Karlsruher Institut für Technologie

Institute for Theoretical Physics

Vacuum stability in the NMSSM

Based on work in collaboration with Fabio Campello and Georg Weiglein

ITP seminar

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Collaborative Research Center TRR 257
Particle Physics Phenomenology after the Higgs Discovery

Vacuum stability in BSM theories

The core concept: In models with extended scalar sectors the EW vacuum might not correspond to the global minimum of the potential.

→ The EW vacuum is not stable

→ If the EW vacuum is short-lived[1] a parameter point is unphysical

→ Constraints on the parameter space of the model

The **outline** of my presentation:

1. Supersymmetry and vacuum stability
2. Technical details of our analysis
3. Results: Application to the NMSSM

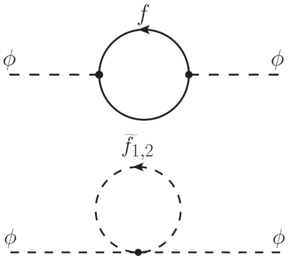
[1] in comparison to the age of the Universe

Supersymmetry (Susy)

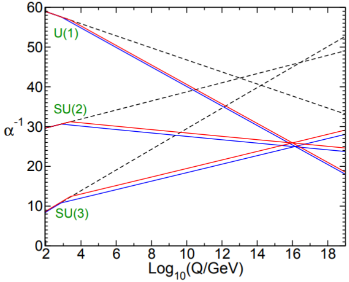
Supersymmetry combines **bosonic** and **fermionic** degrees of freedom into a combined superspace formalism.

Pick your favourite **motivation**, a personal selection:

Hierarchy problem



Gauge-coupling unification



Supergravity (Sugra)

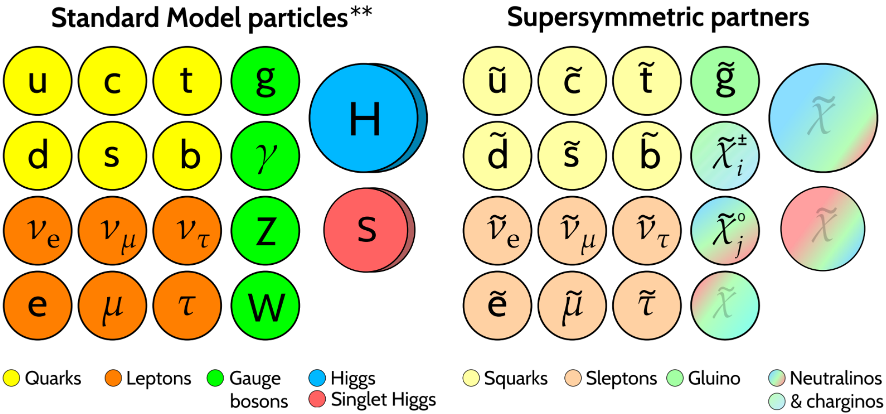
$$\{Q_a, Q^\dagger_{\dot{b}}\} = -2\sigma^\mu_{a\dot{b}} P_\mu$$

Local Susy implies the existence of gravity

There are more: Dark matter candidates, Coleman-Mandula theorem, . . .

The NMSSM

NMSSM: Next-to Minimal Supersymmetric Standard Model



[Slide from Christoph]

The NMSSM

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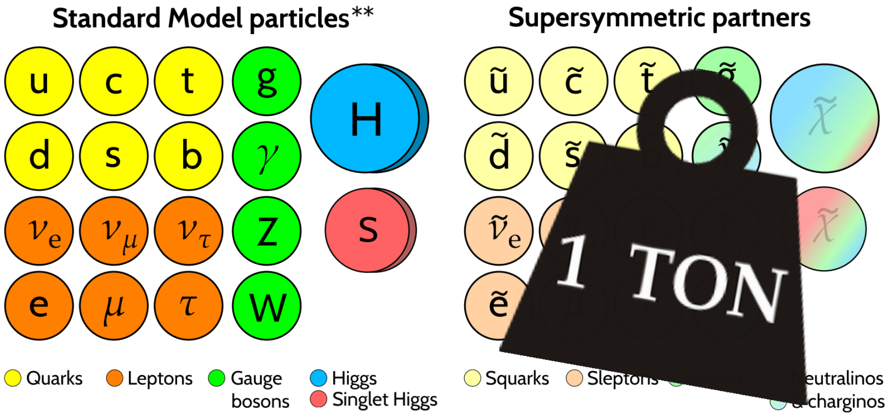

[Slide from Christoph]

Scalar potential of the NMSSM

Susy gives a recipe for constructing the scalar potential:

$$V = F + D + V_{\text{soft}}$$

Scalar potential of the NMSSM

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$$V = F + D + V_{\text{soft}}$$

F-term contributions from the Superpotential:

$$W = \frac{1}{3}\kappa S^3 + \lambda S H_u \cdot H_d + y_t Q_L \cdot H_u \bar{t}_R + y_b H_d \cdot Q_L \bar{b}_R + y_\tau H_d \cdot L_L \bar{\tau}_R$$

$$F = \sum_\phi |\partial_x W|^2 \ , \quad \phi \in \{h_u^0, h_u^+, h_d^0, h_d^-, \tilde{t}_L, \tilde{b}_L, \tilde{\tau}_L, \tilde{\nu}_L, \tilde{t}_R^*, \tilde{b}_R^*, \tilde{\tau}_R^*\}$$

Scalar potential of the NMSSM

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D-term contributions from the gauge structure:

$$D = D_{U(1)_Y} + D_{SU(2)_L} + D_{SU(3)_c}$$

$$D_{U(1)_Y} = \frac{g_1^2}{8}\Big(\sum_{\phi} Y_\phi |\phi|^2\Big)^2$$

$$D_{SU(2)_L} = \frac{g_2^2}{8}\sum_{\Phi_i}\sum_{\Phi_j} 2(\Phi_i^\dagger \Phi_j)(\Phi_j^\dagger \Phi_i) - (\Phi_i^\dagger \Phi_i)(\Phi_j^\dagger \Phi_j)$$

$$D_{SU(3)_c} = \frac{g_3^2}{6}\Big(|\tilde{t}_L|^2 - |\tilde{t}_R|^2 + |\tilde{b}_L|^2 - |\tilde{b}_R|^2\Big)^2$$

Scalar potential of the NMSSM

Susy gives a recipe of how to construct the scalar potential:

$$V = F + D + V_{\text{soft}}$$

Contributions from soft Susy breaking:

$$\begin{aligned} V_{\textsf{soft}} = {} & m_S^2 s^\dagger s + m_{H_u}^2 h_u^\dagger h_u + m_{H_d}^2 h_d^\dagger h_d + \left(A_\lambda s h_u \cdot h_d + \frac{1}{3} A_\kappa s^3 + \text{h.c.}\right) \\ & + m_{Q_3}^2 \tilde{Q}_L^\dagger \tilde{Q}_L + m_{L_3}^2 \tilde{L}_L^\dagger \tilde{L}_L + m_{U_3}^2 |\tilde{t}_R|^2 + m_{D_3}^2 |\tilde{b}_R|^2 + m_{E_3}^2 |\tilde{\tau}_R|^2 \\ & + \left(y_t A_t \tilde{t}_R^* \tilde{Q}_L \cdot h_u + y_b A_b \tilde{b}_R^* h_d \cdot \tilde{Q}_L + y_\tau A_\tau \tilde{\tau}_R^* h_d \cdot \tilde{L}_L + \text{h.c.}\right) \ , \end{aligned}$$

Scalar potential of the NMSSM

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Susy gives a recipe of how to construct the scalar potential:

$$V = F + D + V_{\text{soft}}$$

Physical EW vacuum (the one we want to be in):

$$H_d = \begin{pmatrix} (v_d + h_d + \mathrm{i}a_d)/\sqrt{2} \\ h_d^+ \end{pmatrix} \ , \quad H_d = \begin{pmatrix} h_u^+ \\ (v_u + h_u + \mathrm{i}a_u)/\sqrt{2} \end{pmatrix} \ , \quad s = (v_s + h_s + \mathrm{i}a_s)/\sqrt{2}$$

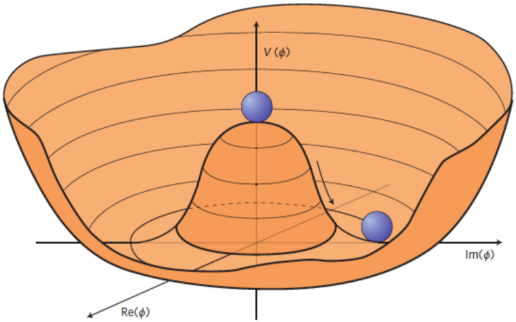
$$\text{with} \quad v_d, v_u, v_s \in \mathbb{R} \quad \text{and} \quad v^2 = v_d^2 + v_u^2 = 246^2 \ \text{GeV}^2$$

BUT: V is a function in many (field) dimensions with loads of parameters

$\rightarrow$ In general there can be several local (dangerous) minima below the EW minimum

Vacuum stability: SM vs. Susy

SM[2]



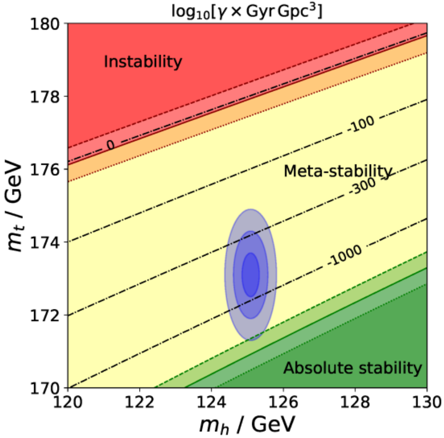
[1504.07217]

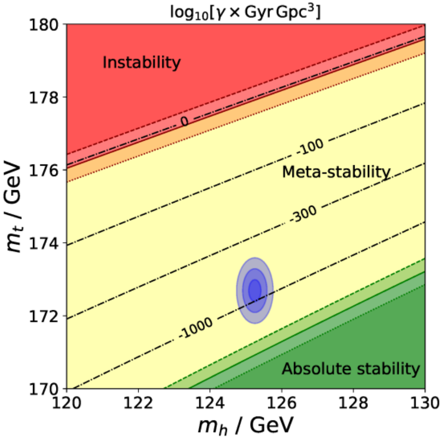
Susy

[2] The SM might have issues with vacuum instabilities at very large energy scales/in the early universe.

Intermezzo: SM

Today there was an update on the arXiv





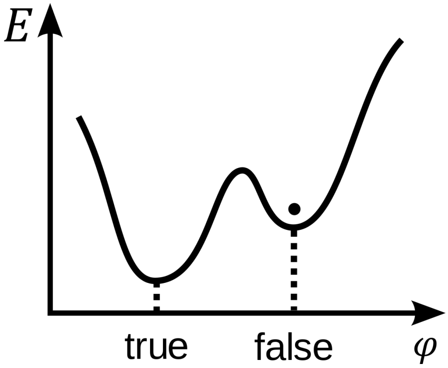
$V_{\rm SM} \approx \lambda(\mu)(\phi^\dagger\phi)^2$ for $\phi \gg v_{\rm EW}$

[Addendum 2023: 1707.09301]

Vacuum stability analysis

Step 1: Is the EW vacuum meta-stable?

→ Determination of all stationary points of the scalar potential below the EW minimum.



Vacuum stability analysis

Step 1: Is the EW vacuum meta-stable?

→ Determine all stationary points of the scalar potential below the EW minimum.

Step 2: What is the lifetime of the EW vacuum?

→ Compute the vacuum decay rates for each possible transition

$$\frac{\Gamma}{V} = K \mathrm{e}^{-S_E}$$

S_E: Euclidean bounce action

Vacuum stability analysis

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Step 1: Is the EW vacuum meta-stable?

$\rightarrow$ Determine all stationary points of the scalar potential below the EW minimum.

Step 2: What is the lifetime of the EW vacuum?

$\rightarrow$ Compute the vacuum decay rates for each possible transition

Step 3: Is the EW vacuum sufficiently long-lived?

$\rightarrow$ Compare the smallest inverse decay rate (lifetime) to the age of the universe

$S_E < 390$: Short-lived EW vacuum, unphysical

$390 < S_E < 440$: Uncertain fate of the EW vacuum

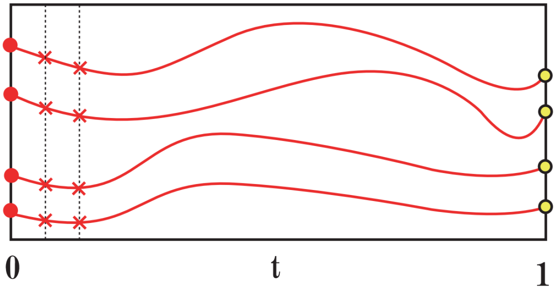
$S_E > 440$: Long-lived (meta-stable) EW vacuum, physical

[W. Hollik, G. Weiglein, J. Wittbrodt: 1812.04644]

Homotopy continuation method

Solutions of a system of polynomial equations can be found using homotopy continuation methods.
$\rightarrow$ Can be used to determine all stationary points $\vec{F}(\vec{z}) = 0$ of tree-level scalar potentials

$$\vec{H}(\vec{z},t) = (1-t)\cdot\vec{F}(\vec{z}) + t\cdot\vec{g}(\vec{z}) = 0 \ , \quad \text{solutions of } \vec{g}(\vec{z}) = 0 \text{ are known.}$$

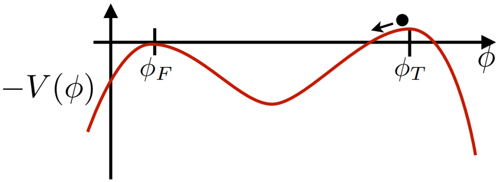


[1310.6515]

We use `HOM4PS2 v.2`: Polyhedral homotopy continuation; best (?) algorithm, closed source...

The bounce solution: 1 field

The bounce solution is the key object describing the vacuum transition.



[1109.4189]

$$\frac{d^2\vec{\phi}}{d\rho^2} + \frac{3}{\rho}\frac{d\vec{\phi}}{d\rho} = \vec{\nabla}_\phi V(\vec{\phi})$$

$$\text{with } \rho = r^2 - t^2$$

Boundary conds.:

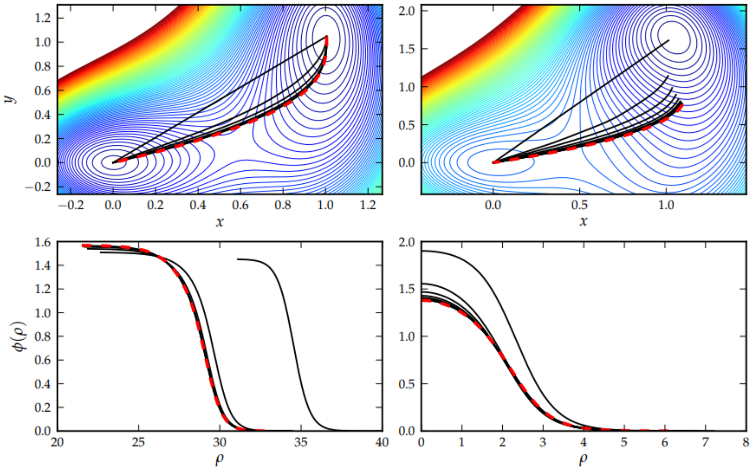
$$\vec{\phi}(\infty) = \vec{\phi}_F \ , \qquad \left.\frac{d\vec{\phi}}{d\rho}\right|_{\rho=0} = 0$$

In one dimensions this can be solved using the **overshoot-undershoot method**.

Bounce action: $S_E = \frac{2\pi^2}{\Gamma(2)} \int \rho^3 d\rho \left[\frac{1}{2}(\dot{\vec{\phi}}(\rho))^2 + \Delta V(\vec{\phi}(\rho))\right]$

The bounce solution: > 1 fields

The main obstacle in many dimensions is **finding the correct tunneling path** in field space.



[1109.4189]

Many public codes: `cosmoTransitions`, `AnyBubble`, `FindBounce`, `SimpleBounce`, `BSMPT-v.3`, ...

The bounce solution: > 1 fields

In our analysis we use three different methods:

Method 1: Straight path approximation
Compute the bounce action assuming that the true tunneling path is well approximated by a straight path connecting the true and false minima (`EVADE`).

Method 2: Path deformation algorithm
Iterative procedure that deforms the straight path into the correct path by minimizing *perpendicular forces* along the path (`cosmoTransitions`).

Method 3: Using neural network to solve differential equations
Transform the problem of solving a system of differential equations into a minimization problem. Then use NN to minimize the loss function.

Method 2 and 3 are planned to be implemented into a future version of `EVADE`.

Method 3

System of coupled differential equations:

$$F_m(x_i, \phi_m(x_i), \nabla_j^1 \phi_m(x_i), \ldots, \nabla_j^p \phi_m(x_i)) = 0$$

Create a loss function L for neural network with set of weights and biases $\{w, b\}$, simplest way: "squared sum" of (F_m + boundary conditions):

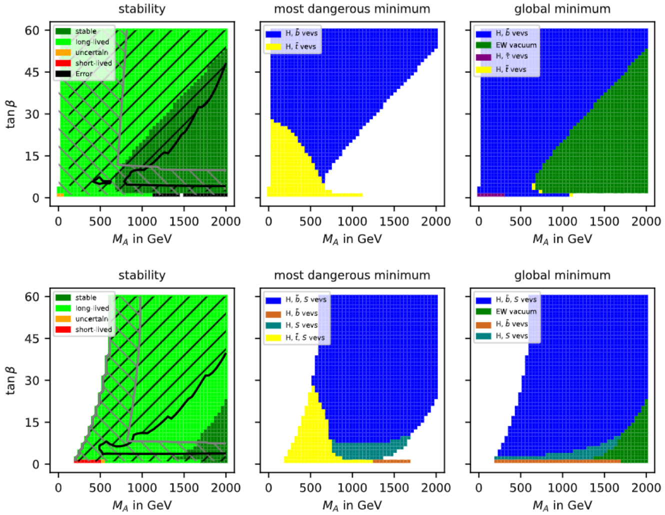
$$L(\{w,b\}) = \sum_{m,x_i} \left[F_m(x_i, \phi_m(x_i), \nabla_j^1 \phi_m(x_i), \ldots, \nabla_j^p \phi_m(x_i))\right]^2 + \sum_\alpha [\mathrm{BC}]_\alpha^2$$

Applied to calculation of bounce solution e.g. in [1902.05563]

So far we have still **issues** applying this to realistic models: discretization of field space problematic in thin-wall limit, convergence to trivial solution, choice of suitable architecture of NN

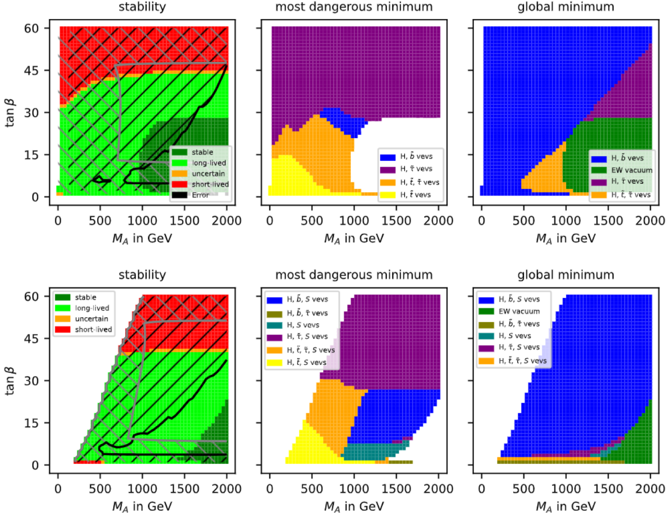
M_h^{125} benchmark scenario

Top: NMSSM, bottom: NMSSM



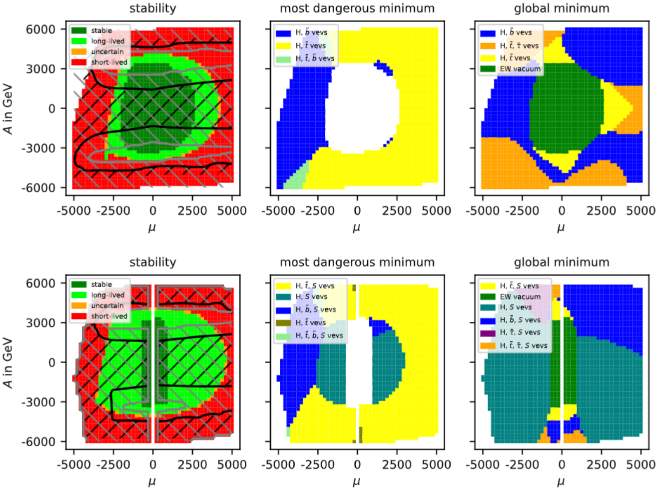
$M_h^{125}(\tilde{\tau})$ benchmark scenario

Top: NMSSM, bottom: NMSSM



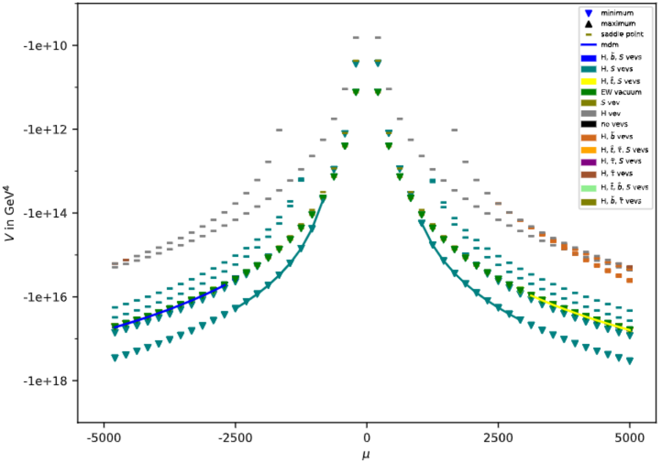
$M_h^{125(\text{tri})}$ benchmark scenario

Top: NMSSM, bottom: NMSSM



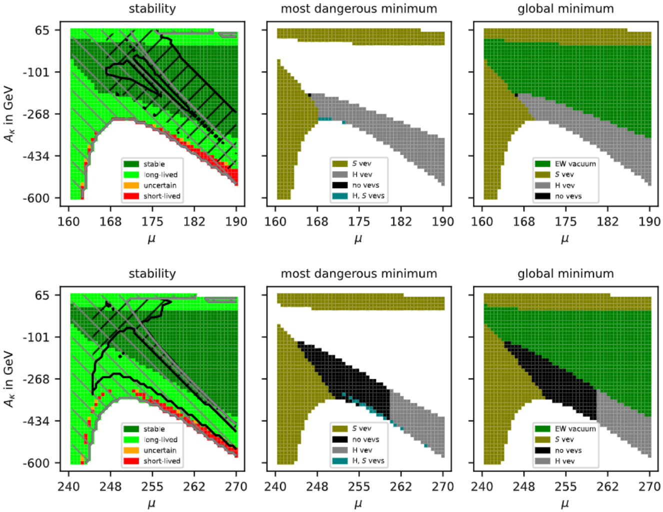
$M_h^{125(\mathrm{tri})}$ benchmark scenario

Top: NMSSM, bottom: NMSSM



NMSSM align. wo. dcpl.

Top: $M_A = 400$ GeV, bottom: $M_A = 600$ GeV



Conclusions so far

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The EW vacuum is in large parts of the parameter space **meta-stable**.
$\rightarrow$ at $T = 0$ not an issue, but could yield cosmological issues

Meta-stable EW vacua are in most cases **sufficiently long-lived**.
$\rightarrow$ demanding a global EW minimum is too restrictive

Analysis of vacuum is difficult but worth the effort.
$\rightarrow$ **constrains the way in which BSM theories might manifest themselves at colliders**

We found parameter space regions in which the **straight path approximation is not sufficient**
$\rightarrow$ `cosmoTransitions` works great for a small number of fields
$\rightarrow$ NN works for > 20 fields but so far not reliable enough

Thanks!