Charged Higgs Bosons in the Complex MSSM

DAO Thi Nhung

ITP Research Seminar

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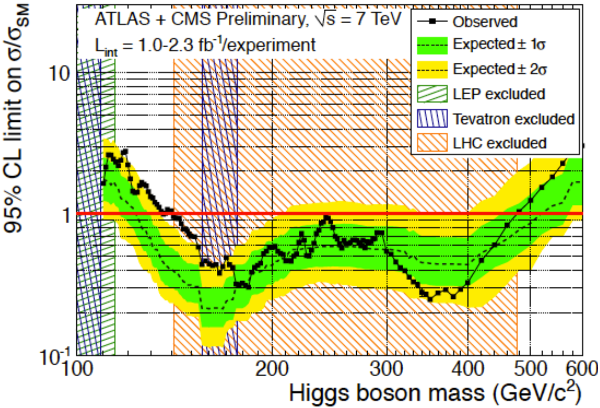
Outline

1. Overview
   - Charged Higgs Bosons Searches
   - Complex MSSM
2. Calculation Framework
3. Decay width and CP asymmetry in $H^\pm \to h_1 W^\pm$
4. Production rate and CP asymmetry in $pp \to W^\mp H^\pm$
5. Conclusions

Where charged Higgs bosons come from?

- What is the mechanism to generate particle masses? The Higgs mechanism is confirmed when Higgs bosons are found.
- No theoretical and experimental constraint on the number of Higgs bosons

$$\text{SM: one Higgs doublets } \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{EWSB} h^0$$



How many Higgs bosons?

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$$\text{SM: one Higgs doublets } \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \xrightarrow{EWSB} h^0$$

- Two-Higgs-Doublet-Model (2HDM) is the simple extension of the SM

$$\begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \text{ and } \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix} \xrightarrow{EWSB} h^0, H^0 \text{ (CP even)}, A^0 \text{ (CP odd)}, H^\pm$$

  - Type I: fermions couple to one doublet → No FCNC at tree-level
  - Type II: down-type fermions couple to H_1, up-type fermions to H_2 (the MSSM) → No FCNC at tree-level
  - Type III: fermions couple to both H_1 and H_2

$H^\pm$ → existence of an extra non-trivial Higgs multiplet

The Minimal Supersymmetry Standard Model (MSSM) Higgs sector

is more predictive in comparison with 2HDM type II:

- Two Higgs doublets with opposite hypercharge are required (analytic superpotential, anomaly free)
- Quartic coupling in Higgs potential is fixed

$$
\begin{aligned}
V \quad = \quad & m_1^2 H_{ui}^* H_{ui} + m_2^2 H_{di}^* H_{di} + \epsilon^{ij} (m_{12}^2 H_{ui} H_{dj} + \text{ H.c}) \\
& + \frac{g_1^2 + g_2^2}{8} (H_{ui}^* H_{ui} - H_{di}^* H_{di})^2 + \frac{g_2^2}{2} |H_{ui}^* H_{di}|^2
\end{aligned}
$$

- Predict a light neutral Higgs boson: $m_{h^0} < 140$ GeV
- Number of Higgs-sector parameters are reduced

$$
M_{H^\pm} (M_{A^0}), \tan\beta = \frac{v_2}{v_1}
$$

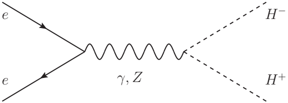
other good points of the MSSM: solve hierarchy problem, unification, darkmatter candidate, ...

Charged Higgs searches at colliders

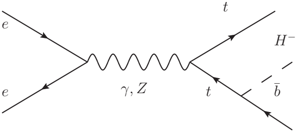
e^-e^+ colliders (LEP, future ILC,...)

- Two main production processes

1) $e^-e^+ \rightarrow H^-H^+$



2) $e^-e^+ \rightarrow H^-t\bar{b}$



- subsequence decays:
  - $M_{H^\pm} < m_t - m_b$: $H^- \rightarrow \tau\bar{\nu}_\tau$, $H^- \rightarrow \bar{c}s$
  - $M_{H^\pm} > m_t + m_b$: $H^- \rightarrow \bar{t}b$, $H^- \rightarrow hW$

[HDECAY, FEYNHIGGS, CPsuperH, FHOLD]

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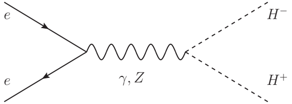

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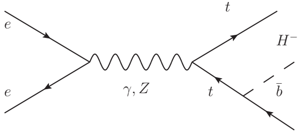

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[HDECAY, FEYNHIGGS, CPsuperH, FHOLD]

LEP (192-209 GeV) analysis based on process 1):

- $M_{H^\pm} \geq 78.6$ GeV at 95% C.L for general 2HDM
- within MSSM, $M^2_{H^\pm} = M^2_A + M^2_W$, $M_A \geq 93.4$ GeV
  $\to M_{H^\pm} \geq 120$ GeV

Charged Higgs searches at colliders

Hadron colliders (Tevatron, LHC)

- Production processes
  1. $pp \to H^- t$ [Plehn, Shou, M. Beccaria, Elber]
     largest production rate
  2. $pp \to H^- tb$ [Stirling, Wu Peng, Dittmaier]
     (with tagged bottom quark)
  3. $pp \to H^\pm W^\mp$ [Kniehl, O. Brein, Hollik, Gao, Rauch, Yang]
  4. $pp \to H^- H^+$ [Eichten, Plehn, Hollik, Kniehl]

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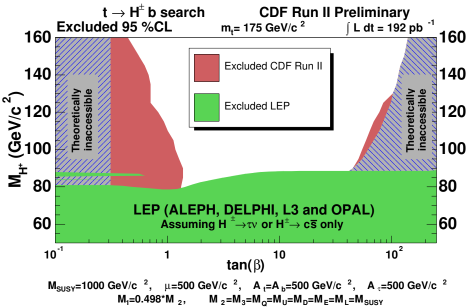
CDF analysis based on:

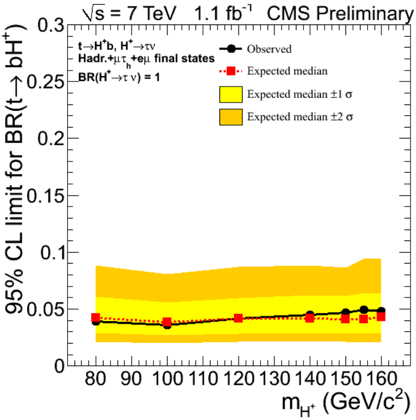
- $gg \to t\bar{t} \to (H^+ \bar{b})\bar{t}$
- $M_{H^\pm} < m_t - m_b$
- important coupling Htb

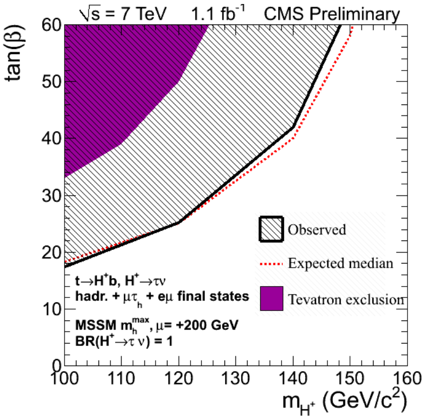
  Phys.Rev.Lett.96:042003



LHC results

- $gg \to t\bar{t} \to (H^+\bar{b})\bar{t}$
- $M_{H^\pm} < m_t - m_b$





Complex Phases in the MSSM

- The superpotential

$$W = (H^d.\tilde{L})h^e\tilde{e}_R + (H^d.\tilde{Q})h^d\tilde{d}_R + (H^u.\tilde{Q})h^u\tilde{u}_R + \mu H^d.H^u$$

- soft-SUSY breaking sector

$$\begin{aligned}\mathcal{L}_{soft} &= -\frac{1}{2}M_3\tilde{g}\tilde{g} - \frac{1}{2}M_2\tilde{W}_i\tilde{W}_i - \frac{1}{2}M_1\tilde{B}\tilde{B} - M_{\tilde{f}}^2\tilde{f}^*\tilde{f} \\ &+ m_{12}^2 H^d H^u + (H^d.\tilde{L})A_l\tilde{e}_R + (H^u.\tilde{Q})A_u\tilde{u}_R + (H^d.\tilde{Q})A_d\tilde{d}_R + \text{ H.c}\end{aligned}$$

- h_{ij}^f, μ, M_i, A_{ij}^f and m_{12}^2 contain complex phases (69 phases, 41 physical phases)
- Number of physical phases can be reduced by using constrains like: FCNC, GUT relation ...
  - Phases may give large impact on charged Higgs productions and decays :

$$\boxed{\phi_\mu,\ \phi_i,\ \phi_\tau,\ \phi_b,\ \phi_t}$$

CP violating phases enter mass matrices

- The sfermion mass matrices

$$M_{\tilde{f}} = \begin{pmatrix} M^2_{\tilde{f},L} + m_f^2 + M_Z^2 \cos 2\beta (I_3^f - Q_f s_w^2) & m_f X_f^* \\ m_f X_f & M^2_{\tilde{f}_R} + m_f^2 + M_Z^2 \cos 2\beta Q_f s_w^2 \end{pmatrix}$$

  where $X_f = A_f - \mu^* \{\cot\beta, \tan\beta\}$, $\cot\beta$ for up-type squarks, $\tan\beta$ for d-type squarks and charged sleptons $\Rightarrow$ important for the third generation

- The chargino mass matrix

$$X = \begin{pmatrix} M_2 & \sqrt{2}\sin\beta M_W \\ \sqrt{2}\cos\beta M_W & \mu \end{pmatrix}$$

- The neutralino mass matrix

$$Y = \begin{pmatrix} M_1 & 0 & -M_Z s_w \cos\beta & M_Z s_w \sin\beta \\ 0 & M_2 & M_Z c_w \cos\beta & M_Z c_w \sin\beta \\ -M_Z s_w \cos\beta & M_Z c_w \cos\beta & 0 & -\mu \\ M_Z s_w \sin\beta & M_Z c_w \sin\beta & -\mu & 0 \end{pmatrix}$$

- The Higgs sector is CP conserving at tree-level, complex phases enter at least one-loop level

CP problem and solutions in MSSM

Problem

- large SUSY-loops contribution to EDMs exceed the experimental bounds

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Several remendies to overcome this problem

- Irrelevant: the first generation of sleptons and the two first generations of squarks are very heavy
- Approximate CP asymmetry: phases are small $\mathcal{O}(10^{-2})$
- Internal cancelations: phases can be large, masses of sparticles in the reach of accelerators

Our purposes:

- Studying effects of complex phases of μ and soft SUSY breaking parameters in
  - $H^{\pm} \to W^{\pm} h_1$ Hollik, D.T.N JHEP 1101:060,2011
  - $pp \to H^{\pm} W^{\mp}$ Hollik, L.D. Ninh, D.T.N Phys.Rev.D38:075003
  - $pp \to H^{-} tb$ Hollik, L.D. Ninh, D.T.N to be appear
- Studying the CP violating asymmetry arising from those phases

$$\delta^{CP} = \frac{\sigma^{+} - \sigma^{-}}{\sigma^{+} + \sigma^{-}}.$$

where σ^{+} can be cross section or decay width of the process, σ^{-} is for the charge conjugate process

$$\delta^{CP} \neq 0 \begin{cases} \text{weak phases (complex couplings)} \\ \text{strong phases (loop,}\gamma_5\text{)} \end{cases}$$

- Calculations are done at one-loop level

NLO Calculation Framework

Singularities:

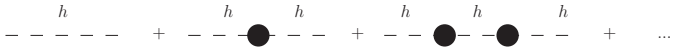
- UV divergencies are cured by renormalization,
  - OS-scheme for e, W-boson, M_{H^+}, m_t
  - DR-scheme for α_s, $\tan\beta$, m_b, $H^\pm$ wave function
- Soft singularities are cancelled between the virtual part and the gluon and photon radiations (phase space slicing and dipole subtraction methods)
- Collinear singularities of the type $\alpha_s \ln(m_b)$ and $\alpha \ln(m_b)$ are absorbed into running m_b, (anti-) bottom PDFs
- Threshold singularities are cured by using complex masses
- Anomalous three-point singularities are integrable
- The top-quark resonance in $g/\gamma b \to W^\mp H^\pm b$ are subtracted in a gauge-invariant way.

Tools:

- FeynArts 3.4, FormCalc 6.0, LoopTools 2.3, LoopInts (Ninh)
- Phase-space intergrators: VEGAS, BASES

Calculations relate to internal and external neutral Higgs bosons,

- No-mixing loop-corrected propagator



$$= \frac{-i}{p^2 - m_h^2 + \hat{\Sigma}_{hh}(p^2)} = \frac{-i{\color{blue}Z}}{p^2 - \underbrace{(m_h^2 + \delta m^2)}_{{\color{red}M_{h_1}^2}}}$$

  in on-shell scheme, ${\color{blue}Z} = 1$ and $\delta m^2 = 0$
  in $\overline{\mathrm{DR}}$ scheme, ${\color{blue}Z} = \left(1 + \frac{d}{dp^2}\mathrm{Re}\,\hat{\Sigma}_{hh}(p^2)\right)^{-1}\Big|_{p^2 = M_{h_1}^2}$ and $\delta m^2 = -\hat{\Sigma}_{hh}(M_{h_1}^2)$

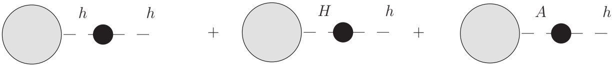
  - For external line:

$$\langle i|T|\phi_h(x)\rangle \to {\color{blue}Z}^{1/2}\langle i|T|\phi_h(x)\rangle$$

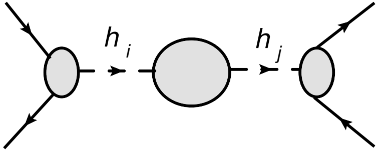
  - For internal line: $\boxed{\frac{-i}{p^2 - m_h^2}}$, $\frac{-i}{p^2 - {\color{red}M_{h_1}^2}}$ (to ensure finiteness (UV, IR) and gauge invariance)

- Three particles mixing

$${\color{blue}Z}^{1/2}\left(\langle i|T|\phi_h\rangle + \langle i|T|\phi_H\rangle \underbrace{\langle\phi_H|\phi_h\rangle}_{{\color{blue}Z_{Hh}}} + \langle i|T|\phi_A\rangle \underbrace{\langle\phi_A|\phi_h\rangle}_{{\color{blue}Z_{Ah}}}\right)$$



Neutral Higgs propagator resummation



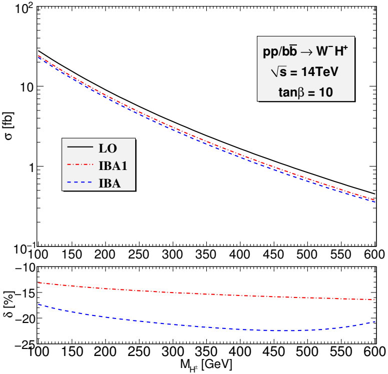
$$\mathcal{A}(p^2) = \sum_{ij} \Gamma_i \, \Delta_{ij}(p^2) \, \Gamma_j, \quad i = h, H, A,$$

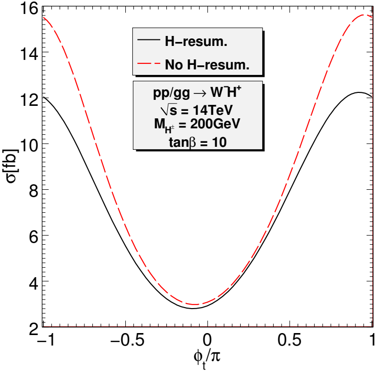
$\Gamma_{i,j}$ are one-particle irreducible Higgs vertices.

$$\begin{aligned} \Delta(p^2) &= i[p^2 - \mathcal{M}(p^2)]^{-1}, \\ \mathcal{M}(p^2) &= \begin{pmatrix} m_h^2 - \hat{\Sigma}_{hh}(p^2) & -\hat{\Sigma}_{hH}(p^2) & -\hat{\Sigma}_{hA}(p^2) \\ -\hat{\Sigma}_{hH}(p^2) & m_H^2 - \hat{\Sigma}_{HH}(p^2) & -\hat{\Sigma}_{HA}(p^2) \\ -\hat{\Sigma}_{hA}(p^2) & -\hat{\Sigma}_{HA}(p^2) & m_A^2 - \hat{\Sigma}_{AA}(p^2) \end{pmatrix}. \end{aligned}$$

- m_i $(i = h, H, A)$ are the lowest-order Higgs-boson masses
- $\hat{\Sigma}_{ij}$ the renormalized self-energies, $\hat{\Sigma}_{h/HA}$ vanish in real MSSM,
- Loop-corrected masses are obtained by diagonalizing $\mathcal{M}(p^2)$
- To avoid double counting, we have to discard all $h_i h_j$ self-energies diagrams in NLO EW corrections

Higgs mixing resummation effects





- less than 10% in subprocesses $b\bar{b} \to W^\mp H^\pm$
- large effects (30% at $\phi_t = \pm\pi$) in subprocesses $gg \to W^\mp H^\pm$

The decay width of $H^- \to h_1 W^-$

$$\Gamma(H^- \to W^- h_1) = \frac{\lambda^{3/2}(M_{H^-}^2, M_W^2, M_{h_1}^2)}{16\pi M_{H^-}^3 M_W^2} \times |M_{h_1}|^2$$

- The Born result: $\Gamma^{(0)}$
  $$M_{h_1} = M_h^{\text{tree}}$$
- The Born improved result: $\Gamma_{\mathbf{Z}}^{(0)}$
  $$M_{h_1} = \sum_i \mathbf{Z}_{1i} M_i^{\text{tree}}, \quad i = h, H, A.$$
- The one-loop improved result: $\Gamma_{\mathbf{Z}}^{(0+1+2)}$
  $$M_{h_1} = \sqrt{Z_{H^-H^+}}\Big[\sum_i \mathbf{Z}_{1i} M_i^{\text{tree}} + \mathbf{Z}_{11}\delta M_h\Big]$$

  $\Gamma_{\mathbf{Z}}^{(0+1)}$: no δM_h^2 (s)top/(s)bottom

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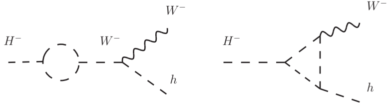

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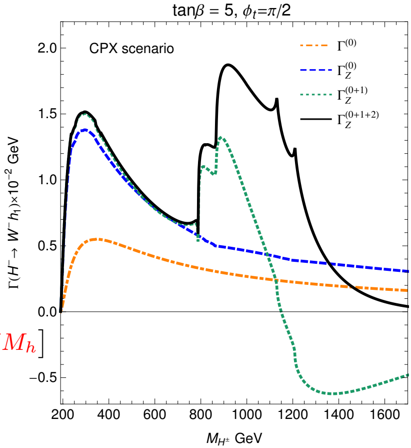
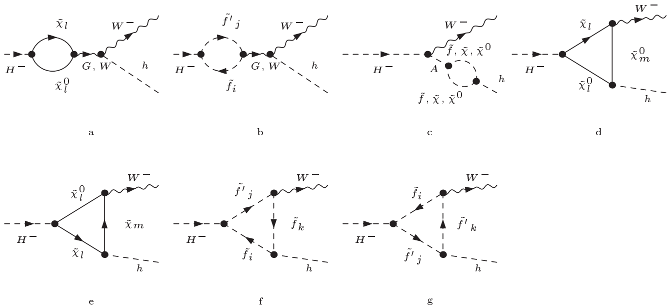


15 % at $M_{H^-} = 300$ GeV

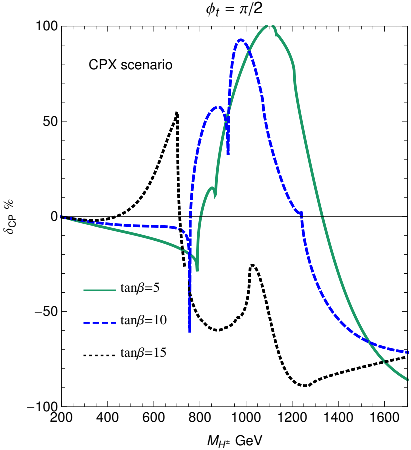
$$\mathcal{A}_{CP} = \frac{\Gamma(H^- \to W^- h_1) - \Gamma(H^+ \to W^+ h_1)}{\Gamma(H^- \to W^- h_1) + \Gamma(H^- \to W^- h_1)}$$

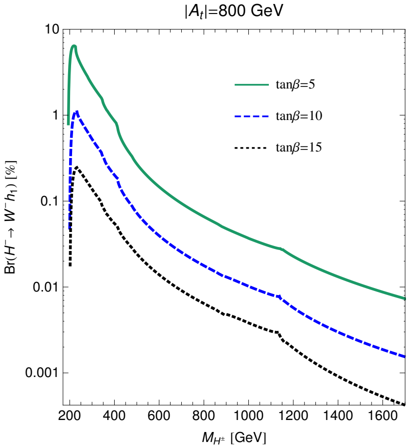
- The strong phases come from one-loop integral, particles inside loops can be on-shell
- The weak phases come from complex couplings in the following diagrams



CP asymmetry in the decays $H^{\pm} \to h_1 W^{\pm}$

$$\mathcal{A}_{CP} = \frac{\Gamma(H^- \to W^- h_1) - \Gamma(H^+ \to W^+ h_1)}{\Gamma(H^- \to W^- h_1) + \Gamma(H^- \to W^- h_1)}$$





CP asymmetry large but difficult to observe due to small branching ratio

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3. Decay width and CP asymmetry in $H^{\pm} \to h_1 W^{\pm}$
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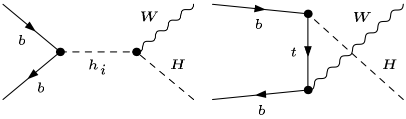
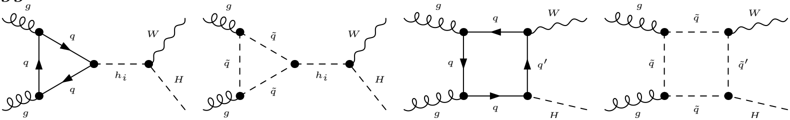
5. Conclusions

The lowest order: $b\bar{b} \to H^{\pm}W^{\mp}$, $gg \to H^{\pm}W^{\mp}$

- $b\bar{b} \to H^{\pm}W^{\mp}$



- $gg \to H^{\pm}W^{\mp}$



Importances

- effective bottom-Higgs couplings:

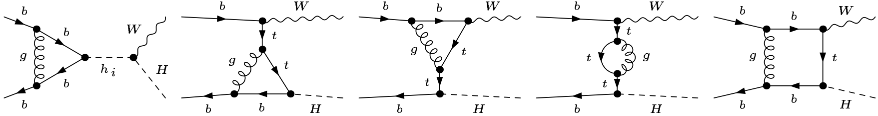
$$\lambda_{b\bar{b}h/H/A} \propto \frac{m_b^{\overline{\mathrm{DR}}}(\mu_R)(1-\Delta_b/t_\beta/t_\alpha)}{1+\Delta_b}$$

$$\lambda_{b\bar{t}H^+} \propto \left(\frac{m_t}{\tan\beta}P_L + \frac{m_b^{\overline{\mathrm{DR}}}(\mu_R)(1-\Delta_b/t_\beta^2)}{1+\Delta_b}\tan\beta P_R\right)$$

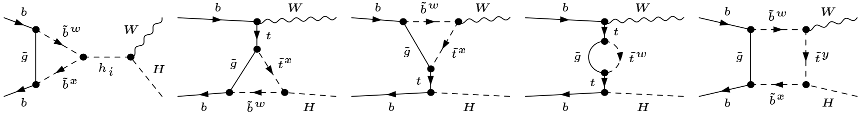
- neutral Higgs propagator: $\mathcal{A}(p^2) = \sum_{ij}\Gamma_i\,\Delta_{ij}(p^2)\,\Gamma_j, \quad i = h, H, A,$

Virtual contributions:

- SM-QCD

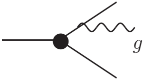


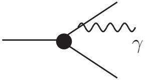
- SUSY-QCD



- EW part consists of 352 self-energies + 440 triangles + 153 boxes

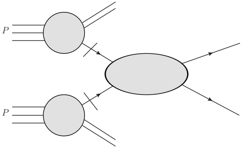
Real contributions:





Gluon induce, photon induce:

Drell-Yan process



$$\sigma^{pp} = \sum_{i,j} \int \mathrm{d}x_1 \mathrm{d}x_2 [F_i^p(x_1, \mu_F) F_j^p(x_2, \mu_F) \hat{\sigma}^{ij}(\mu_R) + i \leftrightarrow j)],$$

PDF: MRST2004qed

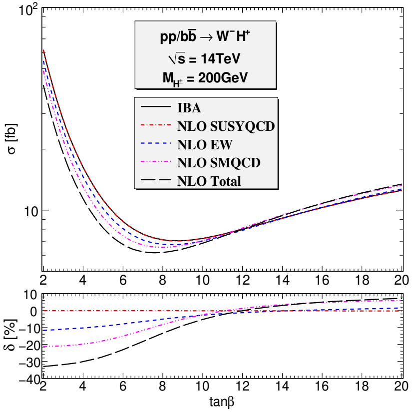
CP asymmetry

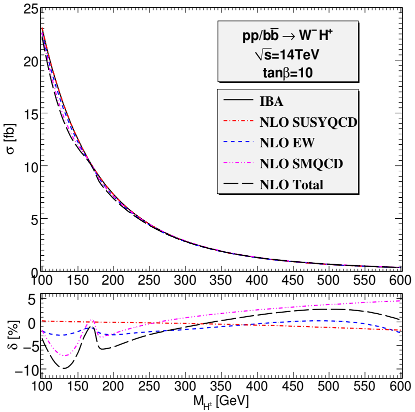
$$\delta_{pp}^{\text{CP violation}} = \frac{\sigma(pp \to W^- H^+) - \sigma(pp \to W^+ H^-)}{\sigma(pp \to W^- H^+) + \sigma(pp \to W^+ H^-)}.$$

Use CPX scenario for numerical studies

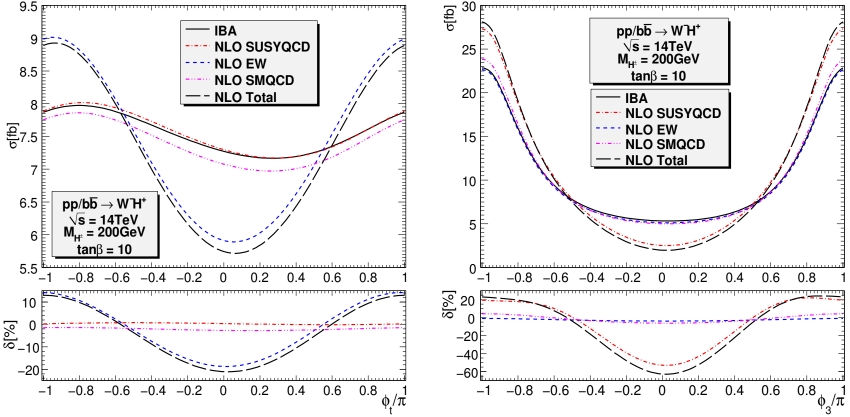
NLO corrections to $b\bar{b} \to H^{\pm}W^{\mp}$

Relative corrections: $\delta = \frac{\sigma_{NLO} - \sigma_{IBA}}{\sigma_{IBA}}$

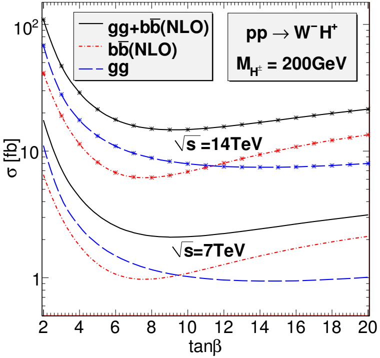


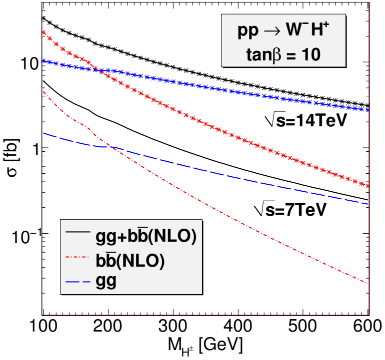


NLO corrections to $b\bar{b} \to H^{\pm}W^{\mp}$

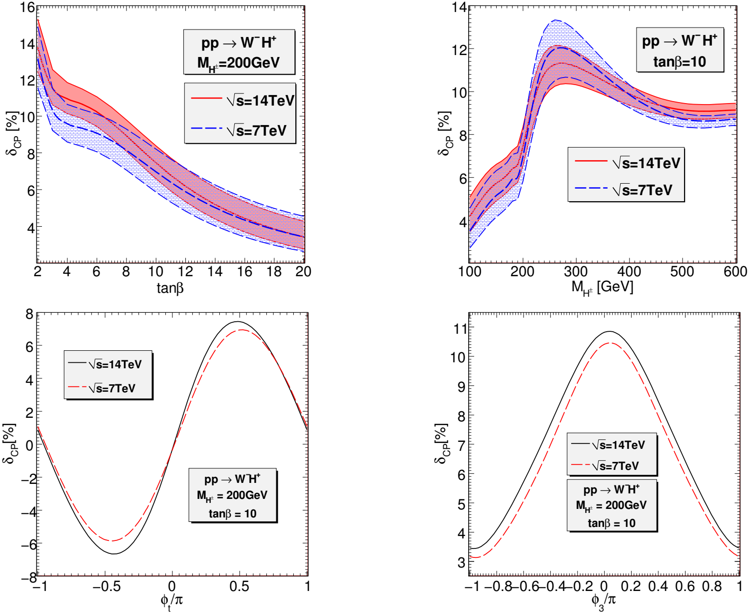


Total hardronic cross section

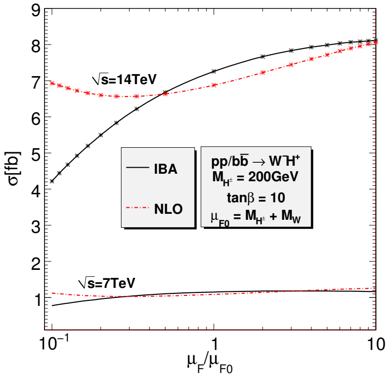


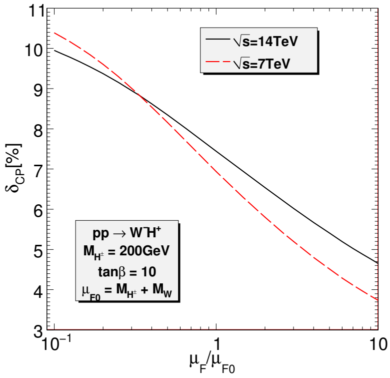


CP asymmetry



- $\delta = [|\sigma(\mu_{F0}/2) - \sigma(\mu_{F0})| + |\sigma(2\mu_{F0}) - \sigma(\mu_{F0})|]/\sigma(\mu_{F0})$
- $\mu_R = \mu_F,\, \mu_{F0} = M_W + M_{H^\pm}$





- NLO: $\delta = 9\%(9\%)$
- IBA: $\delta = 14\%(7\%)$
- 14 TeV $\delta = 24\%$
- 7 TeV: $\delta = 34\%$

Conclusions

- $H^\pm \to h_1 W^\pm$, $pp \to W^\mp H^\pm$ have been studied in general complex MSSM
- CP violating effects on the decay width and production rate are significantly large
- NLO corrections are important for the search of charged Higgs bosons and significantly reduce scale dependence
- Large CP asymmetry is mainly induced from ϕ_t, ϕ_μ, ϕ_3
- The effective bottom-Higgs couplings have significant effects on $b\bar{b}$ annihilation
- The Higgs mixing resummation gives large effects on gg fusion, and CP asymmetry
- Decay width, production rates and CP asymmetry strongly depend on $\tan\beta$, $M_{H^\pm}$, ϕ_t, ϕ_3

Conclusions

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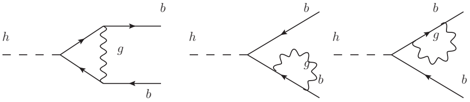
THANK YOU FOR YOUR ATTENTION

BACK UP

The effective bottom-Higgs couplings: $\mathcal{O}(\alpha_s \ln m_b/\mu_R)$ SM-QCD corrections

An example
SM-QCD correction to $hb\bar{b}$



On-shell scheme for b quark:

$\Gamma^{(1)} = \Gamma^{(0)}(m_b) \times \frac{2\alpha_s(m_h)}{3\pi}\left(\frac{9}{2} + 6\ln\frac{m_b}{m_h}\right)$

$\Gamma^{(0)} \propto m_b^2,$ m_b : pole mass

$\alpha_s(m_h) = \frac{\pi}{\beta_0 \ln(m_h/\Lambda_{\mathrm{QCD}})}$

$\Lambda_{\mathrm{QCD}} = 0.5$ GeV$, m_h \to \infty$:

$$\Gamma^{(1)} \approx -\frac{8}{7}\Gamma^{(0)}$$

- $\Gamma^{(1)}, \Gamma^{(0)}$: unreliable
- Higher order terms are important
- On-shell scheme is not good (spoil perturbative expansion)

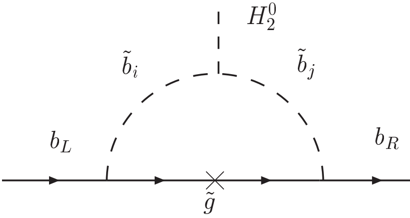
$\overline{\mathrm{MS}}$ scheme:
sum the leading contributions $\alpha_s^n \ln^n(m_b/\mu_R)$ to running $m_b^{\overline{\mathrm{MS}}}(\mu_R)$

$$\Gamma^{(1)} = \Gamma^{(0)}(m_b^{\overline{\mathrm{MS}}}) \times \frac{17\alpha_s(m_h)}{3\pi}$$

$$m_b^{\overline{\mathrm{MS}}}(\mu_R) = m_b\left[1+\frac{\alpha_s}{3\pi}\left(4-6\ln\frac{m_b}{\mu_R}\right)\right]$$

in SUSY: $\overline{\mathrm{MS}} \to \overline{\mathrm{DR}}$

$$m_b^{\overline{\mathrm{DR}}}(\mu_R) = m_b\left[1+\frac{\alpha_s}{3\pi}\left(5-6\ln\frac{m_b}{\mu_R}\right)\right]$$



The effective Yukawa Lagrangian:

$$\mathcal{L}_Y^{\text{eff}} = -Y_b \bar{b}_R \mathcal{H}_1 \cdot Q_L - Y_b \frac{\Delta m_b}{\tan\beta} b_R \tilde{\mathcal{H}}_2 \cdot Q_L + \cdots$$

$$\mathcal{H}_1 = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1 - i\chi_1) \\ -\phi_1^- \end{pmatrix}, \quad \mathcal{H}_2 = \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2 + i\chi_2) \end{pmatrix}, \tilde{\mathcal{H}}_2 = i\sigma_2 \mathcal{H}_2$$

Relation between m_b and Y_b reads

$$Y_b = \frac{m_b}{v\cos\beta(1+\Delta m_b)}, \quad \Delta m_b = \Delta m_b^{SQCD} + \Delta m_b^{SEW}$$

$$\begin{aligned}\Delta m_b^{SQCD} &= \tfrac{2\alpha_s}{3\pi} M_3^* \mu^* \tan\beta C_0(0,0,0,M_{\tilde{b}_1}^2, M_{\tilde{b}_2}^2, M_{\tilde{g}}^2)\\ \Delta m_b^{SEW} &= \tfrac{\alpha_t}{4\pi} A_t^* \mu^* \tan\beta C_0(0,0,0,M_{\tilde{t}_1}^2, M_{\tilde{t}_2}^2, |\mu|^2) + \cdots\end{aligned}$$

- Δm_b in general is complex, due to non-vanishing phases $(\phi_3, \phi_t, \phi_\mu)$
- if Δm_b close to -1, Y_b can be very large
- $\phi_1, \phi_2, \chi_1, \chi_2, \phi_1^-, \phi_2^+ \to h, H, A, G, H^\pm, G^\pm$ to get the effective couplings
- To avoid double counting, we have to subtract the Δm_b-related corrections in one-loop calculation

The effective bottom-Higgs couplings

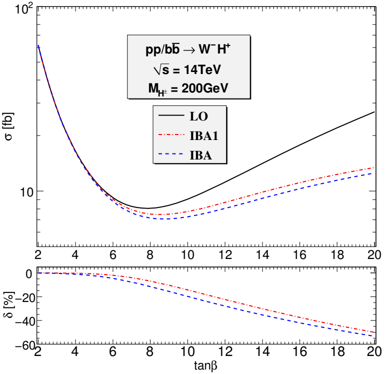
include the subleading contribution: $\Delta m_b \to \Delta_b = \frac{\Delta m_b}{1+\Delta_1}$

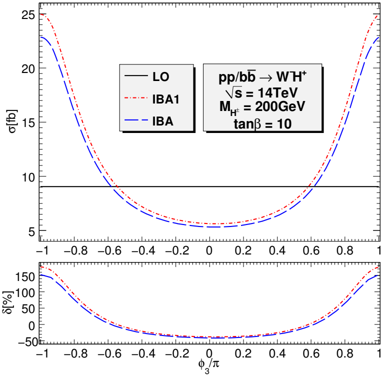
$$\Delta_1 = -\frac{2\alpha_s}{3\pi} M_3^* A_b C_0(0,0,0,M^2_{\tilde{b}_1}, M^2_{\tilde{b}_2}, M^2_{\tilde{g}})$$

$$\begin{aligned}
\bar{\lambda}_{b\bar{b}h} &= \frac{iem_b^{\overline{\mathrm{DR}}}}{2s_W M_W}\frac{\sin\alpha}{\cos\beta}\left(\Delta_b^1 P_L + \Delta_b^{1*} P_R\right), \\
\bar{\lambda}_{b\bar{b}H} &= \frac{-iem_b^{\overline{\mathrm{DR}}}}{2s_W M_W}\frac{\cos\alpha}{\cos\beta}(\Delta_b^2 P_L + \Delta_b^{2*} P_R), \\
\bar{\lambda}_{b\bar{b}A} &= \frac{em_b^{\overline{\mathrm{DR}}}}{2s_W M_W}\tan\beta(\Delta_b^3 P_L - \Delta_b^{3*} P_R), \\
\bar{\lambda}_{b\bar{t}H^+} &= \frac{ie}{\sqrt{2}s_W M_W}\left(\frac{m_t}{\tan\beta}P_L + m_b^{\overline{\mathrm{DR}}}\tan\beta\Delta_b^{3*}P_R\right), \\
\bar{\lambda}_{t\bar{b}H^-} &= \frac{ie}{\sqrt{2}s_W M_W}\left(m_b^{\overline{\mathrm{DR}}}\tan\beta\Delta_b^3 P_L + \frac{m_t}{\tan\beta}P_R\right),
\end{aligned}$$

where

$$\Delta_b^1 = \frac{1-\Delta_b/(\tan\beta\tan\alpha)}{1+\Delta_b}, \Delta_b^2 = \frac{1+\Delta_b\tan\alpha/\tan\beta}{1+\Delta_b}, \Delta_b^3 = \frac{1-\Delta_b/(\tan\beta)^2}{1+\Delta_b}.$$





- $\delta = (\sigma_{\rm IBA} - \sigma_{\rm LO})/\sigma_{\rm LO}$
- 50% at $\tan\beta = 20$
- even 150% at $\phi_3 = \pm\pi$

CPX scenario: maximize CP violating effects

$$|\mu| = 2\ \text{TeV}, |M_2| = 200\ \text{GeV},\ |M_3| = 1\ \text{TeV},\ |A_t| = |A_b| = |A_\tau| = 900\ \text{GeV},$$

$$M_{\tilde{Q}} = M_{\tilde{D}} = M_{\tilde{U}} = M_{\tilde{L}} = M_{\tilde{E}} = M_{\text{SUSY}} = 500\ \text{GeV}$$

$$|M_1| = 5/3 \tan^2\theta_W |M_2|, \quad \phi_t = \phi_b = \phi_\tau = \phi_3 = \phi_1 = \frac{\pi}{2},$$

Renormalization scale and factorization scale: $\mu_R = \mu_F = \mu_{F0} = M_W + M_{H^\pm}$