tilde{\Phi}\right)^{\dagger}\left(D_{\mu} \tilde{\Phi}\right)\right]-\mu^{2} \operatorname{Tr}\left[\tilde{\Phi}^{\dagger} \tilde{\Phi}\right]-\lambda \operatorname{Tr}\left[\left(\tilde{\Phi}^{\dagger} \tilde{\Phi}\right)\right]^{2} \tag{2.19}
\end{equation*}
$$

Compared to (2.4) the covariant derivative becomes

$$
\begin{equation*}
D_{\mu} \tilde{\Phi}=\partial_{\mu} \tilde{\Phi}-i g W_{\mu}^{a} \frac{\sigma^{a}}{4} \tilde{\Phi}+i g^{\prime} \frac{1}{2} B_{\mu} \tilde{\Phi} \sigma_{3} \tag{2.20}
\end{equation*}
$$

In the $B_{\mu}$ term an additional $\sigma_{3}$ occurs that originates from the different hypercharges $\left(Y=-\frac{1}{2}\right.$ and $\left.Y=\frac{1}{2}\right)$ of $\Phi^{C}$ and $\Phi$. The Lagrangian is still invariant under the $\operatorname{SU}(2)_{L}$ transformation but not under the $\mathrm{SU}(2)_{R}$ transformation,

$$
\begin{equation*}
D_{\mu}\left(\tilde{\Phi} U_{R}^{\dagger}\right) \neq\left(D_{\mu} \tilde{\Phi}\right) U_{R}^{\dagger} \tag{2.21}
\end{equation*}
$$

because $\left[\sigma_{3}, U_{R}\right] \neq 0$ in general. So the $S U(2)_{R+L}$ symmetry is broken if $g^{\prime} \neq 0$. Furthermore, the Yukawa couplings are not $S U(2)_{R+L}$ invariant because $m_{u} \neq m_{d}$. So the $S U(2)_{R}$ is only an approximate symmetry.

### 2.1.4. Partial wave unitarity in the SM

In the SM without the Higgs Boson an inconsistency in the description at high energies occurs: Perturbative unitarity is violated in processes that involve external longitudinally polarized vector bosons [21] (for reviews, see [7, 15, 19, 22]). Omitting the contribution of the Higgs boson the Feynman diagrams for the elastic scattering of $W$ bosons are shown in Fig. 2.1.
The scattering amplitude grows with $E^{2}$ at energies much higher than the mass of the $W$ bosons. This occurs because each longitudinal polarization vector gives a factor

$$
\begin{equation*}
\epsilon_{L} \xrightarrow{p_{0} \gg M_{W}} \frac{p}{M_{W}}+\frac{M_{W}}{2 p_{0}}(-1,0,0,1) . \tag{2.22}
\end{equation*}
$$





Figure 2.1.: Feynman diagrams for $W^{+} W^{-} \rightarrow W^{+} W^{-}$scattering without a Higgs boson.

The $E^{4}$ dependence cancels when summed over all three diagrams, leaving only an $E^{2}$ dependence. So for high energies (i.e. all terms with $\frac{M_{W}}{E}$ are dropped) the amplitude is

$$
\begin{equation*}
\mathcal{M}\left(W_{L}^{+} W_{L}^{-} \rightarrow W_{L}^{+} W_{L}^{-}\right) \xrightarrow{E>M_{W}} \frac{g^{2}}{4 M_{W}^{2}}(s+t) . \tag{2.23}
\end{equation*}
$$

The dependence on $E^{2}$ (or $s$ ) of the amplitude violates partial wave unitarity. The amplitude can be expanded in partial waves

$$
\begin{equation*}
\mathcal{M}=16 \pi \sum_{l=0}^{\infty}(2 l+1) P_{l}(\cos \theta) a_{l} \tag{2.24}
\end{equation*}
$$

where $a_{l}$ is the partial wave amplitude of the $l$-th wave. For $2 \rightarrow 2$ processes the differential cross section in the centre-of-mass system (CMS) is given by

$$
\begin{equation*}
\left(\frac{d \sigma}{d \Omega}\right)_{C M S}=\frac{|\mathcal{M}|^{2}}{64 \pi^{2} s} \tag{2.25}
\end{equation*}
$$

Using (2.24) and integrating over the solid angle $\Omega$ leads to

$$
\begin{equation*}
\sigma=\frac{16 \pi}{s} \sum_{l=0}^{\infty}(2 l+1)\left|a_{l}\right|^{2} \tag{2.26}
\end{equation*}
$$

The optical theorem on the other hand leads to the identity

$$
\begin{equation*}
\sigma=\frac{1}{s} \operatorname{Im}(\mathcal{M}(\theta=0)) \tag{2.27}
\end{equation*}
$$

Combining (2.26) and (2.27), it follows that

$$
\begin{equation*}
\left|a_{l}\right|^{2}=\operatorname{Im}\left(a_{l}\right) \quad \Longrightarrow \quad \operatorname{Re}\left(a_{l}\right)^{2}+\left(\operatorname{Im}\left(a_{l}\right)-\frac{1}{2}\right)^{2}=\frac{1}{4} \tag{2.28}
\end{equation*}
$$

This means that the partial wave amplitude is constrained to lie in a unitary circle with radius $\frac{1}{2}$ in the complex plane. So for unitarity reasons

$$
\begin{equation*}
\operatorname{Re}\left(a_{l}\right)<\frac{1}{2} \tag{2.29}
\end{equation*}
$$



Figure 2.2.: The Feynman diagrams for the process $W^{+} W^{-} \rightarrow W^{+} W^{-}$involving a Higgs boson.
must be fulfilled. In $W_{L}^{+} W_{L}^{-}$-scattering the $0-\mathrm{th}{ }^{11}$ partial wave is given by

$$
\begin{equation*}
a_{0} \xrightarrow{E \gg M_{W}} \frac{s}{32 \pi v^{2}} . \tag{2.30}
\end{equation*}
$$

In accordance with (2.29), unitarity is lost at

$$
\begin{equation*}
\sqrt{s}=4 \sqrt{\pi} v \approx 1.7 \mathrm{TeV} \tag{2.31}
\end{equation*}
$$

Even though this is only a tree-level result, it is not possible to determine whether unitarity can be restored by including higher-order corrections, since the theory becomes strongly interacting and thus pertubation theory is not longer reliable.
In the SM, unitarity can be restored by taking the Higgs boson into account. The corresponding Feynman diagrams can be found in Fig. 2.2. With the Higgs contribution the amplitude becomes

$$
\begin{equation*}
\mathcal{M} \xrightarrow{E \gg M_{W}} \frac{g^{2}}{4 M_{W}^{2}}\left[s+t-\frac{s^{2}}{s-M_{H}^{2}}-\frac{t^{2}}{t-M_{H}^{2}} \cdot\right] \tag{2.32}
\end{equation*}
$$

and the 0 -th partial wave therefore yields

$$
\begin{equation*}
a_{0} \xrightarrow{E \gg} M_{W} \frac{M_{H}^{2}}{8 \pi v^{2}} . \tag{2.33}
\end{equation*}
$$

For

$$
\begin{equation*}
M_{H} \lesssim 870 \mathrm{GeV} \tag{2.34}
\end{equation*}
$$

the unitarity condition (2.29) is fulfilled. $2^{2}$

### 2.1.5. The fine-tuning problem

Even though the SM explains the electroweak precision data very well, there are still some unsolved problems. For example, electroweak symmetry breaking is not generated dynamically but is instead introduced ad hoc through a scalar field. Furthermore, the SM gives no candidate for cold dark matter. There is also the unanswered question

[^1]

Figure 2.3.: Feynman graphs for the dominant contributions to the Higgs mass at oneloop level
about the unification of all four fundamental forces cf. Ref. [23, 24]. Besides these reasons, a strong motivation for physics beyond the Standard Model is the fine-tuning problem, which originates from the radiative corrections to the Higgs boson mass. At one-loop the contributing Feynman graphs can be found in Figure 2.3. If these divergent contributions are regularized by a cut-off $\Lambda$, the physical Higgs mass becomes

$$
\begin{equation*}
M_{H}^{2}=\left(M_{H}^{0}\right)^{2}+\frac{3 \Lambda^{2}}{8 \pi^{2} v^{2}}\left(M_{H}^{2}+2 M_{W}^{2}+M_{Z}^{2}-4 m_{t}^{2}\right) . \tag{2.35}
\end{equation*}
$$

Here only the dominant contributions in the scale $\Lambda$ are kept $3 M_{H}^{0}$ denotes the bare mass contained in the unrenormalized Lagrangian. Consequently, the Higgs mass is quadratically divergent. Therefore, one would expect the Higgs mass to be of the same order as the cut-off. This is called the hierarchy problem. But the Higgs mass should be of the order of the electroweak symmetry breaking scale, $v \approx 246 \mathrm{GeV}$. This can be achieved through renormalization by adding appropriate counterterms. Supposing that the SM is valid up to some high-scale as e.g. the GUT scale $M_{G U T}=10^{16} \mathrm{GeV}$ or the Planck scale $M_{P}=10^{19} \mathrm{GeV}$, fine-tuning will be necessary. This means $M_{H}^{0}$ must be adjusted to the 16th digit for the Higgs mass to be $O(v)$ if the cut-off is of the order of the GUT scale cf. Ref. [19]. There is no symmetry of the SM that guarantees this cancellation. Attempts to solve this problems include for example, supersymmetric extensions [7, 26- 32] of the Standard Model, which introduce bosonic partners for fermions and fermionic partners for bosons. This additional symmetry reduces the divergence to a logarithmic divergence. Another possibility is to lower the scale $\Lambda$, as in models with extra dimensions [33, 34]. In the following, an alternative solution in which the Higgs boson is realized as a pseudo-Nambu-Goldstone boson will be discussed.

[^2]
### 2.2. Composite Higgs models

Composite Higgs models are models where a light Higgs emerges as a bound state of a strongly interacting sector. The Higgs boson is realized as a pseudo Nambu-Goldstone boson $4^{4}$ and is therefore naturally lighter than the other usual resonances of the strong sector [8-14]. In such models the hierarchy problem is solved because the Higgs mass arises only at loop level and is protected by an approximate global symmetry. The Higgs mass is not sensitive to virtual effects above the compositeness scale [15]. In the next part of this section, the model-independent scenario of a strongly-interacting light Higgs (SILH) of Ref. [35] will be discussed. Afterwards two concrete models [16, 17] will be presented.

### 2.2.1. The strongly-interacting light Higgs (SILH)

The composite Higgs boson emerges as a pseudo Nambu-Goldstone boson of an enlarged global symmetry $\mathcal{G}$. This symmetry $\mathcal{G}$ is broken at a scale $f$ to a subgroup $\mathcal{H}$. It must be possible to embed the SM electroweak group $S U(2)_{L} \times U(1)_{Y}$ in $\mathcal{H}$. The $\mathcal{G} / \mathcal{H}$ coset space must hence contain at least one $S U(2)_{L}$ doublet, which could be identified with the Higgs doublet. Concrete models are, for example, the custodially symmetric $S O(5) / S O(4)$ (discussed later) or the Littlest Higgs model based on $S U(5) / S O(5)$ [36]. The Higgs potential vanishes at tree level, but $\mathcal{G}$ is broken by the couplings of the SM fields to the strong sector (only $S U(2)_{L} \times U(1)_{C}$ has to be preserved), so that loops of SM fermions and gauge bosons generate a Higgs potential at one-loop. The new strong sector is parameterized by two parameters, namely the coupling $g_{\rho}$ with $g_{S M} \lesssim g_{\rho} \lesssim 4 \pi$ ( $g_{S M}$ stands for a generic SM coupling) and $m_{\rho}$, the mass of the heavy resonances of the strong sector. For the effective low-energy Lagrangian these heavy resonances will be integrated out. These parameters are related through

$$
\begin{equation*}
m_{\rho}=g_{\rho} f . \tag{2.36}
\end{equation*}
$$

The Higgs gets at one-loop a much lighter mass than the heavy resonances

$$
\begin{equation*}
M_{H} \sim g_{S M} v . \tag{2.37}
\end{equation*}
$$

[^3]With these assumptions, a low-energy effective Lagrangian can be constructed cf. Ref. [35]. The Lagrangian, which contains six-dimensional operators, takes the form

$$
\begin{align*}
\mathcal{L}_{S I L H}= & \frac{c_{H}}{2 f^{2}} \partial^{\mu}\left(\Phi^{\dagger} \Phi\right) \partial_{\mu}\left(\Phi^{\dagger} \Phi\right)+\frac{c_{T}}{2 f^{2}}\left(\Phi^{\dagger} \overleftrightarrow{D^{\mu}} \Phi\right)\left(\Phi^{\dagger} \overleftrightarrow{D_{\mu}} \Phi\right) \\
& -\frac{c_{6} \lambda}{f^{2}}\left(\Phi^{\dagger} \Phi\right)^{3}+\left(\frac{c_{y} y_{f}}{f^{2}} \Phi^{\dagger} \Phi \bar{f}_{L} \Phi f_{R}+h . c .\right) \\
& +\frac{i c_{W} g}{2 m_{\rho}^{2}}\left(\Phi^{\dagger} \sigma^{i} \overleftrightarrow{D^{\mu}} \Phi\right)\left(D^{\nu} W_{\mu \nu}\right)^{i}+\frac{i c_{B} g^{\prime}}{2 m_{\rho}^{2}}\left(\Phi^{\dagger} \overleftrightarrow{D^{\mu}} \Phi\right)\left(\partial^{\nu} B_{\mu \nu}\right)  \tag{2.38}\\
& +\frac{i c_{H W} g}{16 \pi^{2} f^{2}}\left(D^{\mu} \Phi\right)^{\dagger} \sigma^{i}\left(D^{\nu} \Phi\right) W_{\mu \nu}^{i}+\frac{i c_{H B} g^{\prime}}{16 \pi^{2} f^{2}}\left(D^{\mu} \Phi\right)^{\dagger}\left(D^{\nu} \Phi\right) B_{\mu \nu} \\
& +\frac{c_{\gamma} g^{2}}{16 \pi^{2} f^{2}} \frac{g^{2}}{g_{\rho}^{2}} \Phi^{\dagger} \Phi B_{\mu \nu} B^{\mu \nu}+\frac{c_{g} g_{S}^{2}}{16 \pi^{2} f^{2}} \frac{y_{t}^{2}}{g_{\rho}^{2}} \Phi^{\dagger} \Phi G_{\mu \nu}^{a} G^{a \mu \nu}
\end{align*}
$$

The $B^{\mu \nu} W^{\mu \nu}$ and $G^{a \mu \nu}$ denote the gauge field strengths of the corresponding gauge fields. $\overleftrightarrow{D_{\mu}}$ is defined as follows:

$$
\begin{equation*}
\Phi^{\dagger} \overleftrightarrow{D_{\mu}} \Phi=\Phi^{\dagger} D_{\mu} \Phi-\left(D_{\mu} \Phi^{\dagger}\right) \Phi \tag{2.39}
\end{equation*}
$$

The coefficients are expected to be of order one. The coefficient $c_{T}$ vanishes in every model that preserves custodial symmetry. The $\rho$ parameter [37]

$$
\begin{equation*}
\rho=\frac{M_{W}^{2}}{M_{Z}^{2} \cos ^{2} \Theta_{W}}=1 \tag{2.40}
\end{equation*}
$$

is experimentally measured up to a precision of $\sim 0.3 \%$ [38]. In order to fulfill this relation $c_{T}$ has to be very small. In the following $\mathrm{c}_{T}$ will always be set to zero.
Only the operators proportional to the coefficients $c_{H}, c_{6}$ and $c_{y}$ give sizeable contributions to the Higgs couplings. The operators with coefficients $c_{W}$ and $c_{B}$ are suppressed by a factor $\frac{g}{m_{\rho}^{2}}=\frac{g}{g_{\rho}^{2} f^{2}}$ and therefore are of a similar size as a one-loop correction in a strongly-coupled theory with $g_{\rho} \gg g$. This also holds true for the operators with the coefficients $c_{\gamma}$ and $c_{g}$. As the operators with the coefficients $c_{H W}$ and $c_{H B}$ are suppressed by a factor $\left(\frac{g}{f^{2}}\right)^{2}$, they are also of one-loop order.
The Feynman rules of the leading order operators can be calculated by inserting

$$
\begin{equation*}
\Phi=\frac{v}{\sqrt{2}}\binom{0}{\frac{H}{v}+1} \tag{2.41}
\end{equation*}
$$

into (2.38). The operator $\left(\partial^{\mu}\left(\Phi^{\dagger} \Phi\right)\right)^{2}$ gives rise to additional terms of the form

- $\partial_{\mu} H \partial^{\mu} H$
- $\partial_{\mu} H \partial^{\mu} H H$
- $\partial_{\mu} H \partial^{\mu} H H^{2}$.

The first is a kinetic term, which has to be absorbed through a field redefinition in the kinetic term of the SM part of the Lagrangian. The other two terms are interactions between three and four Higgs fields. But as they depend on the derivatives of the Higgs field (or on the momentum in momentum space) it is easier to take them into account through a field redefinition as well

$$
\begin{equation*}
H \rightarrow H-\frac{c_{H} \xi}{2}\left(H+\frac{H^{2}}{v}+\frac{H^{3}}{3 v^{2}}\right) \tag{2.42}
\end{equation*}
$$

This result was obtained through an expansion to first order in $\xi=\frac{v^{2}}{f^{2}}$. The results of this subsection can therefore only be used for small values of $\xi$. For high values of $\xi$ a resummation must be done. With the redefinition of (2.42) we have for (2.41)

$$
\begin{equation*}
\Phi=\frac{v}{\sqrt{2}}\binom{0}{\frac{1}{v}\left(H-\frac{c_{H} \xi}{2}\left(H+\frac{H^{2}}{v}+\frac{H^{3}}{3 v^{2}}\right)\right)+1} . \tag{2.43}
\end{equation*}
$$

The operator $\left(\Phi^{\dagger} \Phi\right)^{3}$ gives an additional contribution to the Higgs potential. Calculating the vacuum expectation value (VEV) of the whole potential with (2.43) leads to a redefinition of the VEV [39] while keeping its value fixed to that known by measurements

$$
\begin{equation*}
\frac{v^{2}}{2}=\frac{\mu^{2}}{2 \lambda}\left(1-\frac{3}{4} c_{6} \xi\right)=\frac{(246 \mathrm{GeV})^{2}}{2} \tag{2.44}
\end{equation*}
$$

Due to the field redefinition (2.42) and the new SILH operators, new mass terms for the fermions and the Higgs boson arise. As with the vacuum expectation value, they have to be absorbed by a redefinition, while fixing their values to their experimental ones

$$
\begin{align*}
m_{f} & =\frac{1}{\sqrt{2}} y_{f} v\left(1+\frac{\xi}{2} c_{y}\right)  \tag{2.45}\\
\frac{M_{H}^{2}}{2} & =\frac{(\sqrt{2 \lambda} v)^{2}}{2}\left(1-c_{H} \xi+\frac{3}{4} c_{6} \xi\right) \tag{2.46}
\end{align*}
$$

Taking all this into account leads to

$$
\begin{align*}
\mathcal{L}_{\text {SILH }}= & \xi\left(-\frac{M_{H}^{2}}{2 v}\left[\left(c_{6}-3 \frac{c_{H}}{2}\right) H^{3}+\left(6 c_{6}-\frac{25 c_{H}}{3}\right) \frac{H^{4}}{4 v}+\ldots\right]\right. \\
& +\frac{m_{f}}{v} \bar{f} f\left[\left(c_{y}+\frac{c_{H}}{2}\right) H+\left(3 c_{y}+c_{H}\right) \frac{H^{2}}{2 v}+\ldots\right] \\
& \left.-c_{H} M_{W}^{2}\left(\frac{H}{v}+\frac{2 H^{2}}{v^{2}}+\ldots\right) W_{\mu}^{+} W^{-\mu}-\frac{c_{H} M_{Z}^{2}}{2}\left(\frac{H}{v}+\frac{2 H^{2}}{v^{2}}+\ldots\right) Z_{\mu} Z^{\mu}+\ldots\right) . \tag{2.47}
\end{align*}
$$

The ellipses denote terms in higher order in $H$. So the Higgs-coupling to vector bosons $H V V$, to fermions $H f \bar{f}$ and the trilinear Higgs self-couplings $H^{3}$ read in terms of the
corresponding SM couplings

$$
\begin{array}{rlrl}
g_{H V V} & =g_{H V V}^{S M}\left(1-\frac{c_{H}}{2} \xi\right) & \text { with } g_{H W W}^{S M}=g M_{W} \text { an } \\
g_{H f f}=g_{H f f}^{S M}\left(1-\xi \frac{c_{H}}{2}+\xi c_{y}\right) & \text { with } g_{H f f}^{S M}=\frac{g m_{f}}{2 M_{W}} \\
g_{H H H}=g_{H H H}^{S M}\left(1+\xi c_{6}-\xi \frac{3 c_{H}}{2}\right) & & \text { with } g_{H H H}^{S M}=\frac{3 g M_{H}^{2}}{2 M_{W}} . \tag{2.50}
\end{array}
$$

Since the coupling of the Higgs boson to gauge bosons is modified, the Higgs boson cannot fully unitarize the longitudinally polarized gauge boson scattering amplitude any more. In Ref. [35] the high-energy limit of the scattering amplitudes is calculated. For example, the authors obtained for longitudinal $W^{ \pm}$scattering

$$
\begin{equation*}
\mathcal{M}\left(W_{L}^{ \pm} W_{L}^{ \pm} \rightarrow W_{L}^{ \pm} W_{L}^{ \pm}\right) \approx-\frac{c_{H} s}{f^{2}} \tag{2.51}
\end{equation*}
$$

So there is still an $E_{C M S}^{2}$ dependence, but it is suppressed by a factor of $1 / f^{2}$. This means that the scale at which the amplitude does not fulfill the unitarity condition of (2.29) is shifted to higher energies. The growth with energy of the amplitude is strictly only valid up to the energy scale of the cut-off of the effective theory, which corresponds to $m_{\rho}$. The description above the cut-off depends on the specific model. But unitarity can eventually be restored by the exchange of the heavier resonances. The same energy behaviour is expected for the process $W_{L}^{+} W_{L}^{-} \rightarrow H H$. But this will be investigated in detail in chapter 3 and 4 .

### 2.2.2. Holographic Higgs

The Holographic Higgs Model is the four-dimensional description of a five-dimensional theory in Anti de-Sitter (AdS) space-time. The AdS space-time is slightly curved even in absence of matter. The curvature is hyperbolic 5 The five dimensional space-time metric with curvature radius $\frac{1}{k}$ is given by [16, 34, 41]

$$
\begin{equation*}
d s^{2}=\frac{1}{(k z)^{2}}\left(\eta_{\mu \nu} d x^{\mu} d x^{\nu}\right)-d z^{2}=g_{M N} d x^{M} d x^{N} \tag{2.52}
\end{equation*}
$$

with $M, N=\{0 \ldots 5\}$. The fifth dimension is denoted by the coordinate $z$ and has got two boundaries, one at $z=L_{0}=\frac{1}{k} \sim \frac{1}{M_{\text {Planck }}}$ (UV boundary) and the other at $z=L_{1} \sim \mu_{I R} \sim \frac{1}{\mathrm{TeV}}$ (IR boundary). As gauge symmetry of the bulk ${ }^{6}$ the $S U(3)_{C} \times$ $S O(5) \times U(1)_{B-L}$ was proposed by [16, 17] which is reduced to the SM gauge group $S U(3)_{C} \times S U(2)_{L} \times U(1)_{Y}$ on the IR boundary. On the UV boundary it is broken to $S U(3)_{C} \times S O(4) \times U(1)_{B-L}$ in the model of Ref. [16] whereas in the model of Ref. [17] it is broken to $S U(3)_{C} \times O(4) \times U(1)_{B-L}$. These are the minimal possible scenarios, if the global symmetry is assumed to contain the SM symmetry group, the Higgs boson

[^4]is a pseudo Nambu-Goldstone boson and $S O(3)$ acts as a custodial symmetry group. 7 The five-dimensional model can be described by a four-dimensional effective theory. In the four-dimensional effective description the heavier resonances of the stronglyinteracting sector - the Kaluza-Klein states - are integrated out. As long as the global $S O(5)$ symmetry is exact the Higgs field is an exact Goldstone boson and hence its potential vanishes in all orders of perturbation theory. In order to obtain a Higgs potential at loop order there must be $S O(5)$ violating terms of SM fields. The potential can be generated by SM fermions or gauge bosons. Since gauge bosons do not tend to misalign the vacuum (they can only align the vacuum in the $S U(2) \times U(1)$ preserving direction), electroweak symmetry breaking has to be triggered by fermions in particular by the top quark, whose contribution to the Higgs mass is dominant due to its large mass. But the gauge bosons nevertheless give a contribution to the Higgs potential. The terms, which violate the the $S O(5)$ symmetry and therefore lead to the generation of the Higgs potential, are the linear couplings of the fermions to fermionic composite operators $\mathcal{O}$ of the strongly-interacting sector with the same $S U(3)_{C} \times S U(2)_{L} \times U(1)_{Y}$ quantum numbers of the SM fermion to which it couples [15]
\[

$$
\begin{equation*}
\lambda_{L} \bar{f}_{L} \mathcal{O}_{R}+\lambda_{R} \bar{f}_{R} \mathcal{O}_{L} \tag{2.53}
\end{equation*}
$$

\]

where $\lambda_{L, R}$ are matrices in flavour space. The four Goldstone bosons that arise through the symmetry breaking $\mathrm{SO}(5) \rightarrow \mathrm{SO}(4)^{8}$ can be parameterized by

$$
\begin{equation*}
\Sigma=\langle\Sigma\rangle e^{\frac{\pi}{f}}, \quad\langle\Sigma\rangle=(0,0,0,0,1), \quad \Pi=-i T^{\hat{a}} h^{\hat{a}} \sqrt{2} \tag{2.54}
\end{equation*}
$$

where $T^{\hat{a}}$ denotes the broken generators and $\hat{a}=1,2,3,4$. The scalar field is taken to be dimensionless, so that

$$
\begin{equation*}
\Sigma^{2}=1 \tag{2.55}
\end{equation*}
$$

The couplings between the Higgs boson and the gauge fields can be obtained from the kinetic term

$$
\begin{equation*}
\mathcal{L}_{k i n}=\frac{f^{2}}{2}\left(D_{\mu} \Sigma\right)\left(D^{\mu} \Sigma\right) \tag{2.56}
\end{equation*}
$$

In the unitary gauge where $\Sigma=(\sin h / f, 0,0,0, \cos h / f)\left(\right.$ with $\left.h=\sqrt{\left(h^{\hat{a}}\right)^{2}}\right)$ this becomes

$$
\begin{equation*}
\mathcal{L}_{k i n}=\frac{1}{2} \partial_{\mu} h \partial^{\mu} h+M_{W}^{2}(h)\left(W_{\mu} W^{\mu}+\frac{1}{2 \cos \theta_{W}} Z_{\mu} Z^{\mu}\right) \quad \text { with } \quad M_{W}(h)=\frac{g f}{2} \sin \frac{h}{f} . \tag{2.57}
\end{equation*}
$$

To calculate the Higgs couplings to the gauge fields, $M_{W}$ must be expanded in $h=$ $\langle h\rangle+H$. The couplings are then found to be

$$
\begin{equation*}
g_{H V V}=g_{H V V}^{S M} \sqrt{1-\xi}, \quad g_{H H V V}=g_{H H V V}^{S M}(1-2 \xi) \tag{2.58}
\end{equation*}
$$

with $\xi$ defined as

$$
\begin{equation*}
\xi=\frac{v^{2}}{f^{2}}=\sin ^{2} \frac{\langle h\rangle}{f} \tag{2.59}
\end{equation*}
$$

[^5]The couplings of the fermions to the Higgs boson depend on the representation of the fermions. In $\mathrm{MCHM}_{4}$ (the model of Ref. [16]), the fermions are in the spinorial representation of $S O(5)$. In $\mathrm{MCHM}_{5}$ (the model of Ref. [17]), the fermions are in the fundamental representation. In Ref. [17] another model, $\mathrm{MCHM}_{10}$, is discussed, where the fermions are in the antisymmetric representation, but since the couplings are modified in the same way as in $\mathrm{MCHM}_{5}$ this model will not be discussed separately here.
$\mathrm{MCHM}_{4}$
The fermion masses and couplings to the Higgs boson are given by

$$
\begin{equation*}
\mathcal{L}_{y u k}=-m_{f}(h) \bar{f} f \quad \text { with } \quad m_{f}=y_{f} f \sin \frac{h}{f} \tag{2.60}
\end{equation*}
$$

The parameter $y_{f}$ depends on the couplings $\lambda_{R}$ and $\lambda_{L}$ of Eq.(2.53). For the light fermions these couplings need to be very small to explain their light masses. In order to have a large top mass, these couplings must be much stronger for the third generation, so the right-handed top is mostly composite 9 The masses of the fermions and the couplings to the Higgs boson can be calculated by performing an expansion in $h$ around the VEV. This results in

$$
\begin{equation*}
g_{H f f}=g_{H f f}^{S M} \sqrt{1-\xi} \tag{2.61}
\end{equation*}
$$

The Higgs potential is given by

$$
\begin{equation*}
V(h)=\alpha \cos \frac{h}{f}-\beta \sin ^{2} \frac{h}{f} \tag{2.62}
\end{equation*}
$$

where $\alpha$ and $\beta$ are integrals of self-energies of SM-fields - in particular of the top quark and the $\mathrm{SU}(2)_{L}$ gauge bosons. These self-energies encode the effects of strong dynamics. They can be computed in terms of the 5 d propagators and hence cannot be calculated perturbatively in 4 d . The potential has its minimum at $\cos h / f=-\alpha /(2 \beta)$ and hence

$$
\begin{equation*}
\sqrt{\xi}=\sin \frac{\langle h\rangle}{f}=\sqrt{1-\left(\frac{\alpha}{2 \beta}\right)^{2}} \tag{2.63}
\end{equation*}
$$

The effects of gauge fields only enter in $\beta$. Consequently, the gauge fields only contribute to the $\sin ^{2}$ operator with an overall positive coefficient $\beta_{\text {gauge }}<0$ and hence the gauge fields tend to align the vacuum in the $\mathrm{SU}(2)_{L}$ preserving direction. A misalignment of the vacuum can only come from top loops and only if $\alpha$ is comparable in size to $\beta$. To calculate the Higgs self-couplings the potential has to be expanded around the VEV [43]

$$
\begin{equation*}
V(H)=V(\langle h\rangle)+\frac{1}{2} M_{H}^{2} H^{2}+g_{H H H}^{S M} \sqrt{1-\xi} H^{3}+g_{H^{4}}^{S M}\left(1-\frac{7}{3} \xi\right) H^{4}, \tag{2.64}
\end{equation*}
$$

with the Higgs mass given by

$$
\begin{equation*}
M_{H}^{2}=\frac{4 \beta^{2}-\alpha^{2}}{2 \beta f^{2}} . \tag{2.65}
\end{equation*}
$$

[^6]The SM coupling $g_{H H H}^{S M}$ can be found in Eq. (2.50) and

$$
\begin{equation*}
g_{H^{4}}^{S M}=\frac{M_{H}^{2}}{8 v^{2}} . \tag{2.66}
\end{equation*}
$$

If the Higgs couplings to the fermions and gauge bosons are expanded to first order, the coefficients in the couplings of the model-independent description can be determined. It is found that $c_{H}=1, c_{y}=0$ and $c_{6}=1.10$ With the full couplings, however, it is also possible to look at high values for $\xi$.
$\mathrm{MCHM}_{5}$
In $\mathrm{MCHM}_{5}$, where the fermions are in the fundamental representation of $S O(5)$, the Yukawa coupling of the fermions to the Higgs boson becomes

$$
\begin{equation*}
\mathcal{L}_{y u k}=-m_{f}(h) \bar{f} f \quad \text { with } \quad m_{f}=y_{f} f \sin \frac{h}{f} \cos \frac{h}{f} . \tag{2.67}
\end{equation*}
$$

Again the masses of the fermions and the couplings to the Higgs boson can be calculated by performing an expansion in $h$ around the VEV. This results in

$$
\begin{equation*}
g_{H f f}=g_{H f f}^{S M} \frac{1-2 \xi}{\sqrt{1-\xi}} \tag{2.68}
\end{equation*}
$$

The Higgs potential also changes compared to $\mathrm{MCHM}_{4}$. We have

$$
\begin{equation*}
V(h)=\alpha \sin ^{2} \frac{h}{f}-\beta \sin ^{2} \frac{h}{f} \cos ^{2} \frac{h}{f} . \tag{2.69}
\end{equation*}
$$

Here again $\alpha$ and $\beta$ are integral functions of form factors encoding the strong dynamics, but here gauge fields only contribute to $\alpha$. For $\beta>|\alpha|$ the electroweak symmetry is broken and the minimum of the potential is given by

$$
\begin{equation*}
\sqrt{\xi}=\sin \frac{\langle h\rangle}{f}=\sqrt{\frac{\beta-\alpha}{2 \beta}} \tag{2.70}
\end{equation*}
$$

If $\beta<|\alpha|$, the minimum is at $\cos \frac{\langle h\rangle}{f}=0$, which corresponds to $\xi=1$. But then the masses of the fermions of Eq. (2.67) vanish. So $0 \leq \xi<1$ must be fulfilled. Expansion of $h$ in $H$ around the VEV leads to

$$
\begin{equation*}
V(H)=V(\langle h\rangle)+\frac{1}{2} M_{H}^{2} H^{2}+g_{H^{3}}^{S M}\left(\frac{1-2 \xi}{\sqrt{1-\xi}}\right) H^{3}+g_{H^{4}}^{S M}\left(\frac{1-\frac{28}{3} \xi(1-\xi)}{1-\xi}\right) H^{4} \tag{2.71}
\end{equation*}
$$

with

$$
\begin{equation*}
M_{H}^{2}=\frac{2\left(\beta^{2}-\alpha^{2}\right)}{\beta f^{2}} . \tag{2.72}
\end{equation*}
$$

This corresponds to $c_{y}=1$ in (2.49) and $c_{6}=0$ in (2.50). The coefficient $c_{H}$ is of course the same as in $\mathrm{MCHM}_{4}$, because the coupling of the Higgs boson to the gauge bosons

[^7]does not depend on the representation of the fermions. The parameter space for the Higgs mass and the compositeness parameter $\xi$ is constrained. In direct Higgs searches at LEP and Tevatron a Higgs mass of $\sim 80-115 \mathrm{GeV}$ and of $\sim 162-167 \mathrm{GeV}$ is excluded for small values of $\xi$. However, for $\xi \gtrsim 0.7$ smaller values of $M_{H}$ are also possible. In $\mathrm{MCHM}_{5}$ an additional region with $110 \mathrm{GeV} \lesssim M_{H} \lesssim 200 \mathrm{GeV}$ is excluded by the Tevatron for large $\xi$. Electroweak precision data give further constraints. There are three contributions to oblique parameters [37]. The one concerning the $T$ parameter will vanish if custodial symmetry is preserved as assumed in the last subsection. The contribution to the $S$ parameter gives a lower bound on the masses of the heavier resonances $m_{\rho} \geq 2.5 \mathrm{TeV}$, and hence imposes a lower bound for the cut-off of the effective theory. A third contribution arises because there are no longer complete cancellations between the Higgs and the gauge boson contributions to the $S$ and $T$ parameters as in the SM. They both become logarithmically divergent [44]. This gives rise to an upper bound, $\xi \lesssim 0.3$, which can be relaxed by a factor of approximately 2 if a partial cancellation of about $50 \%$ with contributions from new states is allowed ${ }^{111}$

### 2.2.3. Partially composite top

In the last subsection, the fermions of new strongly interacting sector are only taken into account through form factors. Here we consider fermions with masses lighter than the cut-off of the effective theory, which enter the Lagrangian explicitly. As the top quark will be much more composit ${ }^{12}$ than the other quarks (because of its much larger mass), only direct couplings of the third generation to the composite sector will be taken into account. In Refs. [46 48] a model is described with new vectorlike fermions that transform in the fundamental representation of $S O(5)$. These new fermionic states can be described as an $S O(4) \cong S U(2)_{L} \times S U(2)_{R}$-bidoublet formed by Q and X and a $\mathrm{SO}(4)$-singlet $T .13$ Denoting the vector-like resonances of composite fermions transforming in the fundamental representation of $S O(5)$ by $\Psi$ we have

$$
\Psi=\left(\begin{array}{c}
Q \\
X \\
T
\end{array}\right), \quad Q=\binom{\tilde{T}}{\tilde{B}}, \quad X=\binom{X_{\frac{5}{3}}}{X_{\frac{2}{3}}}
$$

where $Q$ has the same quantum numbers as the $\operatorname{SM}$ doublet $q_{L}, T$ has the same quantum numbers as $t_{R}$ and $X$ is an exotic fermion with hypercharge $Y_{X}=\frac{7}{6}$. The Lagrangian reads

$$
\begin{align*}
\mathcal{L}= & i \bar{q}_{L} \not D q_{L}+i \bar{t}_{R} D t_{R}+i \bar{b}_{R} D D b_{R}+\bar{\Psi}\left(i \not D-m_{\Psi}\right) \Psi+ \\
& \frac{1}{2} f^{2}\left(D_{\mu} \Sigma\right)\left(D^{\mu} \Sigma\right)-y f(\bar{\Psi} \cdot \Sigma)\left(\Sigma^{\dagger} \cdot \Psi\right)  \tag{2.73}\\
& -\left(m_{L} \bar{q}_{L} Q_{R}+m_{R} \bar{T}_{L} t_{R}\right) .
\end{align*}
$$

[^8]The elementary SM fermions do not have a mass term, but obtain their mass through mixing with the new fermions. The fermions of the new sector have a mass term and a coupling to the Goldstone field $\Sigma$. Due to the linear couplings of the top to the new fermions a mixing occurs [50]. With (2.73) and $\Sigma=\left(\frac{1}{\sqrt{2}} \sin \frac{h}{f}, 0,0, \frac{1}{\sqrt{2}} \sin \frac{h}{f}, \cos \frac{h}{f}\right)$ the mass terms of the Lagrangian can be written down:

$$
-\mathcal{L}_{\text {mass }}=\overline{\left(\begin{array}{c}
t_{L}  \tag{2.74}\\
Q_{L}^{u} \\
X_{L}^{u} \\
T_{L}
\end{array}\right)}\left(\begin{array}{cccc}
0 & m_{L} & 0 & 0 \\
0 & m_{\Psi}+\frac{s_{\alpha}^{2}}{2} f y & \frac{s_{\alpha}^{\alpha}}{2} f y & c_{\alpha} \frac{v}{\sqrt{2}} y \\
0 & \frac{s_{\alpha}^{2}}{2} f y & m_{\Psi}+\frac{s_{\alpha}^{2}}{\alpha} f y & c_{\alpha} \frac{v}{\sqrt{2}} y \\
m_{R} & c_{\alpha} \frac{v}{\sqrt{2}} y & c_{\alpha} \frac{v}{\sqrt{2}} y & m_{\Psi}+c_{\alpha}^{2} f y
\end{array}\right)\left(\begin{array}{c}
t_{R} \\
Q_{R}^{u} \\
X_{R}^{u} \\
T_{R}
\end{array}\right)+\text { h.c. }
$$

where $s_{\alpha}$ denotes here $\sin \alpha=\frac{v}{f}$. A diagonalization can be performed by mixing elementary tops with fermions of the new sector. For $v=0$, this diagonalization can be done rather easily. Since $v=0$ means before electroweak symmetry breaking, the top still does not get a mass, but mixes with the quarks of the new sector

$$
\begin{align*}
q_{L} \rightarrow \cos \theta_{L} q_{L}-\sin \theta_{L} Q_{L} & Q_{L} \rightarrow \sin \theta_{L} q_{L}+\cos \theta_{L} Q_{L}
\end{align*} \quad \text { with } \tan \theta_{L}=\frac{m_{L}}{m_{\Psi}}
$$

The mass $\tilde{m}_{\Psi}$ is defined as $m_{\Psi}+f y$. For large $\theta_{L}\left(\theta_{R}\right)$ the left-handed (right-handed) top is thus mostly composite. For $v=0$ the masses of the other quarks are

$$
\begin{equation*}
m_{Q}=\frac{m_{\Psi}}{\cos \theta_{L}}, \quad m_{X}=m_{\Psi}, \quad m_{T}=\frac{\tilde{m}_{\Psi}}{\cos \theta_{R}} \tag{2.76}
\end{equation*}
$$

Constraints on the allowed parameter space come from the $S$ and $T$ parameters and the process $Z \rightarrow \bar{b}_{L} b_{L}$. In Refs. [46, 47], it was found that the allowed parameter space is rather small. Nevertheless, for large $f$ nearly every parameter constellation is possible. But for $f<500 \mathrm{GeV}$ only $\sin \left(\theta_{L}\right)=0.1$ and $\mathrm{m}_{\Psi} \sim-y f$ or $\sin \theta_{L} \sim 1$, which means a mostly composite left-handed top, are allowed. In these parameter regions $f$ can be as low as $\sim 400 \mathrm{GeV}$. A very composite left-handed top, however, is disfavoured by flavour physics [47]. The allowed parameter space is much larger if one assumes a second multiplet of new fermions below the cut-off [46].

## CHAPTER 3

## Higgs pair production at the LHC

To reconstruct the Higgs potential the Higgs self-couplings have to be measured. This can be done using multi-Higgs production processes - especially Higgs pair production for the trilinear coupling. The most important processes at the LHC for Higgs pair production are gluon fusion [51], vector boson fusion [52-55] and double Higgs-strahlung off W and Z bosons [56]. These three processes will be investigated in $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ in the following chapter, cf. also Ref. [43].

### 3.1. Gluon fusion

The $g g \rightarrow H H$ process proceeds at leading order through heavy quark loops - mainly top and bottom loops. The amplitudes for the gluon fusion process are in principle the same as in the SM with modified Higgs couplings. But there is an additional Feynman diagram arising due to a new coupling of two Higgs bosons to two fermions ${ }^{1}$ This coupling can be derived from Eq. (2.60) and from Eq. (2.67) by expanding the fermionic mass to $O\left(H^{2}\right)$. This yields

$$
\begin{array}{ll}
\mathrm{MCHM}_{4}: & g_{H H f f}=\frac{m_{f}}{2 v^{2}} 2 \xi  \tag{3.1}\\
\mathrm{MCHM}_{5}: & g_{H H f f}=\frac{m_{f}}{2 v^{2}} 8 \xi .
\end{array}
$$

As this coupling grows linearly with $\xi$ it can play an important role for large values of $\xi$. In the SM limit it vanishes as expected. The sign of the new coupling is opposite to the coupling $g_{H f f}$. The same result for the couplings can be obtained if the Lagrangian (2.47) of the model-independent description and the coefficients $c_{y}$ and $c_{H}$ as determined for $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ are used.

[^9]

Figure 3.1.: Generic Feynman diagrams of the process $g g \rightarrow H H$

The Feynman diagrams for the process can be found in Fig. 3.1. In principle there are 2 possible permutations of external lines for the triangle diagrams and 6 for the box diagrams. But as the permutations not shown in Fig. 3.1]simply correspond to reversed directions of the arrows in the fermion lines they can be taken into account by using a factor of 2. First the traces of the matrix elements were calculated with FeynCalc [57]. Two linearly independent tensor structures arise, which can be found in Appendix A in Eqs. (A.1) and (A.2). Due to the relations given in (A.3), the expressions (A.1) and (A.2) can be used to project out the scalar form factors corresponding to these two tensor structures. Afterwards, the loop integrals were expressed by the usual tensor integrals, which were then reduced to scalar integrals with Passarino-Veltman reduction [58].
The matrix element with generic couplings then yields

$$
\mathcal{M}=\mathcal{M}_{\triangle}+\mathcal{M}_{\triangle, \text { new }}+\mathcal{M}_{\square}
$$

with

$$
\begin{align*}
& \mathcal{M}_{\triangle}=g_{s}^{2} g_{H f f} g_{H H H} \frac{1}{s-M_{H}^{2}} F_{\triangle} A_{1}^{\mu \nu} \epsilon_{\mu}\left(p_{1}\right) \epsilon_{\nu}\left(p_{2}\right) \delta_{a b}  \tag{3.2}\\
& \mathcal{M}_{\triangle, \text { new }}=g_{s}^{2} g_{H H f f} F_{\triangle} A_{1}^{\mu \nu} \epsilon_{\mu}\left(p_{1}\right) \epsilon_{\nu}\left(p_{2}\right) \delta_{a b} \\
& \mathcal{M}_{\square} \quad=g_{s}^{2} g_{H f f}^{2}\left(F_{\square} A_{1}^{\mu \nu}+G_{\square} A_{2}^{\mu \nu}\right) \epsilon_{\mu}\left(p_{1}\right) \epsilon_{\nu}\left(p_{2}\right) \delta_{a b} .
\end{align*}
$$

The color indices are denoted by $a, b=1, \ldots, 8, \epsilon_{\mu}$ and $\epsilon_{\nu}$ are the polarization vectors of the gluons. The triangle form factor $F_{\triangle}$ and the box form factors $F_{\square}$ and $G_{\square}$ can be found in Appendix A. The strong coupling constant is $\alpha_{s}=\frac{g_{s}^{2}}{4 \pi}$. The other couplings are model-dependent and can be found in Eq. (3.1) and the previous chapter. The results were checked against the results of Ref. [59] for the SM limit $\xi \rightarrow 0$. In (3.2) the factor of 2 due to the the permutation of the diagrams is cancelled by a factor of one half, which arises through the colour structure of the diagrams,

$$
\begin{equation*}
\operatorname{Tr}\left(T^{a} T^{b}\right)=\frac{1}{2} \delta_{a b}, \tag{3.3}
\end{equation*}
$$

where $T^{a}$ are the $S U(3)$-generators.
To derive the cross section it is more convenient to express the remaining angular integration of the phase space integral through an integration over the Mandelstam variable $t$

$$
\begin{equation*}
t=\frac{1}{2}\left(-s-s \sqrt{1-\frac{4 M_{H}^{2}}{s}} \cos \theta\right)+M_{H}^{2} \tag{3.4}
\end{equation*}
$$

The integration limits need to be changed accordingly. Additionally, as $u$ also depends on $\cos \theta$ it must be replaced through the relation

$$
\begin{equation*}
s+t+u=2 M_{H}^{2} \tag{3.5}
\end{equation*}
$$

The cross section is given by

$$
\begin{equation*}
\hat{\sigma}(g g \rightarrow H H)=\int_{t_{1}}^{t_{2}} \mathrm{dt} \frac{1}{256 \pi} \frac{1}{2 s^{2}}\left|\sum_{t, b} \mathcal{M}\right|^{2} \tag{3.6}
\end{equation*}
$$

where the sum runs over the top and bottom quarks. For the full process $p p \rightarrow$ $H H+X$ the partonic cross section has to be convoluted with the parton density functions (PDFs) of the gluon taken at a typical scale Q

$$
\begin{align*}
\sigma(p p \rightarrow H H) & =\int_{4 M_{H}^{2} / s}^{1} \mathrm{~d} \tau \int_{\tau}^{1} \frac{\mathrm{dx}}{x} f^{g}\left(\frac{\tau}{x}, Q^{2}\right) f^{g}\left(x, Q^{2}\right) \hat{\sigma}(\hat{s}=\tau s)  \tag{3.7}\\
& =\int_{4 M_{H}^{2} / s}^{1} \mathrm{~d} \tau \frac{\mathrm{~d} \mathcal{L}^{g g}}{\mathrm{~d} \tau} \hat{\sigma}(\hat{s}=\tau s) .
\end{align*}
$$

The parton distribution functions $f^{g}$ give the probability density for a parton state as a function the momentum fraction carried by the parton in the proton at a scale Q . The parton in this case is a gluon, which is denoted by the index $g$ in $f^{g}$. So all momenta (and therefore the Mandelstam variables) in the amplitude have to be replaced by the corresponding partonic variables, i.e. $s \rightarrow \hat{s}=\tau s$ and $t, u \rightarrow \hat{t}, \hat{u}$. To produce on-shell Higgs bosons the gluons must have a momentum of at least $2 M_{H}$, i.e. $\hat{s} \geq 4 M_{H}^{2}$. The integration is done numerically. Consequently, everything was implemented in a FORTRAN program, in which the numerical integration was done by the Monte Carlo integration routine VEGAS2. Monte Carlo integration is an integration with random numbers in an interval $[0: 1]^{n}$ (for an $n$ dimensional integration). In principle in Monte Carlo integration the value of the integrand function is taken for a large amount of random numbers given by a random number generator. The integral is given by the average of the integrands with these random numbers. This method is much faster than other methods for multidimensional integrals.
The VEGAS routine is an advanced version of Monte Carlo integration. To get an exacter value for the integration it does an importance sampling. This means that the random numbers are not uniformly distributed but more random numbers are taken in the region where the integrand gives the largest contribution. In order to achieve that a probability distribution is modelled, which maps the integrand as exactly as

[^10]possible. This is done by the VEGAS routine by subdividing the integration space in a rectangular grid and performing the calculations in each subspace. The grid is adjusted in every iterative step, according to where the integral receives dominant contributions 61].
VEGAS can only evaluate integrals in the intervals $[0: 1]^{n}$. So an integral with different limits can only be solved if a substitution is done that ensures the limits to be between 0 and 1. It is very useful to do the substitution in such a way that it resembles the integrand, because then it is easier for the VEGAS-routine to find the most important integration region. If this is done in the right way, the integration becomes exacter and converges much faster. In my program this is done by the substitution
\[

$$
\begin{align*}
\tau & =\left(\frac{4 M_{H}^{2}}{s}\right)^{x_{1}}  \tag{3.8}\\
x & =\left(\frac{4 M_{H}^{2}}{s}\right)^{x_{1} x_{2}} \tag{3.9}
\end{align*}
$$
\]

where $x_{1}$ and $x_{2}$ are the random numbers given by VEGAS.
For the parton distribution functions CTEQ6L1 [62] was used. These PDFs are valid up to leading order. For the parton distribution functions and $\alpha_{s}$ the scale was set to

$$
Q=\sqrt{\hat{s}} .
$$

For the strong coupling constant $\alpha_{s}$ and the scalar integrals FORTRAN routines from the program HPAIR were used $\sqrt[3]{ }$ For the quarks in the loop only the top and the bottom quark were taken into account. The contribution of the top loop is by far the dominant one due to the large coupling of the Higgs boson to the tops.
The QCD corrections to this process are rather large. Since the Composite Higgs model does not change the QCD corrections as only the Higgs couplings are modified they can be taken over directly from the SM calculation. As the QCD corrections show little variation with $M_{H}$ they can be taken into account by simply multiplying the leading-order cross section with a $K$ factor of $\sim 2$ [63].

## 3.2. $W$ and $Z$ fusion

The Feynman diagrams for this process can be found in Fig. 3.2. The $W$ and $Z$ fusion cross sections were calculated with Madgraph/Madevent [64] after implementing the Composite Higgs model in two new model files - one for $\mathrm{MCHM}_{4}$ and one for $\mathrm{MCHM}_{5}$ - and introducing the compositeness parameter $\xi$. In order to have the possibility of varying the triple Higgs coupling a parameter $\kappa$ was introduced. In Madgraph/Madevent new variables can be declared in the file VariableName.dat. In the program couplings.f, the Higgs couplings were modified with the newly introduced parameter $\xi$ according to the Composite Higgs models. The triple Higgs coupling was additionally

[^11]multiplied with $\kappa$, which parametrizes triple Higgs coupling variations in terms of the composite Higgs self-coupling.
As a typical scale for the process
$$
Q=M_{V}
$$
was chosen [65]. QCD corrections are much smaller for this process than in gluon fusion, so that the scale dependence of this process should be smaller. As a PDF set CTEQ6L1 was chosen as in the gluon fusion process.
As emphasized in Ref. [66], the cross section can increase with rising energy in the highenergy limit if the couplings of the Higgs boson to the vector bosons are modified with respect to the SM. As the structure of the propagator for the longitudinally polarized vector bosons is responsible for the increase with the energy, only this contribution is calculated $\sqrt[4]{4}$ The longitudinal polarization vectors are
\[

\epsilon_{L}(k)=\frac{1}{M_{W}^{2}}\left($$
\begin{array}{c}
|\vec{k}|  \tag{3.10}\\
0 \\
0 \\
k_{0}
\end{array}
$$\right) \quad with \quad k=\left($$
\begin{array}{c}
k_{0} \\
0 \\
0 \\
|\vec{k}|
\end{array}
$$\right)
\]

In Fig. 3.2 in the second diagram an interchange of the two external $W$ boson lines must be taken into account. The amplitude is calculated in the c.m. frame of the $W$ bosons. This yield\{ $5^{5}$

$$
\begin{align*}
\mathcal{M}_{L L}= & G_{F} \frac{s}{\sqrt{2}}\left\{\left(1+\beta_{W}^{2}\right)\left[(1-2 \xi)+\frac{A \cdot \lambda_{H H H}^{S M}}{\left(s-M_{H}^{2}\right) / M_{Z}^{2}}\right]\right. \\
& \left.+\frac{1-\xi}{\beta_{W} \beta_{H}}\left[\frac{\left(1-\beta_{W}^{4}\right)+\left(\beta_{W}-\beta_{H} \cos \theta\right)^{2}}{\cos \theta-x_{W}}-\frac{\left(1-\beta_{W}^{4}\right)+\left(\beta_{W}+\beta_{H} \cos \theta\right)^{2}}{\cos \theta+x_{W}}\right]\right\} \\
& \text { with } \quad \beta_{W / H}=\sqrt{1-\frac{4 M_{W / H}^{2}}{s}} \quad \text { and } \quad x_{W}=\frac{1-\frac{2 M_{H}^{2}}{s}}{\beta_{W} \beta_{H}} . \tag{3.11}
\end{align*}
$$

The Higgs production angle is denoted by $\theta$. The contribution of the diagram with the triple Higgs coupling $\lambda_{H H H}^{S M}=\frac{3 M_{H}^{2}}{M_{Z}^{2}}$ is modified by a factor of $A=1-\xi$ in $\mathrm{MCHM}_{4}$ and


Figure 3.2.: Generic Feynman diagrams for the process $q q \rightarrow q q H H$.

[^12]by a factor of $A=1-2 \xi$ in $\mathrm{MCHM}_{5}$. The high energy behaviour can easily be found. With $\beta_{W, H} \xrightarrow{s \rightarrow \infty} 1$ and $x_{W} \xrightarrow{s \rightarrow \infty} 1$ the amplitude becomes
\[

$$
\begin{equation*}
\mathcal{M} \xrightarrow{s \rightarrow \infty}-\sqrt{2} G_{F} s \xi . \tag{3.12}
\end{equation*}
$$

\]

The amplitude therefore increases linearly with the energy. This is the same high energy behaviour as for longitudinal gauge boson scattering, which can be found in Ref. [35, 66].

### 3.3. Double Higgs-strahlung off $W$ and $\boldsymbol{Z}$ bosons

The Feynman diagrams for this process can be found in Fig.3.3. As for the $W$ and $Z$ fusion, this process was calculated with Madgraph/Madevent. This time as a typical scale

$$
Q=\sqrt{\left(M_{V}+2 M_{H}\right)^{2}}, \quad V=Z, W^{ \pm}
$$

was chosen.
An analytical formula for the differential cross section of $e^{+} e^{-} \rightarrow$ ZHH can be found in Appendix B. The interference structure of Higgs-strahlung at the LHC is the same as for Higgs-strahlung at an $e^{+} e^{-}$-collider. The cross section for the process $q \bar{q} \rightarrow$ ZHH/WHH can be evaluated, mutatis mutandis, at the quark level for the LHC in the same way as for $e^{+} e^{-}$-collisions, just the couplings have to be adjusted properly [68]. The interference structure hence does not change.




Figure 3.3.: Generic Feynman diagrams for the process $q \bar{q} \rightarrow Z H H / W H H$.

### 3.4. Results

The Composite Higgs pair production cross sections as functions of the Higgs mass for three representative values of $\xi$ can be found in Fig. 3.4 for $\mathrm{MCHM}_{4}$ and in Fig. 3.5 for $\mathrm{MCHM}_{5}$. For comparison the SM cross sections are also shown. They correspond to $\xi=0$. For all plots in this chapter a c.m. energy of $\sqrt{s}=14 \mathrm{TeV}$ was assumed. The top mass was set to 173.3 GeV . The gluon fusion process is by far the dominant process, followed by the $W / Z$ fusion. The behaviour of the cross sections with varying $\xi$ and the sensitivity to $\lambda_{\text {Hнн }}$ can be understood by examining the interference structure of the contributing diagrams.


Figure 3.4.: Higgs pair production cross sections as a function of the Higgs boson mass in the $\mathrm{SM}\left(\xi=0\right.$, upper left) and $\mathrm{MCHM}_{4}$ with $\xi=0.2$ (upper right), 0.5 (bottom left) and 0.8 (bottom right). Arrows indicate the change in the cross section for a variation of $\lambda_{H H H}$ from 0.5 to 1.5 times its value in the corresponding model. Some arrows are rescaled as indicated by appropriate factors to make them visible.

## gg Fusion

Due to the new diagram involving the $H H q \bar{q}$ coupling the cross section increases with rising $\xi$. The reason is that this diagram is not suppressed by a Higgs propagator as is the other triangle diagram involving the triple Higgs coupling. For $\mathrm{MCHM}_{5}$ this leads to an increase compared to the SM cross section of up to a factor of 30 . In $\mathrm{MCHM}_{5}$ the increase in the cross section is stronger than in the $\mathrm{MCHM}_{4}$. On the one hand this is due to the larger $H H q \bar{q}$ coupling, on the other hand the other diagrams would also be larger than the corresponding diagrams in $\mathrm{MCHM}_{4}$ for $\xi \gtrsim 2 / 3$. In $\mathrm{MCHM}_{4}$ the increase is only due to the additional new diagram, because all other diagrams are only modified by a factor of $(1-\xi)$ compared to the SM.
In $\mathrm{MCHM}_{5}$ the diagrams with the triple Higgs coupling and the box diagrams vanish completely for $\xi=0.5$, because the triple Higgs coupling and the coupling of one Higgs to two fermions is zero. Accordingly, the sensitivity to the triple Higgs coupling is also zero.
The arrows in the plots indicate a variation of $\lambda_{H H H}$ from 0.5 pointing to 1.5 times its value in the corresponding model. The curve itself has of course the coupling as expected in the model. In some of the plots, the arrows are rescaled as indicated in


Figure 3.5.: As in Fig. 3.4 but for $\mathrm{MCHM}_{5}$
order to make them visible.
The direction of the arrows in the gluon fusion curves can be explained by destructive interferences. The triangle diagram has an additional minus sign compared to the other diagrams and therefore interferes destructively. This is the case for all values of $\xi$ even in $\mathrm{MCHM}_{5}$, where the triple Higgs coupling changes its sign for $\xi>0.5$. But since $g_{H H H}$ and $g_{H f \bar{f}}$ have the same modification factor, the resulting modification factor in the triangle involving the triple Higgs coupling is quadratic and therefore cannot change the relative sign.
In $\mathrm{MCHM}_{4}$ the sensitivity to a variation of the triple Higgs coupling decreases with growing $\xi$. This is due to the rising dominance of the $H H q \bar{q}$-diagram and to the decreasing triple Higgs coupling. In $\mathrm{MCHM}_{5}$ the sensitivity also decreases for increasing $\xi$. But compared to $\mathrm{MCHM}_{4}$ the sensitivity for $\xi=0.8$ is a bit larger because of the larger triple Higgs coupling, cf. Eqs. (2.64) and (2.71).
$W / Z$ fusion
The cross section for $W / Z$ fusion increases with rising $\xi$ up to a factor of 5 compared to the SM. The reason for this is that for $\xi<0.5$ the cross section increases due to the destructive interference between the diagram with the $V V H H$ coupling and the $u$ - and $t$-channel diagrams, which get smaller with growing $\xi$. For $\xi=0.5$ the $V V H H$ coupling is zero. For $\xi>0.5$ the $V V H H$ coupling changes its sign and the interference becomes constructive.
The diagram involving the triple Higgs coupling interferes destructively with the $u$ - and
$t$-channel diagrams in the SM case and $\mathrm{MCHM}_{4}$. In $\mathrm{MCHM}_{5}$ this situation changes for $\xi>0.5$ because the triple Higgs coupling then changes sign and so the relative sign between the triple Higgs coupling diagram and the $u$ - and $t$ - channel diagram becomes constructive. This can be seen in Fig. 3.5, as the direction of the arrow changes for $\xi=0.8$. In $\mathrm{MCHM}_{4}$ the direction of the arrow does not change as anticipated.
The sensitivity to a variation of the triple Higgs coupling becomes smaller with growing $\xi$ due to the increasing dominance of the strong sector: In the scattering amplitude there is no complete cancellation of the terms only involving gauge couplings $\sim s$ anymore due to the modified $g_{V V H}$ and $g_{V V H H}$ couplings. This behaviour can also be derived from formula (3.11). Hence in composite Higgs models double Higgs production in $W_{L} W_{L}$ fusion becomes strong [35, 66].
The $W$ fusion dominates the $Z$ fusion by a factor of $\sim 2.2-2.8$ for $\xi=0-0.8$. The ratio depends on $\xi$ as due to the modification of the couplings in minimal Composite Higgs models with respect to the SM the diagrams among each other play a more or less important role. For example the $u$ - and $t$-channel diagrams, which have an extra vector boson propagator, can become more important. This of course can lead to a different ratio between $W$ and $Z$ fusion.

WHH/ZHH
The cross sections here are much smaller than for the gluon fusion and the $W$ and $Z$ fusion. For the minimal Composite Higgs models they even become smaller than for the SM.
In the SM all diagrams interfere constructively. Consequently, the arrow which indicates the variation of the triple Higgs coupling, points upwards. For non-zero $\xi$ and $\xi<0.5$ all couplings become smaller and consequently the cross section is reduced as well. For $\xi>0.5$ the $H H V V$ coupling changes its sign and the corresponding diagram interferes destructively. This leads to a change of direction of the arrow in $\mathrm{MCHM}_{4}$. In $\mathrm{MCHM}_{5}$ the triple Higgs coupling also changes its sign, so that the interference between these two diagrams is still constructive. The direction of the arrow does not change. The cross section increases for $\xi>0.5$ for both models. This is because the contribution of the HHVV diagram is increasing again. In $\mathrm{MCHM}_{5}$ the contribution from the diagram with the triple Higgs coupling also increases. The cross section for $\xi=0.8$ is therefore larger than in $\mathrm{MCHM}_{4}$, whereas for $\xi=0.2$ and $\xi=0.5$ the cross section in $\mathrm{MCHM}_{5}$ is smaller than in $\mathrm{MCHM}_{4}$ due to the smaller triple Higgs coupling of $\mathrm{MCHM}_{5}$ for these values of $\xi$.
The ratio of the $W H H$ process to the $Z H H$ process is $\sim 1.6-2.2$ for $\xi=0-0.8$.

### 3.4.1. Sensitivities

In this part two questions will be discussed:

- Can the Composite Higgs model be distinguished from the SM in Higgs pair production processes?
- What are the chances of extracting the triple Higgs coupling in MCHM?

To find the answer to these questions sensitivity regions are constructed for various final states. These final states were obtained by multiplying the cross sections with the corresponding branching ratios. This is a good approximation if the Higgs width is very small, which is the case in the investigated mass range and for the investigated models. The branching ratios were obtained by means of the program HDECAY [69, 70]. Since in $\mathrm{MCHM}_{4}$ all modified couplings are changed by the same factor the branching ratios are the same as in the SM. The SM branching ratios as a function of the Higgs mass can be found in Fig. 3.6, As it can be inferred from the figure, the Higgs boson dominantly decays into $b \bar{b}$ until the gauge boson threshold is reached. Above $M_{H} \sim 140 \mathrm{GeV}$ the dominant decay channel is $H \rightarrow W^{+} W^{-}$followed by $H \rightarrow Z^{0} Z^{0}$.


Figure 3.6.: The branching ratios in the SM and $\mathrm{MCHM}_{4}$ as a function of $M_{H}$.

However, in $\mathrm{MCHM}_{5}$ the branching ratios change considerably. They can be found in Fig. 3.7 as functions of $\xi$ for two representative values of $M_{H}$. The branching ratios into fermions are now governed by the modification factor of the Higgs coupling to fermions. For $0 \leq \xi \leq 0.5$ they decrease with rising $\xi$. At $\xi=0.5$ they vanish completely. For $\xi>0.5$ they increase again with rising $\xi$. For $\xi$ near 0.5 due to the vanishing of the branching ratios into fermions, the branching ratio into $W$ bosons is the largest. Even the branching ratio into $\gamma \gamma$ can be as large as a few percent. Above the gauge boson threshold the decays are dominated by $W^{+} W^{-}$followed by $Z Z$ final states. However, near the technicolor limit $\xi \rightarrow 1$ the gauge boson couplings become very small and therefore the branching ratios into gauge bosons. The fermion modification factors, however, become very large. Therefore, the branching ratio into $b \bar{b}$ dominates. More details to the branching ratios in the minimal Composite Higgs models can be found in Refs. [43, 45].
For the production processes only the gluon fusion will be discussed as due to its much larger cross section it is the most promising of measuring Higgs pair production. For Higgs masses below $\sim 140 \mathrm{GeV}$ the branching ratio for the decay $H \rightarrow b \bar{b}$ is the largest. If the Higgs pair decays into four bottom quarks, however, the signal will be overwhelmed by the large QCD background [71]. The $b \bar{b} b \bar{b}$ final state will therefore not be investigated. For low Higgs masses the Higgs decay into $b \bar{b}$ has to be combined with a rare decay of the second Higgs boson in order to reduce the background. For the Higgs mass range $>140 \mathrm{GeV}$ the $W$ boson branching ratio is the largest. This decay


Figure 3.7.: The branching ratios in $\mathrm{MCHM}_{5}$ as a function of $\xi$ for $M_{H}=120 \mathrm{GeV}$ (left) and $M_{H}=180 \mathrm{GeV}$ (right).
seems to be promising in the SM for the extraction of $\lambda_{H H H}$ cf. Refs. [72, 73]. As possible final states $W^{+} W^{-} W^{+} W^{-}, b \bar{b} \tau^{+} \tau^{-}, b \bar{b} \mu^{+} \mu^{-}$and $b b \gamma \gamma \gamma$ were investigated.
The following analysis addresses the question in which parameter region it is in principle possible to measure a difference between the Composite Higgs model and the SM. Subsequently the parameter regions are identified where the trilinear Higgs selfcoupling can be measured in a specific minimal Composite Higgs model. We assume here that a Higgs boson has been found and realized as a Composite Higgs state. The analysis represents a first estimate, which has to be backed by sophisticated experimental simulations taking into account background processes and detector effects. The results of this analysis, however, show in which parameter region such an analysis is worth doing.

Can Composite Higgs pair production be distinguished from the SM case?
In order to answer this question sensitivity regions in the $M_{H}-\xi$ plane were constructed. These can be found in Figs. 3.8 and 3.9 for $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ respectively. The criterion used to identify the regions was

$$
\begin{equation*}
S^{S M}+\beta \sqrt{S^{S M}}<S^{M C H M} \quad \text { or } \quad S^{S M}-\beta \sqrt{S^{S M}}>S^{M C H M} \quad(\beta=1,2,3,5), \tag{3.13}
\end{equation*}
$$

where $S^{S M}$ denotes the number of signal events in the SM and $S^{M C H M}$ the corresponding number in the minimal Composite Higgs models. The square roots of the signal events quantify the statistical fluctuation. The number of signal events was obtained by multiplying the cross section of the process $p p \rightarrow g g \rightarrow H H+X$ with the branching ratios for the investigated final states and the integrated luminosity, which is here assumed to be $300 \mathrm{fb}^{-1}$. This integrated luminosity can be achieved after several years of running of the LHC at high luminosity [74, 75]. In $\mathrm{MCHM}_{4}$ the most promising decay channels for the deviation of $\mathrm{MCHM}_{4}$ from the SM are $W^{+} W^{-} W^{+} W^{-}$for $M_{H} \gtrsim$ 140 GeV and $b \bar{b} \tau^{+} \tau^{-}$for $M_{H} \lesssim 140 \mathrm{GeV}$. For higher values of $\xi$ in both channels an even larger mass region is possible $\left(M_{H} \lesssim 150 \mathrm{GeV}\right.$ for $b \bar{b} \tau^{+} \tau^{-}$and $M_{H} \gtrsim 120 \mathrm{GeV}$ for $\left.W^{+} W^{-} W^{+} W^{-}\right)$. The $\mathrm{MCHM}_{4}$ can be distinguished from the SM at $5 \sigma$ in the $b \bar{b} \tau^{+} \tau^{-}$ channel for $\xi>0.2$ and in the $W^{+} W^{-} W^{+} W^{-}$channel for $\xi \geq 0.1$.


Figure 3.8.: Areas in the $M_{H}-\xi$ plane for the deviation of the SM Higgs pair production via gluon fusion from the one in the $\mathrm{MCHM}_{4}$. From dark blue to light blue the regions correspond to $5,3,2,1 \sigma$ deviation. The final states are from left top to right bottom $b \bar{b} \mu^{+} \mu^{-}, b \bar{b} \gamma \gamma, b \bar{b} \tau^{+} \tau^{-}$and $W^{+} W^{-} W^{+} W^{-}$.

Since the branching ratio $H \rightarrow \mu^{+} \mu^{-}$is very small, there is no hope of distinguishing $\mathrm{MCHM}_{4}$ from the SM in the $b \bar{b} \mu^{+} \mu^{-}$channel. The branching ratio of $H \rightarrow \gamma \gamma$ is also relatively small, but a bit bigger than $H \rightarrow \mu^{+} \mu^{-} \sqrt[6]{ }$ However, the region where the $\mathrm{MCHM}_{4}$ is distinguishable from the SM is rather small at least for $5 \sigma$. The $3 \sigma$ region is a bit larger (starting from $\xi \gtrsim 0.6$ for $M_{H} \lesssim 140 \mathrm{GeV}$ ). Altogether, as long as $\xi$ is not too small $\mathrm{MCHM}_{4}$ can be distinguished from the SM with $5 \sigma$ over the whole of the considered mass range.
In $\mathrm{MCHM}_{5}$ (Fig 3.9) the situation is different. Due to the modified branching ratios and the much larger cross section with respect to the SM, the sensitivity regions differ strongly from $\mathrm{MCHM}_{4}$. The larger cross section in $\mathrm{MCHM}_{5}$ leads to larger sensitivity regions for the rare decay channels. Even the $\bar{b} \mu^{+} \mu^{-}$channel is now possible for $M_{H} \lesssim 100 \mathrm{GeV}$ and $0.3 \lesssim \xi \lesssim 0.5$, for very high $\xi$ over the whole considered mass range. This results from an interplay between the much higher cross section with

[^13]

Figure 3.9.: As in Fig. 3.8 but for $\mathrm{MCHM}_{5}$.
respect to the SM for high values of $\xi$ and the larger branching ratio of $b \bar{b}$ even for high masses for $\xi \gtrsim 0.7$. A very similar behaviour due to the same reasons can be found in the plots for $b \bar{b} \gamma \gamma$ and $b \bar{b} \tau^{+} \tau^{-}$. The sensitivity region for the $W^{+} W^{-} W^{+} W^{-}$ final state is extended in $\mathrm{MCHM}_{5}$ to the whole mass range for $0.3 \lesssim \xi \lesssim 0.6$. This is because the fermionic branching ratios vanish for $\xi=0.5$ and the $W^{+} W^{-}$branching ratio therefore becomes much larger, cf. Fig. 3.7.
For $\xi \rightarrow 0.5$ the fermionic branching ratios vanish. Therefore below and above $\xi=0.5$ there must be a region where the composite Higgs and the SM rates must be the same and therefore the sensitivity vanishes. The blobs in the $b \bar{b} \tau^{+} \tau^{-}$channel are due to this reason. With a finer grid they tend to form a connected line extending down to $M_{H}=80 \mathrm{GeV}$. The region between these lines (dots) is distinguishable from the SM because the number of events is much smaller (and zero for $\xi=0.5$ ) than the one expected from SM calculations. This can also be seen in the $b \bar{b} \mu^{+} \mu^{-}$and $b \bar{b} \gamma \gamma$ channel around $\xi=0.5$. The SM rates are in this decay channels very small so that it can not be distinguished from the composite Higgs rate with $5 \sigma$ anymore for $\xi \approx 0.5$. The blobs of low-sensitivity in the $W^{+} W^{-} W^{+} W^{-}$final state result from the falling $W^{+} W^{-}$ branching ratio with growing $\xi$.

Can the trilinear Higgs coupling be measured in $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ ?
We assume here that the Higgs boson has already been found, the couplings to gauge bosons and fermions are already known and EWSB is realized in the framework of a Composite Higgs model. 7 To investigate if the trilinear Higgs self-coupling can be extracted from Higgs pair production in gluon fusion sensitivity areas in the $M_{H}-\xi$ plane were constructed, where the cross section with vanishing Higgs self-coupling deviates by more than $5,3,2,1 \sigma$ at $\int \mathcal{L}=300 \mathrm{fb}^{-1}$ from the composite Higgs process with non-zero Higgs self-interaction strength. In Fig. 3.10 they can be found for $\mathrm{MCHM}_{4}$, in Fig. 3.11 for $\mathrm{MCHM}_{5}$. Only the final states $b \bar{b} \gamma \gamma, b \bar{b} \tau^{+} \tau^{-}$and $W^{+} W^{-} W^{+} W^{-}$are presented here, because for the $b \bar{b} \mu^{+} \mu^{-}$final state the prospects to measure the trilinear coupling are rather bad.


Figure 3.10.: Areas in the $M_{H}-\xi$ with sensitivity to non-vanishing $\lambda_{H H H}$ in $\mathrm{MCHM}_{4}$ for the gluon fusion process with subsequent decay. From dark blue to light blue the regions correspond to $5,3,2,1 \sigma$. The final states are from left to right $b \bar{b} \gamma \gamma, b \bar{b} \tau^{+} \tau^{-}$and $W^{+} W^{-} W^{+} W^{-}$. The assumed integrated luminosity is $\int \mathcal{L}=300 \mathrm{fb}^{-1}$.

In $\mathrm{MCHM}_{4}$, there is sensitivity in the $b \bar{b} \tau^{+} \tau^{-}$final state for $M_{H} \lesssim 150 \mathrm{GeV}$ and in the $W^{+} W^{-} W^{+} W^{-}$final state for $M_{H} \gtrsim 110 \mathrm{GeV}$. For the $b \bar{b} \tau^{+} \tau^{-}$final state the $5 \sigma$ region only includes the region $\xi \lesssim 0.7$. Since the diagram with the $H H q \bar{q}$ coupling, which is linear in $\xi$, gives the dominant contribution for high $\xi$, the sensitivity to a non-vanishing $\lambda_{\text {Hн }}$ becomes smaller. This can be anticipated from Fig. 3.4, where the arrows indicating a variation in $\lambda_{H H H}$ became smaller with rising $\xi$. In the $b \bar{b} \gamma \gamma$ final state there is no $5 \sigma$ region. Only in a small region between $100 \mathrm{GeV} \lesssim M_{H} \lesssim$ 140 GeV and $\xi \lesssim 0.2$ can a non-vanishing triple Higgs coupling be distinguished from the anticipated coupling with $3 \sigma$. However, this region is complementary to the one in Fig. 3.8 where the sensitivity to a deviation from the SM prediction was investigated. In the $b \bar{b} \tau^{+} \tau^{-}$and $W^{+} W^{-} W^{+} W^{-}$final states the areas have a large overlap with those from Fig. 3.8, In summary, one can say that there is sensitivity to a non-vanishing $\lambda_{H H H}$ over the whole mass range, but for large $\xi(\xi \gtrsim 0.7)$ only if $M_{H} \gtrsim 130 \mathrm{GeV}$. In $\mathrm{MCHM}_{5}$, the situation is quite different due to the changed branching ratios with respect to the SM. For the $b \bar{b} \gamma \gamma$ and $b \bar{b} \tau^{+} \tau^{-}$final states, there is sensitivity for large $\xi$ over the whole of the considered mass range due to the enhanced fermionic branching

[^14]

Figure 3.11.: As in Fig. 3.10 but for $\mathrm{MCHM}_{5}$
ratios for large $\xi$. For $\xi=0.5$ the fermionic branching ratios vanish due to the vanishing $g_{H f f}$ coupling as does $\lambda_{H H H}$. Accordingly, there is no sensitivity in the region around $\xi=0.5$ in every investigated decay channel over the whole mass range.
Still, for $M_{H} \lesssim 150 \mathrm{GeV}$ there is a sensitivity region in the $b \bar{b} \tau^{+} \tau^{-}$channel as long as $\xi \lesssim 0.4$ or $\xi \gtrsim 0.6$. The same holds for the $W^{+} W^{-} W^{+} W^{-}$final state in the mass range $M_{H} \gtrsim 110 \mathrm{GeV}$.
The question arises if the triple Higgs coupling might be measurable within these models with higher precision. To give an answer, sensitivity areas for a variation of $\lambda_{H H H}$ of $+30 \% 8$ were constructed. The trilinear Higgs coupling is hence varied as

$$
\begin{array}{ll}
\lambda_{H H H}^{\prime}=1.3 \sqrt{1-\xi} \lambda_{H H H}^{S M} & \text { for } \mathrm{MCHM}_{4} \\
\lambda_{H H H}^{\prime}=1.3 \frac{1-2 \xi}{\sqrt{1-\xi}} \lambda_{H H H}^{S M} &  \tag{3.14}\\
\text { for } \mathrm{MCHM}_{5}
\end{array}
$$

The plots can be found in Fig. 3.12 for $\mathrm{MCHM}_{4}$ and in Fig. 3.13 for $\mathrm{MCHM}_{5}$. To achieve considerable sensitivity regions here the integrated luminosity needs to be higher.




Figure 3.12.: Areas in the $M_{H}-\xi$ plane with sensitivity to a $30 \%$ variation of $\lambda_{H H H}$ in $\mathrm{MCHM}_{4}$ for the gluon fusion process with subsequent decay of the Higgs pair. The regions correspond from dark blue to light blue to $5,3,2,1$ $\sigma$. The final states are from left to right $b \bar{b} \gamma \gamma$ at $\int \mathcal{L}=3000 \mathrm{fb}^{-1}$ and $b \bar{b} \tau^{+} \tau^{-}, W^{+} W^{-} W^{+} W^{-}$at $\int \mathcal{L}=600 \mathrm{fb}^{-1}$.

[^15]It was chosen $\int \mathcal{L}=600 \mathrm{fb}^{-1}$ for the $b \bar{b} \tau^{+} \tau^{-}$and $W^{+} W^{-} W^{+} W^{-}$final states. For comparison the $b \bar{b} \gamma \gamma$ final states are also shown but with $\int \mathcal{L}=3000 \mathrm{fb}^{-1}$, which might be possible with a SLHC. The sensitivity areas shrink considerably. For the $b \bar{b} \gamma \gamma$ final


Figure 3.13.: As in Fig. 3.12 but for $\mathrm{MCHM}_{5}$.
state - even though the integrated luminosity was assumed to be very high - there can only be some hope in $\mathrm{MCHM}_{5}$ and for very large values of $\xi$. The sensitivity region in the decay $b \bar{b} \tau^{+} \tau^{-}$also becomes smaller. In $\mathrm{MCHM}_{4}$ only in a region with $M_{H} \lesssim 130$ GeV and $\xi$ up to $\sim 0.3$ the process is sensitive to a $30 \%$ deviation of $\lambda_{H H H}$ at $5 \sigma$. In $\mathrm{MCHM}_{5}$ there is only a small region for low values of $\xi$. The region for large $\xi$ also shrinks, but not as much as the region for low $\xi$. The $W^{+} W^{-} W^{+} W^{-}$final state for $M_{H} \gtrsim 120 \mathrm{GeV}$ is the most promising process. Here the sensitivity regions shrink, but not greatly.
Altogether, a vanishing $\lambda_{H H H}$ might be accesssible over nearly the whole possible parameter space, whereas the prospects of measuring $\lambda_{H H H}$ with $30 \%$ accuracy are much less promising. This has been found in the SM case before [72, 73, 76].

## Feasibility

As mentioned at the beginning of this section, this analysis reveals the regions in the parameter space, where the trilinear Higgs self-coupling might be extracted from Higgs pair production in gluon fusion. For a full analysis the background reactions and detector properties have to be taken into account. Although we have not performed such an analysis here a qualitative answer to the question can be given nevertheless. In Refs. [71, 76] a full analysis was done for the SM for the mass regime where the branching ratio into $b$ quarks is the largest. In Refs. [72, 73] this is done for large Higgs masses where the Higgs boson decays mostly into $W^{+} W^{-}$. It was found that good signal-background ratios for the SM can only be obtained in the decay channel $H H \rightarrow b \bar{b} \gamma \gamma$ for low Higgs masses and $H H \rightarrow W^{+} W^{-} W^{+} W^{-}$for large Higgs masses. But as in the investigated Composite Higgs models the cross section is much larger than the SM cross section, a better signal-background ratio might be possible. So even in the decay channels $b \bar{b} \tau^{+} \tau^{-}$and $b \bar{b} \mu^{+} \mu^{-}$. It was found in this section, however, that only in the $b \bar{b} \tau^{+} \tau^{-}$channel there is a realistic chance to measure the triple Higgs coupling for small Higgs masses.

In a full analysis of the $W^{+} W^{-} W^{+} W^{-}$decay channel, with subsequent decay $W^{+} W^{-} W^{+} W^{-} \rightarrow\left(j j l^{ \pm} \nu\right)\left(j j l^{\prime \pm} \nu\right)$, it was found in Ref. [73] that a vanishing Higgs coupling can be excluded at $95 \%$ C.L. or better in the mass range $150 \mathrm{GeV} \leq M_{H} \leq$ 200 GeV . The main background originates from $W^{ \pm} W^{+} W^{-} j j$ production followed by the $t \bar{t} W^{ \pm}$background, where one top quark decays leptonically and the other hadronically. The backgrounds in the Composite Higgs models change only if an intermediate Higgs is involved. Such backgrounds would be the subleading electroweak process of Higgs-strahlung off a $W$ or $Z$ boson, where the Higgs boson would decay subsequently into $W^{+} W^{-}$. But in $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ the Higgs couplings to gauge bosons are modified by a factor of $\sqrt{1-\xi}$ with respect to the SM, so the background would be smaller than in the SM. A Higgs boson with couplings to $t \bar{t}$ does not present any danger, since the top quark pair threshold is not reached, so that such processes do not play a role. The dominant contributions in the backgrounds will always come from processes involving at least two QCD couplings. But there are no such background processes involving at least one Higgs. So a rough estimate of the background processes for $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ would give the same result as for the SM. Since the signal process of Higgs pair production in gluon fusion with subsequent decay of the Higgs bosons in $4 W$ exceeds the SM process for all $\xi$ in $\mathrm{MCHM}_{4}$ and in $\mathrm{MCHM}_{5}$ as long as $\xi$ is not too larg ${ }^{9}$ the prospects of measuring the triple Higgs coupling are encouraging. The next step is now to perform a realistic analysis including background processes, signal cuts and detector effects.

[^16]
## CHAPTER 4

## Higgs pair production at $e^{+} e^{-}$-colliders

As a complementary collider to the LHC, a new linear $e^{+} e^{-}$-collider, the International Linear Collider (ILC), is in the planning stages. Due to clean signatures and the wellknown centre-of-mass energy for every process, which does not depend on PDFs as for proton-proton collisions, a measurement of the Higgs properties could be possible with very high precision at such a collider [78 82]. Additionally, the number of background events would be strongly reduced compared to the jet-rich LHC environment, especially for the $b \bar{b}$ final state, which has the largest branching ratio for $M_{H} \lesssim 140 \mathrm{GeV}$.
The ILC in the TESLA design [79, 80] is planned to be built for a centre-of-mass energy of $\sqrt{s}=500 \mathrm{GeV}$ with a possible upgrade to $\sqrt{s}=1 \mathrm{TeV}$. Within the first four years a luminosity of $500 \mathrm{fb}^{-1}$ will be accumulated. Later, a luminosity of $1000 \mathrm{fb}^{-1}$ is expected to be achieved [81, 82]. In the CLIC design [83] the centre-of-mass energy is 3 to 5 TeV with integrated luminosities of 3 to $5 \mathrm{ab}^{-1}$.
For the TESLA design, Higgs pair production via double Higgs-strahlung off $Z$ bosons is investigated in the first part of this chapter. In the second part, the production of two Higgs bosons in W boson fusion is studied.

### 4.1. Double Higgs-strahlung off $Z$ bosons

The diagrams which contribute to double Higgs-strahlung are depicted in Fig. 4.1. The differential cross section for the process $e^{+} e^{-} \rightarrow Z H H$ can be found in Appendix B. The numerical calculation was performed by Madgraph/Madevent with the new model files used for the double Higgs-strahlung and W boson fusion at the LHC. To understand the interference structure, an own Fortran routine for the cross section was used. The cross sections in dependence on the Higgs mass can be found in Fig. 4.2 for $\mathrm{MCHM}_{4}$ and in Fig. 4.3 for $\mathrm{MCHM}_{5}$. The cross section is shown for two collider energies: $\sqrt{s}=500 \mathrm{GeV}$ and $\sqrt{s}=1 \mathrm{TeV}$. The cross section can be increased by a factor of 2 if polarized electrons and positrons are used. The degree of polarization for




Figure 4.1.: Generic Feynman diagrams for the process $e^{+} e^{-} \rightarrow Z H H$
electrons is at least $80 \%$ and could be up to $90 \%$ at the ILC. However, the degree of polarization for the positrons is only $60 \%$ [84]. The cross sections in the plots are for unpolarized beams.
The behaviour of the cross sections in Fig. 4.2 and 4.3 is nearly the same as was seen in the double Higgs-strahlung at the LHC. The SM cross section is larger than the cross sections for $\xi \neq 0$ due to constructive interference of all diagrams - at least for an energy of $\sqrt{s}=500 \mathrm{GeV}$. The size of the couplings diminishes for $\xi<0.5$ and therefore the cross sections decrease as well. For $\xi>0.5$ the $H H Z Z$ coupling changes its sign but the absolute value increases again and so does the cross section. In $\mathrm{MCHM}_{5}$, the diagram involving $\lambda_{H H H}$ has the same modification factor as the $H H Z Z$ coupling and hence also increases for $\xi>0.5$. Therefore, the cross section for $\xi=0.8$ is bigger in $\mathrm{MCHM}_{5}$ than in $\mathrm{MCHM}_{4}$, where the HHH coupling becomes smaller with increasing $\xi$. This behaviour cannot only be found for the curves with $\sqrt{s}=500 \mathrm{GeV}$ but also for the ones with $\sqrt{s}=1 \mathrm{TeV}$. However, for the $\sqrt{s}=1 \mathrm{TeV}$ cross section not all diagrams interfere constructively. The diagrams with the additional $Z$ propagator can change their sign, depending on the centre-of-mass energy. But since the diagram with the $H H Z Z$ vertex is dominant for $\xi$ as long as $\xi$ is not near 0.5 , it also dominates the interference structure (at least for $\xi=0.2$ and $\xi=0.8$ ).
In the SM, the cross section for $\sqrt{s}=500 \mathrm{GeV}$ is larger for $M_{H} \lesssim 140 \mathrm{GeV}$ than the cross section for $\sqrt{s}=1 \mathrm{TeV}$ due to the scaling behaviour of the $s$-channel $Z$-exchange. This still holds for $\xi=0.2$. For $\xi=0.5$ and $\xi=0.8$ the cross sections for $\sqrt{s}=1 \mathrm{TeV}$ become larger than the cross sections for $\sqrt{s}=500 \mathrm{GeV}$. The difficulty here is that the energy dependence cannot be easily seen with the help of the cross section formula, since a three-particle phase space is involved. However, the energy dependence must come from the diagrams with the additional $Z$ propagator and the one involving the $H H Z Z$ coupling. As can be found in Appendix B, the diagram involving the triple Higgs coupling is suppressed by an extra Higgs propagator i.e. the squared diagram involving the $H H H$ coupling is multiplied by an additional (Higgs propagator) ${ }^{2}$ compared to the squared diagram with the $H H Z Z$ coupling.
The impact of this can also be seen in the directions of the arrows that indicate the change of the cross section if $\lambda_{H H H}$ is changed by $\pm 50 \%$ in terms of the Composite Higgs trilinear self-coupling. The direction of the arrows depends on the interference structure of the diagram with the triple Higgs coupling and the other diagrams, especially the one with the $H H Z Z$ coupling, since the interference between either of them is much larger than the interference term of the diagrams with $\lambda_{H H H}$ and the ones with the additional $Z$ propagator. For $\xi=0.5$, the $H H Z Z$ coupling is zero, as is $\lambda_{H H H}$ in $\mathrm{MCHM}_{5}$, so that there is no sensitivity at all to $\lambda_{\text {HHH }}$. For $\mathrm{MCHM}_{4}$, this means that


Figure 4.2.: The cross section for the process $e^{+} e^{-} \rightarrow Z H H$ as a function of the Higgs mass in the SM (upper left), and the $\mathrm{MCHM}_{4}$ with $\xi=0.2$ (upper right), $\xi=0.5$ (bottom left) and $\xi=0.8$ (bottom right) for two different collider energies $\sqrt{s}=500 \mathrm{GeV}$ (red curve) and $\sqrt{s}=1 \mathrm{TeV}$ (blue curve). Arrows indicate the change in the cross section for a variation of $\lambda_{H H H}$ from 0.5 to 1.5 of its value corresponding to the used model.
the direction of the arrow for $\xi=0.5$ is given by the interference structure between the diagrams with the additional $Z$ propagator and, of course, the one with the trilinear Higgs coupling. For an energy of $\sqrt{s}=500 \mathrm{GeV}$ the interference is constructive, though for an energy of $\sqrt{s}=1 \mathrm{TeV}$ the interference is destructive. For larger energies the sensitivity to a variation in $\lambda_{H H H}$ becomes smaller, since the corresponding diagram is suppressed by a Higgs propagator whereas the diagram involving the $H H Z Z$ coupling is not suppressed by an additional propagator and hence dominates for large energies. As the $Z$ propagator has a different energy structure than the Higgs propagator, the diagram with the additional $Z$ propagator also dominates the diagram with the triple Higgs coupling for large energies. An explanation for the directions of the other arrows can be found in section 3.4.
The energy behaviour of the cross section is shown in Fig. 4.4 for $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ for $M_{H}=120 \mathrm{GeV}$. For larger $M_{H}$ the shape of the curves does not change but they are shifted to higher energies. The maximal cross section for the SM and for $\xi=0.2$ is reached at $\sqrt{s} \sim 2 M_{H}+M_{Z}+200 \mathrm{GeV}$. For $\xi=0.5$ and $\xi=0.8$, the cross section seems to increase with energy. For the SM and $\xi=0.2$, the cross section peaks around $\sqrt{s} \approx 530 \mathrm{GeV}$ and then decreases. As can be anticipated from the curve in the


Figure 4.3.: As in Fig. 4.2 but for the $\mathrm{MCHM}_{5}$.
$\mathrm{MCHM}_{5}$ for $\xi=0.5$, where only the diagrams with an additional $Z$ propagator are non-zero, these diagrams lead to an increase of the cross section with $\sqrt{s}$. So does the diagram with the $H H Z Z$ coupling. In the SM, however, the interference between these diagrams becomes destructive for large energies and so the cross section decreases. For $\xi>0.5$, however, the diagram involving $g_{H H Z Z}$ changes its sign, so the interference between these diagrams becomes constructive for large energies. Therefore, the cross section increases with energy, unlike in the SM. In $\mathrm{MCHM}_{4}$, the curve has a small peak for $\xi=0.5$ whereas in $\mathrm{MCHM}_{5}$ there is no peak. The reason for this is that in $\mathrm{MCHM}_{5}$ only the diagrams with the additional $Z$ propagator are non-zero and these diagrams increase with the centre-of-mass energy. However, in the $\mathrm{MCHM}_{4}$ not only the diagrams with the additional $Z$ propagator but also the one with the trilinear Higgs coupling is non-zero. The peak is caused by the diagram with $\lambda_{H H H}$, which decreases with the energy.
One question is still open: Does the growth with energy in the Composite Higgs model with large $\xi$ lead to problems with partial wave unitarity? To answer this question the cross sections in these cases are studied for larger energies. In Fig. 4.5 it can be seen that the cross sections for large energies become constant. In both models, the cross section for large energies is nearly the same since the high energy behaviour is given by the diagrams with the additional $Z$ propagator and the one with the $H H Z Z$ coupling. In both models these diagrams receive the same modification factors in the Composite Higgs models. Hence the cross sections for large energies are nearly the same in both models.


Figure 4.4.: Energy dependence of the cross section of the process $e^{+} e^{-} \rightarrow Z H H$ for the SM (red), $\xi=0.2$ (blue), $\xi=0.5$ (black) and $\xi=0.8$ (green). The plot on the left is calculated in the $\mathrm{MCHM}_{4}$, the plot on the right is in the $\mathrm{MCHM}_{5}$. The Higgs mass is assumed to be 120 GeV .


Figure 4.5.: High energy behaviour of the cross section of the process $e^{+} e^{-} \rightarrow Z H H$ in the $\mathrm{MCHM}_{4}$ (left) and the $\mathrm{MCHM}_{5}$ (right) for $\xi=0.5$ (red) and $\xi=0.8$ (blue).

To investigate the prospects of measuring the trilinear Higgs self-coupling in double Higgs-strahlung, sensitivity areas in the $M_{H}-\xi$ plane are constructed in Fig. 4.6 for $\mathrm{MCHM}_{4}$ and in Fig. 4.7 for the $\mathrm{MCHM}_{5}$. The sensitivity areas show the prospects of measuring a non-vanishing $\lambda_{H H H}$ for two final states: $b \bar{b} b \bar{b}$ and $W^{+} W^{-} W^{+} W^{-}$. Below the gauge boson threshold the dominating decay channel is $H \rightarrow b \bar{b}$. For $M_{H_{-}} \gtrsim$ 140 GeV the Higgs boson decays dominantly via $W^{+} W^{-} W^{+} W^{-}$. At the ILC, $b \bar{b} b \bar{b}$ backgrounds are much smaller than at the LHC. However, the cross sections for Higgs pair production are also smaller than at the LHC. Hence combining the $b \bar{b}$ decay channel with one of the rare Higgs decays would lead to a too small number of events for the assumed energies and luminosities. Experimental simulations taking into account detector properties have demonstrated that for Higgs masses in the intermediate range and an integrated luminosity of $\int \mathcal{L}=1000 \mathrm{fb}^{-1}$ the triple Higgs coupling may be determined with an accuracy of $\sim 20 \%$ in the SM [79, 85].


Figure 4.6.: Areas in the $M_{H}-\xi$ plane with sensitivity to a non-vanishing $\lambda_{H H H}$ for the process $e^{+} e^{-} \rightarrow Z H H$ with subsequent decay for $\mathrm{MCHM}_{4}$. The regions correspond from dark blue to light blue to $5,3,2,1 \sigma$. The final states are $b \bar{b} b \bar{b}$ on the left and $W^{+} W^{-} W^{+} W^{-}$on the right at $\int \mathcal{L}=500 \mathrm{fb}^{-1}$

In the sensitivity plots, a collider energy of $\sqrt{s}=500 \mathrm{GeV}$ is assumed, since for 1 TeV the prospects of measuring the triple Higgs coupling are worse due to smaller sensitivities cf. Figs. 4.2 and 4.3. For luminosities of $500 \mathrm{fb}^{-1}$ and $1000 \mathrm{fb}^{-1}$ no regions with $5 \sigma$ sensitivity can be found. Also for polarized cross sections the prospects are not significantly better. The reason is that for low values of $\xi$ the cross section decreases with energy. For higher values of $\xi$, even though the cross section increases with energy, the sensitivity to $\lambda_{H H H}$ is diluted by the other diagrams since the diagram involving the triple Higgs coupling decreases with energy. The sensitivity areas are shown for an integrated luminosity of $500 \mathrm{fb}^{-1}$. They slightly increase for an integrated luminosity of $1000 \mathrm{fb}^{-1}$.
The behaviour of the sensitivity regions for $\mathrm{MCHM}_{4}$ can be explained as follows: For $M_{H} \gtrsim 160 \mathrm{GeV}$ there is no sensitivity in the $b \bar{b} b \bar{b}$ final state due to the small branching ratio in this mass range. The same holds for the $W^{+} W^{-} W^{+} W^{-}$final state for $M_{H} \lesssim$ 100 GeV . The non-sensitivity regions for $0.5 \lesssim \xi \lesssim 0.6$ for both final states are due to the small cross section in this region. The cross section is minimal there since for $\xi=0.5$ the diagram involving the $H H Z Z$ coupling vanishes. For $\xi \geq 0.5$ the $H H Z Z$ coupling becomes larger again but the contributions of the other diagrams decrease with rising $\xi$, so the minimal cross section is reached for a value of $\xi$ which is slightly larger than 0.5 . For a finer grid, these regions tend to form a connected line.
In $\mathrm{MCHM}_{5}$, the mass range for the sensitivity areas are also dictated by the branching ratios: For the final state $b \bar{b} b \bar{b}$ there is no sensitivity for $M_{H} \gtrsim 160 \mathrm{GeV}$ for $\xi<0.5$ and for $W^{+} W^{-} W^{+} W^{-}$the sensitivity vanishes for $M_{H} \lesssim 100 \mathrm{GeV}$. For $\xi=0.5$, the triple Higgs coupling is zero, so around this value there is no sensitivity at all. For large Higgs masses, these non-sensitivity regions are a slightly larger since the cross


Figure 4.7.: Areas in the $M_{H}-\xi$ plane with sensitivity to a non-vanishing $\lambda_{H H H}$ for the process $e^{+} e^{-} \rightarrow Z H H$ with subsequent decay for $\mathrm{MCHM}_{5}$. The regions correspond from dark blue to light blue to $5,3,2,1 \sigma$. The final states are $b \bar{b} b \bar{b}$ on the left and $W^{+} W^{-} W^{+} W^{-}$on the right at $\int \mathcal{L}=500 \mathrm{fb}^{-1}$.
section for large Higgs masses and $\xi \sim 0.5$ is very small because the assumed collider energy is only slightly bigger than the threshold energy of $\sqrt{s}=2 M_{H}+M_{Z}$ (e.g. for a Higgs mass of 200 GeV the threshold energy would be $\sqrt{s} \approx 490 \mathrm{GeV}$ ).
The cross section is minimal for a $\xi$ value which is slightly larger than 0.5 since for $\xi=0.5$ only the diagrams involving the additional $Z$ propagator are non-zero. But these diagrams become smaller with rising $\xi$. The other diagrams, however, increase for $\xi \geq 0.5$. This interplay leads to the region between $\xi \sim 0.5$ and $\xi \sim 0.6$ in the contour plot for the $W^{+} W^{-} W^{+} W^{-}$final state, where the sensitivity with rising $\xi$ is larger in the beginning, than drops and finally increases again. The mass region with sensitivity in the $W^{+} W^{-} W^{+} W^{-}$final state for large $\xi$ becomes smaller due to the decrease of the branching ratio $H \rightarrow W^{+} W^{-}$for large $\xi$. The branching ratio $H \rightarrow b \bar{b}$ for large $\xi$ and for large Higgs masses increases with rising $\xi$. Accordingly, there is sensitivity in the $b \bar{b} b \bar{b}$ channel for large $\xi$ over the whole mass range.
Altogether, the triple Higgs coupling can be measured over the whole parameter space except in a region between $0.5 \lesssim \xi \lesssim 0.6$ in $\mathrm{MCHM}_{4}$ and $0.45 \lesssim \xi \lesssim 0.6$ in $\mathrm{MCHM}_{5}$ using an $e^{+} e^{-}$-collider with a centre-of-mass energy of $\sqrt{s}=500 \mathrm{GeV}$ for an integrated luminosity of $500 \mathrm{fb}^{-1}$. The regions will shrink if a more detailed analysis, taking into account background processes and detector properties, is done.

### 4.2. W boson fusion

$W$ boson fusion, together with double Higgs-strahlung, is the most important Higgs pair production process at $e^{+} e^{-}$-colliders. This section concentrates on $W$ fusion. $Z$ fusion is not taken into account here, since the SM cross section for $Z$ fusion is about eight


Figure 4.8.: Generic Feynman diagrams for the process $e^{+} e^{-} \rightarrow \bar{\nu} \nu H H$.
times smaller than the one of $W$ boson fusion, as the electron- $Z$ couplings are smaller [67]. The ratio, however, could slightly change in Minimal Composite Higgs models, but it is not expected that this change would make the cross sections comparably large. Additionally, in contrast to the LHC, the final states for both processes can be distinguished in an $e^{+} e^{-}$-collider. The Feynman diagrams for this process can be found in Fig. 4.8. The numerical calculation of the cross sections for $W$ boson fusion was done with Madgraph/Madevent. As for double Higgs-strahlung unpolarized cross sections are investigated. However, with $100 \%$ polarized electron and positron beams the cross section can be increased by a factor of 4 . The dependence of the cross sections on


Figure 4.9.: The cross section for the process $e^{+} e^{-} \rightarrow \bar{\nu} \nu H H$ as a function of the Higgs mass in the SM (upper left), and the $\mathrm{MCHM}_{4}$ with $\xi=0.2$ (upper right), $\xi=0.5$ (lower left) and $\xi=0.8$ (lower right) for two different collider energies $\sqrt{s}=500 \mathrm{GeV}$ (red curve) and $\sqrt{s}=1 \mathrm{TeV}$ (blue curve). Arrows indicate the change in the cross section for a variation of $\lambda_{H H H}$ from 0.5 to 1.5 times its value in the corresponding model.


Figure 4.10.: As in Fig 4.9 but for $\mathrm{MCHM}_{5}$.
the Higgs mass for two different collider energies can be found in Fig. 4.9 for the SM and in $\mathrm{MCHM}_{4}$ for three representative values of $\xi$. In Fig. 4.10 the corresponding plots for $\mathrm{MCHM}_{5}$ can be found. The interference structure is the same as at the LHC. In chapter 3 , explanations for the behaviour of the cross sections with rising $\xi$ can be found, as well as explanations for the length and direction of arrows indicating the change in the cross section for a variation in the triple Higgs coupling of $0.5 \lambda_{H H H}$ to $1.5 \lambda_{H H H}$, where $\lambda_{H H H}$ denotes the coupling in the SM for the SM process and the MCHM coupling for the Composite Higgs pair production process. The cross section for $W$ boson fusion is slightly larger than the one for double Higgs-strahlung as long as a collider energy of $\sqrt{s}=1 \mathrm{TeV}$ is assumed. Due to its scaling behaviour, double Higgsstrahlung dominates over the $W$ boson fusion process at lower energies and for very low values of $\xi$ and $M_{H}$. In $W$ fusion the cross sections increase with rising centre-of-mass energy as anticipated from formula (3.12). The length of the arrows, however, and hence the sensitivity to $\lambda_{H H H}$ becomes smaller for larger energies. The reason for this is that the diagram involving the triple Higgs coupling is suppressed by an additional Higgs propagator and therefore does not increase with the energy, whereas the other two diagrams do. Consequently, for large energies the $u$ - and $t$ - channel diagrams and the diagram involving the $H H W W$ coupling dominate.
The energy behaviour of the cross section can be found in Fig.4.11. As anticipated from Eq. (3.12), the cross section increases with rising centre-of-mass energy. The increase is stronger for larger $\xi$. In the Composite Higgs models the amplitude increases in contrast to the SM with $\mathcal{M} \sim \xi s$. This behaviour was also checked for even larger
energies, as in the plots of Fig. 4.11. The same results were found in Ref. [66] for $W$ boson fusion at the LHC. Due to the modified Higgs couplings to gauge bosons the composite Higgs can only partly unitarize this scattering amplitude. The loss of perturbative unitarity is postponed to larger energies compared to the case without a Higgs boson. The Minimal Composite Higgs models, however, are only valid up to the cut-off of the effective theory, which should be smaller than the unitarity bound.
In order to investigate the prospects of measuring $\lambda_{H H H}$ in $W$ fusion, sensitivity areas in the $M_{H}-\xi$ plane were constructed. The contour plots in Fig. 4.12 for $\mathrm{MCHM}_{4}$ and in Fig. 4.13 for $\mathrm{MCHM}_{5}$ show whether a triple Higgs coupling equal to zero can be distinguished from a non-zero $\lambda_{H H H}$, where $\lambda_{H H H}$ denotes the triple Higgs coupling in the corresponding model, in two different final states - $b \bar{b} b \bar{b}$ and $W^{+} W^{-} W^{+} W^{-}$. In the upper plot an integrated luminosity of $500 \mathrm{fb}^{-1}$ is assumed, the plots below correspond to a luminosity of $2000 \mathrm{fb}^{-1}$. This factor of 4 in the luminosity can also be achieved if polarized cross sections are used instead of unpolarized cross sections. The sensitivity areas are always shown for a centre-of-mass energy of $\sqrt{s}=1 \mathrm{TeV}$, since for $\sqrt{s}=500$ GeV the cross sections are very small.
In both the $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ the $5 \sigma$ sensitivity areas become much larger if the luminosity is increased. Due to the small branching ratios, the cross sections for $W$ boson fusion are too small in the $b \bar{b} b \bar{b}$ final state for $M_{H} \gtrsim 160 \mathrm{GeV}$ (in $\mathrm{MCHM}_{5}$ only for small $\xi$ ) and for the $W^{+} W^{-} W^{+} W^{-}$decay channel for $M_{H} \lesssim 110 \mathrm{GeV}$ to have any sensitivity at all to a non-vanishing $\lambda_{H H H}$.
In $\mathrm{MCHM}_{4}$, the sensitivity decreases with rising $\xi$ since the triple Higgs coupling is modified by a factor of $\sqrt{1-\xi}$ with respect to the SM and hence becomes smaller with rising $\xi$. This means that for a luminosity of $2000 \mathrm{fb}^{-1}$ in the $b \bar{b} b \bar{b}$ channel deviations from a vanishing $\lambda_{H H H}$ can only be measured up to $\xi \approx 0.7$ and in the $W^{+} W^{-} W^{+} W^{-}$ final state up to $\xi \approx 0.75$ with $5 \sigma$. For a luminosity of $500 \mathrm{fb}^{-1}$ this border lies at even smaller $\xi$. In the $W^{+} W^{-} W^{+} W^{-}$final state the maximal $\xi$ of the $5 \sigma$ region becomes smaller with rising $M_{H}$. The reason for this behaviour is the branching ratio into $W$


Figure 4.11.: Energy behaviour of the cross section of the process $e^{+} e^{-} \rightarrow \bar{\nu} \nu H H$ in the $\mathrm{MCHM}_{4}$ (left plot) and the $\mathrm{MCHM}_{5}$ (right plot). The red curve corresponds to the SM, the blue to $\xi=0.2$, the black to $\xi=0.5$ and the green to $\xi=0.8$.
bosons, which is largest for $M_{H} \approx 170 \mathrm{GeV}$.
In $\mathrm{MCHM}_{5}$, there is no sensitivity for $\xi \approx 0.5$ since $\lambda_{H H H}=0$ for $\xi=0.5$. In the $W^{+} W^{-} W^{+} W^{-}$final state, there is only a small $5 \sigma$ region for $\xi>0.5$ for an integrated luminosity of $500 \mathrm{fb}^{-1}$. The triple Higgs coupling is only large enough for $\xi>0.8$ to show deviations from the case with a vanishing $\lambda_{H H H}$. The mass range of this $5 \sigma$ region is determined by the branching ratio $H \rightarrow W^{+} W^{-}$, which is largest in this region. For $\xi>0.5$ the mass range for the sensitivity in the $W^{+} W^{-} W^{+} W^{-}$decay channel becomes smaller with rising $\xi$, whereas in the $b \bar{b} b \bar{b}$ final state the range becomes larger. This is due to the branching ratios: The $b \bar{b}$ branching ratio is larger for very high $\xi$ than the branching ratio $H \rightarrow W^{+} W^{-}$, even for large Higgs masses. The latter branching ratio diminishes with rising $\xi$ and vanishes at $\xi=1$.
Altogether, the sensitivity areas can be enlarged very much if the luminosity is increased or if the electron and positron beams are polarized. For a luminosity of $2000 \mathrm{fb}^{-1}$, the triple Higgs coupling can be measured over the whole mass range for $\xi \lesssim 0.7$ in $\mathrm{MCHM}_{4}$ and in $\mathrm{MCHM}_{5}$ for $\xi \lesssim 0.35$ and $\xi \gtrsim 0.65$.


Figure 4.12.: Sensitivity areas in the $M_{H}-\xi$ plane for the measurement of a nonvanishing $\lambda_{H H H}$ in the process $e^{+} e^{-} \rightarrow \bar{\nu} \nu H H$ in $\mathrm{MCHM}_{4}$ with a centre-of-mass energy of $\sqrt{s}=1 \mathrm{TeV}$. The regions correspond from dark blue to light blue to $5,3,2,1 \sigma$. The final states are $b \bar{b} b \bar{b}$ on the left and $W^{+} W^{-} W^{+} W^{-}$on the right with $\int \mathcal{L}=500 \mathrm{fb}^{-1}$ for the two upper plots and $\int \mathcal{L}=2000 \mathrm{fb}^{-1}$ for the two lower plots.


Figure 4.13.: Sensitivity areas in the $M_{H}-\xi$ plane for the measurement of a nonvanishing $\lambda_{H H H}$ in the process $e^{+} e^{-} \rightarrow \bar{\nu} \nu H H$ in $\mathrm{MCHM}_{5}$ with a centre-of-mass energy of $\sqrt{s}=1 \mathrm{TeV}$. The regions correspond from dark blue to light blue to $5,3,2,1 \sigma$. The final states are $b \bar{b} b \bar{b}$ on the left and $W^{+} W^{-} W^{+} W^{-}$on the right with $\int \mathcal{L}=500 \mathrm{fb}^{-1}$ for the two upper plots and $\int \mathcal{L}=2000 \mathrm{fb}^{-1}$ for the two lower plots.

## CHAPTER 5

## The Higgs boson in a model with partially composite top

In this chapter, a modified Composite Higgs model is investigated, in which the top quark is partially composite. A description of the properties of the used model as well as the notation was given in subsection [2.2.3. The first section of this chapter deals with the diagonalization of the mass matrix of Eq. (2.74). To study the properties of the Higgs boson and the new fermions, single Higgs production via gluon fusion is investigated as an example in the second section. The main focus is on the last section of this chapter, where Higgs pair production via gluon fusion is discussed.

### 5.1. Diagonalization of the mass matrix

As mentioned in subsection 2.2.3, the masses of the SM fermions are generated through a mixing with fermions of the new strong sector. Since the top quark mass is much larger than the masses of the other SM quarks, the mixing between the top and the new fermions is much stronger than the mixing between the new particles and the other quarks. So the discussion will be constrained to the top sector.
In order to do calculations involving the new fermions or the top, the quark mass eigenstates are needed, so the mass matrix (2.74) must be diagonalized. Since the mass matrix $M$ maps the space of right-handed fermions to the space of left-handed fermions, a diagonalization always has to be done with two different matrices, where one of them (here $V$ ) rotates the left-handed quarks and the other one (here $U$ ) the right-handed quarks, hence

$$
\begin{equation*}
V^{\dagger} M U=M_{\text {diag }} . \tag{5.1}
\end{equation*}
$$

As $V$ and $U$ should only rotate the states they have to be unitary This means

$$
\begin{equation*}
V^{\dagger} V=V V^{\dagger}=\mathbb{1} \quad \text { and } \quad U^{\dagger} U=U U^{\dagger}=\mathbb{1} \tag{5.2}
\end{equation*}
$$

So for the fermionic interaction states $\Psi_{L}$ and $\Psi_{R}$ we have

$$
\begin{align*}
\bar{\Psi}_{L} M \Psi_{R} & =\left(\bar{\Psi}_{L} V\right)\left(V^{\dagger} M U\right)\left(U^{\dagger} \Psi_{R}\right)  \tag{5.3}\\
& =\bar{\Psi}^{\prime}{ }_{L} M_{\text {diag }} \Psi_{R}^{\prime},
\end{align*}
$$

where the fermionic interaction states are rotated to the mass eigenstates $\Psi_{L}^{\prime}$ and $\Psi_{R}^{\prime}$

$$
\begin{align*}
& \Psi_{L}^{\prime}=V^{\dagger} \Psi_{L}  \tag{5.4}\\
& \Psi_{R}^{\prime}=U^{\dagger} \Psi_{R} \tag{5.5}
\end{align*}
$$

This rotation does not change the kinetic term and therefore the propagators, because

$$
\begin{equation*}
i \bar{\Psi} \not \partial \Psi=i\left(\bar{\Psi}_{L} \not \partial \Psi_{L}+\bar{\Psi}_{R} \not \partial \Psi_{R}\right)=i\left(\bar{\Psi}_{L}^{\prime} V^{\dagger} \not \partial V \Psi_{L}^{\prime}+\bar{\Psi}_{R}^{\prime} U^{\dagger} \not \partial U \Psi_{R}^{\prime}\right)=i \bar{\Psi}^{\prime} \not \partial \Psi^{\prime} \tag{5.6}
\end{equation*}
$$

with the unitarity conditions of (5.2). The rotation matrices are needed to derive the coupling matrices for the couplings of the fermions to the Higgs boson in the mass basis. With

$$
\begin{equation*}
H \bar{\Psi}_{L} G \Psi_{R}=H \bar{\Psi}_{L}^{\prime} V^{\dagger} G U \Psi_{R}^{\prime}=H \bar{\Psi}_{L}^{\prime} G^{\prime} \Psi_{R}^{\prime} \tag{5.7}
\end{equation*}
$$

the new coupling matrix $G^{\prime}$ in the basis of the fermion mass eigenstates can be calculated by multiplication of the coupling matrix $G$ of the interaction states with the rotation matrices $V^{\dagger}$ and $U$. The coupling matrices can be found in Appendix C. Since the fermions are in the fundamental representation of $S O(5)$, the couplings $g_{H H H}, g_{H V V}$ and $g_{H H V V}$ are of course modified in the same way as in $\mathrm{MCHM}_{5}$. Altogether, all Higgs couplings in the mass eigenstates can be found if the matrices $V$ and $U$ are known. So the question is how to find $U$ and $V$. In subsection 2.2 .3 and Refs. [46, 47] ${ }^{2}$ these matrices have already been found before electroweak symmetry breaking, which means the VEV is equal to zero. However, this also has to be done for $v \neq 0$. First, an analytical approximation will be discussed. Later, a numerical method will be explained.

## Analytical method

An exact analytical way to diagonalize the mass matrix and to find the rotation matrices $V$ and $U$ was not found. But as the diagonalization could easily be done for $v=0$, an expansion in $\epsilon=\sqrt{\xi}=\frac{v}{f}$ can be done. Such an expansion is very useful because one can for example gain some insight into what exactly happens with the masses and how the couplings behave. Furthermore, the expansion can be used for small values of $\epsilon$, which are preferred by electroweak precision data.

[^17]The expansion is done in first order in $\epsilon=\frac{v}{f}$. As an ansatz for the matrices

$$
V=\left(\begin{array}{cccc}
\cos \theta_{L}+v_{1} \epsilon & \sin \theta_{L}+v_{2} \epsilon & v_{3} \epsilon & v_{4} \epsilon  \tag{5.8}\\
-\sin \theta_{L}+v_{5} \epsilon & \cos \theta_{L}+v_{6} \epsilon & v_{7} \epsilon & v_{8} \epsilon \\
v_{9} \epsilon & v_{10} \epsilon & 1+v_{11} \epsilon & v_{12} \epsilon \\
v_{13} \epsilon & v_{14} \epsilon & v_{15} \epsilon & 1+v_{16} \epsilon
\end{array}\right)
$$

and

$$
U=\left(\begin{array}{cccc}
\cos \theta_{R}+u_{1} \epsilon & u_{2} \epsilon & u_{3} \epsilon & \sin \theta_{R}+u_{4} \epsilon  \tag{5.9}\\
u_{5} \epsilon & 1+u_{6} \epsilon & u_{7} \epsilon & u_{8} \epsilon \\
u_{9} \epsilon & u_{10} \epsilon & 1+u_{11} \epsilon & u_{12} \epsilon \\
-\sin \theta_{R}+u_{13} \epsilon & u_{14} \epsilon & u_{15} \epsilon & \cos \theta_{R}+u_{16} \epsilon
\end{array}\right)
$$

can be taken. There are 32 parameters $v_{1} \ldots v_{16}$ and $u_{1} \ldots u_{16}$ that have to be determined. Also, the mass matrix has to be expanded to first order in $\epsilon$. Therefore, the VEV $v$ was expressed through $v=\epsilon f$. Additionally, $s_{\alpha}=\epsilon$ and $c_{\alpha}=1+O\left(\epsilon^{2}\right)$ have to be set. In order to restrict the 32 parameters, the unitarity conditions of (5.2) can be used. For each of the two matrices this fixes ten parameters, because $U^{\dagger} U$ is a symmetric matrix and therefore ten parameters are needed to set $U^{\dagger} U=\mathbb{1}$. So every matrix still has six free parameters. However, Eq. (5.1) is not yet fulfilled. Consequently, another condition is that the result of the multiplication $V^{\dagger} M U$ has to be diagonal. A diagonal $4 \times 4$ matrix has four free parameters. Hence another twelve parameters are constrained. So (5.1) together with the conditions in (5.2) constrains all free parameters.
The matrix multiplications were done with Mathematica. The masses of the new quarks were found to be

$$
\begin{equation*}
m_{Q}=\frac{m_{\Psi}}{\cos \theta_{L}}, \quad \quad m_{X}=m_{\Psi}, \quad m_{T}=\frac{\tilde{m}_{\Psi}}{\cos \theta_{R}} \tag{5.10}
\end{equation*}
$$

Note that this is the same result as obtained for $v=0$. However, in first order in $\epsilon$ the top quark also gets a mass

$$
\begin{equation*}
m_{t}=\frac{1}{\sqrt{2}} \epsilon f y \sin \theta_{L} \sin \theta_{R} \tag{5.11}
\end{equation*}
$$

This result corresponds to the one in Ref. [47]. In order to ensure that $m_{t}=173.3 \mathrm{GeV}$ and that $y$ is not larger than $4 \pi$ (so that it is still possible to perform perturbation theory) $\sin \theta_{L}$ or $\sin \theta_{R}$ must not be too small. This indicates that either the righthanded top or the left-handed top is mostly composite, which means that the mass eigenstate of the left-handed/right-handed top quark consists mainly of contributions from fermions of the new sector. Even though the top quark only gets its mass in first order in $\epsilon$, a finite mass for the top quark can still be guaranteed in the SM limit $f \rightarrow \infty$ since the $f$ cancels with the one in $\epsilon=\frac{v}{f}$.
Setting the top mass to $m_{t}=173.3 \mathrm{GeV}$ determines one of the parameters. The others are still free but constrained by electroweak precision data. As free parameters $\sin \theta_{L}$, $y, f$ and the ratio $\tilde{m}_{\Psi} / m_{\Psi}$ are chosen. The parameter $\sin \theta_{R}$ will always be determined through the top mass (5.11). The rotated coupling matrices in first order in $\epsilon$ can be found in Appendix [D. The full result for the rotation matrices $V$ and $U$ are not presented here, since they are very long and do not provide more to the understanding.

Another way to diagonalize the mass matrix analytically is to take the squared mass matrix

$$
\begin{align*}
& M_{\text {diag }}^{2}=V^{\dagger} M U U^{\dagger} M^{\dagger} V=V^{\dagger} M M^{\dagger} V \\
& M_{\text {diag }}^{2}=U^{\dagger} M^{\dagger} V V^{\dagger} M U=U^{\dagger} M^{\dagger} M U . \tag{5.12}
\end{align*}
$$

These relations allow us to find the matrices $V$ and $U$ as matrices of the eigenvectors of the matrix $M^{\dagger} M$ (or $M M^{\dagger}$ ) and the quark masses by taking the square root of the eigenvalues of these matrices. However, calculating the eigenvalues of these matrices leads to very long and complicated relations. The obtained top mass in dependence of the free parameters cannot be solved analytically for one of these variables. This would not allow the top mass to be set to 173.3 GeV except when the equation is solved numerically.

## Numerical method

Numerically the mass matrix was diagonalized by a singular value decomposition with LAPACK [87], a linear algebra package for FORTRAN. In order to do this, three routines of the LAPACK package are needed. First, the routine DGEBRD reduces the mass matrix to a bidiagonal matrix. The basis changing matrices for this procedure can only be given back completely if another routine named DORGBR is called (twice, i.e. one time for each matrix). These matrices and the bidiagonal matrix is needed as an input for the routine DBDSQR, which computes the singular value decomposition of a bidiagonal matrix. Also the matrices noted earlier with $V$ and $U$ can be obtained with this routine. The correct application of this procedure was checked against some examples calculated with the singular value decomposition function of Mathematica. The singular value decomposition does not order the singular values in a particular way. The convention of LAPACK is to order the largest singular value in the first position and from there on the singular values are ordered with decreasing size on the diagonal. Consequently, the masses must be ordered in the way corresponding to the fermions, which means the lightest mass has to stand in the first matrix element, since it should denote the top mass. This is achieved with an auxiliary matrix $O$. An estimation of how the others have to be ordered can be found by looking at the analytic results in first order in $\epsilon$. If the allowed parameter space where $\tilde{m}_{\Psi} \ll m_{\Psi}$ is chosen, then $m_{T} \ll m_{Q}, m_{X}$ must hold. The diagonal mass matrix has to be multiplied with

$$
O=\left(\begin{array}{llll}
0 & 1 & 0 & 0  \tag{5.13}\\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0
\end{array}\right)
$$

so that

$$
\begin{equation*}
M_{\text {diag }}^{\text {new }}=O^{\dagger} M_{\text {diag }} O . \tag{5.14}
\end{equation*}
$$

With $O^{\dagger} O=O O^{\dagger}=\mathbb{1}$, the original mass matrix can be obtained by

$$
\begin{equation*}
V O O^{\dagger} M_{\text {diag }} O O^{\dagger} U^{\dagger}=M \tag{5.15}
\end{equation*}
$$

Consequently, the matrices $V$ and $U$ have to be multiplied with

$$
\begin{align*}
V_{\text {new }} & =V O  \tag{5.16}\\
U_{\text {new }} & =U O . \tag{5.17}
\end{align*}
$$

If the masses were to be placed in a different order then of course the matrix $O$ has to be changed.
Another problem is how to set the top mass to its value of 173.3 GeV . The dependence on the parameters of the model is not known if the matrix diagonalization is done numerically. As already mentioned, one of the parameters is not free but has to be chosen in a way which fixes the top mass to its measured value. This was done by fitting the value for $\sin \theta_{R}$ by a bisection method until the top mass is fixed to its experimental value.
This works in the following way: In the beginning an interval of values for $\sin \theta_{R}$ is chosen. Then the singular value decomposition is performed for both the minimal and the maximal value of this interval. The value of the top mass corresponding to $\sin \theta_{R}$ in the middle of the interval is also computed. Now a comparison has to be made: If the obtained mass value of the middle of the $\sin \theta_{R}$ interval is smaller than the top mass, then either the maximal or minimal $\sin \theta_{R}$ value will be replaced by the value of the middle of the $\sin \theta_{R}$ interval, depending on which one also corresponds to a lower mass value than the top mass. If the mass value of the middle of the interval is larger than the top mass, then of course the $\sin \theta_{R}$ value which also gives a larger top mass is replaced. So a new interval is chosen at every step of this procedure. This procedure will last as long as a given precision for the top mass is not yet reached. Here a precision 0.05 GeV is assumed. For the lower and upper bound of the interval, one has to be careful because in the parameter space where $\tilde{m}_{\Psi} \ll m_{\Psi}$, the interval for $\sin \theta_{R}$ has to be chosen negative (e.g. [-1:0] seems sensible). Since $y$ will be negative, this ensures the top mass to be positive. This can be seen from formula (5.11) for the first order top mass.
To assure the correctness of this procedure, the following checks were done

- As already mentioned, the LAPACK procedure was checked against the same calculation in Mathematica.
- The SM limit was checked. For $f \rightarrow \infty$ the masses of the new fermions should go to $\infty$. Indeed, a check for very large $f$ shows that the masses of the new fermions get very large.
- For very large $f$, the results of the numerical method and the first order calculation should lead to the same results. This could be confirmed.
- For $f \rightarrow \infty, g_{H t t}$ should have the SM value, which is indeed the case.

Of course, the masses of the fermions should not be too high, because they have to be below the cut-off of the theory. So large masses, that of instance arise for $f \rightarrow \infty$, are of purely theoretical interest. In such a case heavy resonances of the strongly-interacting sector also have to be taken into account in principle.


Figure 5.1.: Generic Feynman diagram for the process $g g \rightarrow H$. The $m$ indicates that four particles can run in the triangle: The top and the three new fermions.

### 5.2. Single Higgs production in gluon fusion

The production of one Higgs boson in gluon fusion is the most important Higgs discovery channel at the LHC, since it provides the largest cross section (for reviews, see Refs. [19, 88]). This still holds in $\mathrm{MCHM}_{5}$ as long as $\xi$ is not near 0.5 , where the $g_{H f \bar{f}}$ coupling and therefore also the cross section vanishes [45]. The question is whether the effects of the new quarks in a model with partially composite top can be seen in such a process. In a recent study [89], it was found that in the low energy limit Higgs production via gluon fusion does not depend on any parameter other than $f$. This will be examined here in the context of the full amplitude including the new fermions. For the process only two diagrams contribute. The second one, arising through a permutation of the external gluon lines, is equivalent to the first one and can therefore be taken into account by a factor of 2. The Feynman diagram can be found in Fig. 5.1. Not only the top quark but also the new quarks can run in the triangle. Their couplings to the gluons are the same as for the top quark, since they have the same quantum numbers $\sqrt[3]{ }$ Apart from the new fermions, that have to be taken into account, only the coupling $g_{H f f}$ changes with respect to the SM. The calculation of the amplitude of the triangle is nearly the same as for the triangle in chapter 3 but there is now only one particle in the final state which means that the phase space integral can be integrated out easily. For the SM this process can be found in Ref. [90]. The partonic cross section is given by

$$
\begin{equation*}
\sigma(g g \rightarrow H)=\frac{\sigma_{0}}{M_{H}^{2}} \delta\left(\hat{s}-M_{H}^{2}\right) \tag{5.18}
\end{equation*}
$$

with

$$
\begin{equation*}
\sigma_{0}=\left.\left.\frac{G_{F} \alpha_{s}^{2}}{288 \sqrt{2} \pi} \frac{9}{16}\right|_{f=t, Q, X, T} A_{f}\left(\tau_{f}\right)\right|^{2} \tag{5.19}
\end{equation*}
$$

and with $\tau_{f}=\frac{M_{H}^{2}}{4 m_{f}^{2}}$.
For the fermion loops the top as well as the new fermions $Q, X$ and $T$ are taken into

[^18]account in the form factor $A_{f}$, which reads
\[

$$
\begin{equation*}
A_{f}(\tau)=2[\tau+(\tau-1) f(\tau)] \frac{1}{\tau^{2}} \frac{g_{H f f} v}{m_{f}} \tag{5.20}
\end{equation*}
$$

\]

The function $f(\tau)$ is given by

$$
f(\tau)= \begin{cases}\arcsin ^{2} \sqrt{\tau} & \tau \leq 1  \tag{5.21}\\ -\frac{1}{4}\left[\log \frac{1+\sqrt{1-\tau^{-1}}}{1-\sqrt{1-\tau^{-1}}}-i \pi\right]^{2} & \tau>1\end{cases}
$$

For the coupling $g_{H f f}$ the corresponding matrix element of the coupling matrix $G_{H f f}$ must be inserted. Denoting the gluon luminosity with

$$
\begin{equation*}
\frac{d \mathcal{L}^{g g}}{d \tau}=\int_{\tau}^{1} \frac{\mathrm{~d} x}{x} f^{g}\left(\frac{\tau}{x}, Q^{2}\right) f^{g}\left(x, Q^{2}\right) \tag{5.22}
\end{equation*}
$$

the cross section of the full process $p p \rightarrow g g \rightarrow H$ is given by

$$
\begin{equation*}
\sigma(p p \rightarrow H)=\sigma_{0} \tau_{H} \frac{d \mathcal{L}^{g g}}{d \tau_{H}} \tag{5.23}
\end{equation*}
$$

with $\tau_{H}=\frac{M_{H}^{2}}{s}$. There, $s$ denotes the collider energy squared which is taken to be $(14 \mathrm{TeV})^{2}$.

## Results

In $\mathrm{MCHM}_{5}$, there are no new fermions with masses beyond the cut-off. The fermions of the strongly interacting sector are integrated out and are only taken into account through modified couplings. If the new fermions are integrated out there is no dependence on the new parameters which arise in the model with partially composite top. Thus, the dependence of the cross section on these new parameters, as well as the behaviour of the cross sections of the model with partially composite top compared to $\mathrm{MCHM}_{5}$, is investigated. For the parton distribution functions, CTEQ6L1 is used. A scale

$$
Q=M_{H}
$$

was assumed, as proposed in Ref. [90]. In order to take into account the QCD corrections, the cross section was multiplied by a $K$ factor of 1.6 [90]. The QCD corrections do not change with respect to the SM in the models discussed here, since the virtual corrections stay the same due to the same couplings of the new fermions to the gluons as for the SM quarks. The real corrections involve only quarks of the proton and are therefore the same as in the SM. A centre-of-mass energy of $\sqrt{s}=14 \mathrm{TeV}$ was used. The parameters of the model are always chosen in a way that they fulfill the electroweak precision tests. In Ref. [47] the corresponding allowed parameter regions were derived.
A comparison of the cross sections of $\mathrm{MCHM}_{5}$ and the model with partially composite top can be found in Fig. 5.2. In these plots the blue curves correspond to the SM or $\mathrm{MCHM}_{5}$ where the new fermions are integrated out. The red curve corresponds to


Figure 5.2.: Plots for the comparison of the $\mathrm{MCHM}_{5}$ (blue dots), where the new fermions are integrated out, and a model, where the new fermions are taken into account explicitly (red line). The parameters for this model are here $y=-10.2, \tilde{m}_{\Psi} / m_{\Psi}=0.1$ and $\sin \theta_{L}=0.1$. The masses of the new fermions $m_{Q}, m_{X}$ and $m_{T}$, are $8973 \mathrm{GeV}, 8843 \mathrm{GeV}$ and 4063 GeV for $\xi=0.1$ and for $\xi=0.2$, they are $6459 \mathrm{GeV}, 6255 \mathrm{GeV}$ and 2875 GeV and for $\xi=0.3$, they are $5403 \mathrm{GeV}, 5107 \mathrm{GeV}$ and 2347 GeV .
the model where the new fermions emerge explicitly. The cross sections decrease with increasing $\xi$ because the coupling $g_{H f \bar{f}}$ decreases for $\xi<0.5$ which corresponds to the examined parameter region.
The SM limit of the new model $(f \rightarrow \infty)$ corresponds very well with the SM case and the two curves overlap. The masses of the new fermions become very large in the limit $f \rightarrow \infty$ and therefore the new fermions do not play a role. The coupling of the Higgs boson to $t \bar{t}$ also becomes the same as in the SM.
For $\xi \neq 0$, it was found in Ref. [89] that for low energies the new fermions can be integrated out. The effects of the strong dynamics were encoded in a modification factor of the Higgs coupling to the fermions of $\left(1-\frac{3}{2} \xi\right)$. This is the same coupling modification factor as in $\mathrm{MCHM}_{5}$ for the $g_{H f \bar{f}}$ coupling expanded in first order in $\xi$. As can be found in Fig. 5.2, the curves for $\xi \neq 0$ where the new fermions are taken into account explicitly coincides very well with the one where they are integrated out. Hence the same conclusions as in Ref. [89] can be drawn, even for the LHC energy of $\sqrt{s}=14 \mathrm{TeV}$. If the new fermions are integrated out a change in the masses and in their couplings to the Higgs boson and therefore in the new parameters of the model
should not lead to large deviations in the cross section, as in the case where they are integrated out the cross section only depends on $\xi$.

| $\sin \theta_{L}$ | $y$ | $\tilde{m}_{\Psi} / m_{\Psi}$ | $m_{Q}[\mathrm{GeV}]$ | $m_{X}[\mathrm{GeV}]$ | $m_{T}[\mathrm{GeV}]$ | $\sigma[\mathrm{pb}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | - | - | $>$ cut-off | $>$ cut-off | $>$ cut-off | 12.086 |
| 0.1 | -10.2 | 0.1 | 5842 | 5595 | 2571 | 12.408 |
| 0.1 | -10.2 | 0.05 | 5351 | 5298 | 1260 | 12.402 |
| 0.1 | -12.2 | 0.1 | 6719 | 6684 | 1187 | 12.286 |
| 0.1 | -12.2 | 0.05 | 6355 | 6327 | 599 | 12.227 |
| 0.95 | 10.2 | 5 | 3685 | 1285 | 6514 | 12.155 |

Table 5.1.: Cross sections for different model parameters for the process $p p \rightarrow H+X$.
In the Table 5.1, the cross section for different parameters are listed. For all cross sections $\xi=0.25$ and $M_{H}=120 \mathrm{GeV}$ was chosen. The first row of data is calculated in $\mathrm{MCHM}_{5}$ where the new fermions are integrated out. The table shows that shifting the parameters can lead to very different particle masses, but the cross section does not vary much. This confirms once more that the new fermions can be integrated out for this process.

### 5.3. Higgs pair production

In the Higgs pair production process in the model with a partially composite top, there are some changes compared to $\mathrm{MCHM}_{5}$. Since the coupling of the Higgs boson to the fermions is now a non-diagonal matrix, the Higgs can couple to two different fermions, which leads to new types of diagrams. The triangle diagrams do not change with respect to $\mathrm{MCHM}_{5}$, only that in the loop the top quark as well as the new quarks can run. But in the box diagrams not only one fermion but also two different fermions can run. The Feynman diagrams can be found in Fig. 5.3. The masses $m_{1}$ and $m_{2}$ should indicate the different fermions. $m_{1}$ and $m_{2}$ can even stand for the same masses, because of course the processes, where only one kind of fermion runs in the loop, are also possible. The amplitudes for these diagrams were calculated with FeynCalc analogous to the ones in section 3.1. All field contractions are again taken into account by multiplying the amplitude with a factor 2 as the diagrams with the fermion lines in the opposite direction give the same contribution. The tensor structure does not change, because it was derived with the help of Ward identities. Therefore, the same projectors can be used. The form factors can be found Appendix E However, one thing is a bit tricky: Since the coupling matrix is not symmetric, one needs to pay attention to the non-diagonal matrix elements:

$$
\begin{align*}
H \bar{\Psi}_{L, i} G_{i j} \Psi_{R, j}+H \bar{\Psi}_{R, i}\left(G^{\dagger}\right)_{i j} \Psi_{L, j} & =H\left(\bar{\Psi}_{i} G_{i j}\left(\frac{1+\gamma_{5}}{2}\right) \Psi_{j}+\bar{\Psi}_{i} G_{j i}\left(\frac{1-\gamma_{5}}{2}\right) \Psi_{j}\right) \\
& =\underbrace{H \bar{\Psi}_{i}\left(\frac{G_{i j}}{2}+\frac{G_{j i}}{2}\right) \Psi_{j}}_{(1)}+\underbrace{H \bar{\Psi}_{i}\left(\frac{G_{i j}}{2}-\frac{G_{j i}}{2}\right) \gamma_{5} \Psi_{j}}_{(2)} \tag{5.24}
\end{align*}
$$

As can be seen here, for the non-diagonal matrix elements a new Lorentz structure of the couplings arises. The first part (1) gives a structure as expected: The coupling matrix for that part is symmetric. But the second part (2) has a different structure: The coupling gets an additional $\gamma_{5}$ and is not symmetric anymore. It differs by a minus sign depending on whether the fermion $i$ or $j$ is incoming or outgoing 4 This coupling is illustrated in Fig. [5.4. The form factors for the coupling structure of part (2) were calculated separately, since the additional $\gamma_{5}$ changes the traces that have to be calculated for the fermion loop. Only diagrams where the $\gamma_{5}$ coupling is present in both Higgs vertices to two fermions give a contribution, because diagrams with only one $\gamma_{5}$ are cancelled by the corresponding diagrams with the fermion lines in the opposite direction. As explained above for the coupling without $\gamma_{5}$ they are taken into account by a factor of 2. The form factors can also be found in Appendix E. They look very similar to the other form factors for the same projectors but some terms have a different sign. For the diagrams with two $\gamma_{5}$ matrices an extra minus sign is always needed because each diagram involves a vertex where particle $i$ is incoming and $j$ outgoing and vice versa.
To calculate the cross section numerically, a FORTRAN program was written, which performs the mass matrix calculation and the phase space integration. For details on the phases space integration see section 3.1. As PDF set CTEQ6L1 was again used. The QCD corrections do not change with respect to the SM and are taken into account by a $K$ factor of 2 [63]. Again a centre-of-mass energy of $\sqrt{s}=14 \mathrm{TeV}$ is assumed. The same scale as in chapter 3 for gluon fusion was assumed. Here LoopTools [91] was used for the scalar integrals.

## Results

Unfortunately, results for very large $f$ or low Higgs masses $M_{H}$ (depending on the choice of the other values) cannot be presented here, since there are still some numerical problems in the program which means that the Monte Carlo integration does not


Figure 5.3.: Generic Feynman diagrams for the process $g g \rightarrow H H$. The masses $m_{1}$ and $m_{2}$ in the box diagrams denote that it is possible that two different fermions are in the loop. Here, $m_{1}$ and $m_{2}$ stand for every possible combination of the masses $m_{t}, m_{Q}, m_{X}$ and $m_{T}$. Also $m$ also stands for these masses.

[^19]| $\sin \theta_{L}$ | $y$ | $\tilde{m}_{\Psi} / m_{\Psi}$ | $m_{Q}[\mathrm{GeV}]$ | $m_{X}[\mathrm{GeV}]$ | $m_{T}[\mathrm{GeV}]$ | $\sigma[\mathrm{fb}]$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| - | - | - | $>$ cut-off | $>$ cut-off | $>$ cut-off | 65.164 |
| 0.1 | -10.2 | 0.1 | 5842 | 5595 | 2571 | 40.179 |
| 0.1 | -10.2 | 0.05 | 5351 | 5298 | 1260 | 40.306 |
| 0.1 | -12.2 | 0.1 | 6719 | 6684 | 1187 | 49.238 |
| 0.1 | -12.2 | 0.05 | 6355 | 6327 | 599 | 49.952 |
| 0.95 | 10.2 | 5 | 3685 | 1285 | 6514 | 58.510 |

Table 5.2.: Cross sections for different model parameters for the process $p p \rightarrow H H+X$.
converge. In the beginning the value of the error to the cross section seems to converge, but suddenly the error becomes large and the value for the cross section jumps. This problem could be solved for some parameters by the introduction of a small $\epsilon$ for the integration limits of the integration over the Bjorken variable $x$, which means that the random numbers given by the Vegas routine are substituted in such a way that they lie in $[\epsilon, 1-\epsilon]$. Usually, $\epsilon$ is chosen $O\left(10^{-8}\right)$. The numerical problem is caused in the form factors $G_{\square}$ and $G_{\square, 5}$. Presumably, the problem lies in the mass hierarchies since it only emerges for light Higgs masses or large $f$. For large $f$, the masses of the new fermions are very high, which leads to a large mass hierarchy between the Higgs mass and the masses of the new fermions.
To start with, the question of whether the effects of the new fermions in the loop can be seen is investigated. For that purpose, a table of some values of the cross section for different parameters is helpful. The Higgs mass for the cross sections in Table 5.2 is chosen to be $M_{H}=180 \mathrm{GeV}$ and $\xi=0.25$. As it can be found in Table 5.2, the parameters of the model affect the cross section much more than in the single Higgs production. But the reason does not seem to be the change in the masses but rather the change in the couplings. A shift in the parameter $\tilde{m}_{\Psi} / m_{\Psi}$ does not change the cross section very much, but the masses, especially $m_{T}$, change. However, $\tilde{m}_{\Psi} / m_{\Psi}$ does not change the coupling $g_{H f f}$ as much as a change in the other parameters would do. This can be anticipated from the coupling matrix $G_{H f \bar{f}}$ in first order in $\frac{v}{f}$ of Appendix D, since most of the matrix elements do not depend on $\tilde{m}_{\Psi} / m_{\Psi}$ in first order, especially not $g_{H t t}$, which has the largest effect on the cross section. Shifting the value of the coupling $y$ has a much larger effect on the cross section. Examining the allowed parameter


Figure 5.4.: Illustration of part (2) of Eq. (5.24). This part of the coupling of the Higgs to two different fermions depends on the direction of the fermion lines.
space where $\sin \theta_{L} \sim 1$, the cross section changes significantly compared to case where $\sin \theta_{L}=0.1$. Every parameter set chosen in this table shows that the cross section in the model with partially composite top is smaller than the cross section where the new fermions are integrated out (cf. the first row of Table 5.2). For comparison, the cross section was also calculated for the fictitious case in which the coupling matrix $G_{H f \bar{f}}$ is diagonal in order to get an answer to the question of whether the diagrams with two different fermions have an effect on the cross section. Indeed, if in this case the parameters are chosen in the same way as in the second row of the table ( $y=-10.2$, $\sin \theta_{L}=0.1, \tilde{m}_{\Psi} / m_{\Psi}=0.1, \xi=0.25$ and $\left.M_{H}=180 \mathrm{GeV}\right)$ the cross section more than doubles. The fact that the cross section becomes smaller if the diagrams with two different fermions in the loop are also taken into account is mainly due to the diagrams with the coupling with the new Lorentz structure (the form factors $F_{\square, 5}$ and $G_{\square, 5}$ ).
Another interesting question is the behaviour of the cross section with respect to $\xi$. In Fig. 5.5 cross sections as a function of $M_{H}$ are shown for two values of $\xi$. In chapter 3, it was found that the cross section in $\mathrm{MCHM}_{5}$ increases with $\xi$ due to the new triangle diagram with the new $g_{H H f \bar{f}}$ coupling. The cross section in the model with partially composite top should show the same behaviour. Indeed, in Fig. 5.5 the cross section increases if $\xi$ is shifted from 0.1 to 0.25 . But as already seen in the table, the cross sections are smaller in the model with partially composite top than in $\mathrm{MCHM}_{5}$. As expected, the cross sections are larger than the SM cross section (which would correspond to $\xi=0$ ).
The arrows in Fig. 5.5 indicate the behaviour of the cross section under a change in the triple Higgs coupling. The arrows point from the value of the cross section with $0.5 \lambda_{H H H}$ to the value of the cross section for $1.5 \lambda_{H H H}$. The direction of the arrows can be explained by the interference structure: As already described for the SM and $\mathrm{MCHM}_{5}$ in section 3.4, the diagram involving the triple Higgs coupling interferes destructively with the other diagrams and so the arrow points downwards. For the cross section in the model with partially composite top, the situation is of course more complicated, since all the diagrams must in principle be taken into account to answer this question. But since the top loop is the most important one, and the signs of the top couplings do not change compared to the $\mathrm{MCHM}_{5}$, the interference structure stays the same. The arrows become smaller with increasing $\xi$. This is due to the increasing importance of the triangle diagram with the $g_{H H f \bar{f}}$ coupling. This coupling also gets larger with rising $\xi=\frac{v^{2}}{f^{2}}$ in the model with partially composite top because the coupling matrix scales with $\frac{1}{f}$.
In Fig. [5.6 the effects on the cross section for a variation of $\lambda_{H H H}$, which is given by $\lambda_{H H H}^{\prime}=\kappa \lambda_{H H H}$, can be seen. The slope of the curves for different parameters and even for the case where the new fermions are integrated out is nearly the same. Of course, for smaller cross sections the change in per cent with varying $\kappa$ is more important than for larger cross sections. This behaviour can be explained with the help of the results of single Higgs production. There, the cross section varies only slightly with the parameters (except for $\xi$ ). The structure of the Feynman diagrams for this process are nearly the same as for the Feynman diagram involving the trilinear Higgs coupling in Higgs pair production. The amplitude is only multiplied by an additional Higgs propagator and the triple Higgs coupling. But the structure involving the new fermions
does not change. According to this, the behaviour of the cross section with a variation of $\kappa$ should be the same for the curves with different parameters at a given value of $\xi$. This is the reason why the shape of the curves are nearly the same for every parameter set. The shift in the curves can be explained by the other Feynman diagrams, especially the box diagrams of the Higgs pair production process. The same behaviour will be found for other values of $\xi$.
The prospects of measuring e.g. a non-vanishing Higgs coupling should lie between the SM and $\mathrm{MCHM}_{5}$ for the corresponding value of $\xi$. As is shown in Fig. 5.5, the cross sections for the model with partially composite top are always smaller for the investigated parameters than in $\mathrm{MCHM}_{5}$ but larger than in the SM. The slope of the curve showing the cross section with varying $\kappa$ is the same. The effects on the variation of $\lambda_{H H H}$ of the cross section in the investigated model are therefore larger than in $\mathrm{MCHM}_{5}$ but smaller than in the SM. The prospects of measuring a non-vanishing $\lambda_{H H H}$ for the small values of $\xi$, which are preferred in this model by electroweak precision tests, are very good in $\mathrm{MCHM}_{5}$ (Fig. 3.11) and even better in the SM for $H H \rightarrow b \bar{b} \tau^{+} \tau^{-}$and $H H \rightarrow W^{+} W^{-} W^{+} W^{-}$. So the prospects of measuring a non-vanishing $\lambda_{H H H}$ should be as good as in $\mathrm{MCHM}_{5}$ in the model with partially composite top. However, taking all background processes into account might lead to a worse result because the cross section is slightly smaller than in the $\mathrm{MCHM}_{5}$ and so the background to signal ratio could be reduced.


Figure 5.5.: These plots give a comparison between the $\mathrm{MCHM}_{5}$ (red curve), where the fermions of the strongly interacting sector are integrated out, and the model with partially composite top (blue curve) for the parameters $y=$ $-10.2, \tilde{m}_{\Psi} / m_{\Psi}=0.1$ and $\sin \theta_{L}=0.1$, and the $\operatorname{SM}$ (black curve). In the upper plot $\xi=0.1$, in the lower plot $\xi=0.25$. The masses of the new fermions, $m_{Q}, m_{X}$ and $m_{T}$, are for $\xi=0.1$ given by $8416 \mathrm{GeV}, 8369 \mathrm{GeV}$ and 1946 GeV and for $\xi=0.25$, they are $5842 \mathrm{GeV}, 5595 \mathrm{GeV}$ and 2571 GeV . The arrows indicate the change in the cross section for a variation of $\lambda_{H H H} 0.5$ to 1.5 times its value in the corresponding model. For the plot for $\xi=0.1$ the arrows are scaled by a factor of 0.5 .


Figure 5.6.: Variation of the cross section for a modified trilinear coupling according to $\lambda_{H H H}^{\prime}=\kappa \lambda_{H H H}$ for $\xi=0.25$. The upper plot shows the cross section for varying $\kappa$ for the parameter sets of $y=-10.2, \sin \theta_{L}=0.1$ and $\tilde{m}_{\Psi} / m_{\Psi}=$ 0.1 (black line), $y=-12.2, \sin \theta_{L}=0.1$ and $\tilde{m}_{\Psi} / m_{\Psi}=0.1$ (red line), $y=10.2, \sin \theta_{L}=0.95$ and $\tilde{m}_{\Psi} / m_{\Psi}=5$ (green line) and for comparison in $\mathrm{MCHM}_{5}$ (blue line). The value $\kappa=1$ is marked by a dashed line, as it belongs to the cross section with a $\lambda_{H H H}$ given by the model. The lower plot shows the variation of the cross section in percent. $0 \%$ corresponds to no variation of the cross section. The colours code is the same as in the upper plot.

## CHAPTER 6

## Summary

The understanding of the origin of electroweak symmetry breaking is still an outstanding issue. In the SM, electroweak symmetry breaking is explained by the Higgs mechanism. A short review of it was given in chapter 2. Even though the SM is in very good agreement with all measurements so far, there are still some important unanswered questions related to the size of the Higgs mass of the Higgs boson in the context of the SM being valid up to some high mass scale as e.g. $M_{G U T}=10^{16} \mathrm{GeV}$.
To solve this and other problems alternatives to the SM have been developed. In this work a model is discussed where the Higgs boson is not a fundamental scalar particle anymore but a bound state of a strongly-interacting sector. Thus, the Higgs boson emerges as a pseudo Nambu-Goldstone boson of an enlarged global symmetry with a mass arising through loops of fermions and gauge bosons which can naturally be kept small. An introduction into the model was given in the second part of chapter 2 based on a minimal approach leading to two different models - $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$. For low energies the particle content of these models is the same as in the SM but the Higgs couplings are modified in terms of a new parameter $\xi$ measuring the degree of compositeness.
For an understanding of electroweak symmetry breaking the Higgs potential must be reconstructed. This involves the measurement of Higgs self-couplings. The quartic Higgs self-coupling is neither accessible at the LHC nor the ILC, in contrary to the trilinear Higgs self-coupling. The focus of this work is therefore on the measurement of the trilinear Higgs self-interaction, which is accessible in Higgs pair production processes. They have been discussed in the framework of $\mathrm{MCHM}_{4}$ and $\mathrm{MCHM}_{5}$ in chapter 3 for the LHC. In that context, Higgs pair production via gluon fusion, via vector boson fusion and via double Higgs-strahlung off vector bosons was studied. For the gluon fusion process, sensitivity areas for the deviation of the minimal Composite Higgs models with respect to the SM in the $M_{H}-\xi$ plane were constructed for various final states. In the same way the sensitivity to a non-vanishing triple Higgs coupling $\lambda_{H H H}$ was investigated. Especially for the $W^{+} W^{-} W^{+} W^{-}$final state, the results were very promising.

This analysis concentrated on the identification of parameter regions in the $M_{H}-\xi$ plane, where the trilinear Higgs self-coupling can be extracted in the framework of a minimal Composite Higgs model. The results now have to be backed by a more sophisticated analysis taking into account all background processes as well as detector properties.
In chapter 4, Higgs pair production at an $e^{+} e^{-}$-collider was discussed. A centre-ofmass energy of 500 GeV and of 1 TeV was assumed. This is the energy region of the ILC TESLA design. In this context, the cross sections for double Higgs-strahlung off $Z$ bosons and $W$ fusion were presented. For the double Higgs-strahlung process, it was found that the cross sections are smaller than in the SM for low $\xi$ but if $\xi \gtrsim 0.5$ the cross sections become larger for large energies. They also show a different energy behaviour as in the SM case for large $\xi$. In double Higgs-strahlung, the prospects of measuring a non-vanishing trilinear Higgs coupling for a collider energy of $\sqrt{s}=500$ GeV are rather good in the final states $b \bar{b} b \bar{b}$ and $W^{+} W^{-} W^{+} W^{-}$even for a luminosity of $500 \mathrm{fb}^{-1}$ which is hoped to be achieved in an early stage at the ILC. However, a larger centre-of-mass energy of 1 TeV would not allow to measure the triple Higgs-coupling due to the decreasing contribution of the Feynman diagram involving the trilinear Higgs self-coupling with the energy. In $W$ boson fusion, the cross sections increase with rising $\xi$. The energy behaviour also changes compared to the SM: The amplitude is now $\sim s$, due to the modification of the Higgs couplings to the gauge bosons. The triple Higgs coupling can be measured in $W$ boson fusion for an integrated luminosity of $2000 \mathrm{fb}^{-1}$ and a collider energy of 1 TeV over the whole mass range for $\xi \lesssim 0.7$ in $\mathrm{MCHM}_{4}$ and in $\mathrm{MCHM}_{5}$ for $\xi \lesssim 0.35$ and $\xi \gtrsim 0.65$.
A model, where the new fermions of the strongly interacting sector have masses below the cut-off of the effective theory, is investigated in chapter 5. For the calculation of single Higgs production and Higgs pair production in gluon fusion the new fermions have to be taken into account explicitly. The results of both processes are compared to the results obtained where the new fermions are integrated out. For single Higgs production, the cross sections have been found to be nearly the same in both cases whereas for Higgs pair production the cross section depends on the choice of the model parameters. This process is still under investigation. A variation in the triple Higgs coupling shows similar effects as in the $\mathrm{MCHM}_{5}$.
With the caveat of a lacking background study in the Minimal Composite Higgs models but backed by SM studies, a non-vanishing triple Higgs coupling can be measured at the LHC as soon as the full centre-of-mass energy and a luminosity of $300 \mathrm{fb}^{-1}$ is reached. At a future $e^{+} e^{-}$collider, a measurement of a non-zero $\lambda_{H H H}$ is possible over nearly the whole considered parameter space in double Higgs-strahlung for $\sqrt{s}=500 \mathrm{GeV}$ and an integrated luminosity of $500 \mathrm{fb}^{-1}$ and in $W$ fusion for $\sqrt{s}=1 \mathrm{TeV}$ and a luminosity of $2000 \mathrm{fb}^{-1}$.

## APPENDIX A

## Gluon fusion: Form factors

## Notation

$p_{1}$ and $p_{2}$ are the four-momenta of the gluons; $p_{3}$ and $p_{4}$ are the four-momenta of the Higgs bosons. The Mandelstam variables are defined as

$$
s=\left(p_{1}+p_{2}\right)^{2} \quad t=\left(p_{1}+p_{3}\right)^{2} \quad u=\left(p_{2}+p_{3}\right)^{2}
$$

The scalar integrals are defined as

$$
\begin{aligned}
C_{i j} & =\int \frac{\mathrm{d}^{4} q}{(2 \pi)^{4}} \frac{1}{\left(q^{2}-m^{2}\right)\left(\left(q+p_{i}\right)^{2}-m^{2}\right)\left(\left(q+p_{i}+p_{j}\right)^{2}-m^{2}\right)} \\
D_{i j k} & =\int \frac{1}{(2 \pi)^{4}} \frac{\mathrm{~d}^{4} q}{\left(q^{2}-m^{2}\right)\left(\left(q+p_{i}\right)^{2}-m^{2}\right)\left(\left(q+p_{i}+p_{j}\right)^{2}-m^{2}\right)\left(\left(q+p_{i}+p_{j}+p_{k}\right)^{2}-m^{2}\right)}
\end{aligned}
$$

The analytic expressions can be found in Refs. [58, 92].

Tensor basis and projectors
The tensor basis was obtained by reducing the general tensor structure consisting of $g^{\mu \nu}, p_{1}^{\nu} p_{2}^{\mu}, p_{1}^{\nu} p_{3}^{\mu}, p_{2}^{\mu} p_{3}^{\nu}$ and $p_{3}^{\mu} p_{3}^{\nu}$ with the help of Ward identities. Through an appropriate linear combination the two remaining tensor structures were rewritten to the ones of Ref. 59]

$$
\begin{align*}
A_{1}^{\mu \nu}= & \frac{1}{\sqrt{2}}\left(g^{\mu \nu}-\frac{p_{1}^{\nu} p_{2}^{\mu}}{\left(p_{1} \cdot p_{2}\right)}\right)  \tag{A.1}\\
A_{2}^{\mu \nu}= & \frac{1}{\sqrt{2}}\left(g^{\mu \nu}+\frac{p_{3}^{2} p_{1}^{\nu} p_{2}^{\mu}}{p_{T}^{2}\left(p_{1} \cdot p_{2}\right)}-\frac{2\left(p_{3} \cdot p_{2}\right) p_{1}^{\nu} p_{3}^{\mu}}{p_{T}^{2}\left(p_{1} \cdot p_{2}\right)}-\frac{2\left(p_{3} \cdot p_{1}\right) p_{3}^{\nu} p_{2}^{\mu}}{p_{T}^{2}\left(p_{1} \cdot p_{2}\right)}+\frac{2 p_{3}^{\mu} p_{3}^{\nu}}{p_{T}^{2}}\right)  \tag{A.2}\\
& \text { with } \quad p_{T}^{2}=2 \frac{\left(p_{1} \cdot p_{3}\right)\left(p_{2} \cdot p_{3}\right)}{\left(p_{1} \cdot p_{2}\right)}-p_{3}^{2} \\
& \text { and } \quad A_{1} \cdot A_{2}=0 \quad \text { and } \quad A_{1} \cdot A_{1}=A_{2} \cdot A_{2}=1 \tag{A.3}
\end{align*}
$$

Triangle form factor

$$
\begin{equation*}
F_{\triangle}=2\left(2 m+\left(4 m^{3}-s m\right) C_{12}\right) \tag{A.4}
\end{equation*}
$$

Box form factors

$$
\begin{align*}
F_{\square}= & 4+8 m^{2} C_{12}-2\left(m^{2} s+2 m^{2} M_{H}^{2}-8 m^{4}\right)\left(D_{123}+D_{213}+D_{132}\right) \\
& +\frac{1}{s}\left(2 M_{H}^{2}-8 m^{2}\right)\left[-\left(t u-M_{H}^{4}\right) D_{132}+2\left(t-M_{H}^{2}\right) C_{13}+2\left(u-M_{H}^{2}\right) C_{23}\right] \\
G_{\square}= & \frac{1}{t u-M_{H}^{4}}\left[2\left(8 m^{2}-t-u\right)\left(t u-M_{H}^{4}\right) m^{2}\left(D_{123}+D_{213}+D_{132}\right)\right. \\
& +\left(u^{2}+M_{H}^{4}-8 u m^{2}\right)\left(s C_{12}+2\left(u^{2}-M_{H}^{2}\right) C_{23}-s u D_{123}\right) \\
& +\left(t^{2}+M_{H}^{4}-8 t m^{2}\right)\left(s C_{12}+2\left(t^{2}-M_{H}^{2}\right) C_{13}-s t D_{213}\right) \\
& \left.-\left(t^{2}+u^{2}-2 M_{H}^{4}\right)\left(t+u-8 m^{2}\right) C_{34}\right] \tag{A.5}
\end{align*}
$$

## APPENDIX B

## Differential cross section for double Higgs-strahlung off $Z$ bosons

Coupling factors

$$
\begin{align*}
f_{H Z Z} & =\frac{2 M_{Z}^{2}}{v} \sqrt{1-\xi}  \tag{B.1}\\
f_{H H Z Z} & =\frac{2 M_{Z}^{2}}{v^{2}}(1-2 \xi)  \tag{B.2}\\
f_{H H H} & =\frac{3 M_{H}^{2}}{v} \sqrt{1-\xi}  \tag{B.3}\\
f_{H H H} & =\frac{3 M_{H}^{2}}{v} \frac{(1-2 \xi)}{\sqrt{1-\xi}} \tag{B.4}
\end{align*} \quad \text { in } \mathrm{MCHM}_{4} \quad \text { in } \mathrm{MCHM}_{5}
$$

The differential cross section of the process

$$
e^{-}\left(k_{-}\right)+e^{+}\left(k_{+}\right) \rightarrow H\left(p_{1}\right)+H\left(p_{2}\right)+Z\left(p_{3}\right)
$$

is 67, 93, 94]

$$
\begin{equation*}
\frac{d \sigma}{d E_{1} d E_{2}}=\frac{G_{F} M_{Z}^{2}}{384 \pi^{3} \sqrt{2}} \frac{1}{\left(s-M_{Z}^{2}\right)^{2}}\left(a_{e}^{2}+v_{e}^{2}\right) \frac{1}{2} \mathcal{A} \tag{B.5}
\end{equation*}
$$

with

$$
\begin{align*}
\mathcal{A}= & f_{H Z Z}^{4} A_{a a}+f_{H H Z Z}^{2} A_{b b}+f_{H Z Z}^{2} f_{H H H}^{2} A_{c c} \\
& +f_{H Z Z}^{2} f_{H H Z Z} A_{a b}+f_{H Z Z}^{3} f_{H H H} A_{a c}+f_{H Z Z} f_{H H H} f_{H H Z Z} A_{b c} \tag{B.6}
\end{align*}
$$

and

$$
\begin{align*}
A_{a a} & =d_{1}^{2} d_{4}+\frac{d_{2}}{M_{Z}^{4}}+\frac{2}{M_{Z}^{2}} d_{1} d_{3}  \tag{B.7}\\
A_{b b} & =d_{4}  \tag{B.8}\\
A_{c c} & =d_{4} \frac{1}{\left(2 p_{1} \cdot p_{2}+M_{H}^{2}\right)^{2}}  \tag{B.9}\\
A_{a b} & =2 d_{1} d_{4}+2 \frac{d_{3}}{M_{Z}^{2}}  \tag{B.10}\\
A_{a c} & =\frac{A_{a b}}{\left(2 p_{1} \cdot p_{2}+M_{H}^{2}\right)}  \tag{B.11}\\
A_{b c} & =\frac{2 d_{4}}{\left(2 p_{1} \cdot p_{2}+M_{H}^{2}\right)}, \tag{B.12}
\end{align*}
$$

with

$$
\begin{aligned}
d_{1}= & \frac{1}{2 p_{1} \cdot p_{3}+M_{H}^{2}}+\frac{1}{2 p_{2} \cdot p_{3}+M_{H}^{2}} \\
d_{2}= & \frac{\left(E_{1}^{2}-M_{H}^{2}\right)\left\{-M_{H}^{2} M_{Z}^{2}+\left(p_{2} \cdot p_{3}\right)^{2}\right\}}{M_{Z}^{2}\left(2 p_{2} \cdot p_{3}+M_{H}^{2}\right)^{2}}+\frac{\left(E_{2}^{2}-M_{H}^{2}\right)\left\{-M_{H}^{2} M_{Z}^{2}+\left(p_{1} \cdot p_{3}\right)^{2}\right\}}{M_{Z}^{2}\left(2 p_{1} \cdot p_{3}+M_{H}^{2}\right)^{2}} \\
& +2 \frac{\left(E_{1} E_{2}-p_{1} \cdot p_{2}\right)\left\{-M_{Z}^{2} p_{1} \cdot p_{2}+\left(p_{1} \cdot p_{3}\right)\left(p_{2} \cdot p_{3}\right)\right\}}{M_{Z}^{2}\left(2 p_{2} \cdot p_{3}+M_{H}^{2}\right)\left(2 p_{1} \cdot p_{3}+M_{H}^{2}\right)} \\
d_{3}= & \frac{M_{Z}^{2}\left(p_{1} \cdot p_{2}-E_{1} E_{2}\right)+p_{2} \cdot p_{3}\left(E_{1} E_{3}-p_{1} \cdot p_{3}\right)}{M_{Z}^{2}\left(2 p_{2} \cdot p_{3}+M_{H}^{2}\right)} \\
& +\frac{M_{Z}^{2}\left(p_{1} \cdot p_{2}-E_{1} E_{2}\right)+p_{1} \cdot p_{3}\left(E_{2} E_{3}-p_{2} \cdot p_{3}\right)}{M_{Z}^{2}\left(2 p_{1} \cdot p_{3}+M_{H}^{2}\right)} \\
d_{4}= & 2+\frac{E_{3}}{M_{Z}^{2}} .
\end{aligned}
$$

The axial and vector current couplings are

$$
\begin{equation*}
a_{e}=-1 \quad v_{e}=4 \sin ^{2} \theta_{W}-1 \tag{B.13}
\end{equation*}
$$

where $\theta_{W}$ denotes the Weinberg angle.

## APPENDIX C

## Coupling matrices

The coupling matrices of the Higgs bosons to the fermions can be derived from the Lagrangian (2.73) by an expansion in $h$ around $v$. As for the Higgs pair production the coupling matrix of one Higgs boson to two fermions, as well as the coupling matrix of two Higgs bosons to two fermions is needed they are given here. The coupling matrix of one Higgs boson to two fermions can also be found in Ref. [48]. The notation here is the same as in subsection [2.2.3.

The coupling matrix $G_{H f f}$
The Lagrangian of the couplings of one Higgs boson to two fermions is

$$
-\mathcal{L}^{H f f}=y H\left(\begin{array}{c}
t_{L}  \tag{C.1}\\
Q_{L}^{u} \\
X_{L}^{u} \\
T_{L}
\end{array}\right) \underbrace{\left(\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & s_{\alpha} c_{\alpha} & s_{\alpha} c_{\alpha} & \frac{1-2 s_{\alpha}^{2}}{\sqrt{2}} \\
0 & s_{\alpha} c_{\alpha} & s_{\alpha} c_{\alpha} & \frac{1-2 s_{\alpha}^{2}}{\sqrt{2}} \\
0 & \frac{1-2 s_{\alpha}^{2}}{\sqrt{2}} & \frac{1-2 s_{\alpha}^{2}}{\sqrt{2}} & -2 s_{\alpha} c_{\alpha}
\end{array}\right)}_{G_{H f \bar{f}}}\left(\begin{array}{c}
t_{R} \\
Q_{R}^{u} \\
X_{R}^{u} \\
T_{R}
\end{array}\right)+\text { h.c. . }
$$

Note that as long as no transformation to mass eigenstates is done, the coupling $g_{H t \bar{t}}=$ $G_{H f \bar{f}}(1,1)=0$. This is not surprising, as before the rotation to mass eigenstates the fundamental top (the top quark with no mixing) does not have a mass and therefore no coupling to the Higgs boson is expected.

The coupling matrix $G_{H H f \bar{f}}$
The Lagrangian of the couplings of two Higgs bosons to two fermions is

$$
-\mathcal{L}^{H H f f}=\frac{y}{f} H^{2}\left(\begin{array}{c}
t_{L}  \tag{C.2}\\
Q_{L}^{u} \\
X_{L}^{u} \\
T_{L}
\end{array}\right) \underbrace{\left(\begin{array}{cccc}
0 & 0 & 0 & 0 \\
0 & 1-2 s_{\alpha}^{2} & 1-2 s_{\alpha}^{2} & -4 \frac{c_{\alpha} s_{\alpha}}{\sqrt{2}} \\
0 & 1-2 s_{\alpha}^{2} & 1-2 s_{\alpha}^{2} & -4 \frac{c_{\alpha}^{2} s_{\alpha}}{\sqrt{2}} \\
0 & -4 \frac{c_{\alpha} s_{\alpha}}{\sqrt{2}} & -4 \frac{c_{\alpha} \alpha_{\alpha}}{\sqrt{2}} & -2\left(1-2 s_{\alpha}^{2}\right)
\end{array}\right)}_{G_{H H f \bar{f}}}\left(\begin{array}{c}
t_{R} \\
Q_{R}^{u} \\
X_{R}^{u} \\
T_{R}
\end{array}\right)+h . c .
$$

Note that this coupling will vanish in the SM limit $f \rightarrow \infty$, because it is $\sim \frac{1}{f}$.

## APPENDIX D

## Coupling matrices in $\mathrm{O}(\epsilon)$

The coupling matrices in first order are

$$
G_{H f \bar{f}}=\left(\begin{array}{cccc}
\frac{y \sin \theta_{L} \sin \theta_{R}}{\sqrt{2}} & v y A & -m_{L} v y \cos \theta_{L} B & -\frac{m_{L} y \cos \theta_{L} \cos \theta_{R}}{\sqrt{2} m_{\Psi}}  \tag{D.1}\\
-\frac{y \cos \theta_{L} \sin \theta_{R}}{\sqrt{2}} & -v y \cos \theta_{L} C & v y \cos \theta_{L} D & \frac{y \cos \theta_{L} \cos \theta_{R}}{\sqrt{2}} \\
-\frac{y \sin \theta_{R}}{\sqrt{2}} & -v y \cos \theta_{L} E & \frac{m_{R}^{2} v\left(2 m_{\Psi}+f y\right)}{2 f m_{\Psi}\left(m_{R}^{2}+f y\left(2 m_{\Psi}+f y\right)\right)} & \frac{y \cos \theta_{R}}{\sqrt{2}} \\
\frac{1}{2} y v F & \frac{y}{\sqrt{2}} & \frac{y}{\sqrt{2}} & v y \cos \theta_{R} G
\end{array}\right)
$$

where $A-G$ are functions of $f, y, m_{L}, m_{R}$ and $m_{\Psi}$ and are too long to be written down here. The first matrix element, which corresponds to the coupling of the Higgs bosons to two tops, is in first order in $\epsilon=\frac{v}{f}$ given by $g_{H t \bar{t}}=\frac{m_{t}}{v}$ as in the SM. For the coupling of two fermions to two Higgs bosons only the diagonal elements of the matrix are written down here, since the other matrix elements are not needed for the Higgs pair production process

$$
\begin{align*}
G_{H H f \bar{f}}(1,1) & =\frac{m_{L} m_{R} v y\left(\frac{2 f y}{m_{R}^{2}+\left(m_{\Psi}+f y\right)^{2}}-\frac{\frac{f y}{m_{\Psi}}+\frac{f m_{\Psi} y}{m_{L}^{2}+m_{\Psi}^{2}}+4}{m_{\Psi}+f y}\right) \cos \theta_{L} \cos \theta_{R}}{\sqrt{2} f^{2} m_{\Psi}} \\
G_{H H f \bar{f}}(2,2) & =\frac{y \cos \theta_{L}}{f}  \tag{D.2}\\
G_{H H f \bar{f}}(3,3) & =\frac{y}{f} \\
G_{H H f \bar{f}}(4,4) & =-\frac{2 y \cos \theta_{R}}{f} .
\end{align*}
$$

As expected these couplings vanish in the SM limit $f \rightarrow \infty$. The coupling $g_{H H t \bar{t}}$ only arises in first order in $\frac{v}{f}$.

## APPENDIX E

## Gluon fusion in a model with partially composite top: Form factors

The scalar integrals are here defined by

$$
\begin{align*}
A_{0}\left(m^{2}\right) & =\int \frac{\mathrm{d}^{4} q}{(2 \pi)^{4}} \frac{1}{q^{2}-m^{2}}  \tag{E.1}\\
B_{0}\left(p_{1}^{2}, m_{1}^{2}, m_{2}^{2}\right) & =\int \frac{\mathrm{d}^{4} q}{(2 \pi)^{4}} \frac{1}{\left(q^{2}-m_{1}^{2}\right)\left(\left(q+p_{1}\right)^{2}-m_{2}^{2}\right)}  \tag{E.2}\\
C_{0}\left(p_{1}^{2}, p_{2}^{2},\left(p_{1}+p_{2}\right)^{2}, m_{1}^{2}, m_{2}^{2}, m_{3}^{2}\right) & =\int \frac{\mathrm{d}^{4} q}{(2 \pi)^{4}} \frac{1}{\left(q^{2}-m_{1}^{2}\right)\left(\left(q+p_{1}\right)^{2}-m_{2}^{2}\right)\left(\left(q+p_{1}+p_{2}\right)^{2}-m_{3}^{2}\right)} \tag{E.3}
\end{align*}
$$

$$
\begin{align*}
& D_{0}\left(p_{1}^{2}, p_{2}^{2}, p_{3}^{2},\left(p_{1}+p_{2}+p_{3}\right)^{2},\left(p_{1}+p_{2}\right)^{2},\left(p_{2}+p_{3}\right)^{2}, m_{1}^{2}, m_{2}^{2}, m_{3}^{3}, m_{4}^{4}\right)= \\
& \int \frac{1}{(2 \pi)^{4}} \frac{\mathrm{~d}^{4} q}{\left(q^{2}-m_{1}^{2}\right)\left(\left(q+p_{1}\right)^{2}-m_{2}^{2}\right)\left(\left(q+p_{1}+p_{2}\right)^{2}-m_{3}^{2}\right)\left(\left(q+p_{1}+p_{2}+p_{3}\right)^{2}-m_{3}^{2}\right)} . \tag{E.4}
\end{align*}
$$

The tensor basis and the form factor for the triangles can be found in Appendix A. For the boxes, the form factors are

$$
\begin{aligned}
F_{\square}= & 2\left[2 s+4 m_{1}^{2} s C_{0}\left(0,0, s, m_{1}^{2}, m_{1}^{2}, m_{1}^{2}\right)+\right. \\
& \left(m_{1}^{2} M_{H}^{2}+2 m_{1} m_{2} M_{H}^{2}+m_{2}^{2} M_{H}^{2}-M_{H}^{4}-m_{1}^{2} t-2 m_{1} m_{2} t-m_{2}^{2} t+M_{H}^{2} t\right) \\
& C_{0}\left(M_{H}^{2}, 0, t, m_{1}^{2}, m_{2}^{2}, m_{2}^{2}\right) \\
& +\left(m_{1}^{2} M_{H}^{2}+2 m_{1} m_{2} M_{H}^{2}+m_{2}^{2} M_{H}^{2}-M_{H}^{4}-m_{1}^{2} t-2 m_{1} m_{2} t-m_{2}^{2} t+M_{H}^{2} t\right) \\
& C_{0}\left(M_{H}^{2}, 0, t, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right) \\
& +\left(m_{1}^{2} M_{H}^{2}+2 m_{1} m_{2} M_{H}^{2}+m_{2}^{2} M_{H}^{2}-M_{H}^{4}-m_{1}^{2} u-2 m_{1} m_{2} u-m_{2}^{2} u+M_{H}^{2} u\right) \\
& C_{0}\left(M_{H}^{2}, 0, u, m_{1}^{2}, m_{2}^{2}, m_{2}^{2}\right)
\end{aligned}
$$

$$
\begin{align*}
& +\left(m_{1}^{2} M_{H}^{2}+2 m_{1} m_{2} M_{H}^{2}+m_{2}^{2} M_{H}^{2}-M_{H}^{4}-m_{1}^{2} u-2 m_{1} m_{2} u-m_{2}^{2} u+M_{H}^{2} u\right) \\
& C_{0}\left(M_{H}^{2}, 0, u, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right) \\
& +\left(-3 m_{1}^{2} M_{H}^{4}-6 m_{1} m_{2} M_{H}^{4}-3 m_{2}^{2} M_{H}^{4}+M_{H}^{6}+m_{1}^{4} s+2 m_{1}^{3} m_{2} s+2 m_{1}^{2} m_{2}^{2} s\right. \\
& +2 m_{1} m_{2}^{3} s+m_{2}^{4} s+m_{1}^{2} M_{H}^{2} t+4 m_{1} m_{2} M_{H}^{2} t+m_{2}^{2} M_{H}^{2} t-m_{1} m_{2} t^{2}+m_{1}^{2} M_{H}^{2} u \\
& \left.+4 m_{1} m_{2} M_{H}^{2} u+m_{2}^{2} M_{H}^{2} u+m_{1}^{2} t u+m_{2}^{2} t u-M_{H}^{2} t u-m_{1} m_{2} u^{2}\right) \\
& D_{0}\left(M_{H}^{2}, 0, M_{H}^{2}, 0, t, u, m_{1}^{2}, m_{2}^{2}, m_{2}^{2}, m_{1}^{2}\right) \\
& +\left(2 m^{4} s+4 m_{1}^{3} m_{2} s+2 m_{1}^{2} m_{2}^{2} s-2 m_{1}^{2} M_{H}^{2} s-m_{1} m_{2} s^{2}\right) \\
& D_{0}\left(M_{H}^{2}, M_{H}^{2}, 0,0, s, t, m_{1}^{2}, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right) \\
& +\left(2 m_{1}^{4} s+4 m_{1}^{3} m_{2} s+2 m_{1}^{2} m_{2}^{2} s-2 m_{1}^{2} M_{H}^{2} s-m_{1} m_{2} s^{2}\right) \\
& \left.D_{0}\left(M_{H}^{2}, M_{H}^{2}, 0,0, s, u, m_{1}^{2}, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)\right] / s \tag{E.5}
\end{align*}
$$

$$
\begin{aligned}
G_{\square}= & {\left[4 s A_{0}\left(m_{1}^{2}\right)-4 s A_{0}\left(m_{2}^{2}\right)+\left(-4 m_{1}^{2} s+4 m_{2}^{2} s\right) B_{0}\left(0, m_{1}^{2}, m_{2}^{2}\right)+\right.} \\
& \left(4 m_{1}^{4} s+8 m_{1}^{3} m_{2} s-8 m_{1} m_{2}^{3} s-4 m_{2}^{4} s-2 M_{H}^{4} s+4 m_{1} m_{2} s t+4 m_{2}^{2} s t-\right. \\
& \left.s t^{2}+4 m_{1} m_{2} s u+4 m_{2}^{2} s u-s u^{2}\right) C_{0}\left(0,0, s, m_{1}^{2}, m_{1}^{2}, m_{1}^{2}\right)+ \\
& \left(-2 m_{1}^{4} M_{H}^{2}-4 m_{1}^{3} m_{2} M_{H}^{2}+4 m_{1} m_{2}^{3} M_{H}^{2}+2 m_{2}^{4} M_{H}^{2}+2 m_{1}^{4} t+4 m_{1}^{3} m_{2} t-\right. \\
& 4 m_{1} m_{2}^{3} t-2 m_{2}^{4} t+m_{1}^{2} M_{H}^{2} t-m_{2}^{2} M_{H}^{2} t-m_{1}^{2} t^{2}+m_{2}^{2} t^{2}+m_{1}^{2} M_{H}^{2} u-m_{2}^{2} M_{H}^{2} u- \\
& \left.m_{1}^{2} t u+m_{2}^{2} t u\right) C_{0}\left(0, t, M_{H}^{2}, m_{2}^{2}, m_{2}^{2}, m_{1}^{2}\right)+ \\
& \left(2 m_{1}^{4} M_{H}^{2}+4 m_{1}^{3} m_{2} M_{H}^{2}-4 m_{1} m_{2}^{3} M_{H}^{2}-2 m_{2}^{4} M_{H}^{2}-m_{1}^{2} M_{H}^{2} t+m_{2}^{2} M_{H}^{2} t-2 m_{1}^{4} u-\right. \\
& 4 m_{1}^{3} m_{2} u+4 m_{1} m_{2}^{3} u+2 m_{2}^{4} u-m_{1}^{2} M_{H}^{2} u+m_{2}^{2} M_{H}^{2} u+m_{1}^{2} t u-m_{2}^{2} t u+m_{1}^{2} u^{2}- \\
& \left.m_{2}^{2} u^{2}\right) C_{0}\left(0, u, M_{H}^{2}, m_{1}^{2}, m_{1}^{2}, m_{2}^{2}\right)+ \\
& \left(-2 m_{1}^{4} M_{H}^{2}-4 m_{1}^{3} m_{2} M_{H}^{2}+4 m_{1} m_{2}^{3} M_{H}^{2}+2 m_{2}^{4} M_{H}^{2}+2 M_{H}^{6}+2 m_{1}^{4} t+4 m_{1}^{3} m_{2} t-\right. \\
& 4 m_{1} m_{2}^{3} t-2 m_{2}^{4} t-3 m_{1}^{2} M_{H}^{2} t-8 m_{1} m_{2} M_{H}^{2} t-5 m_{2}^{2} M_{H}^{2} t-2 M_{H}^{4} t+3 m_{1}^{2} t^{2}+ \\
& \left.8 m_{1} m_{2} t^{2}+5 m_{2}^{2} t^{2}+2 M_{H}^{2} t^{2}-2 t^{3}+m_{1}^{2} M_{H}^{2} u-m_{2}^{2} M_{H}^{2} u-m_{1}^{2} t u+m_{2}^{2} t u\right) \\
& C_{0}\left(M_{H}^{2}, 0, t, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)+ \\
& \left(-2 m_{1}^{4} M_{H}^{2}-4 m_{1}^{3} m_{2} M_{H}^{2}+4 m_{1} m_{2}^{3} M_{H}^{2}+2 m_{2}^{4} M_{H}^{2}+m_{1}^{2} M_{H}^{2} t-m_{2}^{2} M_{H}^{2} t+\right. \\
& 2 m_{1}^{4} u+4 m_{1}^{3} m_{2} u-4 m_{1} m_{2}^{3} u-2 m_{2}^{4} u+m_{1}^{2} M_{H}^{2} u-m_{2}^{2} M_{H}^{2} u-m_{1}^{2} t u+m_{2}^{2} t u- \\
& \left.m_{1}^{2} u^{2}+m_{2}^{2} u^{2}\right) C_{0}\left(M_{H}^{2}, 0, u, m_{1}^{2}, m_{2}^{2}, m_{2}^{2}\right)+ \\
& \left(-4 m_{1}^{4} M_{H}^{2}-8 m_{1}^{3} m_{2} M_{H}^{2}+8 m_{1} m_{2}^{3} M_{H}^{2}+4 m_{2}^{4} M_{H}^{2}+2 M_{H}^{6}+2 m_{1}^{2} M_{H}^{2} t-\right. \\
& 2 m_{2}^{2} M_{H}^{2} t+4 m_{1}^{4} u+8 m_{1}^{3} m_{2} u-8 m_{1} m_{2}^{3} u-4 m_{2}^{4} u-2 m_{1}^{2} M_{H}^{2} u- \\
& 8 m_{1} m_{2} M_{H}^{2} u-6 m_{2}^{2} M_{H}^{2} u-2 M_{H}^{4} u-2 m_{1}^{2} t u+2 m_{2}^{2} t u+2 m_{1}^{2} u^{2}+8 m_{1} m_{2} u^{2}+ \\
& \left.6 m_{2}^{2} u^{2}+2 M_{H}^{2} u^{2}-2 u^{3}\right) C_{0}\left(M_{H}^{2}, 0, u, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)+ \\
& \left(4 m_{1}^{2} M_{H}^{4}+8 m_{1} m_{2} M_{H}^{4}+4 m_{2}^{2} M_{H}^{4}-2 M_{H}^{4} t-2 m_{1}^{2} t^{2}-4 m_{1} m_{2} t^{2}-2 m_{2}^{2} t^{2}+\right. \\
& \left.t^{3}-2 M_{H}^{4} u+t^{2} u-2 m_{1}^{2} u^{2}-4 m_{1} m_{2} u^{2}-2 m_{2}^{2} u^{2}+t u^{2}+u^{3}\right) \\
& \left.m_{1}^{2}, m_{2}^{2}, m_{1}^{2}\right)+
\end{aligned}
$$

$$
\begin{align*}
& \left(2 m_{1}^{4} M_{H}^{4}+4 m_{1}^{3} m_{2} M_{H}^{4}+4 m_{1}^{2} m_{2}^{2} M_{H}^{4}+4 m_{1} m_{2}^{3} M_{H}^{4}+2 m_{2}^{4} M_{H}^{4}-2 m_{1}^{6} s-\right. \\
& 4 m_{1}^{5} m_{2} s+2 m_{1}^{4} m_{2}^{2} s+8 m_{1}^{3} m_{2}^{3} s+2 m_{1}^{2} m_{2}^{4} s-4 m_{1} m_{2}^{5} s-2 m_{2}^{6} s+2 m_{1}^{4} M_{H}^{2} t- \\
& 4 m_{1}^{2} m_{2}^{2} M_{H}^{2} t+2 m_{2}^{4} M_{H}^{2} t-m_{1}^{2} M_{H}^{4} t-m_{2}^{2} M_{H}^{4} t-m_{1}^{4} t^{2}+2 m_{1}^{2} m_{2}^{2} t^{2}- \\
& m_{2}^{4} t^{2}+2 m_{1}^{4} M_{H}^{2} u-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} u+2 m_{2}^{4} M_{H}^{2} u-m_{1}^{2} M_{H}^{4} u-m_{2}^{2} M_{H}^{4} u- \\
& 4 m_{1}^{4} t u-4 m_{1}^{3} m_{2} t u-4 m_{1} m_{2}^{3} t u-4 m_{2}^{4} t u+m_{1}^{2} t^{2} u+m_{2}^{2} t^{2} u-m_{1}^{4} u^{2}+ \\
& \left.2 m_{1}^{2} m_{2}^{2} u^{2}-m_{2}^{4} u^{2}+m_{1}^{2} t u^{2}+m_{2}^{2} t u^{2}\right) D_{0}\left(M_{H}^{2}, 0, M_{H}^{2}, 0, t, u, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}, m_{2}^{2}\right)+ \\
& \left(4 m_{1}^{4} M_{H}^{4}+8 m_{1}^{3} m_{2} M_{H}^{4}+4 m_{1}^{2} m_{2}^{2} M_{H}^{4}+2 m_{1}^{2} M_{H}^{6}-2 m_{1}^{6} s-4 m_{1}^{5} m_{2} s\right. \\
& +2 m_{1}^{4} m_{2}^{2} s+8 m_{1}^{3} m_{2}^{3} s+2 m_{1}^{2} m_{2}^{4} s-4 m_{1} m_{2}^{5} s-2 m_{2}^{6} s+2 m_{1}^{4} M_{H}^{2} s- \\
& m_{2}^{2} M_{H}^{4} s-m_{1}^{4} s^{2}-8 m_{1}^{4} M_{H}^{2} t-16 m_{1}^{3} m_{2} M_{H}^{2} t-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} t-3 m_{1}^{2} M_{H}^{4} t+ \\
& 8 m_{1} m_{2}^{3} s t+5 m_{2}^{4} s t+M_{H}^{4} s t+4 m_{1}^{4} t^{2}+8 m_{1}^{3} m_{2} t^{2}+2 m_{1}^{2} m_{2}^{2} t^{2}- \\
& 4 m_{1} m_{2} s t^{2}-4 m_{2}^{2} s t^{2}+s t^{3}-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} u-3 m_{1}^{2} M_{H}^{4} u+m_{2}^{4} s u+ \\
& \left.2 m_{1}^{2} M_{H}^{2} t u-m_{2}^{2} s t u+m_{1}^{2} t^{2} u+2 m_{1}^{2} m_{2}^{2} u^{2}+m_{1}^{2} t u^{2}\right) \\
& D_{0}\left(M_{H}^{2}, M_{H}^{2}, 0,0, s, t, m_{1}^{2}, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)+ \\
& \left(4 m_{1}^{4} M_{H}^{4}+8 m_{1}^{3} m_{2} M_{H}^{4}+4 m_{1}^{2} m_{2}^{2} M_{H}^{4}+2 m_{1}^{2} M_{H}^{6}-2 m_{1}^{6} s-4 m_{1}^{5} m_{2} s\right. \\
& +2 m_{1}^{4} m_{2}^{2} s+8 m_{1}^{3} m_{2}^{3} s+2 m_{1}^{2} m_{2}^{4} s-4 m_{1} m_{2}^{5} s-2 m_{2}^{6} s+2 m_{1}^{4} M_{H}^{2} s- \\
& m_{2}^{2} M_{H}^{4} s-m_{1}^{4} s^{2}-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} t-3 m_{1}^{2} M_{H}^{4} t+m_{2}^{4} s t+2 m_{1}^{2} m_{2}^{2} t^{2}- \\
& 8 m_{1}^{4} M_{H}^{2} u-16 m_{1}^{3} m_{2} M_{H}^{2} u-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} u-3 m_{1}^{2} M_{H}^{4} u+8 m_{1} m_{2}^{3} s u+ \\
& 5 m_{2}^{4} s u+M_{H}^{4} s u+2 m_{1}^{2} M_{H}^{2} t u-m_{2}^{2} s t u+m_{1}^{2} t^{2} u+4 m_{1}^{4} u^{2}+8 m_{1}^{3} m_{2} u^{2}+ \\
& \left.2 m_{1}^{2} m_{2}^{2} u^{2}-4 m_{1} m_{2} s u^{2}-4 m_{2}^{2} s u^{2}+m_{1}^{2} t u^{2}+s u^{3}\right) \\
& \left.\left.D_{0}\left(M_{H}^{2}, M_{H}^{2}, 0,0, s, u, m_{1}^{2}, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)\right)\right] /\left(M_{H}^{4}-t u\right) \tag{E.6}
\end{align*}
$$

The form factors are all finite, since the divergent one- and two-point scalar integrals $A_{0}$ and $B_{0}$ cancel if all combinations for the fermions are taken into account. For $m_{1}=m$ and $m_{2}=m\left(\right.$ E.5) and (E.6) become the same as $F_{\square}$ and $G_{\square}$ in Appendix A. The form factors for part (2) of the couplings where each Higgs coupling gets a $\gamma_{5}$ matrix are:

$$
\begin{aligned}
F_{\square, 5}= & 2\left[2 s+4 m_{1}^{2} s C_{0}\left(0,0, s, m_{1}^{2}, m_{1}^{2}, m_{1}^{2}\right)+\right. \\
& \left(m_{1}^{2} M_{H}^{2}-2 m_{1} m_{2} M_{H}^{2}+m_{2}^{2} M_{H}^{2}-M_{H}^{4}-m_{1}^{2} t+2 m_{1} m_{2} t-m_{2}^{2} t+M_{H}^{2} t\right) \\
& C_{0}\left(M_{H}^{2}, 0, t, m_{2}^{2}, m_{2}^{2}, m_{1}^{2}\right) \\
& +\left(m_{1}^{2} M_{H}^{2}-2 m_{1} m_{2} M_{H}^{2}+m_{2}^{2} M_{H}^{2}-M_{H}^{4}-m_{1}^{2} t+2 m_{1} m_{2} t-m_{2}^{2} t+M_{H}^{2} t\right) \\
& C_{0}\left(M_{H}^{2}, 0, t, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right) \\
& +\left(m_{1}^{2} M_{H}^{2}-2 m_{1} m_{2} M_{H}^{2}+m_{2}^{2} M_{H}^{2}-M_{H}^{4}-m_{1}^{2} u+2 m_{1} m_{2} u-m_{2}^{2} u+M_{H}^{2} u\right) \\
& C_{0}\left(M_{H}^{2}, 0, u, m_{1}^{2}, m_{1}^{2}, m_{2}^{2}\right) \\
& +\left(m_{1}^{2} M_{H}^{2}-2 m_{1} m_{2} M_{H}^{2}+m_{2}^{2} M_{H}^{2}-M_{H}^{4}-m_{1}^{2} u+2 m_{1} m_{2} u-m_{2}^{2} u+M_{H}^{2} u\right) \\
& C_{0}\left(M_{H}^{2}, 0, u, m_{1}^{2}, m_{2}^{2}, m_{2}^{2}\right)
\end{aligned}
$$

$$
\begin{align*}
& +\left(-3 m_{1}^{2} M_{H}^{4}+6 m_{1} m_{2} M_{H}^{4}-3 m_{2}^{2} M_{H}^{4}\right. \\
& +M_{H}^{6}+m_{1}^{4} s-2 m_{1}^{3} m_{2} s+2 m_{1}^{2} m_{2}^{2}+2 m_{1} m_{2}^{3} s+m_{2}^{4} s \\
& +m_{1}^{2} M_{H}^{2} t-4 m_{1} m_{2} M_{H}^{2} t+m_{2}^{2} M_{H}^{2} t+m_{1} m_{2} t^{2} \\
& +m_{1}^{2} M_{H}^{2} u-4 m_{1} m_{2} M_{H}^{2} u+m_{2}^{2} M_{H}^{2} u+m_{1}^{2} t u+m_{2}^{2} t u \\
& \left.-M_{H}^{2} t u+m_{1} m_{2} u^{2}\right) D_{0}\left(M_{H}^{2}, 0, M_{H}^{2}, 0, t, u, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}, m_{2}^{2}\right) \\
& +\left(2 m^{4} s-4 m_{1}^{3} m_{2} s+2 m_{1}^{2} m_{2}^{2} s-2 m_{1}^{2} M_{H}^{2} s+m_{1} m_{2} s^{2}\right) \\
& D_{0}\left(M_{H}^{2}, 0,0, M_{H}^{2}, t, s, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}, m_{1}^{2}\right) \\
& +\left(2 m_{1}^{4} s-4 m_{1}^{3} m_{2} s+2 m_{1}^{2} m_{2}^{2} s-2 m_{1}^{2} M_{H}^{2} s+m_{1} m_{2} s^{2}\right) \\
& \left.D_{0}\left(M_{H}^{2}, 0,0, u, s, M_{H}^{2}, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}, m_{1}^{2}\right)\right] / s \tag{E.7}
\end{align*}
$$

$$
\begin{aligned}
G_{\square, 5}= & {\left[4 s A_{0}\left(m_{1}^{2}\right)-4 s A_{0}\left(m_{2}^{2}\right)+\left(-4 m_{1}^{2} s+4 m_{2}^{2} s\right) B_{0}\left(0, m_{1}^{2}, m_{2}^{2}\right)+\right.} \\
& \left(4 m_{1}^{4} s-8 m_{1}^{3} m_{2} s+8 m_{1} m_{2}^{3} s-4 m_{2}^{4} s-2 M_{H}^{4} s-4 m_{1} m_{2} s t+4 m_{2}^{2} s t-\right. \\
& \left.s t^{2}-4 m_{1} m_{2} s u+4 m_{2}^{2} s u-s u^{2}\right) C_{0}\left(0,0, s, m_{1}^{2}, m_{1}^{2}, m_{1}^{2}\right)+ \\
& \left(-2 m_{1}^{4} M_{H}^{2}+4 m_{1}^{3} m_{2} M_{H}^{2}-4 m_{1} m_{2}^{3} M_{H}^{2}+2 m_{2}^{4} M_{H}^{2}+2 m_{1}^{4} t-4 m_{1}^{3} m_{2} t+\right. \\
& 4 m_{1} m_{2}^{3} t-2 m_{2}^{4} t+m_{1}^{2} M_{H}^{2} t-m_{2}^{2} M_{H}^{2} t-m_{1}^{2} t^{2}+m_{2}^{2} t^{2}+m_{1}^{2} M_{H}^{2} u-m_{2}^{2} M_{H}^{2} u- \\
& \left.m_{1}^{2} t u+m_{2}^{2} t u\right) C_{0}\left(0, t, M_{H}^{2}, m_{2}^{2}, m_{2}^{2}, m_{1}^{2}\right)+ \\
& \left(2 m_{1}^{4} M_{H}^{2}-4 m_{1}^{3} m_{2} M_{H}^{2}+4 m_{1} m_{2}^{3} M_{H}^{2}-2 m_{2}^{4} M_{H}^{2}-m_{1}^{2} M_{H}^{2} t+m_{2}^{2} M_{H}^{2} t-2 m_{1}^{4} u+\right. \\
& 4 m_{1}^{3} m_{2} u-4 m_{1} m_{2}^{3} u+2 m_{2}^{4} u-m_{1}^{2} M_{H}^{2} u+m_{2}^{2} M_{H}^{2} u+m_{1}^{2} t u-m_{2}^{2} t u+m_{1}^{2} u^{2}- \\
& \left.m_{2}^{2} u^{2}\right) C_{0}\left(0, u, M_{H}^{2}, m_{1}^{2}, m_{1}^{2}, m_{2}^{2}\right)+ \\
& \left(-2 m_{1}^{4} M_{H}^{2}+4 m_{1}^{3} m_{2} M_{H}^{2}-4 m_{1} m_{2}^{3} M_{H}^{2}+2 m_{2}^{4} M_{H}^{2}+2 M_{H}^{6}+2 m_{1}^{4} t-4 m_{1}^{3} m_{2} t+\right. \\
& 4 m_{1} m_{2}^{3} t-2 m_{2}^{4} t-3 m_{1}^{2} M_{H}^{2} t+8 m_{1} m_{2} M_{H}^{2} t-5 m_{2}^{2} M_{H}^{2} t-2 M_{H}^{4} t+3 m_{1}^{2} t^{2}- \\
& \left.8 m_{1} m_{2} t^{2}+5 m_{2}^{2} t^{2}+2 M_{H}^{2} t^{2}-2 t^{3}+m_{1}^{2} M_{H}^{2} u-m_{2}^{2} M_{H}^{2} u-m_{1}^{2} t u+m_{2}^{2} t u\right) \\
& C_{0}\left(M_{H}^{2}, 0, t, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)+ \\
& \left(-2 m_{1}^{4} M_{H}^{2}+4 m_{1}^{3} m_{2} M_{H}^{2}-4 m_{1} m_{2}^{3} M_{H}^{2}+2 m_{2}^{4} M_{H}^{2}+m_{1}^{2} M_{H}^{2} t-m_{2}^{2} M_{H}^{2} t+\right. \\
& 2 m_{1}^{4} u-4 m_{1}^{3} m_{2} u+4 m_{1} m_{2}^{3} u-2 m_{2}^{4} u+m_{1}^{2} M_{H}^{2} u-m_{2}^{2} M_{H}^{2} u-m_{1}^{2} t u+m_{2}^{2} t u- \\
& \left.m_{1}^{2} u^{2}+m_{2}^{2} u^{2}\right) C_{0}\left(M_{H}^{2}, 0, u, m_{1}^{2}, m_{2}^{2}, m_{2}^{2}\right)+ \\
& \left(-4 m_{1}^{4} M_{H}^{2}+8 m_{1}^{3} m_{2} M_{H}^{2}-8 m_{1} m_{2}^{3} M_{H}^{2}+4 m_{2}^{4} M_{H}^{2}+2 M_{H}^{6}+2 m_{1}^{2} M_{H}^{2} t-\right. \\
& 2 m_{2}^{2} M_{H}^{2} t+4 m_{1}^{4} u-8 m_{1}^{3} m_{2} u+8 m_{1} m_{2}^{3} u-4 m_{2}^{4} u-2 m_{1}^{2} M_{H}^{2} u+ \\
& 8 m_{1} m_{2} M_{H}^{2} u-6 m_{2}^{2} M_{H}^{2} u-2 M_{H}^{4} u-2 m_{1}^{2} t u+2 m_{2}^{2} t u+2 m_{1}^{2} u^{2}-8 m_{1} m_{2} u^{2}+ \\
& \left.6 m_{2}^{2} u^{2}+2 M_{H}^{2} u^{2}-2 u^{3}\right) C_{0}\left(M_{H}^{2}, 0, u, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)+ \\
& \left(4 m_{1}^{2} M_{H}^{4}-8 m_{1} m_{2} M_{H}^{4}+4 m_{2}^{2} M_{H}^{4}-2 M_{H}^{4} t-2 m_{1}^{2} t^{2}+4 m_{1} m_{2} t^{2}-2 m_{2}^{2} t^{2}+\right. \\
& \left.t^{3}-2 M_{H}^{4} u+t^{2} u-2 m_{1}^{2} u^{2}+4 m_{1} m_{2} u^{2}-2 m_{2}^{2} u^{2}+t m_{1}^{2}+m_{2}^{3}\right) \\
& \left.C_{1}^{2}\right)+
\end{aligned}
$$

$$
\begin{align*}
& \left(2 m_{1}^{4} M_{H}^{4}-4 m_{1}^{3} m_{2} M_{H}^{4}+4 m_{1}^{2} m_{2}^{2} M_{H}^{4}-4 m_{1} m_{2}^{3} M_{H}^{4}+2 m_{2}^{4} M_{H}^{4}-2 m_{1}^{6} s+\right. \\
& 4 m_{1}^{5} m_{2} s+2 m_{1}^{4} m_{2}^{2} s-8 m_{1}^{3} m_{2}^{3} s+2 m_{1}^{2} m_{2}^{4} s+4 m_{1} m_{2}^{5} s-2 m_{2}^{6} s+2 m_{1}^{4} M_{H}^{2} t- \\
& 4 m_{1}^{2} m_{2}^{2} M_{H}^{2} t+2 m_{2}^{4} M_{H}^{2} t-m_{1}^{2} M_{H}^{4} t-m_{2}^{2} M_{H}^{4} t-m_{1}^{4} t^{2}+2 m_{1}^{2} m_{2}^{2} t^{2}- \\
& m_{2}^{4} t^{2}+2 m_{1}^{4} M_{H}^{2} u-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} u+2 m_{2}^{4} M_{H}^{2} u-m_{1}^{2} M_{H}^{4} u-m_{2}^{2} M_{H}^{4} u- \\
& 4 m_{1}^{4} t u+4 m_{1}^{3} m_{2} t u+4 m_{1} m_{2}^{3} t u-4 m_{2}^{4} t u+m_{1}^{2} t^{2} u+m_{2}^{2} t^{2} u-m_{1}^{4} u^{2}+ \\
& \left.2 m_{1}^{2} m_{2}^{2} u^{2}-m_{2}^{4} u^{2}+m_{1}^{2} t u^{2}+m_{2}^{2} t u^{2}\right) D_{0}\left(M_{H}^{2}, 0, M_{H}^{2}, 0, t, u, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}, m_{2}^{2}\right)+ \\
& \left(4 m_{1}^{4} M_{H}^{4}-8 m_{1}^{3} m_{2} M_{H}^{4}+4 m_{1}^{2} m_{2}^{2} M_{H}^{4}+2 m_{1}^{2} M_{H}^{6}-2 m_{1}^{6} s+4 m_{1}^{5} m_{2} s\right. \\
& +2 m_{1}^{4} m_{2}^{2} s-8 m_{1}^{3} m_{2}^{3} s+2 m_{1}^{2} m_{2}^{4} s+4 m_{1} m_{2}^{5} s-2 m_{2}^{6} s+2 m_{1}^{4} M_{H}^{2} s- \\
& m_{2}^{2} M_{H}^{4} s-m_{1}^{4} s^{2}-8 m_{1}^{4} M_{H}^{2} t+16 m_{1}^{3} m_{2} M_{H}^{2} t-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} t-3 m_{1}^{2} M_{H}^{4} t- \\
& 8 m_{1} m_{2}^{3} s t+5 m_{2}^{4} s t+M_{H}^{4} s t+4 m_{1}^{4} t^{2}-8 m_{1}^{3} m_{2} t^{2}+2 m_{1}^{2} m_{2}^{2} t^{2}+ \\
& 4 m_{1} m_{2} s t^{2}-4 m_{2}^{2} s t^{2}+s t^{3}-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} u-3 m_{1}^{2} M_{H}^{4} u+m_{2}^{4} s u+ \\
& \left.2 m_{1}^{2} M_{H}^{2} t u-m_{2}^{2} s t u+m_{1}^{2} t^{2} u+2 m_{1}^{2} m_{2}^{2} u^{2}+m_{1}^{2} t u^{2}\right) \\
& D_{0}\left(M_{H}^{2}, M_{H}^{2}, 0,0, s, t, m_{1}^{2}, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)+ \\
& \left(4 m_{1}^{4} M_{H}^{4}-8 m_{1}^{3} m_{2} M_{H}^{4}+4 m_{1}^{2} m_{2}^{2} M_{H}^{4}+2 m_{1}^{2} M_{H}^{6}-2 m_{1}^{6} s+4 m_{1}^{5} m_{2} s\right. \\
& +2 m_{1}^{4} m_{2}^{2} s-8 m_{1}^{3} m_{2}^{3} s+2 m_{1}^{2} m_{2}^{4} s+4 m_{1} m_{2}^{5} s-2 m_{2}^{6} s+2 m_{1}^{4} M_{H}^{2} s- \\
& m_{2}^{2} M_{H}^{4} s-m_{1}^{4} s^{2}-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} t-3 m_{1}^{2} M_{H}^{4} t+m_{2}^{4} s t+2 m_{1}^{2} m_{2}^{2} t^{2}- \\
& 8 m_{1}^{4} M_{H}^{2} u+16 m_{1}^{3} m_{2} M_{H}^{2} u-4 m_{1}^{2} m_{2}^{2} M_{H}^{2} u-3 m_{1}^{2} M_{H}^{4} u-8 m_{1} m_{2}^{3} s u+ \\
& 5 m_{2}^{4} s u+M_{H}^{4} s u+2 m_{1}^{2} M_{H}^{2} t u-m_{2}^{2} s t u+m_{1}^{2} t^{2} u+4 m_{1}^{4} u^{2}-8 m_{1}^{3} m_{2} u^{2}+ \\
& \left.2 m_{1}^{2} m_{2}^{2} u^{2}+4 m_{1} m_{2} s u^{2}-4 m_{2}^{2} s u^{2}+m_{1}^{2} t u^{2}+s u^{3}\right) \\
& \left.\left.D_{0}\left(M_{H}^{2}, M_{H}^{2}, 0,0, s, u, m_{1}^{2}, m_{2}^{2}, m_{1}^{2}, m_{1}^{2}\right)\right)\right] /\left(M_{H}^{4}-t u\right) \tag{E.8}
\end{align*}
$$

The matrix elements are

$$
\mathcal{M}=\mathcal{M}_{\triangle}+\mathcal{M}_{\triangle, \text { new }}+\mathcal{M}_{\square}
$$

with

$$
\begin{align*}
\mathcal{M}_{\triangle}= & \sum_{\substack{i=1 \\
m=m_{i}}}^{4} g_{s}^{2} G_{H f f}(i, i) g_{h h h} \frac{1}{s-M_{H}^{2}} F_{\triangle} A_{1}^{\mu \nu} \epsilon_{\mu}\left(p_{1}\right) \epsilon_{\nu}\left(p_{2}\right) \delta_{a b} \\
\mathcal{M}_{\triangle, \text { new }}= & \sum_{\substack{i=1 \\
m=m_{i}}}^{4} g_{s}^{2} G_{H H f f}(i, i) F_{\triangle} A_{1}^{\mu \nu} \epsilon_{\mu}\left(p_{1}\right) \epsilon_{\nu}\left(p_{2}\right) \delta_{a b} \\
\mathcal{M}_{\square}= & \sum_{\substack{i=1 \\
m_{1}=m_{i}}}^{4} \sum_{\substack{j=1 \\
m_{2}=m_{j}}}^{4}\left[g_{s}^{2}\left(\frac{G_{H f f}(i, j)+G_{H f f}(j, i)}{2}\right)^{2}\right. \\
& \left(F_{\square} A_{1}^{\mu \nu}+G_{\square} A_{2}^{\mu \nu}\right)+g_{s}^{2}\left(\frac{G_{H f f}(i, j)-G_{H f f}(j, i)}{2}\right)^{2} \\
& \left.\left(F_{\square, 5} A_{1}^{\mu \nu}+G_{\square, 5} A_{2}^{\mu \nu}\right)\right] \epsilon_{\mu}\left(p_{1}\right) \epsilon_{\nu}\left(p_{2}\right) \delta_{a b} . \tag{E.9}
\end{align*}
$$

$G_{H f f / H H f f}(\mathrm{i}, \mathrm{j})$ is a matrix element of the coupling matrix in the basis of the fermion mass eigenstates. The sums in (E.9) are needed because it is essential to take into account all combinations of the fermions $t, Q, X$ and $T$ in the fermion loop.

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## Zusammenfassung

Das Standardmodell der Elementarteilchenphysik beruht auf einer $S U(3)_{C} \times S U(2)_{L} \times$ $U(1)_{Y}$-Eichtheorie. Die $S U(3)_{C}$ ist die Farbsymmetrie der Quantenchromodynamik, die $S U(2)_{L} \times U(1)_{Y}$ beschreibt den elektroschwachen Sektor. Die $S U(2)_{L} \times U(1)_{Y}$ wird spontan zur elektromagnetischen Gruppe $U(1)_{e m}$ gebrochen. Im Standardmodell wird die elektroschwache Symmetriebrechung auf die einfachst mögliche Art verwirklicht: Ein einzelnes skalares Feld $\Phi$ erhält einen Vakuumerwartungswert ungleich Null. Dafür wird ein $\Phi^{4}$-Potential mit der typischen Minimaxform für das skalare Feld $\Phi$ benötigt. Das skalare Feld transformiert sich unter der fundamentalen Darstellung der $S U(2)_{L}$. Massen für die Eichbosonen werden mittels des skalaren Feldes und seines von Null verschiedenen Vakuumerwartungswertes generiert. Dadurch gehen drei der vier Freiheitsgrade des skalaren Feldes in die longitudinalen Polarisationszustände der massiven Eichbosonen über. Ein Freiheitsgrad jedoch bleibt übrig: Das Higgs Boson. Durch die Einführung des skalaren Feldes und damit des Higgs Bosons bekommen die Eichbosonen nicht nur ihre Masse, sondern es wird auch die Unitarität der Streuung longitudinaler Eichbosonen hergestellt.
Obwohl das Higgs Boson bis jetzt experimentell noch nicht gefunden wurde, konnte das Standardmodell bisher alle elektroschwachen Präzessionsmessungen erklären. Außer der Tatsache, dass das Higgs Boson bisher noch nicht gefunden wurde, gibt es noch weitere Gründe, die für eine Erweiterung des Standardmodell sprechen: Beispielsweise wird im Standardmodell keinerlei Erklärung für die vierte fundamentale Kraft - die Gravitation - geliefert, weswegen das Standardmodell häufig als eine effektive Niederenergietheorie bezeichnet wird. Im ersten Teil des Kapitels 2 wird eine kurze Beschreibung des Standardmodells mit Schwerpunkt auf den Higgssektor gegeben. Am Ende des ersten Teils wird auf ein weiteres Problem des Standardmodells eingegangen, das Hierarchieproblem.
Der zweite Teil des Kapitels 2 beschäftigt sich mit einer Alternative zum Standardmodell, den Composite Higgs Modellen. In den Composite Higgs Modellen ist das Higgs Boson kein fundamentales Teilchen mehr, sondern ein gebundener Zustand eines neuen stark wechselwirkenden Sektors. Das Higgs Boson ist ein Pseudo-Nambu-Goldstone Boson einer näherungsweisen globalen Symmetrie, welche bei einer Skala $f$ gebrochen
wird. Die ungebrochene Symmetrie muss die Standardmodelleichguppe als Untergruppe enthalten. Da das Higgs Boson als Pseudo-Nambu-Goldstone Boson auftritt, ist es ganz natürlich durch einen großen Massenunterschied von den anderen typischen Resonanzen des stark wechselwirkenden Sektors getrennt. Zunächst wurden solche Modelle unabhängig von der zugrunde liegenden Eichsymmetrie im Rahmen einer effektiven Theorie bei niedrigen Energien, bei denen außer dem Higgs Boson alle weiteren Resonanzen des stark wechselwirkenden Sektors ausintegriert werden können, diskutiert. Die effektive Lagrangedichte dieses stark wechselwirkenden leichten Higgs Bosons gewinnt man durch eine Entwicklung erster Ordnung in $\xi=(v / f)^{2}$, wobei $v$ die Skala der elektroschwachen Symmetriebrechung ist und $f$ die Skala des stark wechselwirkenden Sektors. Für $\xi=0$ ist man beim Standardmodell, $\xi \rightarrow 1$ entspricht dem Technicolorlimes. Da diese Lagrangedichte eine Entwicklung in $\xi$ darstellt, ist sie nur für sehr kleine $\xi$ und damit in der Nähe des Standardmodelllimes ausreichend. Für größere $\xi$ ist eine Resummation notwendig, die man erhält, wenn man explizit eine globale Symmetrie zugrunde legt. Dies wurde in Kapitel 2 im Rahmen minimaler Modelle diskutiert. Diese minimalen Modelle haben als globale Symmetriegruppe die $S O(5)$, welche bei einer Skala $f$ zur $S O(4)$ gebrochen wird. Sie unterscheiden sich jedoch durch die Darstellung der Gruppe, in der die Fermionen transformieren. In diesem Kontext wurden das $\mathrm{MCHM}_{4}$ mit Fermionen in der spinoriellen Darstellung und das $\mathrm{MCHM}_{5}$ mit Fermionen in der fundamentalen Darstellung diskutiert. Phänomenologisch unterscheiden sich die bislang genannten Modelle für niedrige Energien vom Standardmodell nur dahingehend, dass die Higgs-Kopplungen modifiziert werden. Am Ende des Kapitels 2 wurde jedoch noch eine Alternative zu diesen Modellen betrachtet, in der die Fermionen des neuen Sektors Massen unterhalb des Abschneideparameters der effektiven Theorie haben, weswegen sie explizit in der Theorie auftreten. Dieses Modell ist eine Erweiterung des $\mathrm{MCHM}_{5}$. Die neuen Fermionen koppeln auch an das Top-Quark, wodurch ein Mischen des Tops mit den neuen Teilchen stattfindet. Dadurch erhält das Top-Quark seine Masse.
Um den Mechanismus der elektroschwachen Symmetriebrechung zu verstehen, muss das Potential bekannt sein. Dieses kann experimentell bestimmt werden, indem man Higgsselbstkopplungen misst. Diese sind in der Multihiggsproduktion zugänglich. Diese Diplomarbeit konzentriert sich auf die Higgspaarproduktion, mit deren Hilfe man die Higgs-Selbstkopplung zwischen drei Higgs Bosonen messen kann. Die Produktion von mehr als zwei Higgs Bosonen führt zu sehr kleinen Wirkungsquerschnitten, die daher nicht am LHC gemessen werden können.
In Kapitel 3 werden die drei wichtigsten Prozesse für Higgspaarproduktion betrachtet: Gluonfusion, Vektorbosonfusion und doppelte Higgsstrahlung. Auch in den betrachteten minimalen Composite Higgs Modellen ist die Gluonfusion wie im Standardmodell der Prozess mit dem höchsten Wirkungsquerschnitt. Dieser vergrößert sich sogar drastisch für große $\xi$. Der Grund hierfür ist, dass eine weiteres Feynmandiagramm hinzukommt, da es im Composite Higgs Modell eine Kopplung gibt, bei der zwei Higgs Bosonen an zwei Fermionen koppeln. Diese wurde im Rahmen dieser Diplomarbeit für die beiden betrachteten minimalen Composite Higgs Modelle hergeleitet. Die Wirkungsquerschnitte der drei verschiedenen Prozesse wurde verglichen. Es zeigt sich, dass sowohl der Wirkungsquerschnitt der Vektorbosonfusion als auch der Glu-
onfusion mit wachsendem $\xi$ ansteigt. Bei der doppelten Higgsstrahlung sind für die betrachteten Werte von $\xi$ die Wirkungsquerschnitte immer kleiner als im Standardmodell. Um dieses Verhalten zu verstehen wurde die Interferenzstruktur des jeweiligen Prozesses diskutiert.
Im Anschluss daran wurden die Aussichten untersucht, das angenommene Modell vom Standardmodell bei einer Luminosität von $300 \mathrm{fb}^{-1}$ in der Gluonfusion zu unterscheiden. Dies wurde für verschiedene Endzustände gemacht. Im $\mathrm{MCHM}_{4}$ ist dies ab $\xi \gtrsim 0.2$ möglich, je nach Higgs Masse entweder im $b \bar{b} \tau^{+} \tau^{-}$oder $W^{+} W^{-} W^{+} W^{-}$Endzustand. Im $\mathrm{MCHM}_{5}$ ist dies mit diesen beiden Endzuständen schon für wesentlich kleinere Werte von $\xi$ möglich.
Um die Aussichten für eine Messung der Triple-Higgs-Selbstkopplung $\lambda_{H H H}$ zu untersuchen, wurde angenommen, dass zur Zeit ihrer Messung alle anderen Kopplungen hinreichend gut bestimmt sind und im Rahmen der betrachteten minimalen Composite Higgs Modelle realisiert sind. Es wurden Sensitivitätsschaubilder in der $M_{H}-\xi$-Ebene erstellt für verschiedene Endzustände und verschiedene Variationen von $\lambda_{H н H}$ : Zum einen die Sensitivität auf nicht-verschwindendes $\lambda_{H H H}$ zum anderen auf eine Bestimmung von $\lambda_{H H H}$ mit $30 \%$ Genauigkeit. Dabei wurde festgestellt, dass eine nichtverschwindende Triple-Higgskopplung über nahezu den gesamten Parameterbereich in verschiedenen Endzuständen gemessen werden kann. Bei einer Bestimmung der Triple-Higgs-Kopplung auf $30 \%$ Genauigkeit sind die Aussichten deutlich schlechter. Auch bei einer höheren integrierten Luminosität $\left(600 \mathrm{fb}^{-1}\right)$ sind die Parameterbereiche vor allem bei niedrigen Higgsmassen deutlich reduziert. Da für eine komplette Analyse auch Untergrundprozesse und Detektoreigenschaften miteinbezogen werden müssen, stellt die durchgeführte Analyse nur die maximalen Parameterbereiche dar. In einem kurzen Abschnitt am Ende des dritten Kapitels wurde eine Abschätzung gegeben, wie sich die Untergründe in den betrachteten Modellen verhalten.
In Kapitel 4 wurde die Higgspaarproduktion an einem zukünftigen $e^{+} e^{-}$-Beschleuniger untersucht. Ein solcher $e^{+} e^{-}$-Beschleuniger wäre zum Beispiel der ILC - ein Linearcollider (der sich gerade in der Planungsphase befindet). Die wichtigsten Higgspaarproduktionsprozesse am $e^{+} e^{-}$-Beschleuniger sind doppelte Higgsstrahlung und $W$-Bosonfusion. Für diese beiden Prozesse wurden die Wirkungsquerschnitte bestimmt. Das Hochenergieverhalten ändert sich gegenüber dem Standardmodell in beiden Fällen. Bei der doppelten Higgsstrahlung kann für große $\xi$ festgestellt werden, dass im Gegensatz zum Standardmodell der Wirkungsquerschnitt mit steigender Schwerpunktsenergie zunimmt, jedoch für hohe Energien wird dieser Anstieg sehr viel schwächer. Bei der $W$-Bosonfusion wurde herausgefunden, dass die Amplitude der longitudinalen W-Bosonen mit Schwerpunktsenergie quadratisch zunimmt. Durch die modifizierten Higgskopplungen kann das Higgs Boson die Unitarität für diesen Prozess nicht mehr vollständig herstellen. Die Energie, bei der die Unitarität verletzt ist, ist jedoch zu höheren Energien als in einem Modell ohne Higgs Boson verschoben. Da die betrachteten Modelle jedoch immer auf effektiven Theorien beruhen, ist die Unitarität nicht verletzt, solange der Abschneideparameter der effektiven Theorie niedriger ist als die Energie, bei der die Unitarität verletzt wird. Diese kann durch die anderen Resonanzen des stark wechselwirkenden Sektors wiederhergestellt werden. Am Ende der jeweiligen Abschnitte wurden noch die Aussichten diskutiert, eine nichtverschwindende Triple-

Higgs-Kopplung zu messen. Diese sind in der doppelten Higgsstrahlung für eine Schwerpunktsenergie von $\sqrt{s}=500 \mathrm{GeV}$ und einer Luminosität von $500 \mathrm{fb}^{-1}$ sehr gut, für höhere Schwerpunktsenergien werden sie schlechter. In der $W$-Fusion kann man für eine sehr hohe Luminosität $\left(2000 \mathrm{fb}^{-1}\right)$ oder polarisierte Elektronen- und Positronenstrahlen bei eine Luminosität von $500 \mathrm{fb}^{-1}$ in den Endzuständen $b \bar{b} b \bar{b}$ und $W^{+} W^{-} W^{+} W^{-}$bei einer Schwerpunktsenergie von $\sqrt{s}=1 \mathrm{TeV}$ über einen großen Parameterbereich ein nichtverschwindendes $\lambda_{H H H}$ messen.
In Kapitel 5 wird Gluonfusion am LHC in einem Composite Higgs Modell untersucht, in dem es neue Fermionen mit Massen unterhalb des Abschneideparameters der effektiven Theorie gibt. Um in diesem Modell Rechnungen durchführen zu können, muss die Fermionen-Massenmatrix diagonalisiert werden. Dies wird im ersten Teil des Kapitels ausführlich erläutert. Der zweite Teil beschäftigt sich mit der Produktion von einem Higgs Boson mittels Gluonfusion. Anhand dieses etwas einfacheren Prozesses konnte überprüft werden, ob die Massendiagonalisierung richtig durchgeführt wurde. Beispielsweise sollte man den gleichen Wirkungsquerschnitt wie im Standardmodell erhalten, wenn in dem neuen Modell die Skala $f \rightarrow \infty$ gesetzt wird. In der Tat war dies der Fall. Desweiteren wurde festgestellt, dass man in der Higgsproduktion mittels Gluonfusion die Effekte der neuen Fermionen nicht direkt sehen kann. Man erhält nahezu den gleichen Wirkungsquerschnitt, wie in dem Fall, in dem die neuen Fermionen ausintegriert werden. Im letzten Teil des Kapitels wird Higgspaarproduktion mit den neuen Fermionen berechnet. Hier konnte festgestellt werden, dass der Wirkungsquerschnitt von den neuen Parameter des Modells abhängt und diese somit nicht einfach ausintegriert werden können. Am Ende des Kapitels wurde die Sensitivität des Wirkungsquerschnitts auf eine Änderung der Triple-Higgs-Kopplung untersucht. Die Wirkungsquerschnitte sind etwas kleiner als im $\mathrm{MCHM}_{5}$, dafür ändern sie sich jedoch im Verhältnis etwas stärker bei einer Variation von $\lambda_{H н н}$.
Insgesamt stehen die Aussichten gut in den untersuchten Modellen die Triple-HiggsSelbstkopplung zu messen und damit einen großen Schritt in Richtung Rekonstruktion des Higgspotentials zu unternehmen. Dafür muss der LHC jedoch bei voller Schwerpunktsenergie und höherer Luminosität laufen.

## Danksagung

In erster Linie möchte ich mich bei Frau Prof. Dr. Margarete Mühlleitner für die sehr intensive Betreuung bedanken. Sie hatte immer ein offenes Ohr für meine Probleme und Fragen und hat sich auch immer viel Zeit genommen, diese mit mir zu diskutieren. Dies gab mir die Möglichkeit sehr viel von ihr zu lernen. Desweiteren hat sie mich immer sehr unterstützt und ermutigt, auch darin eigene Ideen zu entwickeln. Ich danke ihr auch für die Korrekturvorschläge zu dieser Arbeit.
Bei Herrn Prof. Dr. Dieter Zeppenfeld möchte ich mich für seine Bereitschaft bedanken, dass Korreferat zu übernehmen.
Allen Institutsmitgliedern danke ich für eine sehr schöne Zeit und die große Hilfsbereitschaft. Insbesondere bei technischen Fragen haben mir jederzeit Bastian Feigl, Christian Röhr, Florian Geyer und Dr. Christoph Englert geholfen.
Zudem möchte ich mich herzlich bei meinen Zimmerkollegen bedanken für die gute Stimmung und die Diskussionsbereitschaft. Insbesondere konnten mir oft Franziska Schissler und Christian Hangst weiterhelfen. Für die Unterstützung bei der Verwendung von FeynCalc möchte ich Eva Popenda danken. Ein Dankeschön gilt auch Martin Köhl, der mich auf das LAPACK-Paket aufmerksam gemacht hat.
Mein besonderer Dank für die Beantwortung unzähliger Fragen gilt Dr. Michael Rauch und Dr. Heidi Rzehak. Beide haben sich immer sehr viel Zeit genommen meine Fragen zu beantworten oder mit mir meine Probleme zu diskutieren.
Bei Prof. Dr. Tilman Plehn möchte ich mich dafür bedanken, dass er sich einen ganzen Tag Zeit genommen hat, um alle meine Fragen zu beantworten. Für hilfreiche Anmerkungen zur zugehörigen Veröffentlichung und damit auch zu meiner Diplomarbeit möchte ich mich bei Prof. Dr. Jose Espinosa, Prof. Dr. Christophe Grojean und Dr. Michael Spira bedanken.
Für das Korrekturlesen meiner Diplomarbeit bedanke ich mich herzlich bei Christopher Blundell, Florian Geyer, Thorben Graf, Christian Hangst, Christian Röhr, Franziska Schissler und Lisa-Marie Zoller. Ganz besonders möchte ich mich dafür bei Dr. Sophy Palmer und Max Zoller bedanken, die beide die komplette Arbeit gelesen und korrigiert haben.
Meinen Eltern möchte ich für die Unterstützung während meines gesamten Studiums
bedanken.
Zu guter Letzt möchte ich mich bei Max Zoller bedanken, der mich immer unterstützt und ermutigt hat. Außerdem möchte ich ihm für die vielen hilfreichen Diskussionen danken.


[^0]:    Prof. Dr. Margarete Mühlleitner
    Karlsruhe, den 1. Februar 2011

[^1]:    ${ }^{1}$ The 0 -th partial wave gives the strictest bounds 7].
    ${ }^{2}$ The energy bound of Eq. (2.31) and the restriction on the Higgs mass of Eq. (2.34) becomes stricter, if a full coupled-channel computation is done involving $W, Z$ and $H$ cf. Ref. [21].

[^2]:    ${ }^{3}$ The quadratic divergence cannot be cancelled by simply setting the Higgs mass to $M_{H}=$ $\sqrt{4 m_{t}^{2}-M_{Z}^{2}-2 M_{W}^{2}} \sim 320 \mathrm{GeV}$, because by taking more loops into account, the general form of the corrections becomes $\Lambda^{2} \sum_{n=0}^{\infty} c_{n}\left(\lambda_{i}\right) \log ^{n}\left(\frac{\Lambda}{\mu}\right)$, where $\mu$ is the renormalization scale and $c_{n}$ denotes the coefficient, which depends on the couplings. These coefficients cannot be cancelled in all orders, cf. Refs. 19, 25).

[^3]:    ${ }^{4}$ A pseudo Nambu-Goldstone boson gets a mass at loop level in contrast to a Goldstone boson of an exact symmetry

[^4]:    ${ }^{5}$ Often the word warped is used, because the five-dimensional space-time is curved. But the four dimensional universe still appears to be static and flat for an observer on a boundary [40].
    ${ }^{6}$ Bulk is the interval $\left.\mathrm{z} \in\right] L_{0}, L_{1}[$.

[^5]:    ${ }^{7}$ For a non-minimal model see Ref. [42].
    ${ }^{8} \mathrm{Or} \mathrm{SO}(5) \rightarrow \mathrm{O}(4)$ in the model of Ref. 17.

[^6]:    ${ }^{9}$ In the next subsection the new fermionic states will not only be taken into account through form factors but also explicitly through terms such as (2.53). There this compositeness can be seen.

[^7]:    ${ }^{10}$ With these coefficients the mass of the fermions, the Higgs boson and the gauge bosons are still generated for every choice of $\xi$ (see Eq.(2.45), (2.46)).

[^8]:    ${ }^{11}$ For more details see, e.g. [45].
    ${ }^{12}$ The quarks are composite in the sense of a mixing between the SM quarks and the quarks of the strongly interacting sector. "More composite" means a stronger admixture of the heavy particles.
    ${ }^{13}$ Another scenario is, for example, where Q is not a doublet of new fermions but the SM doublet of left-handed top and bottom (cf. Refs. [44, 49]). But as that seems to be excluded by B-physics and contributions to the T parameter, this will not be discussed further here.

[^9]:    ${ }^{1}$ Despite the new coupling the process $\gamma \rightarrow H H$ is still zero as expected due to angular momentum conservation (the photon has spin 1 and the Higgs boson has spin 0).

[^10]:    ${ }^{2}$ The algorithm can be found in Ref. 60].

[^11]:    ${ }^{3}$ HPAIR is a program written by M.Spira and can be found at http://people.web.psi.ch/spira/proglist.html.

[^12]:    ${ }^{4}$ In a lot of papers [53, 65] the whole process is calculated in the effective $W$-approximation anyway. The effective $W$-approximation treats the $W$ bosons as constituents of the proton.
    ${ }^{5}$ For the SM this formula can be found in Ref. 67].

[^13]:    ${ }^{6}$ The decay channel $b \bar{b} \gamma \gamma$ is the most promising for the measurement of the trilinear Higgs coupling in the SM for low Higgs masses 76].

[^14]:    ${ }^{7}$ In Ref. [77], it was found that $\xi$ can be extracted with an accuracy of $O(20 \%)$ at the LHC.

[^15]:    ${ }^{8}$ Choosing - $30 \%$ would lead to quite similar sensitivity areas as was checked during the calculations.

[^16]:    ${ }^{9}$ Since for $\xi$ near 1 the branching ratio into $4 W$ in the $\mathrm{MCHM}_{5}$ becomes much smaller than in the SM.

[^17]:    ${ }^{1} \mathrm{~A}$ proof that it is always possible to find such a biunitary transformation to diagonalize a matrix $M$ can be found in Ref. [86].
    ${ }^{2}$ In Ref. [46] there seems to be a mistake in the signs. But the result of subsection [2.2.3 agrees with Ref. [47].

[^18]:    ${ }^{3}$ The couplings of the fermions to the gluons do not change through the transformation into mass eigenstates. This can be directly inferred from Eq. (5.6), since the kinetic term and the fermion gluon coupling have the same Lorentz structure.

[^19]:    ${ }^{4}$ Such a behaviour is not completely new: This is also the case in the MSSM, where the neutral pseudoscalar Higgs boson and the Goldstone boson couple in this manner to squarks [7].

