Towards QCD *WZjj* @ NLO
– Real Emission Calculation –

Research Seminar

Matthias Kerner

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Outline

1. Catani Seymour Algorithm
2. *WZjj*
3. Random Helicity Summation
4. Numerical problems
5. Outlook and summary

Catani Seymour Subtraction Formalism

Problem: Both contributions in

$$\sigma^{NLO} = \int_{m+1} d\sigma^R + \int_m d\sigma^V$$

are seperately divergent. For a numerical integration these (collinear and soft) singularities have to be cancelled beforehand.

$\rightarrow$ Rewrite σ^{NLO} as

$$\sigma^{NLO} = \int_{m+1} \left[d\sigma^R|_{\epsilon=0} - d\sigma^A|_{\epsilon=0} \right] + \int_m \left[d\sigma^V + \int_1 d\sigma^A \right]_{\epsilon=0}$$

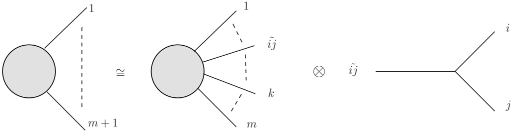
with a function $d\sigma^A$, which can be integrated analytically over a one particle phasespace in $4-2\epsilon$, reproducing the singular behaviour of $d\sigma^R$:

$$d\sigma^A \xrightarrow[\text{region}]{\text{soft/coll.}} d\sigma^R$$

Dipole Factorization

$|\mathcal{M}_{m+1}|^2$ factorizes if partons i, j are collinear or one of them is soft:

$$d\sigma^A \propto \sum_{\substack{\text{pairs}\\ i,j}} \sum_{k \neq i,j} \mathcal{D}_{ij,k}$$



$$\mathcal{D}_{ij,k} = -\frac{1}{2p_a p_i} \underbrace{\frac{1}{x_{ij,k}}}_{\substack{\text{if } i,j, \text{ or } k\\ \text{in initial state}}} \; {}_m\langle\, 1, \ldots, m| \underbrace{\frac{T_k \cdot T_{ij}}{T_{ij}^2} V_{ij,k}}_{\text{spin and colour correlation}} \underbrace{|1, \ldots, m\rangle_m}_{\mathcal{M}_m}$$

$V_{ij,k}$ depends on dipole type.
e.g. initial state $g \to q\bar{q}$ with final state spectator:

$${}_m\langle s|V_k^{ai}|s'\rangle_m = 8\pi\mu^{2\epsilon}\alpha_S T_R[1 - \epsilon - 2x_{ik,a}(1 - x_{ik,a})]\delta_{ss'}$$

- Spectator k needed to get
  - mumentum conservation
  - external onshell particles
  - the right colour correlation
- Momenta of Born matrix elements

$$\tilde{p}_i = \tilde{p}_i(i, j, k; p_1, ..., p_{m+1})$$

have to pass jet definition and cuts

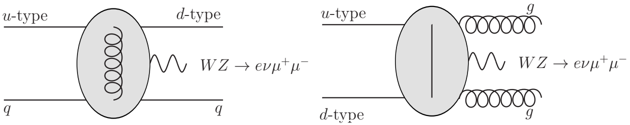
- Some of the kinematics are the same and can be used for different dipoles

$$\text{kin}(\mathcal{D}_{ij,k}) = \text{kin}(\mathcal{D}_{ji,k}), \qquad \text{kin}(\mathcal{D}_k^{ai}) = \text{kin}(\mathcal{D}_{ij}^{a})$$

- 15 different Born kinematics for *WZjj*

WZjj

Two basic types of Feynman diagrams (LO)



Small differences of 4 quark subprocesses depending on q:

- different couplings of u-/d-type quarks
- u-channel only for same quark families

Real Emission:

- 27 different crossings ($\hat{=}$ number of calls to real emission amplitude)
- 522 dipoles
- 260 LO matrix elements have to be computed
- $T_i \cdot T_j$ can't be reduced to Casimir invariants because $n_{\text{partons,LO}} \not\leq 3$

Random Helicity Summation

Number of contributing helicity combinations:

amplitude type	4q	2q2g	4q1g	2q3g
# helicity combinations	4/6	8	8/12	16

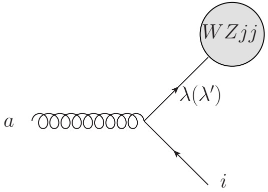
Choose a random helicity combination (for every phasespace point) and use the approximation

$$\sum_{\{\lambda_i\}} |\mathcal{M}|^2_{\{\lambda_i\}} \approx |\mathcal{M}|^2_{\text{rand. hel.}} \cdot \#\ \text{helicity combinations}$$

- Same result as full summation for large number of phasespace points
- Significantly reduced cpu time per phase space point
- Some more phase space points needed to reach a given accuracy of the integration

Polarized Dipoles

Small modifications to dipoles are needed to account for fixed helicities [Czakon et. al., 2009]



e.g. initial state splitting $g \to q\bar{q}$
with final state spectator:

$$\mathcal{D}_k^{ai} = -\frac{8\pi\alpha_s T_R}{2p_a p_i\, x_{ik,a}} \cdot \sum_{\lambda,\lambda'} {}_m\langle\lambda'|\frac{T_k \cdot T_{ai}}{T_{ai}^2}|\lambda\rangle_m$$

$$\cdot\, \delta_{\lambda'\lambda}\delta'_{\lambda\lambda_i}(\delta_{\lambda_a\lambda_i}(1 - 2x_{ik,a}(1 - x_{ik,a})) + (1 - 2\delta_{\lambda_a\lambda_i})x_{ik,a}^2) + \mathcal{O}(\epsilon)$$

with

$$\delta'_{ab} = 1 - \delta_{ab}$$

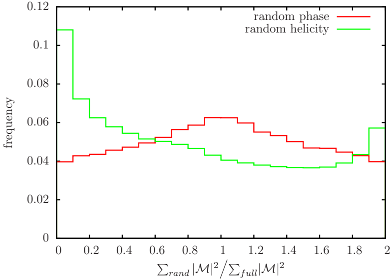
Random helicity summation with random phases

- There can be a big difference of $\mathcal{M}^2$ with different helicity configurations
- A linear combination of multiple helicity states with a random phase Φ can be used as well:

$$\epsilon_\mu(\Phi) = e^{i\Phi}\epsilon_\mu^+ + e^{-i\Phi}\epsilon_\mu^-$$

using

$$\sum_\lambda |\mathcal{M}_\lambda|^2 = \frac{1}{2\pi}\int_0^{2\pi} d\Phi |\mathcal{M}_\Phi|^2$$



- All helicities contribute $\rightarrow$ better approximation of $\sum_\lambda$
- Can't be used with (polarized) dipoles

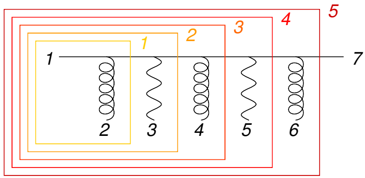
Implementation

- Random helicity summation used for partons
- Random phase used for helicity of charged leptonpair
- No big effort to switch to full helicity summation
- New matrix element routines
  - improved speed and numerical precision compared to MadGraph (see next slides)
  - some speed improvements for full helicity summation
- Only small changes to dipole routines for other *QCD VVjj* processes

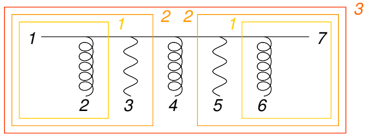
Numerical problems

MadGraph amplitudes turn out to be inaccurate in regions with collinear splittings.
$\rightarrow$ New amplitudes had to be implemented.

MadGraph



own implementation



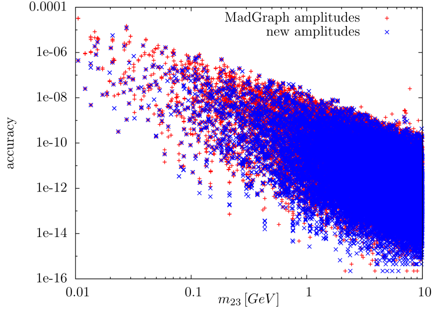
Numerical stability of MadGraph amplitudes:

- partons 1 and 2 are collinear ✓
- partons 6 and 7 are collinear ×

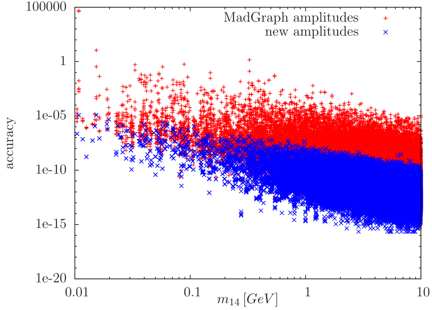
Numerical accuracy

Comparison of matrix elements calculated with double and quad precision

"good" parton pair



"bad" parton pair



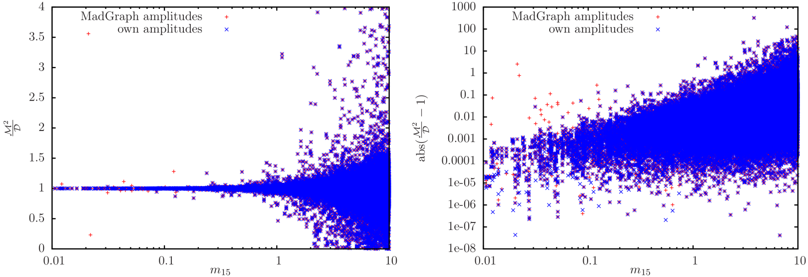
Speed of amplitude calculation

Runtime of amplitude routines in μs

		MadGraph	MadGraph modified[1]	own implementation
2q2g	1 hel	28	13	3.4
	$\sum$ parton hel	114	54	8
4q	1 hel	20-40	7-21	1.8-3.2
	$\sum$ parton hel	60-120	27-43	4.7
2q3g	1 hel	195	100	12
4q1g	1 hel	172	95	12-19
	$\sum$ parton hel	1010	540	52

[1] electroweak parts replaced with own decay currents

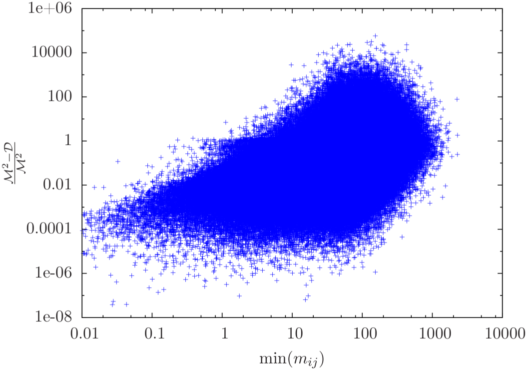
Dipoles in the collinear limit



- Plots only show one subprocess and one emitter pair (but similiar plots for other configurations)
- Summed over spectator partons

Full Subtraction

Subtracted matrix elements after summation over all subprocesses

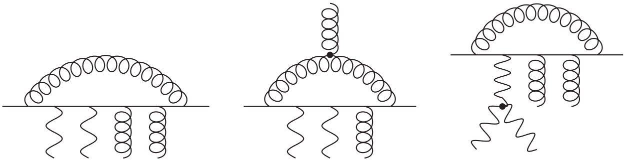


Looks quite good, now → Dipole subtractions works (?)

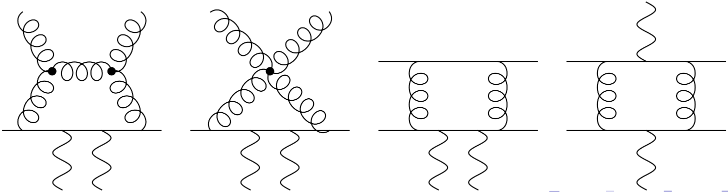
Some more test will be done (last parts implemented one week ago)

Virtual Corrections

- Will be done together with Ninh and Paco
- Up to hexagon loop contributions
- Some parts can be reused from other processes (e.g. $W\gamma\gamma j$) but with modified colour structure



- Some new contributions



Summary

- Real emission (probably) works
- Some more tests needed
- Random helicity summation possible
- New matrix elements
  - numerical improvements
  - faster