Some Useful Formulae

Euler Beta-function:

$$\begin{aligned}
B(x,y) &= \frac{\Gamma(x)\,\Gamma(y)}{\Gamma(x+y)} \\
&= \int_0^\infty dt\; t^{x-1}\;(1+t)^{-x-y} \\
&= 2\int_0^{\pi/2} (\sin\theta)^{2x-1}(\cos\theta)^{2y-1}d\theta \\
&= \int_0^1 dt\; t^{x-1}(1-t)^{y-1} \\
&= r^y(r+1)^x \int_0^1 dt\, \frac{t^{x-1}\,(1-t)^{y-1}}{(r+t)^{x+y}} \qquad \forall r
\end{aligned}$$

Expansion of the Γ-function:

$$\begin{aligned}
\Gamma(\epsilon) &= \frac{1}{\epsilon}\Gamma(1+\epsilon) = \frac{1}{\epsilon}\Big[\Gamma(1) + \epsilon\Gamma'(1) + \cdots\Big] \\
&= \frac{1}{\epsilon} + \Gamma'(1) + \mathcal{O}(\epsilon) = \frac{1}{\epsilon} - \gamma + \mathcal{O}(\epsilon)
\end{aligned}$$

with the Euler constant

$$\gamma = 0.57721\,56649\ldots$$

$$\Gamma(-n+\epsilon) = \frac{(-1)^n}{n!}\left[\frac{1}{\epsilon} + \left(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \gamma\right) + \mathcal{O}(\epsilon)\right]$$

Formulas for 1-loop integrals in Minkowski space ($q^2 = {q^0}^2 - \vec{q}^{\,2}$ throughout):

$$\begin{aligned}
&\mu^{4-d}\int \frac{d^dp}{(2\pi)^d}\,\frac{1}{(-p^2+2p\cdot q+M^2)^a} \\
=\;& \frac{i}{16\pi^2}(4\pi\mu^2)^{\frac{4-d}{2}}\,\frac{\Gamma\left(a-\frac{d}{2}\right)}{\Gamma(a)}\,\frac{1}{(q^2+M^2)^{a-\frac{d}{2}}}
\end{aligned}$$

Successive derivatives with respect to q_μ yield:

$$\begin{aligned}
&\mu^{4-d}\int \frac{d^dp}{(2\pi)^d}\frac{p^\mu}{(-p^2+2p\cdot q+M^2)^a} \\
=\;& \frac{i}{16\pi^2}(4\pi\mu^2)^{\frac{4-d}{2}}\,\frac{\Gamma\left(a-\frac{d}{2}\right)}{\Gamma(a)}\,\frac{q^\mu}{(q^2+M^2)^{a-\frac{d}{2}}} \\
&\mu^{4-d}\int \frac{d^dp}{(2\pi)^d}\frac{p^\mu p^\nu}{(-p^2+2p\cdot q+M^2)^a} \\
=\;& \frac{i}{16\pi^2}(4\pi\mu^2)^{\frac{4-d}{2}}\,\frac{\Gamma\left(a-\frac{d}{2}\right)}{\Gamma(a)}\,\frac{q^\mu q^\nu + \frac{1}{2}g^{\mu\nu}\frac{q^2+M^2}{\frac{d}{2}+1-a}}{(q^2+M^2)^{a-\frac{d}{2}}}
\end{aligned}$$