Renormalization of the THDM and the NLO Corrections to the Decay $H^+ \to W^+ h/H$

Robin Lorenz

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Outline

The Two-Higgs-Doublet Model (THDM)

THDM $=$ SM with two complex scalar SU(2) doublets Φ_1 and Φ_2 with $Y = +1$;
Renormalizable Lagrangian, invariant under $SU_L(2) \times U_Y(1)$:

$$\begin{aligned}\mathcal{L}_{\text{EW}} = \; & \sum_{\psi} \overline{\psi} i \not{D} \psi - \frac{1}{4} W^a_{\mu\nu} W_a^{\mu\nu} - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \mathcal{L}_{\text{Ghost}} \\ & + \sum_{i=1,2} (D_\mu \Phi_i)^\dagger (D^\mu \Phi_i) + \mathcal{L}_{\text{GF}}(\Phi_1, \Phi_2, A^a_\mu, B_\mu) \\ & + \mathcal{L}_{\text{Yuk}}(\Phi_1, \Phi_2, \{\psi\}) \\ & - V(\Phi_1, \Phi_2)\end{aligned}$$

The Physical Degrees of Freedom

- The most general CP-conserving scalar potential (additionally respecting a Z_2-symmetry):

$$
\begin{aligned}
V(\Phi_1, \Phi_2) \;=\; & m_{11}^2 \Phi_1^\dagger \Phi_1 \;+\; m_{22}^2 \Phi_2^\dagger \Phi_2 \;-\; m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1 \right) \\
& + \frac{\lambda_1}{2} \left(\Phi_1^\dagger \Phi_1 \right)^2 + \frac{\lambda_2}{2} \left(\Phi_2^\dagger \Phi_2 \right)^2 + \lambda_3 \Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2 + \lambda_4 \Phi_1^\dagger \Phi_2 \Phi_2^\dagger \Phi_1 \\
& + \frac{\lambda_5}{2} \left(\left(\Phi_1^\dagger \Phi_2 \right)^2 + \left(\Phi_2^\dagger \Phi_1 \right)^2 \right)
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$$

- Diagonalization of the mass matrices:

$$
\begin{pmatrix} \rho_1 \\ \rho_2 \end{pmatrix} = R(\alpha) \begin{pmatrix} H \\ h \end{pmatrix} \quad \rightarrow \quad R(\alpha)^T \, M_\rho \, R(\alpha) = \begin{pmatrix} m_H^2 & 0 \\ 0 & m_h^2 \end{pmatrix}
$$

$$
\begin{pmatrix} \eta_1 \\ \eta_2 \end{pmatrix} = R(\beta) \begin{pmatrix} G^0 \\ A \end{pmatrix} \quad \rightarrow \quad R(\beta)^T \, M_\eta \, R(\beta) = \begin{pmatrix} 0 & 0 \\ 0 & m_A^2 \end{pmatrix}
$$

$$
\begin{pmatrix} \omega_1^+ \\ \omega_2^+ \end{pmatrix} = R(\beta) \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \quad \rightarrow \quad R(\beta)^T \, M_\omega \, R(\beta) = \begin{pmatrix} 0 & 0 \\ 0 & m_{H^\pm}^2 \end{pmatrix}
$$

What have I done?

- Full one-loop electroweak corrections to the decay

$$H^+ \to W^+ \; h/H$$

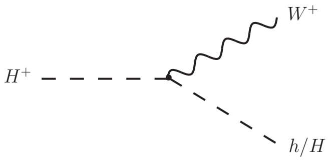

Figure 1 : Tree-level Feynman diagram for the decay $H^+ \to W^+ \; h/H$.

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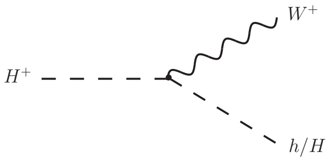

Figure 1 : Tree-level Feynman diagram for the decay $H^+ \to \; W^+ \; h/H$.

- Study of different renormalization schemes for the THDM:

  What are the differences?

  What are the intuitions behind them?

  Is there a universally preferable scheme?

Which Parameters to Renormalize and How?

- Complete set of 12 independent parameters:

$$\{ \; m_{11}, \; m_{22}, \; m_{12}, \; \lambda_1 - \lambda_5, \; v_1, \; v_2, \; g, \; g' \; \}$$

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- Freitas *et al.* [hep-ph/0205281] establish a no-go-theorem for the MSSM.

  $\rightarrow$ What about the general THDM?

Scheme 1: The *Kan* Scheme

The 'Kanemura approach': [Kanemura *et al.*, hep-ph/1502.07716]

$$\sqrt{Z_f} \;=\; R(\delta\theta)^T \; R(\theta)^T \sqrt{Z_\gamma} \; R(\theta) \tag{1}$$

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$$\hat{\Gamma}_f(p^2) \;=\; \sqrt{Z_f}^\dagger \left[\, p^2 \mathbb{1}_{2\times 2} \;-\; (D_f + \delta D_f) \;+\; \Sigma_f \, \right] \sqrt{Z_f} \;, \tag{2}$$

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which yields

$$\delta\alpha \;=\; \frac{1}{2(m_H^2 - m_h^2)} \left[\Sigma_{Hh}(m_h^2) \;+\; \Sigma_{Hh}(m_H^2) \;-\; 2\delta T_{Hh} \right] \tag{3}$$

$$\delta\beta \;=\; -\frac{1}{2\; m_{H^\pm}^2} \left[\Sigma_{G^+H^+}(m_{H^\pm}^2) \;+\; \Sigma_{G^+H^+}(0) \;-\; 2\delta T_{G^+H^+} \right] \tag{4}$$

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In order to circumvent the gauge dependence of β, Kanemura *et al.* suggest:

$$\delta\beta \equiv -\frac{1}{m_{H^\pm}^2}\left[\Sigma_{G^+H^+}(m_{H^\pm}^2) - \delta T_{G^+H^+}\right]\bigg|_{\text{G.I.}} \tag{5}$$

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For the Z_γ this implies

$$\sqrt{Z_\gamma^{Min}} \;=\; \begin{pmatrix} 1 \;+\; \frac{1}{2}\delta Z_1 & 0 \\ 0 & 1 \;+\; \frac{1}{2}\delta Z_2 \end{pmatrix}\;, \tag{7}$$

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The renormalization conditions for the scalar sector:

$$\left\{\hat{\Gamma}_{G^+H^+}(m^2_{H^\pm}) \;\stackrel{!}{=}\; 0\,,\quad \mathrm{Re}\left(\hat{\Gamma}'_{H^+H^+}(m^2_{H^\pm})\right) \;\stackrel{!}{=}\; 1\right\} \quad\Rightarrow\; \text{fix } \delta Z_1 \text{ and } \delta Z_2 \tag{9}$$

$$\delta \mathrm{t}_\beta \;\stackrel{!}{=}\; \frac{\mathrm{t}_\beta}{2}\,(\delta Z_2 - \delta Z_1) \tag{10}$$

$$\hat{\Gamma}_{Hh}(m_H^2) \;\stackrel{!}{=}\; 0 \qquad\Rightarrow\; \text{fixes } \delta\alpha \tag{11}$$

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$$\Gamma_H^{\text{NLO}} = \Gamma_H^{\text{LO}} \Big[1 + 2\,\text{Re}\left(F^{\text{NLO}}_{H^+W^-H}\right) + \delta Z_{WW} + \delta Z_{H^+H^+} + \delta Z_{HH} - \frac{c_{\beta-\alpha}}{s_{\beta-\alpha}} \left(\delta Z_{G^+H^+} + \delta Z_{hH}\right) + 2\frac{\delta g}{g} + 2\frac{c_{\beta-\alpha}}{s_{\beta-\alpha}} \delta(\beta - \alpha) \Big]$$

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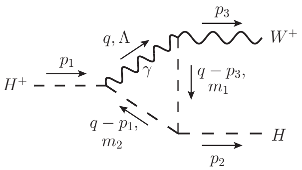
- Problem: IR divergent virtual corrections, e.g. from:



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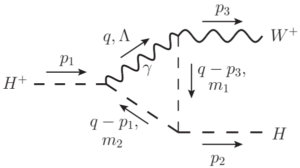

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- Suggestion to circumvent this issue is to use the decay

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The virtual photon corrections form a UV finite subset,

$$\Gamma^{\mathrm{NLO}} = \Gamma^{\mathrm{LO}} + \Gamma^{\mathrm{NLO}}_{\mathrm{virt,QED}} + \Gamma^{\mathrm{NLO}}_{\mathrm{virt,rest}} + \Gamma^{\mathrm{NLO}}_{\mathrm{real}} , \qquad (13)$$

and one can therefore require

$$\Gamma^{\mathrm{NLO}}_{\mathrm{virt,rest}} \stackrel{!}{=} 0 \qquad (14)$$

to fix the angle counterterm.

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- Why not using a vertex which satisfies the following requirements?
  1. Involves only neutral external fields.
  2. Depends on α and/or β.
  3. No external Goldstone bosons are involved.

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- Couplings meeting these criteria:
  - **Yukawa couplings**: none
  - **2 scalars - 2 gauge bosons**: none
  - **2 scalars - 1 gauge boson**: $\lambda_{hA^0Z} = \frac{e}{2s_W c_W} c_{\beta-\alpha}$, $\lambda_{HA^0Z} = -\frac{e}{2s_W c_W} s_{\beta-\alpha}$
  - **1 scalar - 2 gauge bosons**: $\lambda_{HZZ} = \frac{ieM_W}{s_W c_W^2} c_{\beta-\alpha}$, $\lambda_{hZZ} = \frac{ieM_W}{s_W c_W^2} s_{\beta-\alpha}$
  - **Trilinear scalar couplings**: $\lambda_{HA^0A^0}$, $\lambda_{hA^0A^0}$, λ_{hHH} , λ_{hhH} , λ_{hhh} , λ_{HHH}
  - **Quartic scalar couplings**: $\lambda_{A^0A^0A^0A^0}$, $\lambda_{HHA^0A^0}$, $\lambda_{hhA^0A^0}$, $\lambda_{HhA^0A^0}$, λ_{HHHH} , λ_{hhhh} , λ_{hhHH} , λ_{hhhH} , λ_{hHHH}

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• gauge independence • numerically stable	• process dependence • technically involved (especially beyond one-loop level)

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Real Corrections

The IR finite decay width is obtaind by adding corrections from real soft photons:

$$\Gamma^{\text{obs}}_{h/H} \;=\; \Gamma(H^+ \to W^+\ h/H) \;+\; \underbrace{\Gamma(H^+ \to \gamma\ W^+\ h/H)\Big|_{E_\gamma \leq \Delta E}}_{\equiv \Gamma^{\text{soft}}_{\gamma h/H}}\,, \tag{15}$$

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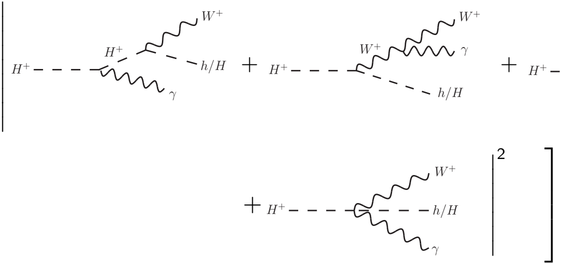
where $\Gamma^{\text{soft}}_{\gamma h/H}$ reads schematically

$$\Gamma^{\text{soft}}_{\gamma h/H} = \frac{1}{2m_{H^\pm}} \int \mathrm{d}\Pi_2 \sum_{\lambda_W} \Bigg[\int_{E_\gamma \leq \Delta E} \frac{\mathrm{d}^3 p_\gamma}{(2\pi)^3} \frac{1}{2E_\gamma} \sum_{\lambda_\gamma} \tag{16}$$



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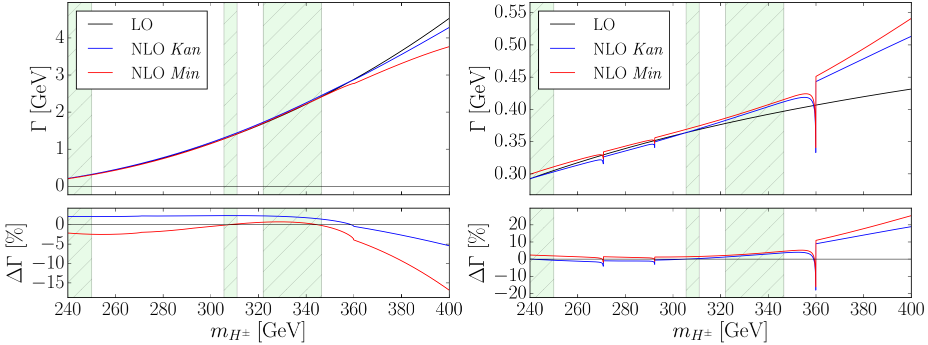

Figure 2 : Partial decay width Γ (upper part) and $\Delta\Gamma = (\Gamma^{\text{NLO}} - \Gamma^{\text{LO}})/\Gamma^{\text{LO}}$ (lower part) of the decays $H^+ \to W^+ h$ (left) and $H^+ \to W^+ H$ (right) at LO and at NLO in the *Kan* and *Min* scheme for the scenarios of the class C_1.

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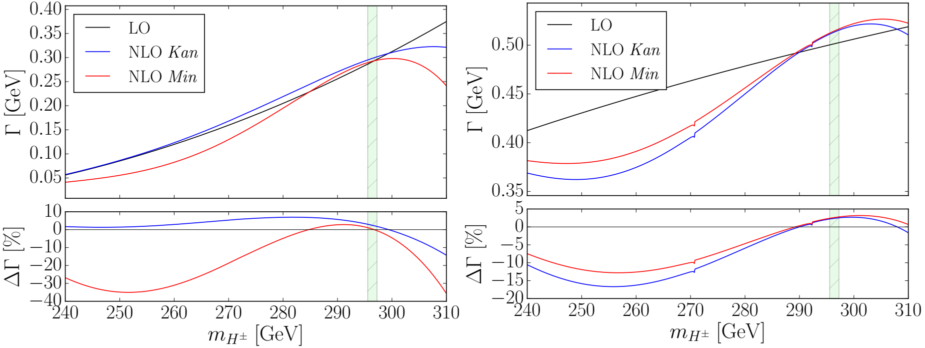

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The *HybMS* Scheme

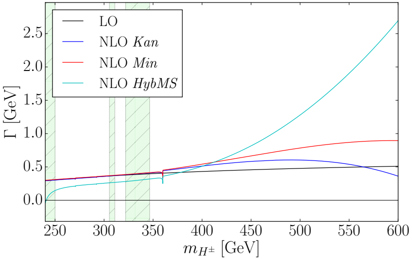

Figure 4 : Comparison of the NLO partial decay width of the decay $H^+ \to W^+ H$ in the *HybMS*, *Kan* and *Min* scheme for the scenarios of class C_1 .

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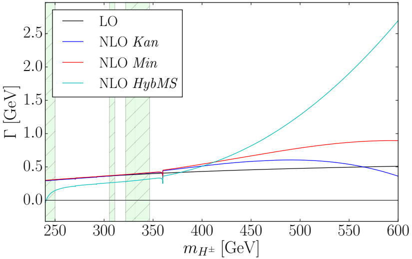

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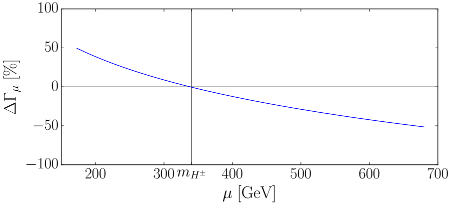


Figure 5 : $\Delta\Gamma_\mu = (\Gamma(\mu) - \Gamma(\mu = m_{H\pm}))/\Gamma(\mu = m_{H\pm})$ in the *Kan* scheme for the scenario of class C_1 with $m_{H\pm} = 340$ GeV, where μ was varied from $m_{H\pm}/2$ to $2m_{H\pm}$.

Conclusion

- NLO corrections are of the order of a few percent to ten percent for parameter regions which are allowed.
- The difference between renormalization schemes suggests that the two-loop calculation is needed to reduce the uncertainty.
- The differences between various schemes became clearer, but so far, no scheme seems to be ideal in all respects, each one of them has its drawbacks. More investigations are needed.

Outlook

- Implement a process-dependent scheme using the decays $A^0 \to \tau^+\tau^-$ and $H \to \tau^+\tau^-$.
- Redo the calculation in general R_ξ gauge and check Kanemura's suggestion to avoid gauge dependence of $\tan(\beta)$
- Check gauge dependence in the *Min* scheme
- Check gauge dependence of the NLO amplitude? How to be sure that one hasn't made a mistake?
- Use extended Slavnov-Taylor identities to examine relation between gauge independence, numerical stability and process independence for the THDM. Is there a universally preferable scheme?

Thank you

for your attention!

Backup: THDMs and FCNCs

$$\mathcal{L}_{Yuk} = - \bar{Q}_L \left(\Gamma_1^d \phi_1 + \Gamma_2^d \phi_2\right) D_R - \bar{Q}_L \left(\Gamma_1^u \tilde{\phi}_1 + \Gamma_2^u \tilde{\phi}_2\right) U_R$$
$$- \bar{L}_L \left(\Gamma_1^l \phi_1 + \Gamma_2^l \phi_2\right) E_R + h.c.$$

- Model I:

  All quarks couple to ϕ_2
  E_R couple to ϕ_2

- Lepton-specific Model:

  All quarks couple to ϕ_2
  E_R couple to ϕ_1

- Model II:

  U_R couple to ϕ_2
  D_R and E_R couple to ϕ_1

- Flipped Model:

  U_R and E_R couple to ϕ_2
  D_R couple to ϕ_1

Backup: On-shell Conditions

1. The real part of the pole of the propagator is given by the physical mass m.
2. The mixing with other fields of the same quantum numbers vanishes on the mass shell, which is defined by $p^2 = m^2$.
3. The field is properly normalized, i.e. the residue of the propagator at the pole is equal to i.

$$G(p^2) = \sqrt{Z_\phi}^* \sqrt{Z_\phi} \int d^4x \; e^{ipx} \; \langle \Omega | T \phi(x) \phi(0) | \Omega \rangle$$

$$= \int d^4x \; e^{ipx} \; \langle \Omega | T \phi_0(x) \phi_0(0) | \Omega \rangle$$

$$=$$

$$= \frac{i}{p^2 - m_0^2 + \Sigma + i\epsilon}$$

Backup: On-shell Conditions

$$\hat{\Gamma}_f(p^2) \;=\; \sqrt{Z_f}^{\dagger} \;\Big[\; p^2 \mathbb{1}_{2\times 2} \;-\; (D_f + \delta D_f) \;+\; \Sigma_f \;\Big]\; \sqrt{Z_f}$$

$$\mathrm{Re}\left(\hat{\Gamma}_{f_1 f_2}(m^2_{f_1})\right) \;\stackrel{!}{=}\; 0, \qquad \mathrm{Re}\left(\hat{\Gamma}_{f_1 f_2}(m^2_{f_2})\right) \;\stackrel{!}{=}\; 0\,,$$

$$\mathrm{Re}\left(\hat{\Gamma}_{f_1 f_1}(m^2_{f_1})\right) \;\stackrel{!}{=}\; 0, \qquad \mathrm{Re}\left(\hat{\Gamma}_{f_2 f_2}(m^2_{f_2})\right) \;\stackrel{!}{=}\; 0\,,$$

$$\mathrm{Re}\left(\frac{\partial \hat{\Gamma}_{f_1 f_1}(p^2)}{\partial p^2}\right)\Bigg|_{p^2 = m^2_{f_1}} \;\stackrel{!}{=}\; 1, \qquad \mathrm{Re}\left(\frac{\partial \hat{\Gamma}_{f_2 f_2}(p^2)}{\partial p^2}\right)\Bigg|_{p^2 = m^2_{f_2}} \;\stackrel{!}{=}\; 1\,.$$

Backup: On-shell Conditions

$$\delta m_{f_i}^2 = \mathrm{Re}\left(\Sigma_{f_i f_i}(m_{f_i}^2) - \delta T_{f_i f_i} \right) \qquad \text{for } i = 1,2\ ,$$

$$\delta Z_{f_i f_i}^{os} = \mathrm{Re}\left(\frac{\partial \Sigma_{f_i f_i}}{\partial p^2} \right)\Bigg|_{p^2 = m_{f_i}^2} \qquad \text{for } i = 1,2\ ,$$

$$\delta Z_{f_i f_j}^{os} = \frac{2}{m_{f_i}^2 - m_{f_j}^2} \left[\Sigma_{f_i f_j}(m_{f_j}^2) - \delta T_{f_i f_j} \right] \qquad \text{for } i,j = 1,2 \text{ and } i \neq j\ .$$

Backup: Tadpole Conditions

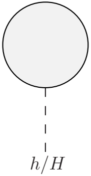
The matrices with tadpole parameters are defined as

$$\delta T \;=\; R(\theta)^T \begin{pmatrix} \frac{\delta T_1}{v_1} & 0 \\ 0 & \frac{\delta T_2}{v_2} \end{pmatrix} R(\theta) \;,$$

where the relation to the physical tadpoles is given through

$$\begin{aligned} -V(\Phi_1, \Phi_2)\big|_{\text{lin. terms}} \quad &= \quad - \; T_1\rho_1 \; - \; T_2\rho_2 \\ &= \quad \underbrace{\left(-c_\alpha T_1 - s_\alpha T_2\right)}_{=: T_H} H \; + \; \underbrace{\left(-c_\alpha T_2 + s_\alpha T_1\right)}_{=: T_h} h \;. \end{aligned}$$

These $T_{h/H}$ are then fixed by the following conditions



$$+ \qquad \delta T_{h/H} \stackrel{!}{=} 0 \;.$$

Backup: The Kanemura Approach

$$\begin{pmatrix} f_1 \\ f_2 \end{pmatrix} = R(\theta)^T \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix} \mapsto R(\theta + \delta\theta)^T \sqrt{Z_\gamma} \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$$

$$\overset{\text{NLO}}{=} \underbrace{R(\delta\theta)^T \; R(\theta)^T \sqrt{Z_\gamma} \; R(\theta)}_{=: \sqrt{Z_f}} R(\theta)^T \begin{pmatrix} \gamma_1 \\ \gamma_2 \end{pmatrix}$$

$$= \sqrt{Z_f} \begin{pmatrix} f_1 \\ f_2 \end{pmatrix}$$

Backup: The Kanemura Approach

$$\begin{pmatrix} H \\ \text{h} \end{pmatrix} \mapsto \begin{pmatrix} 1+\frac{1}{2}\delta Z_{HH} & \delta C_h + \delta\alpha \\ \delta C_h - \delta\alpha & 1+\frac{1}{2}\delta Z_{hh} \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} ,$$

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} \mapsto \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^0G^0} & \delta C_{A^0} + \delta\beta \\ \delta C_{A^0} - \delta\beta & 1+\frac{1}{2}\delta Z_{A^0A^0} \end{pmatrix} \begin{pmatrix} G^0 \\ A^0 \end{pmatrix} ,$$

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} \mapsto \begin{pmatrix} 1+\frac{1}{2}\delta Z_{G^+G^+} & \delta C_{H^+} + \delta\beta \\ \delta C_{H^+} - \delta\beta & 1+\frac{1}{2}\delta Z_{H^+H^+} \end{pmatrix} \begin{pmatrix} G^+ \\ H^+ \end{pmatrix} .$$

Backup: The *Min* Scheme

- The field renormalization matrix in the *Min* Scheme:

$$\sqrt{Z_f^{Min}} \equiv R(\delta\theta)^T \; R(\theta)^T \; \sqrt{Z_\gamma^{\text{Min}}} \; R(\theta)$$

$$= \mathbb{1} + \underbrace{\frac{1}{2}\begin{pmatrix} c_\theta^2 \; \delta Z_1 + s_\theta^2 \; \delta Z_2 & c_\theta s_\theta \left(\delta Z_2 - \delta Z_1\right) + 2\delta\theta \\ c_\theta s_\theta \left(\delta Z_2 - \delta Z_1\right) - 2\delta\theta & s_\theta^2 \; \delta Z_1 + c_\theta^2 \; \delta Z_2 \end{pmatrix}}_{= \delta Z_f^{Min}} .$$

Backup: The *Min* Scheme

- The entries of the wave-function normalization matrix

$$Z_N = \begin{pmatrix} \sqrt{Z_{f_1}} & \sqrt{Z_{f_1}}\ Z_{f_1 f_2} \\ \sqrt{Z_{f_2}}\ Z_{f_2 f_1} & \sqrt{Z_{f_2}} \end{pmatrix}$$

are given by

$$Z_{f_i f_j} = -\frac{\hat{\Sigma}_{f_i f_j}(m_{f_i}^2)}{m_{f_i}^2 - m_{f_j}^2 + \hat{\Sigma}_{f_j f_j}(m_{f_i}^2)} ,$$

$$\sqrt{Z_{f_i}} = \left[1 + \mathrm{Re}\left(\hat{\Sigma}'_{f_i f_i}(p^2) \right) - \mathrm{Re}\left(\frac{\left(\hat{\Sigma}_{f_i f_j}(p^2) \right)^2}{p^2 - m_{f_j}^2 + \hat{\Sigma}_{f_j f_j}(p^2)} \right)' \right]^{-\frac{1}{2}} \Bigg|_{p^2 = m_{f_i}^2} .$$

Backup: Definition of Scenario Classes

Only scenarios respecting

$$m_{H^\pm} \geq M_W + m_H ,$$

were considered. With the help of `ScannerS` [Santos *et al*, arXiv:1301.2599] the following classes of scenarios have been scanned:

Name	Type	$m_{H^\pm}$ [GeV]	m_H [GeV]	m_{A^0} [GeV]	m_{12} [GeV]	tan(β)	$s_{\beta-\alpha}$
C_1	I	[240, 400]	$m_{H^\pm}$ - 110	$m_{H^\pm}$ - 50	$m_{H^\pm}$ - 250	5	0.8
C_2	I	[240, 310]	$m_{H^\pm}$ - 110	$m_{H^\pm}$ - 50	$m_{H^\pm}$ - 250	15	0.95

Table 1 : Definition of two classes of type I scenarios with $m_{H^\pm}$ as only free parameter.