Spontane Symmetriebrechung im SM

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EW gauge-boson sector of the SM

Gauge invariance and renormalizability completely determine the kinetic terms for the gauge bosons

$$\mathcal{L}_{YM} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu}$$

$$\begin{aligned} B^{\mu\nu} &= \partial^\mu B^\nu - \partial^\nu B^\mu \\ W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon^{abc}\, W_{b,\mu}\, W_{c,\nu} \end{aligned}$$

The gauge symmetry does NOT allow any mass terms for $W^\pm$ and Z, i.e. forbidden are terms like

$$\mathcal{L}_{Mass} = \frac{1}{2} m_W^2 W^a_\mu W_a^\mu$$

Spontaneous symmetry breaking

Experimentally, the weak bosons are massive. We give mass to the gauge bosons through the Higgs mechanism: generate mass terms from the kinetic energy term of a scalar douplet field Φ that undergoes spontaneous symmetry breaking.

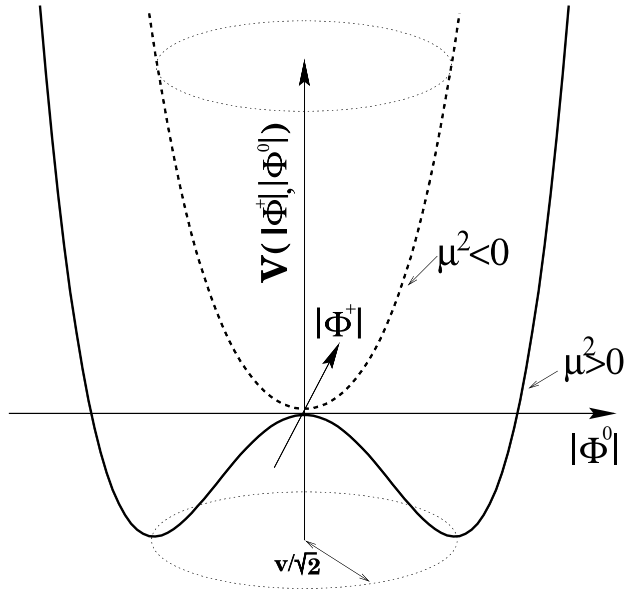
Introduce a complex scalar douplet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}, \qquad Y_\Phi = \frac{1}{2}$$

$$\mathcal{L}_{\text{Higgs}} = (D_\mu \Phi)^\dagger (D^\mu \Phi) - V\left(\Phi^\dagger \Phi\right)$$

$$D^\mu = \partial^\mu - igW_i^\mu \frac{\sigma^i}{2} - ig' Y_\Phi B^\mu$$

$$V\left(\Phi^\dagger\Phi\right) = V_0 - \mu^2 \Phi^\dagger\Phi + \lambda \left(\Phi^\dagger\Phi\right)^2, \qquad \mu^2, \lambda > 0$$



Notice the "wrong" mass sign.

$V\left(\Phi^\dagger\Phi\right)$ is $SU(2)_L \times U(1)_Y$ symmetric.

Spontaneous symmetry breaking

The potential has minima at

$$|\Phi|^2 = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}$$

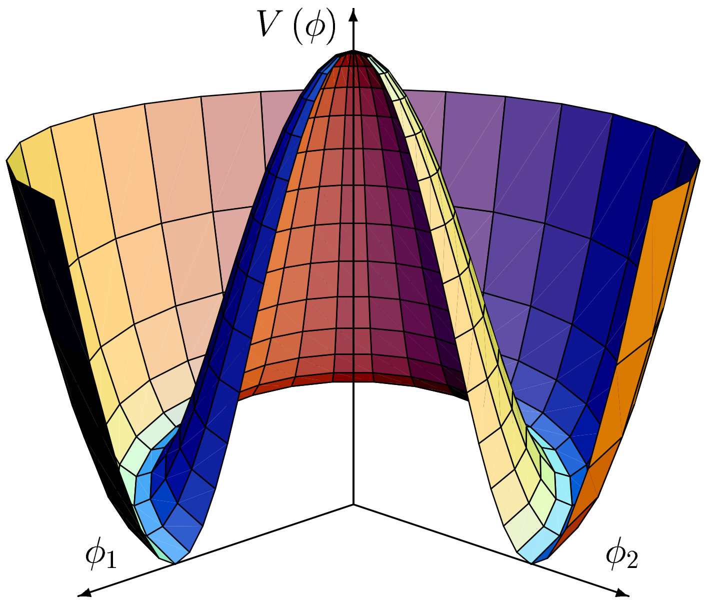
All these minimum configurations (ground states) are connected by gauge transformations, that change the phase of the complex field Φ, without affecting its modulus.

v is called the vacuum expectation value (VEV) of the neutral component of the Higgs doublet.

When the system chooses one of the minimum configurations, this configuration is no longer symmetric under the the gauge symmetry.

This is called spontaneous symmetry breaking.

The Lagrangian is still gauge invariant and all properties connected with it (such as current conservation) still hold!



Expanding Φ around the minimum

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} = \begin{pmatrix} \phi^+ \\ \frac{1}{\sqrt{2}}\left[v + H(x) + i\chi(x)\right] \end{pmatrix} = \frac{1}{\sqrt{2}} \exp\left[\frac{i\sigma_i \theta^i(x)}{v}\right] \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

We can rotate away the fields $\theta^i(x)$ by an $SU(2)_L$ gauge transformation

$$\Phi(x) \to \Phi'(x) = U(x)\Phi(x) = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

where $U(x) = \exp\left[-\frac{i\sigma_i\theta^i(x)}{v}\right]$.

This gauge choice, called unitary gauge, is equivalent to absorbing the Goldstone modes $\theta^i(x)$.

The vacuum state can be chosen to correspond to the vacuum expectation value

$$\Phi_0 = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}$$

Notice that only a scalar field can have a vacuum expectation value.The VEV of a fermion or vector field would break Lorentz invariance.

$\mathbf{SU(2)_L \times U(1)_Y \longrightarrow U(1)_{em}}$

Let's verify that the vacuum state breaks the gauge symmetry.

A state $\tilde{\Phi}$ is invariant under a symmetry operation $\exp(igT^a\theta_a)$ if

$$\exp(igT^a\theta_a)\tilde{\Phi} = \tilde{\Phi}$$

This means that a state is invariant if (just expand the exponent)

$$T^a\tilde{\Phi} = 0$$

For the $SU(2)_L \times U(1)_Y$ case we have

$$\sigma_1\Phi_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} v/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \qquad \text{broken}$$

$$\sigma_2\Phi_0 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} -iv/\sqrt{2} \\ 0 \end{pmatrix} \neq 0 \qquad \text{broken}$$

$\mathbf{SU(2)}_L \times \mathbf{U(1)}_Y \longrightarrow \mathbf{U(1)}_{\text{em}}$

$$\sigma_3 \Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ -v/\sqrt{2} \end{pmatrix} \neq 0 \qquad \text{broken}$$

$$Y \Phi_0 = \mathrm{Y}_\Phi \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = +\frac{1}{2} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \neq 0 \qquad \text{broken}$$

But, if we examine the effect of the electric charge operator $\hat{Q}$ on the (electrically neutral) vacuum state, we have ($\mathrm{Y}_\Phi = 1$)

$$\hat{Q}\Phi_0 = \left(\frac{1}{2}\sigma_3 + Y\right)\Phi_0 = \begin{pmatrix} \mathrm{Y}_\Phi + \frac{1}{2} & 0 \\ 0 & \mathrm{Y}_\Phi - \frac{1}{2} \end{pmatrix} \Phi_0 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

the electric charge symmetry is unbroken!

Consequences for the scalar field H

The scalar potential

$$V\left(\Phi^\dagger\Phi\right) = \lambda\left(\Phi^\dagger\Phi - \frac{v^2}{2}\right)^2$$

expanded around the vacuum state

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v + H(x) \end{pmatrix}$$

becomes

$$V = \frac{\lambda}{4}\left(2vH + H^2\right)^2 = \frac{1}{2}(2\lambda v^2)H^2 + \lambda v H^3 + \frac{\lambda}{4}H^4$$

Consequences:

- the scalar field H gets a mass which is given by the quartic coupling λ

$$m_H^2 = 2\lambda v^2$$

- there is a term of cubic and quartic self-coupling.

Higgs kinetic terms and coupling to W, Z

$$
\begin{aligned}
D^\mu\Phi &= \left(\partial^\mu - igW_i^\mu\frac{\sigma^i}{2} - ig'\frac{1}{2}B^\mu\right)\frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+H(x) \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2\sqrt{2}}\left[g\begin{pmatrix} W_3^\mu & W_1^\mu - iW_2^\mu \\ W_1^\mu + iW_2^\mu & -W_3^\mu \end{pmatrix} + g'B^\mu\right]\begin{pmatrix} 0 \\ v+H \end{pmatrix} \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}(v+H)\begin{pmatrix} g\left(W_1^\mu - iW_2^\mu\right) \\ -gW_3^\mu + g'B^\mu \end{pmatrix}\right] \\
&= \frac{1}{\sqrt{2}}\left[\begin{pmatrix} 0 \\ \partial^\mu H \end{pmatrix} - \frac{i}{2}\left(1+\frac{H}{v}\right)\begin{pmatrix} gv\sqrt{2}W^{\mu +} \\ -\sqrt{g^2+g'^2}vZ^\mu \end{pmatrix}\right]
\end{aligned}
$$

$$
\left(D^\mu\Phi\right)^\dagger D_\mu\Phi = \frac{1}{2}\partial^\mu H\partial_\mu H + \left[\left(\frac{gv}{2}\right)^2 W^{\mu +}W_\mu^- + \frac{1}{2}\frac{\left(g^2+g'^2\right)v^2}{4}Z^\mu Z_\mu\right]\left(1+\frac{H}{v}\right)^2
$$

We have defined

$$W^{\pm}_{\mu} = \frac{1}{\sqrt{2}}\left(W^1_\mu \mp i W^2_\mu\right) ,$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g W^3_\mu - g' B_\mu\right) = \cos\theta_W W^3_\mu - \sin\theta_W B_\mu$$

with

$$\sin\theta_W = \frac{g'}{\sqrt{g^2+g'^2}} , \qquad \cos\theta_W = \frac{g}{\sqrt{g^2+g'^2}} ,$$

The orthogonal combination of the neutral gauge fields is the photon

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}\left(g' W^3_\mu + g B_\mu\right) = \sin\theta_W W^3_\mu + \cos\theta_W B_\mu$$

With these definitions the gauge kinetic term

$$\mathcal{L}^{gauge}_{kin} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^3_{\mu\nu} W^{3,\mu\nu} - \frac{1}{2} W^+_{\mu\nu} W^{\mu\nu -}$$

and the mass term become

$$\mathcal{L}^{gauge}_{kin} = -\frac{1}{4} A_{\mu\nu} A^{\mu\nu} - \frac{1}{4} Z_{\mu\nu} Z^{\mu\nu} - \frac{1}{2} W^+_{\mu\nu} W^{\mu\nu -}$$

$$\mathcal{L}^{gauge}_{mass} = \left(\frac{gv}{2}\right)^2 W^{\mu +} W^-_\mu + \frac{1}{2}\frac{\left(g^2+g'^2\right) v^2}{4} Z^\mu Z_\mu$$

Consequences

- The W and Z gauge bosons have acquired masses

$$m_W^2 = \frac{g^2 v^2}{4} \qquad m_Z^2 = \frac{\left(g^2 + g'^2\right) v^2}{4} = \frac{m_W^2}{\cos^2\theta_W}$$

From the measured value of the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \left(\frac{g}{2\sqrt{2}}\right)^2 \frac{1}{m_W^2} \qquad \Longrightarrow \qquad v = \sqrt{\frac{1}{\sqrt{2}G_F}} \approx 246.22\,\text{GeV}$$

- the photon stays massless
- HWW and HZZ couplings from $2H/v$ term (and $HHWW$ and $HHZZ$ couplings from H^2/v^2 term)

$$\mathcal{L}_{HVV} = \frac{2m_W^2}{v} W_\mu^+ W^{-\mu} H + \frac{m_Z^2}{v} Z^\mu Z_\mu H \equiv g m_W W_\mu^+ W^{-\mu} H + \frac{1}{2}\frac{g m_Z}{\cos\theta_W} Z^\mu Z_\mu H$$

Higgs coupling proportional to mass

- tree-level HVV (V = vector boson) coupling requires VEV!
Normal scalar couplings give $\Phi^\dagger \Phi V$ or $\Phi^\dagger \Phi V V$ couplings only.

Fermion fields of the SM and gauge quantum numbers

				$SU(3)$	$SU(2)$	$U(1)_Y$
$Q^i_L =$	$\begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$\begin{pmatrix} c_L \\ s_L \end{pmatrix}$	$\begin{pmatrix} t_L \\ b_L \end{pmatrix}$	3	2	$\frac{1}{6}$
$u^i_R =$	u_R	c_R	t_R	3	1	$\frac{2}{3}$
$d^i_R =$	d_R	s_R	b_R	3	1	$-\frac{1}{3}$
$L^i_L =$	$\begin{pmatrix} \nu_{eL} \\ e_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\mu L} \\ \mu_L \end{pmatrix}$	$\begin{pmatrix} \nu_{\tau L} \\ \tau_L \end{pmatrix}$	1	2	$-\frac{1}{2}$
$e^i_R =$	e_R	μ_R	τ_R	1	1	-1
$\nu^i_R =$	ν_{eR}	$\nu_{\mu R}$	$\nu_{\tau R}$	1	1	0

Fermion mass generation

A direct mass term is not invariant under SU(2)$_L$ or U(1)$_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d \bar{Q}_L \Phi d_R - \Gamma_d \bar{d}_R \Phi^\dagger Q_L \\
&\quad -\Gamma_u \bar{Q}_L \Phi_c u_R + \text{h.c.} \\
&\quad -\Gamma_e \bar{L}_L \Phi e_R + \text{h.c.} \\
&\quad -\Gamma_\nu \bar{L}_L \Phi_c \nu_R + \text{h.c.}
\end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q, L are left-handed doublet fields and d_R, u_R, e_R, ν_R are right-handed $SU(2)$ -singlet fields.

Notice: neutrino masses can be implemented via Γ_ν term. Since $m_\nu \approx 0$ we neglect it in the following.

Fermion mass generation

A direct mass term is not invariant under $SU(2)_L$ or $U(1)_Y$ gauge transformation

$$m_f \bar{\psi}\psi = m_f \left(\bar{\psi}_R \psi_L + \bar{\psi}_L \psi_R\right)$$

Generate fermion masses through Yukawa-type interactions terms

$$\begin{aligned} \mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij} \bar{Q}_L^{\prime i} \Phi d_R^{\prime j} - \Gamma_d^{ij*} \bar{d}_R^{\prime i} \Phi^\dagger Q_L^{\prime j} \\ & -\Gamma_u^{ij} \bar{Q}_L^{\prime i} \Phi_c u_R^{\prime j} + \text{h.c.} \\ & -\Gamma_e^{ij} \bar{L}_L^i \Phi e_R^j + \text{h.c.} \end{aligned}$$

$$\Phi_c = i\sigma_2 \Phi^* = \frac{1}{\sqrt{2}} \begin{pmatrix} v + H(x) \\ 0 \end{pmatrix}$$

where Q', u' and d' are quark fields that are generic linear combination of the mass eigenstates u and d and Γ_u, Γ_d and Γ_e are 3×3 complex matrices in generation space, spanned by the indices i and j.

$\mathcal{L}_{\text{Yukawa}}$ is Lorentz invariant, gauge invariant and renormalizable, and therefore it can (actually it must) be included in the Lagrangian.

Expanding around the vacuum state

In the unitary gauge we have

$$\bar{Q}_L'^{\,i}\,\Phi\,d_R'^{\,j} = \begin{pmatrix}\bar{u}_L'^{\,i} & \bar{d}_L'^{\,i}\end{pmatrix}\begin{pmatrix}0\\ \frac{v+H}{\sqrt{2}}\end{pmatrix} d_R'^{\,j} = \frac{v+H}{\sqrt{2}}\,\bar{d}_L'^{\,i}\,d_R'^{\,j}$$

$$\bar{Q}_L'^{\,i}\,\Phi_c\,u_R'^{\,j} = \begin{pmatrix}\bar{u}_L'^{\,i} & \bar{d}_L'^{\,i}\end{pmatrix}\begin{pmatrix}\frac{v+H}{\sqrt{2}}\\ 0\end{pmatrix} u_R'^{\,j} = \frac{v+H}{\sqrt{2}}\,\bar{u}_L'^{\,i}\,u_R'^{\,j}$$

and we obtain

$$\begin{aligned}\mathcal{L}_{\text{Yukawa}} &= -\Gamma_d^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{d}_L'^{\,i}\,d_R'^{\,j} - \Gamma_u^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{u}_L'^{\,i}\,u_R'^{\,j} - \Gamma_e^{ij}\,\frac{v+H}{\sqrt{2}}\,\bar{e}_L^{i}\,e_R^{j} + \text{h.c.}\\ &= -\left[M_u^{ij}\,\bar{u}_L'^{\,i}\,u_R'^{\,j} + M_d^{ij}\,\bar{d}_L'^{\,i}\,d_R'^{\,j} + M_e^{ij}\,\bar{e}_L^{i}\,e_R^{j} + \text{h.c.}\right]\left(1+\frac{H}{v}\right)\end{aligned}$$

with mass matrices $M^{ij} = \Gamma^{ij}\frac{v}{\sqrt{2}}$

Diagonalizing M_f

It is always possible to diagonalize M_f^{ij} $(f = u, d, e)$ with a bi-unitary transformation ($U^f_{L/R}$ must be unitary in order to preserve the form of the kinetic terms in the Lagrangian)

$$
\begin{aligned}
f'_{Li} &= \left(U^f_L\right)_{ij} f_{Lj} \\
f'_{Ri} &= \left(U^f_R\right)_{ij} f_{Rj}
\end{aligned}
$$

with U^f_L and U^f_R chosen such that

$$\left(U^f_L\right)^\dagger M_f U^f_R = \text{diagonal}$$

For example:

$$(U^u_L)^\dagger M_u U^u_R = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} \qquad \left(U^d_L\right)^\dagger M_d U^d_R = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix}$$

Mass terms

$$
\begin{aligned}
\mathcal{L}_{\text{Yukawa}} &= -\sum_{f',i,j} M_f^{ij}\, \bar{f}_L'^{\,i}\, f_R'^{\,j} \left(1+\frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_{f,i,j} \bar{f}_L^i \left[\left(U_L^f\right)^\dagger M_f U_R^f\right]_{ij} f_R^j \left(1+\frac{H}{v}\right) + \text{h.c.} \\
&= -\sum_f m_f \left(\bar{f}_L f_R + \bar{f}_R f_L\right) \left(1+\frac{H}{v}\right) = \sum_f m_f\, \bar{f} f \,\left(1+\frac{H}{v}\right)
\end{aligned}
$$

We succeed in producing fermion masses and we got a fermion-antifermion-Higgs coupling proportional to the fermion mass.

The Higgs Yukawa couplings are flavor diagonal: no flavor changing Higgs interactions.

Mass diagonalization and charged current interaction

The charged current interaction is given by

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}'^{\,i}_L\, \not{W}^+\, d'^{\,i}_L + \text{h.c.}$$

After the mass diagonalization described previously, this term becomes

$$\frac{e}{\sqrt{2}\sin\theta_W}\bar{u}^i_L \left[(U^u_L)^\dagger\, U^d_L\right]_{ij} \not{W}^+ d^j_L + \text{h.c.}$$

and we define the Cabibbo-Kobayashi-Maskawa matrix V_{CKM}

$$V_{CKM} = (U^u_L)^\dagger\, U^d_L$$

- V_{CKM} is not diagonal and then it mixes the flavors of the different quarks.

- It is a unitary matrix and the values of its entries must be determined from experiments.

Mass diagonalization and neutral current interaction

The neutral current interaction for e.g. down type quarks is given by

$$\frac{e}{\sin\theta_W\cos\theta_W}(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W)\bar{d}'^{\,i}_L \not{Z}\, d'^{\,i}_L + \frac{e}{\sin\theta_W\cos\theta_W}(+\frac{1}{3}\sin^2\theta_W)\bar{d}'^{\,i}_R \not{Z}\, d'^{\,i}_R$$

After the mass diagonalization described previously, this term becomes

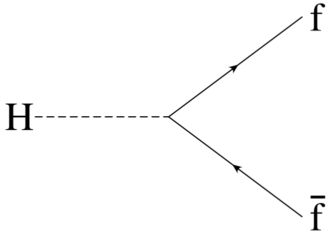
$$\frac{e}{\sin\theta_W\cos\theta_W}(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W)\bar{d}^i_L \left[\left(U^d_L\right)^\dagger U^d_L\right]_{ij} \not{Z}\, d^j_L + \frac{e}{\sin\theta_W\cos\theta_W}(+\frac{1}{3}\sin^2\theta_W)\bar{d}^i_R \left[\left(U^d_R\right)^\dagger U^d_R\right]_{ij} \not{Z}\, d^j_R$$

Now the unitary matrices cancel and the Z interaction is flavor diagonal also in the mass eigenstate basis

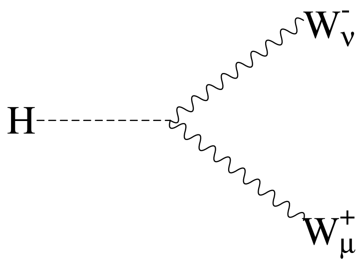
$$\frac{e}{\sin\theta_W\cos\theta_W}(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W)\bar{d}^i_L \not{Z}\, d^i_L + \frac{e}{\sin\theta_W\cos\theta_W}(+\frac{1}{3}\sin^2\theta_W)\bar{d}^i_R \not{Z}\, d^i_R$$

It works the same way for the other flavors. This mechanism is called the GIM mechanism after Glashow, Iliopoulos and Maiani, who invented it.

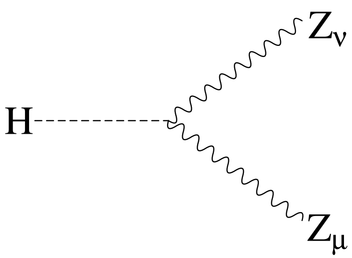
Feynman rules for Higgs couplings



$$-i\frac{m_f}{v}$$



$$i g\, m_W\, g_{\mu\nu}$$



$$i\, g\, \frac{1}{\cos\theta_W}\, m_Z\, g_{\mu\nu}$$

Within the Standard Model, the Higgs couplings are almost completely constrained. The only free parameter (not yet measured) is the Higgs mass

$$m_H^2 = 2\lambda v^2$$