Tales of (Non)decoupling

David López-Val

based in part on work together with
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Institut für Theoretische Physik

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Outline

1. Intuition
2. Getting formal
3. Examples
4. EFT viewpoint
5. Elsewhere
6. Summary
A central idea behind the unification of forces of vastly different strength is that such an apparent hierarchy arises not from the difference of the fundamental coupling constants of the theory but rather from that of the masses of the exchanged particles.

In constructing a viable unified theory, these heavy particles must be incorporated into the structure with due caution. Among the most important requirements are (i) that superheavy particles must effectively decouple at low energies, (ii) that a correct effective light-particle theory must emerge at low energies, and (iii) that the mass hierarchy, arranged at the tree level, should be stable against radiative corrections. As we shall see, these requirements are deeply interrelated. None of them are trivial to satisfy.
A central idea behind the unification of forces of vastly different strength is that such an apparent hierarchy arises not from the difference of the fundamental coupling constants of the theory but rather from that of the masses of the exchanged particles.

In constructing a viable unified theory, these heavy particles must be incorporated into the structure with due caution. Among the most important requirements are (i) that superheavy particles must effectively decouple at low energies, (ii) that a correct effective light-particle theory must emerge at low energies, and (iii) that the mass hierarchy, arranged at the tree level, should be stable against radiative corrections. As we shall see, these requirements are deeply interrelated. None of them are trivial to satisfy.
Motivation (II) - Phenomenological interest

♠ **DECOUPLING** – *a somehow implicit notion*

- Collider physics itself
- Effective field theory
- Model-independent new physics parametrizations (e.g. $S, T, U$ parameters)
Intuition

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- Model-independent new physics parametrizations (e.g. $S$, $T$, $U$ parameters)

♣ **NONDECOUPLING** – – *a somehow implicit notion (as well!!)*

- Precision EW and heavy flavor physics as probes of missing SM building blocks
  - $m_t - m_W$ correlations, $m_H$ blueband
- Precision EW & heavy flavor physics constraints on new physics
Intuition

Keywords

**High scale** \( \Lambda_{\text{UV}} \) **Heavy fields** \( \Phi \) **Large masses** \( M \) \( \tilde{O}(\Lambda_{\text{UV}}) \) \( \mathcal{L}_{\text{UV}}(\Phi, \varphi; g, g^*, m, M) \)

for \( M \to \infty \)

**Low scale** \( \Lambda_{\text{IR}} \) **Light fields** \( \varphi \) **Low masses** \( m \) \( \tilde{O}(\Lambda_{\text{IR}}) \) \( \mathcal{L}_{\text{IR}}(\varphi; g, m) \)

A more formal definition

The property by which field theory amplitudes computed from \( L_{\text{IR}} \) and \( L_{\text{UV}} \) are asymptotically coincident in the heavy mass limit – up to at most a redefinition of the renormalizable parameters in \( L_{\text{IR}} \)

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Tales of (Non)decoupling
Intuition

**Keywords**

**HIGH SCALE** $\Lambda_{UV}$ **HEAVY FIELDS** $\Phi$ **LARGE MASSES** $M$ $\hat{O}(\Lambda_{UV})$ $\mathcal{L}_{UV}(\Phi, \varphi; g, g^*, m, M)$

for $M \to \infty$

**LOW SCALE** $\Lambda_{IR}$ **LIGHT FIELDS** $\varphi$ **LOW MASSES** $m$ $\hat{O}(\Lambda_{IR})$ $\mathcal{L}_{IR}(\varphi; g, m)$

**DECOUPLING**

$\hat{O}(\Lambda_{IR})$ UV–insensitive for $M \to \infty$

heavy field effects undetectable for $M \to \infty$
Intuition

Keywords

**High Scale** $\Lambda_{UV}$ **Heavy Fields** $\Phi$ **Large Masses** $M$ $\hat{O}(\Lambda_{UV})$ $\mathcal{L}_{UV}(\Phi, \varphi; g, g^*, m, M)$

for $M \to \infty$

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A more formal definition

The property by which field theory amplitudes computed from $\mathcal{L}_{IR}$ and $\mathcal{L}_{UV}$ are asymptotically coincident in the heavy mass limit – up to at most a redefinition of the renormalizable parameters in $\mathcal{L}_{IR}$
**Keywords**

**HIGH SCALE** $\Lambda_{UV}$  
**HEAVY FIELDS** $\Phi$  
**LARGE MASSES** $M$  
$O(\Lambda_{UV})$  
$L_{UV}(\Phi, \varphi; g, g^*, m, M)$

for $M \to \infty$

**LOW SCALE** $\Lambda_{IR}$  
**LIGHT FIELDS** $\varphi$  
**LOW MASSES** $m$  
$O(\Lambda_{IR})$  
$L_{IR}(\varphi; g, m)$

♣ $O(\Lambda_{IR})$ are UV–sensitive for $M \to \infty$

♣ NONDECOUPLING

♣ heavy field effects remain for $M \to \infty$

**A more formal definition**

Remainder of the UV-scale dynamics in low-energy observables for asymptotically large heavy field masses
Some questions ahead

- What fundamental field theory mechanisms are responsible for decoupling?
- Which ones underline the breakdown of decoupling conditions?
- In which situations are non-decoupling effects genuinely physical?
- How does nondecoupling impact BSM Higgs physics?
- What is the interplay between nondecoupling and EFTs?
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The notion of heavy field decoupling - a first QFT formalization:

K. Symanzik (1973), *Infrared singularities and small distance behavior analysis*


Phys.Rev. **D11** (1975) 2856

*We examine some problems associated with the low-momentum behavior of gauge theories and other renormalizable field theories. Our main interest is in the infrared structure of unbroken non-Abelian gauge theories and how this is affected by the presence of other heavy fields coupled to the massless gauge fields. It is shown in the context of a simple model of gauge mesons coupled to massive fermions that the heavy fields decouple at low momenta except for their contribution to renormalization effects.*
The Appelquist-Carazzone theorem

♠, Setup: Simplified model of massless gauge bosons coupled to massive fermions:

\[
\mathcal{L}_{\text{full}} = -\frac{1}{4} F^{\mu \nu} F_{\mu \nu} + i \bar{\psi} \not{\partial} \psi + M \bar{\psi} \psi + \mathcal{L}_{\varphi \mu - \psi}
\]

\[
\mathcal{L}_{\text{full}} = \mathcal{L}_{\text{IR}}(\varphi, \{g\}) + \mathcal{L}_{\text{UV}}(\varphi, \{g, g^* M\}) + \mathcal{L}_{\varphi \mu - \psi}
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\[G^{(n)}(p_1, \ldots, p_n; \mu)\]

generic n–point light field Green’s function computed from \(\mathcal{L}_{\text{full}}\)

\[\tilde{G}^{(n)}(p_1, \ldots, p_n; \mu)\]

generic n–point light field Green’s function computed from \(\mathcal{L}_{\text{eff}}\)
The Appelquist-Carazzone theorem

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  generic n–point light field Green’s function computed from \(\mathcal{L}_{\text{eff}}\)

Statement:
In the limit \(\{p_i^2\} \ll \{M^2\}\),

\[
G^{(n)}(p_1, \ldots, p_n; \mu) = Z^{n/2} \tilde{G}^{(n)}(p_1, \ldots, p_n; \mu) + \mathcal{O}(1/M),
\]

\(\mathcal{L}_{\text{eff}} \equiv \mathcal{L}_{\text{IR}}(\{\tilde{g}_i\}), \text{ with } \tilde{g}_i \equiv \tilde{g}_i(\{g_i, M\}) \text{ and } Z \equiv Z(\{g_i, M\})\).
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generic n–point light field Green’s function computed from \( \mathcal{L}_{\text{eff}} \)

\[ \mathcal{G}^{(n)}(p_1, \ldots, p_n; \mu) = \frac{Z^{n/2}}{2} \tilde{\mathcal{G}}^{(n)}(p_1, \ldots, p_n; \mu) + \mathcal{O}(1/M), \]

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Proof keypoints:

♣ SCALING  ♣ RENORMALIZABILITY
EXAMPLE I: Decoupling of $e^-$ in low-energy QED ($p^2 \ll m_e^2$)

\[
\exp(i\Gamma_{\text{eff}}[A_\mu]) = \int [\mathcal{D}\Psi][\mathcal{D}\overline{\Psi}] \exp \left\{ iS_{\text{QED}}[A_\mu, \Psi, \overline{\Psi}] \right\}
\]

\[
= \int [\mathcal{D}\Psi][\mathcal{D}\overline{\Psi}] \exp \left\{ i \int d^4x \left[ -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \overline{\Psi}(i\not{\partial} - m_e) \Psi \right] \right\}
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\[
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### EXAMPLE II

Leading SUSY-QCD corrections to top decay

\[
\Gamma(t \to W^+b) = \Gamma_{\text{born}}(t \to W^+b) \left[ 1 + \# \frac{\alpha_s}{4\pi} \frac{m_t^2}{M_{\text{SUSY}}^2} \right]
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EXAMPLE III: Leading $Z'$ corrections to $Z \to bb$

\[
\Gamma(Z \to bb) = \Gamma_{\text{born}}(Z \to bb) \left[ 1 + \frac{\alpha_{\text{ew}}}{4\pi} \frac{m_t^2}{M_{Z',\text{SUSY}}^2} \right]
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COUNTEREXAMPLE: Leading SM EW corrections to $Z \to bb$

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Generalizing Appelquist-Carazzone

Just found the puzzle!

while GUT fields decouple \( \ldots \)  

heavy SM fields do not

\[ \mathcal{O} \left( \frac{m_t^2}{M_{Z'}^2} \right) \]

Historical landmarks:

- \( m_c \) effects in \( J_\mu^5 @ \mathcal{O}(\alpha_s^2) \): Collins, Wilczek, Zee ['78]
- \( m_t \) correction to \( \delta \rho \): Marciano ['75]; Toussaint ['78]
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Getting formal

Generalizations

Generalizing Appelquist-Carazzone

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Generalized conditions for decoupling

1 A **separably renormalizable light sector**, remaining so at each possible stage of symmetry breaking

2 Asymptotically weak couplings

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Generalizing Appelquist-Carazzone

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Generalized conditions for decoupling

1. **A separably renormalizable light sector**, remaining so at each possible stage of symmetry breaking
2. **Asymptotically weak couplings**

⇒ i) ensures that all $G^n$ remain protected by gauge invariance from receiving contributions from the heavy fields other than $\mathcal{O}(1/M)$ (explicitly decoupling) or removable via renormalization

⇒ ii) ensures the validity of scaling arguments
## A nondecoupling taxonomy

### Intrinsic, in situations beyond the validity range of the AC theorem

- Nonrenormalizable low-scale theory: Higgsless SM
- Asymptotically large couplings: SM in the $m_t \rightarrow \infty$ limit

### Accidental, in situations where AC holds though is apparently violated

- Renormalization scheme-induced: renormalization conditions for the low(high)–scale parameters fixed through high(low)–scale observables
  - Chankowski et al. [hep-ph/0605302]

- UV–scale induced: low–energy parameters must satisfy some relations and cannot be renormalized independently
  - linked gauge couplings from GUTs

- Delayed decoupling genuine UV dynamics slows down the decoupling
  - tanβ-enhanced radiative corrections in the MSSM
  - Haber et al. ['01]
The role of symmetries

الف: Symmetries play a key role in understanding the field theory mechanisms responsible for protecting or breaking decoupling.
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We have seen how superheavy particles in grand unified theories decouple at low momenta. The reason, we believe, is that the gauge invariance $SU(2)_L \otimes U(1)_Y$ of the SM which remains unbroken at the energies of the order of the superheavy particle masses. This explanation was suggested to us by Frank Wilczek. We thank him for a very useful discussion on that point.
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\[ \Rightarrow \text{renormalizability} \]
protects Green’s functions from receiving divergent contributions not absorbable into CTs

\[ \Rightarrow m_H^2 \text{ as the remainder of the would-be UV poles if the Higgs was removed} \]

Additional underlying symmetries lead to tempered nondecoupling

\[ \Rightarrow \text{EXAMPLE: Custodial symmetry leads to Veltman screening} \]

\[ \delta \rho \sim G_F \log \left( \frac{m_H^2}{m_W^2} \right) \]
Sanity checks . . .

Some \textit{sanity} checks - to attest whether a dependence on heavy field dynamics is truly a manifestation of non-decoupling:

<table>
<thead>
<tr>
<th>Genuine or Accidental?</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Do nondecoupling effects persist under renormalization scheme changes?</td>
</tr>
<tr>
<td>2. Are these strictly nonzero in the exact $M \to \infty$ limit or reflect a delayed (though complete) decoupling?</td>
</tr>
<tr>
<td>3. Are these effects compatible with a consistent UV behavior of the high–energy theory (satisfying unitarity and perturbativity constraints)? Do they correspond to experimentally allowed parameter space regions?</td>
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Sample case: $b \rightarrow s\gamma$

- The **GIM mechanism** prevents (suppresses) tree-level (loop-induced) FCNC’s.
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\[ \mathcal{M}(b \to s\gamma) = \sum_{q=u,c,t} \lambda_q F(x_q) \]

where \( \lambda_q = V^*_{qb} V_{qs} \)

\[ \lambda_u + \lambda_c + \lambda_t = 0 \]

- bypasses GIM at one-loop

EW penguin contributing to the $B \to X_S \gamma$ FCNC decay
Sample case: \( b \to s\gamma \)

- **The GIM mechanism** prevents (suppresses) tree-level (loop-induced) FCNC's.

\[
\mathcal{M}(b \to s\gamma) = \sum_{q=u,c,t} \lambda_q F(x_q)
\]

where \( \lambda_q = V_{qb} V_{qs} \)

- **CKM unitarity**
  \[
  \lambda_u + \lambda_c + \lambda_t = 0 \implies \text{tree–level GIM}
  \]

- **\( F(x_q) \sim \frac{m_q^2}{m_W^2} \)**
  \[
  \mathcal{O}\left(\frac{m_t^2}{m_W^2}\right) - \text{bypasses GIM at one-loop}
  \]

- \( m_t^2 \) may be seen as a UV-regulator from the low-energy viewpoint.

EW penguin contributing to the \( B \to X_S \gamma \) FCNC decay.
Scalar potential

\[ V(\Phi_1, \Phi_2) = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - \left[ m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \]

\[ + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \]

\[ + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \text{h.c.} \right], \]
Nondecoupling @ mass spectrum

♠ Scalar potential

\[ V(\Phi_1, \Phi_2) = m^2_{11} \Phi_1^\dagger \Phi_1 + m^2_{22} \Phi_2^\dagger \Phi_2 - \left[ m^2_{12} \Phi_1^\dagger \Phi_2 + \text{h.c.} \right] \]
\[ + \frac{\lambda_1}{2} (\Phi_1^\dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 |\Phi_1^\dagger \Phi_2|^2 \]
\[ + \left[ \frac{\lambda_5}{2} (\Phi_1^\dagger \Phi_2)^2 + \lambda_6 (\Phi_1^\dagger \Phi_1)(\Phi_1^\dagger \Phi_2) + \lambda_7 (\Phi_2^\dagger \Phi_2)(\Phi_1^\dagger \Phi_2) + \text{h.c.} \right], \]

♠ Mass eigenvalues

\[ m^2_{H^\pm} = \frac{m^2_{12}}{s_\beta c_\beta} - \frac{v^2}{2 re} [\lambda_4 + \lambda_5 + \lambda_6 \cot \beta + \lambda_7 \tan \beta] \]
\[ m^2_A = \frac{m^2_{12}}{s_\beta c_\beta} - \frac{v^2}{2 re} [2\lambda_5 + \lambda_6 \cot \beta + \lambda_7 \tan \beta] \]
Nondecoupling @ precision

\[ (\alpha_{em} T)_{2HDM} \supset \frac{G_F}{8\sqrt{2}\pi^2} \left\{ m_{h^\pm}^2 \left[ 1 - \frac{m_{A^0}^2}{m_{h^\pm}^2 - m_{A^0}^2} \ln \frac{m_{H^\pm}^2}{m_{A^0}^2} \right] \right. \]

\[ + \cos^2(\beta - \alpha) m_{h_0}^2 \left[ \frac{m_{A^0}^2}{m_{A^0}^2 - m_{h_0}^2} \ln \frac{m_{A^0}^2}{m_{h_0}^2} - \frac{m_{H^\pm}^2}{m_{H^\pm}^2 - m_{h_0}^2} \ln \frac{m_{H^\pm}^2}{m_{h_0}^2} \right] \]

\[ + \sin^2(\beta - \alpha) m_{h_0}^2 \left[ \frac{m_{A^0}^2}{m_{A^0}^2 - m_{h_0}^2} \ln \frac{m_{A^0}^2}{m_{h_0}^2} - \frac{m_{H^\pm}^2}{m_{H^\pm}^2 - m_{h_0}^2} \ln \frac{m_{H^\pm}^2}{m_{h_0}^2} \right] \} \]
Nondecoupling @ precision

\[
(\alpha_{em} T)_{2\text{HDM}} \supset \frac{G_F}{8\sqrt{2} \pi^2} \left\{ m_H^2 \pm \left[ 1 - \frac{m_{A0}^2}{m_{H0}^2 - m_{H0}^2} \ln \frac{m_{H0}^2}{m_{A0}^2} \right] \right. \\
+ \cos^2(\beta - \alpha) m_{h0}^2 \left[ \frac{m_{A0}^2}{m_{A0}^2 - m_{h0}^2} \ln \frac{m_{A0}^2}{m_{h0}^2} - \frac{m_{H0}^2}{m_{H0}^2} \ln \frac{m_{H0}^2}{m_{h0}^2} \right] \\
+ \sin^2(\beta - \alpha) m_{H0}^2 \left[ \frac{m_{A0}^2}{m_{A0}^2 - m_{H0}^2} \ln \frac{m_{A0}^2}{m_{H0}^2} - \frac{m_{H0}^2}{m_{H0}^2} \ln \frac{m_{H0}^2}{m_{H0}^2} \right] \left\} \right.
\]

\[\left(\alpha_{em} T\right)_{2\text{HDM}} \simeq -\frac{3 G_F m_Z^2 s_w^2}{8\sqrt{2} \pi^2} \xi^2 \log \left( \frac{m_{\Phi}^2}{m_W^2} \right) \text{ for degenerate } m_{\Phi} = H, A, H^\pm \]
An example with a colored scalar partner $\mathcal{L} \supset -\mu_S^2 |S|^2 - \lambda_s \Phi^\dagger \Phi |S|^2$,

$$m_S^2(v) = \frac{\lambda_s v^2}{2} + \mu_S^2$$

$\mathcal{L}_{hSS} \supset \lambda_s v |S|^2 H$

$\lambda_s = \frac{2(m_S^2 - \mu_S^2)}{v^2}$

$\mathcal{L} \supset -\frac{1}{4} g_{hGG} G^{\mu\nu} A^A G_{\mu\nu} H$

with

$$g_{hGG} = -\frac{g_s^2}{4\pi^2} \left[ \sum_S \text{dim}(r_L) C_{2,S}(r_C) \frac{\lambda_s v}{2m_S^2} A_s(\tau_S) \right]$$
An example with a colored scalar partner $L \supset -\mu_S^2 \phi^2 - \lambda_s \phi^\dagger \phi |S|^2$,

$$m_S^2(v) = \frac{\lambda_s v^2}{2} + \mu_S^2$$

$$L_{hSS} \supset \lambda_s v |S|^2 H$$

$$\lambda_s = \frac{2(m_S^2 - \mu_S^2)}{v^2}$$

$$\mathcal{L} \supset -\frac{1}{4} g_{hGG} G_{\mu\nu}^A G_{\mu\nu}^A H$$

$$g_{hGG} = -\frac{g_s^2}{4\pi^2} \left[ \sum_S \dim(r_L) C_{2,S}(r_C) \frac{\lambda_s v}{2m_S^2} A_s(\tau_S) \right]$$

$\rightarrow 0$ if $m_s \rightarrow \infty$ as $\mu_S \rightarrow \infty$ while $\lambda \sim \mathcal{O}(1)$

$g_{hGG} \sim \frac{\lambda v^2}{m_S^2}$

$\rightarrow \text{constant}$ if $m_S \rightarrow \infty$ while $\mu_S \ll m_s$
Nondecoupling @ loop-induced decays

♠ An example with a colored scalar partner \( \mathcal{L} \supset -\mu_S^2 |S|^2 - \lambda_S \Phi^\dagger \Phi |S|^2 \),

\[
m^2_S(v) = \frac{\lambda_S v^2}{2} + \mu_S^2
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\mathcal{L}_{hSS} \supset \lambda_S v |S|^2 H
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\[
\rightarrow 0 \quad \text{if } m_s \to \infty \text{ as } \mu_S \to \infty \text{ while } \lambda \sim O(1)
\]

\[
g_{hGG} \sim \frac{\lambda v^2}{m_S^2}
\]

\[
\rightarrow \text{constant} \quad \text{if } m_S \to \infty \text{ while } \mu_S \ll m_s
\]

♠ Non–decoupling contributions are connected to LETs

\[
\mathcal{L} \supset \frac{\alpha_s}{12\pi v} \left[ s C_2(r_C) \frac{\partial \log(m(v))}{\partial \log(v)} \right] H G^{\mu\nu^A} G_{\mu\nu^A}.
\]
Nondecoupling @ loop corrections

Coupling shift parametrization:
\[ \hat{\kappa}_f \equiv \frac{\Gamma_{hff}^S}{\Gamma_{hff,SM}^S} \equiv (1 + \Delta_f) \]
Nondecoupling @ loop corrections

Coupling shift parametrization:

\[ \hat{\kappa}_f \equiv \frac{\Gamma_{hff}^S}{\Gamma_{hff,SM}^S} \equiv (1 + \Delta_f) \]

Leading quantum effects on Higgs couplings [Kanemura et. al. arXiv:1502.07716]

Coupling shift @ NLO EW

\[ 1 + \Delta_t = \xi \cot \beta - \frac{1}{2} \xi^2 - \frac{1}{96 \pi^2} \sum c\Phi \frac{m_{\phi}^2}{v^2} \left( 1 - \frac{M^2}{m_{\phi}^2} \right)^2 \]

\[ - \frac{1}{12 \pi^2} \cot^2 \beta \sum c\Phi \left[ \frac{m_t^4}{v^2 m_{\phi}^2} + \tan^2 \beta \frac{m_b^2 m_t^2}{v^2 m_{H^\pm}^2} \right] \]

with \( \Phi = A, H, H^\pm \) and \( \xi = \cos(\beta - \alpha) \simeq \frac{\bar{\lambda} v^2}{M^2} \)
Delayed decoupling

$\Gamma(h \rightarrow bb) = \Gamma_{\text{Born}}(h \rightarrow bb) [1 + \Delta_{\text{SUSY-QCD}} + \ldots]$
Delayed decoupling

\[ h \to bb \quad \text{SUSY-QCD corrections} \]

\[ \Gamma(h \to bb) = \Gamma_{\text{Born}}(h \to bb) \left[ 1 + \Delta_{\text{SUSY-QCD}} + \ldots \right] \]

\[ \begin{aligned} \Delta_{\text{SUSY-QCD}} & \sim \frac{\alpha_s}{3\pi} \left[ \frac{-\mu m_{\tilde{g}}}{M_{\text{SUSY}}^2} (\tan \beta + \cot \alpha) + \mathcal{O} \left( \frac{v^2}{M_{\text{SUSY}}^2} \right) \right] \\ & \sim \frac{\alpha_s}{3\pi} (\tan \beta + \cot \alpha) \end{aligned} \]

\[ \begin{aligned} \uparrow \quad \text{Decoupling only appears for } & \{ m_A \simeq M_{\text{SUSY}} \} \gg m_Z \Rightarrow \tan \beta + \cot \alpha \sim \frac{\tan \beta \cos(2\beta) m_Z^2}{m_A^2} \\ \uparrow \quad \text{But DELAYED for } & \tan \beta \gg 1 \end{aligned} \]
Outline

1. Intuition
2. Getting formal
3. Examples
4. EFT viewpoint
5. Elsewhere
6. Summary
Our mindset

High scale
Heavy fields
Large masses
High-scale Lagrangian:

\[ \Lambda_{\text{new}} \]
\[ \Psi \]
\[ M_{\Psi} \sim \mathcal{O}(\Lambda_{\text{new}}) \]

\[ \mathcal{L}_{\text{UV}}(\Psi, \phi; g, g_{\text{UV}}, m, M_{\Psi}) = \mathcal{L}_{\text{SM}}(\phi, g, m) + \mathcal{L}_{\text{new}}(\Psi, g_{\text{UV}}, M_{\Psi}) + \mathcal{L}_{\text{New} - \text{SM}} \]
Our mindset

**EW scale**
- Light fields
- \( \phi \)
- Low masses
- \( m = \{ m_W, m_Z, m_H \} \sim O(\Lambda_{ew}) \)

**Low-scale Lagrangian:**
\[
\mathcal{L}_{\text{IR}}(\phi; g, m) \equiv \mathcal{L}_{\text{SM}}
\]

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- Heavy fields
- \( \Psi \)
- Large masses
- \( M_\Psi \sim O(\Lambda_{\text{new}}) \)

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\]
Our mindset

**EW scale**

Light fields

low masses

Low-scale Lagrangian:

\[ \Lambda_{IR} \]

\[ \phi \]

\[ m = \{m_W, m_Z, m_H\} \sim O(\Lambda_{ew}) \]

\[ L_{IR}(\phi; g, m) \equiv L_{SM} \]

**Renormalization Group**

High scale

Heavy fields

Large masses

High-scale Lagrangian:

\[ \Lambda_{new} \]

\[ \Psi \]

\[ M_{\Psi} \sim O(\Lambda_{new}) \]

\[ L_{UV}(\Psi, \phi, g, g_{UV}, m, M_{\Psi}) = L_{SM}(\phi, g, m) + L_{new}(\Psi, g_{UV}, M_{\Psi}) + L_{New - SM} \]
It is clear in the case of theories without spontaneously broken symmetry one cannot talk about decoupling without the existence of an effective light-particle theory, because in its absence we cannot absorb the large mass effects by redefinition of the parameters of the light theory. These two concepts are, therefore, two sides of one and the same subject.

\[
\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{i_D} \sum_{D>4} \frac{C^{(D)}_{i_D}}{\Lambda^{D-2}} \mathcal{O}^{(D)}_{i_D}
\]
It is clear in the case of theories without spontaneously broken symmetry one cannot talk about decoupling without the existence of an effective light-particle theory, because in its absence we cannot absorb the large mass effects by redefinition of the parameters of the light theory. These two concepts are, therefore, two sides of one and the same subject.

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\mathcal{L} = \mathcal{L}_{\text{SM}} + \sum_{i_D} \sum_{D>4} \frac{C_{i_D}^{(D)}}{\Lambda^{D-2}} \mathcal{O}_{i_D}^{(D)}
\]

Wilson coefficient behavior

- **Decoupling:** \( C_{i_D}^{(D)} \sim \mathcal{O}(1/\Lambda) \)
- **Screening:** \( C_{i_D}^{(D)} \sim \mathcal{O}(\log(\Lambda)) \)
- **Nondecoupling:** \( C_{i_D}^{(D)} \sim \mathcal{O}(\Lambda) \) (or higher)

**Example:** top-quark effect on \( C_{GGH} \)
Nondecoupling @ EFTs

♠ Plain \((linear, d6)\) EFT ENDANGERED when \(m^2_{\text{phys}} \sim M^2_{\text{heavy}} \pm g v^2\)

\(\Rightarrow\) \(v\)-induced terms spoil the \(M_{\text{heavy}} - \Lambda_{\text{EW}}\) separation

\(\Rightarrow\) Operators beyond \(d6\) may become relevant
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♠ Default matching
- Matching scale: $\Lambda = M_{\text{heavy}}$
- Unbroken phase matching: ($\Lambda \gg \Lambda_{\text{EWSB}}$)
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$v$-improved matching:

- Matching scale $\Lambda = M_{\text{phys}}$
- Wilson coefficients written in terms of mass-eigenstate & mixing

Example: $c_H$ in the additional Higgs singlet extension

\[
\frac{\lambda_H^2 v^2}{2 \lambda_2 \Lambda^2} \left[ \partial_\mu (\Phi^\dagger \Phi) \partial^2 \mu (\Phi^\dagger \Phi) \right] \quad \text{VS} \quad \frac{2 (1 - \cos \alpha) v^2}{m_H^2} \left[ \partial_\mu (\Phi^\dagger \Phi) \partial^\mu (\Phi^\dagger \Phi) \right]
\]

May be not that much esoteric after all . . .

. . . but a cross-talk between separated scales
Nondecoupling elsewhere

- May be not that much esoteric after all . . .

. . . but a cross-talk between separated scales

- Criticality
- Turbulence
- Secular effects in orbital motion

- Coupling of short-to-long distance modes in structure formation
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Take-home ideas

- **Decoupling** governed by Appelquist-Carazzone

- Heavy field imprints
  - either **absorbed** in the renormalization of the parameters and wave function of the light fields
  - or **suppressed** by inverse powers of the heavy scales

- Proof keypoints: scaling & renormalizability

- Validity conditions:
  1. **A separably renormalizable light sector**, remaining so at each possible stage of symmetry breaking
  2. **Asymptotically weak couplings**

- Nondecoupling manifests:
  - When the heavy sector is not separably renormalizable
  - When the limit $M_{\text{heavy}} \to \infty$ implies asymptotically large couplings

- Underlying symmetries temper nondecoupling effects

- Nondecoupling pervades BSM Higgs physics - $m_{\text{phys}}^2 \sim M_{\text{heavy}}^2 + \mathcal{O}(\lambda v^2)$

- Nondecoupling effects in the SM EFT may be accounted for via a dedicated ($v$-improved) matching
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