Karlsruher Institut für Technologie

POWHEG & VBFNLO

First steps towards an interface

Forschungsseminar

Franziska Schissler | January 12^{th}, 2012

Outline

1. Motivation
2. The POWHEG-Method
   - Shower algorithm
   - Subtraction formalism
   - POWHEG master formula
3. The POWHEG-BOX
   - Needed ingredients
4. Processes
   - *WW* production
   - *Z* production in vector boson fusion
5. Conclusion & Outlook

Motivation

- Fixed order calculations $\longrightarrow$ predicitions for inclusive quantities
- Shower Monte Carlo (SMC) $\longrightarrow$ exclusive quantities

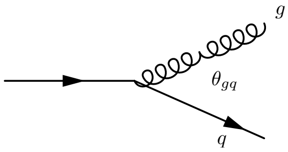
SMC (LO)	NLO
LO (α_s^n) accuracy	NLO ($\alpha_s^{(n+1)}$) accuracy
Large dependence on μ_r/μ_f	reduced dependence
Sudakov suppression of soft/collinear emission	Wrong shapes in the Sudakov region

$\Longrightarrow$ combine both approaches, try to merge benefits

2 matching prescriptions to avoid double counting of radiation:

1. MC@NLO
2. POWHEG (Positive Weight Hardest Emission Generator)

Shower algorithm



$$\frac{1}{(q+g)^2 - m_q^2} = \frac{1}{2E_g E_q (1 - \beta_q \cos\theta_{gq})}$$

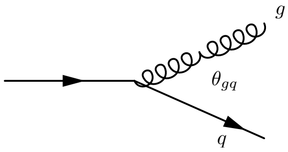
soft divergence: $E_g \to 0$

collinear divergence: $\theta_{gq} \to 0 \, (m_q = 0)$

$\Rightarrow$ initial or final state collinear & soft emission strongly enhanced

- cross section for extra emission factorizes in coll./soft limit
  $\rightarrow$ Shower algorithm to evaluate enhanced contribution at all orders
- description valid from hard scale down to hadronization scale

Shower algorithm



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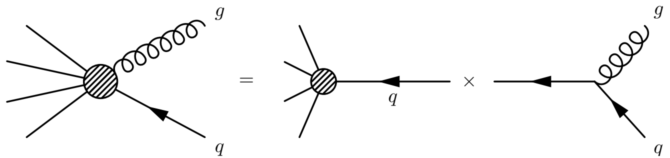
Factorization in the collinear limit



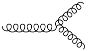
$$|\mathcal{M}_{n+1}|^2 d\Phi_{n+1} \rightarrow |\mathcal{M}_n|^2 d\Phi_n \times \frac{\alpha_s}{2\pi}\,\frac{dt}{t}\,P_{q,qg}(z) dz\,\frac{d\varphi}{2\pi}$$

$$d\Phi_{n+1} \rightarrow d\Phi_n \times d\Phi_r\,(\propto dt\,dz\,d\varphi)$$

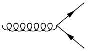
t : ordering variable: $(q+g)^2, p_T^2, E^2\theta^2$

z : Energy fraction of quark $\frac{q^0}{q^0+g^0}$

$P_{q,qg}(z) = C_F \frac{1+z^2}{1-z}$: Altarelli-Parisi splitting kernel

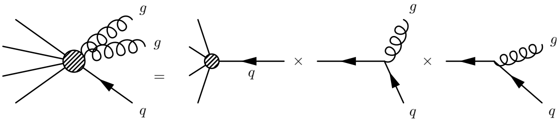


$$P_{g,gg}(z) = C_A\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right)$$



$$P_{g,qq}(z) = T_F\left(z^2 + (1-z)^2\right)$$

Multiple emissions



$$|\mathcal{M}_{n+2}|^2 d\Phi_{n+2} \to |\mathcal{M}_n|^2 d\Phi_n \times \frac{\alpha_s}{2\pi}\frac{dt'}{t'} P(z')dz' \frac{d\varphi'}{2\pi} \times \frac{\alpha_s}{2\pi}\frac{dt}{t} P(z)dz \frac{d\varphi}{2\pi}\theta(t'-t)$$

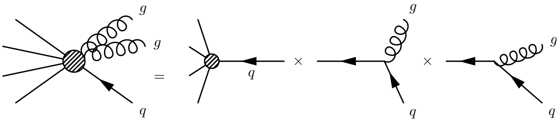
- σ_n for n collinear emissions contains factorized integral

$$\sigma_n \approx \sigma_0 \alpha_s^n \int_{t_0}^{Q^2} \frac{dt_n}{t_n}\frac{dt_{n-1}}{t_{n-1}} \dots \frac{dt_1}{t_1} \times \theta(Q^2 > t_n > t_{n-1} \dots > t_1 > t_0)$$

$$= \sigma_0 \frac{\alpha_s^n}{n!}\left[\log\left(\frac{Q^2}{t_0}\right)\right]^n$$

  - Leading Log approximation
  - Q^2: ordering variable upper cutoff, scale of hard subprocess
  - $t_0 \approx \Lambda_{QCD}$: infrared cutoff

Multiple emissions



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$$\times \frac{\alpha_s}{2\pi}\frac{dt}{t} P(z)dz \frac{d\varphi}{2\pi}\theta(t'-t)$$

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Sudakov form factor

Single emission probability: $d\mathcal{P}_{1e}(t, t+dt) = \frac{\alpha_s(t)}{2\pi}\frac{dt}{t}\int P_{i,jk}(z)dz$

probability of not emitting: $d\mathcal{P}_{0e}(t, t+dt) = 1 - d\mathcal{P}_{1e}(t, t+dt)$

no-emission probability along line i from scale t_1 to t_2 by dividing into subintervals

$$\Rightarrow \mathcal{P}_{0e}(t_1, t_2) = \lim_{N\to\infty}\prod_{n=1}^{N}\left(1 - \sum_{(jk)}\frac{\alpha_s(t_n)}{2\pi}\frac{dt_n}{t_n}\int P_{i,jk}(z)dz\right)$$

$$= \exp\left(-\int_{t_2}^{t_1}\frac{\alpha_s(t)}{2\pi}\frac{dt}{t}\sum_{(jk)}\int P_{i,jk}(z)dz\right) \equiv \Delta_i(t_1, t_2)$$

$\Delta_i(t_1, t_2)$: Sudakov Form Factor

The POWHEG-Method

[Nason 2004],[Frixione, Nason, Oleari 2007]

Subtraction formalism

- NLO cross section for $2 \rightarrow n$ process,

$$d\Phi_{n+1} = d\Phi_n \, d\Phi_{rad}, \; d\Phi_{rad} \propto dt \, dz \, d\varphi$$

$$d\sigma_{NLO} = \Big[\mathcal{B}(\Phi_n) + \underbrace{\overbrace{\mathcal{V}_\text{b}(\Phi_n)}^{\text{UV finite}} + \mathcal{R}(\Phi_{n+1}) d\Phi_{rad}}_{\text{seperately IR divergent}}\Big] d\Phi_n, \;\; \mathcal{V}_\text{b} = 2\text{Re}\left(\mathcal{M}_B^* \mathcal{M}_V\right)$$

- add local Counterterms to cancel IR divergencies (KLN theorem)

$$\mathcal{R}(\Phi_n, \Phi_{rad}) - \mathcal{C}(\Phi_n, \Phi_{rad}) \rightarrow \text{finite}$$

$$\mathcal{V}(\Phi_n) = \mathcal{V}_\text{b}(\Phi_n) + \int \mathcal{C}(\Phi_n, \Phi_{rad}) d\Phi_{rad} \rightarrow \text{finite}$$

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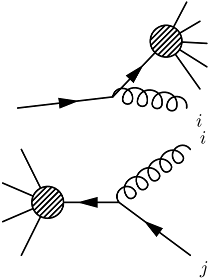
- Counterterms $\mathcal{C}$ have to cancel IR divergencies of the virtual and real amplitude
- soft & collinear divergencies are universal
- Catani-Seymour subtraction (dipole subtraction) [Catani,Seymour 1997]
- FKS subtraction (POWHEG-BOX) [Frixione, Kunszt, Signer 1996],[Frixione 1997]
  - Write Real term $\mathcal{R}$ as sum of terms with AT MOST 1 singular & collinear singularity associated with 1 parton (FKS parton)
  - Singular regions:
    - *i*: final state parton i soft/coll to initial state parton
    - *ij*: final state parton i soft/coll to final state parton j



- $\mathcal{R} = \sum_i \mathcal{R}_i + \sum_{ij} \mathcal{R}_{ij}$
- add Counterterm for every region with specific mapping of momenta

Finite NLO cross section

$$d\sigma_{NLO} = \Big[\mathcal{B}(\Phi_n) + \mathcal{V}_{\mathrm{b}}(\Phi_n) + \mathcal{C}(\Phi_n, \Phi_{rad}) d\Phi_{rad} + \left[\mathcal{R}(\Phi_n, \Phi_{rad}) - \mathcal{C}(\Phi_n, \Phi_{rad})\right] d\Phi_{rad}\Big] d\Phi_n,$$

$$\mathcal{V}(\Phi_n) = \underbrace{\mathcal{V}_{\mathrm{b}}(\Phi_n) + \overbrace{\int \mathcal{C}(\Phi_n, \Phi_{rad}) d\Phi_{rad}}^{\text{divergent}}}_{\text{finite}}$$ → cancel divergencies analytically

$\mathcal{R}(\Phi_n, \Phi_{rad}) - \mathcal{C}(\Phi_n, \Phi_{rad})$ → numerical cancelation

$\overline{B}$-function

$$\overline{\mathcal{B}} = \int d\sigma_{NLO}\, d\Phi_{rad} = \left[\mathcal{B}(\Phi_n) + \mathcal{V}(\Phi_n) + \int \left[\mathcal{R}(\Phi_n, \Phi_{rad}) - \mathcal{C}(\Phi_n, \Phi_{rad})\right] d\Phi_{rad}\right] d\Phi_n$$

Inclusive NLO cross section at fixed underlying Born kinematics

Finite NLO cross section

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$$\mathcal{R}(\Phi_n, \Phi_{rad}) - \mathcal{C}(\Phi_n, \Phi_{rad}) \rightarrow \text{numerical cancelation}$$

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Inclusive NLO cross section at fixed underlying Born kinematics

POWHEG master formula

- first branching at scale t' starting from higher scale t is independent of subsequent ones:

$$d\mathcal{P}_{first} = \Delta_i(t, t')\frac{\alpha_s(t')}{2\pi}\frac{dt'}{t'}\sum_{(jk)}\int P_{i,jk}(z)\,dz\int\frac{d\varphi}{2\pi}$$

- Standard Shower MC's first emission

$$d\sigma_{SMC} = \underbrace{\mathcal{B}(\Phi_n)}_{Born}\,d\Phi_n\left[\Delta_{SMC}(t_0) + \Delta_{SMC}(t)\underbrace{\frac{\alpha_s(t)}{2\pi}\frac{1}{t}P(z)}_{\lim_{k_T\to 0}\mathcal{R}(\Phi_{n+1})/\mathcal{B}(\Phi_n)}\,d\Phi_{rad}^{SMC}\right]$$

- $\Delta_{SMC}(t) = exp\left[-\int d\Phi'_{rad}\frac{\alpha_s(t')}{2\pi}\frac{1}{t'}P(z')\theta(t'-t)\right]$

  SMC Sudakov factor: Probability of not emitting a parton at scale greater than t

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- POWHEG first emission

$$d\sigma_{PWG} = \overline{\mathcal{B}}(\Phi_n)\, d\Phi_n \left[\Delta_{PWG}(\Phi_n, p_T^{min}) + \Delta_{PWG}(\Phi_n, k_T) \frac{\mathcal{R}(\Phi_n, \Phi_{rad})}{\mathcal{B}(\Phi_n)} \theta(k_T - p_T) d\Phi_{rad} \right]$$

- POWHEG Sudakov factor: Probability of not emitting with transverse momentum harder than p_T

$$\Delta_{PWG}(\Phi_n, p_T) = exp \left[- \int d\Phi'_{rad} \frac{\mathcal{R}(\Phi_n, \Phi_{rad})}{\mathcal{B}(\Phi_n)} \theta(k_T(\Phi_n, \Phi'_{rad}) - p_T) \right]$$

- NLO cross section for inclusive quantities
- NLO accuracy preserved in the hard region, $\Delta_{PWG}(\Phi_n, p_T) \approx 1$

$$d\sigma_{PWG} \approx \frac{\overline{\mathcal{B}}(\Phi_n)}{\mathcal{B}(\Phi_n)} \mathcal{R}(\Phi_n, \Phi_{rad}) d\Phi_n d\Phi_{rad} \approx \mathcal{R}(\Phi_n, \Phi_{rad}) \left(1 + \mathcal{O}(\alpha_s)\right) d\Phi_n\, d\Phi_{rad}$$

- LL accuracy of a SMC in soft/collinear limit $k_T \to 0$

$$\frac{\mathcal{R}(\Phi_n, \Phi_{rad})}{\mathcal{B}(\Phi_n)} d\Phi_{rad} \approx \frac{\alpha_s}{2\pi} \frac{1}{t} P(z)\, dt\, dz \frac{d\varphi}{2\pi}, \quad \overline{\mathcal{B}} \approx \mathcal{B}\left(1 + \mathcal{O}(\alpha_s)\right)$$

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The POWHEG-BOX

[Alioli, Nason, Oleari, Re 2010]

- FORTRAN code
- Projection of real cross section into singular regions
- Counter terms (soft/coll. approximation) & radiation phase space
- pdfs
- NLO differential cross section $\longrightarrow$ comparison to VBFNLO
- LesHouches event files (unweighted events)
  - Calculate upper bounds for efficient generation of Sudakov suppressed events
  - Hardest emission according to POWHEG Sudakov

Needed ingredients

obtained from VBFNLO-subroutines, converted into POWHEG-BOX format

1. Flavour structures of Born & Real processes
2. Coupling constants
3. Born phase space
4. Born squared amplitudes $\mathcal{B}, \mathcal{B}_{ij}, \mathcal{B}_{\mu\nu}$
5. Real matrix elements squared
6. Virtual finite part ($2\,Re\left(B^*\ V_{fin}\right)$)
7. Born colour structures in $N_c \to \infty$ limit

Needed ingredients

1. Flavour structures of Born & Real processes
   - find all subprocesses
   - recycle old processes
2. Coupling constants (conversion to POWHEG-format): [Jäger, Zanderighi 2011]
3. Born phase space (PS)
   - 'easy' PS: take VBFNLO one (Jacobian $\overset{!}{\neq}$ 0, 1 to 1 correspondence)
   - 'difficult' PS: produce 'unweighted' events (weight=$\frac{\sigma_{tot}}{\mathcal{M}^2}$)
4. Born squared amplitudes $\mathcal{B}, \mathcal{B}_{ij}, \mathcal{B}_{\mu\nu}$
   - devide into different flavour structures (not calculated in 1 step)
   - Flag `smartsig`
   - $\sum$ helicity combinations ($\mathcal{B} \overset{!}{\neq} 0$)
   - colour-correlated $\mathcal{B}_{ij}$: $\sum_{i,i\neq j} \mathcal{B}_{ij} = C_{f_j}\mathcal{B}$
   - spin-correlated $\mathcal{B}_{\mu\nu}$: $\sum_{\mu,\nu} g_{\mu\nu}\mathcal{B}^{\mu\nu} = -\mathcal{B}$

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5. Real matrix elements squared $\mathcal{R}$
   - Flavour structures, (anti-)particles
   - soft approximation, parton k radiated

$$\mathcal{R} = 4\pi\alpha_s\mu_r^{2\epsilon}\left[\sum_{i\neq j}\mathcal{B}_{ij}\frac{k_i\cdot k_j}{(k_i\cdot k)(k_j\cdot k)} - \mathcal{B}\sum_i\frac{k_i^2}{(k_i\cdot k)^2}C_i\right] + \mathcal{R}^f,$$

   - collinear approximation, final state singularities:

$$\begin{aligned}\mathcal{R}^{gg} &\propto \mathcal{B}_{\mu\nu},\ P_{g,gg}\\ \mathcal{R}^{qg} &\propto \mathcal{B},\ P_{q,qg}\\ \mathcal{R}^{q\bar{q}} &\propto \mathcal{B}_{\mu\nu},\ P_{g,q\bar{q}}\end{aligned}$$

6. Virtual finite part ($2\,Re\left(\mathcal{M}^*_{Born}\mathcal{M}_{V_{fin}}\right)$)

$$\mathcal{V}_{\rm b} = \mathcal{N}\,\frac{\alpha_s}{2\pi}\left[-\sum_{i\in\mathcal{I}}\left(\frac{1}{\epsilon^2}\,C_{f_i} + \frac{1}{\epsilon}\,\gamma_{f_i}\right)\mathcal{B} + \frac{1}{\epsilon}\sum_{\substack{i,j\in\mathcal{I}\\ i\neq j}}\log\frac{2k_i\cdot k_j}{Q^2}\,\mathcal{B}_{ij} + \mathcal{V}_{\rm fin}\right],$$

where

$$\mathcal{N} = \frac{(4\pi)^\epsilon}{\Gamma(1-\epsilon)}\left(\frac{\mu^2}{Q^2}\right)^\epsilon, Q = \mu.$$

$$\mathcal{N}_{\rm VBFNLO} = (4\pi)^\epsilon\,\Gamma(1+\epsilon)\left(\frac{\mu^2}{s}\right)^\epsilon.$$

$$\frac{\mathcal{N}_{\rm VBFNLO}}{\mathcal{N}} = 1 + \epsilon\ln\left(\frac{\mu^2}{s}\right) + \frac{1}{2}\epsilon^2\ln^2\left(\frac{\mu^2}{s}\right) + \frac{\pi^2}{6}\epsilon^2 + \mathcal{O}(\epsilon^3)$$

need ϵ poles of virtual part to match normalization constant.

$$\text{FKS-subtraction} \rightarrow \mathcal{V} = \frac{\alpha_s}{2\pi}\left(\mathcal{Q}\mathcal{B} + \sum_{\substack{i,j\in\mathcal{I}\\ i\neq j}}\mathcal{I}_{ij}\mathcal{B}_{ij} + \mathcal{V}_{\rm fin}\right)$$

WW **production**, $pp \to WW \to l^+ \nu l^- \overline{\nu}$

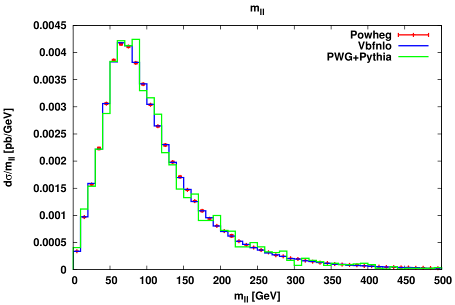
POWHEG: $\sigma = (1.312246 \pm 0.001024)$ pb [Melia, Nason, Röntsch, Zanderighi 2011]

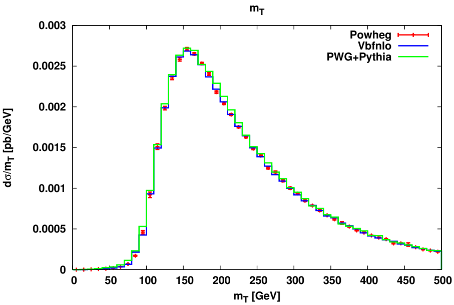
VBFNLO: PS+ME, $(\sigma = 1.313733 \pm 0.000754)$ pb

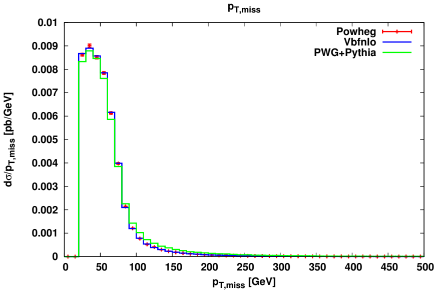
Cuts:

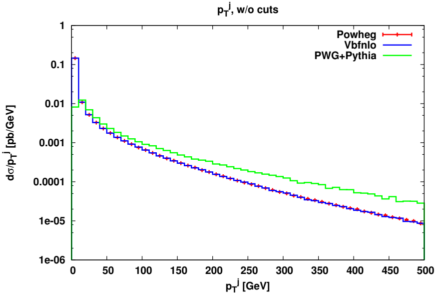
$p_{T,l} > 20$ GeV, $|\eta_l| < 2.5$, $p_{T,miss} > 20$ GeV, $p_{T,j} > 20$ GeV, $|\eta_j| < 3.5$

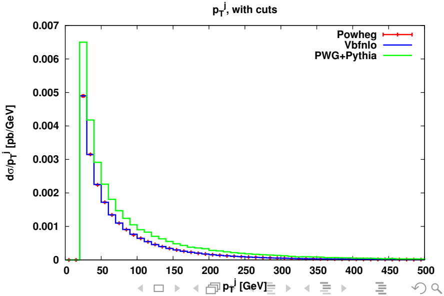
Shower: PYTHIA 6.421 + VBFNLO, [Sjostrand, Mrenna, Skands 2006]






p_T,miss
Powheg
Vbfnlo
PWG+Pythia
dσ/p_T,miss [pb/GeV]
0.01
0.009
0.008
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0 50 100 150 200 250 300 350 400 450 500
p_T,miss [GeV]


p_T^j, w/o cuts
Powheg
Vbfnlo
PWG+Pythia
dσ/p_T^j [pb/GeV]
1
0.1
0.01
0.001
0.0001
1e-05
1e-06
0 50 100 150 200 250 300 350 400 450 500
p_T^j [GeV]

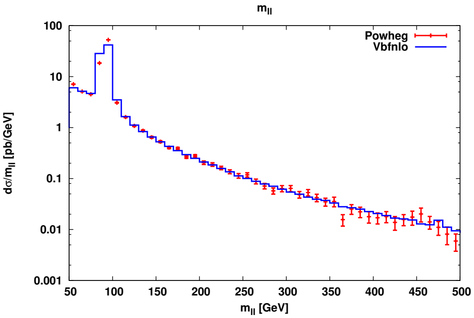

p_T^j, with cuts
Powheg
Vbfnlo
PWG+Pythia
dσ/p_T^j [pb/GeV]
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
0 50 100 150 200 250 300 350 400 450 500
p_T^j [GeV]

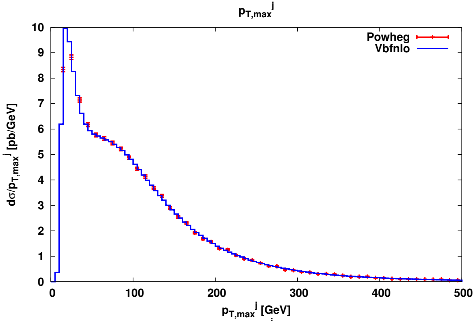
Z in VBF, $pp \to Zjj \to l^+ l^- jj$

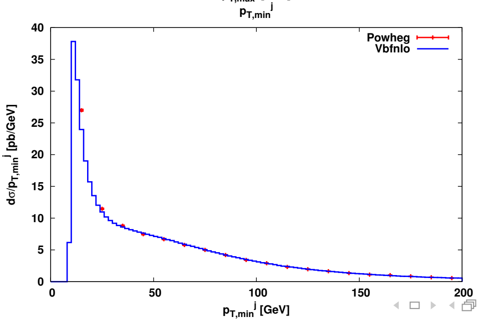
PS: unweighted events, ME: VBFNLO

LO VBFNLO: $\sigma = \left(986.2 \pm 0.4\right)$ fb

LO POWHEG: $\sigma = \left(986.8 \pm 0.5\right)$ fb




$p_{T,max}^j$
Powheg
Vbfnlo
$d\sigma/p_{T,max}^j$ [pb/GeV]
$p_{T,max}^j$ [GeV]


$p_{T,min}^j$
Powheg
Vbfnlo
$d\sigma/p_{T,min}^j$ [pb/GeV]
$p_{T,min}^j$ [GeV]

Conclusion & Outlook

- *WW* production works fine
  - anomalous couplings
  - *WZ*, $W\gamma$
  - Triboson processes
- unweighted events as Phase space generator $\surd$
- Vector boson fusion
  - *Zjj*: LO $\surd$, NLO??
  - *Wjj*: learn more about this
  - plenty of other VBF processes