On one-loop calculation and Landau singularities

LE Duc Ninh

Institut für Theoretische Physik, Karlsruhe

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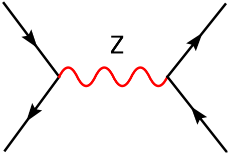
Outline

- A short review: one-loop calculation
- Landau singularities

Tree-level results:

- can be 0: $gg \to H$, $gg \to W^{\mp}H^{\pm}$, ...
- can be ∞: $e^+e^- \to \mu^+\mu^-$, ...



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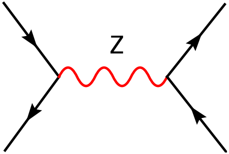

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- can be anything: $\alpha(\mu_R) = ?$, $\alpha_s(\mu_R) = ?$

Why NLO calculations?

- Importance of multiparticle processes at the LHC, linear colliders:
  - Many heavy particles (W, Z, t, $\ldots$) can be simultaneously produced. A heavy particle $\rightarrow$ jets, leptons, photons.
  - Irreducible backgrounds to these signals.
  - Additional jets: increase $S/\sqrt{B}$ ratio.

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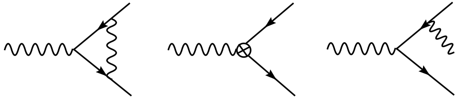
- Importance of NLO corrections:
  - LO predictions suffer from large theoretical (scale, missing higher-order terms) uncertainty.
  - NLO QCD corrections: $\alpha_s(M_Z) \approx 0.1$, $\mathcal{O}(10 \div 100\%)$, decoupling.
  - NLO EW corrections: $\alpha(M_Z) \approx 0.01$, $\mathcal{O}(5 \div 20\%)$, non-decoupling, Sudakov double logs, ...

Going beyond LO

The standard procedure:



$$d\sigma_{NLO} = d\sigma_{virt} + d\sigma_{real}$$

- At NLO, many divergences appear: UV, (IR, collinear,) Landau singularities (more later).
- Renormalisation to regularize UV divergences.
- By adding real radiation, we cancel all soft and some collinear singularities.
- The left-over collinear singularities can be factorized.
  In *pp* processes: these collinear singularities are absorbed into PDFs.
  In e^+e^- processes: initial-state collinear singularities induce large corrections $\alpha \ln(s/m_e^2)$.

NLO calculations: difficulties

- Conceptual problems:
  - Unstable particles, complex-mass scheme at 1-loop [Denner and Dittmaier hep-ph/0605312]: gauge invariant (analytic continuation), unitarity violation of $\mathcal{O}(\alpha^2)$.
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  - complex-mass scheme for the MSSM?
- Technical problems:
  - Large amplitude expressions (many MBs).
  - Numerical cancellation.
  - Time consuming.

Structure of 1-loop amplitudes (Virtual)

One-loop integrals: follow 't Hooft, Passarino, Veltman (1979).
Idea:

$$M(z) = a_i A_i^0 + b_i B_i^0 + c_i C_i^0 + d_i D_i^0 + R$$

a_i, b_i, c_i, d_i, R are rational.
Question: How to get the coefficients and the rational term?

- Feynman diagram approach: do tensor reduction for each diagram (in $D = 4 - 2\epsilon$). Finite terms like $\epsilon \times \frac{1}{\epsilon}$ contribute to the rational term R (a by-product).
- On-shell methods (multiple cuts): $\mathrm{Disc}(\mathrm{LHS}) = \mathrm{Disc}(\mathrm{RHS})$.
- OPP (Ossola, Papadopoulos, Pittau) method (working at the integrand level).

Numerical instability: which method is good?
Gauge-invariance check: at the integrand level??

On-shell methods

The amplitude $M(z)$ is an integral:

$$M(z) = \int d^D q \frac{N(q,z)}{D_1 \dots D_M}, \quad D_i = q_i^2 - m_i^2 + i\epsilon$$

$$= a_i \int \frac{d^D q}{D_1} + b_i \int \frac{d^D q}{D_1 D_2} + c_i \int \frac{d^D q}{D_1 D_2 D_3} + d_i \int \frac{d^D q}{D_1 D_2 D_3 D_4} + R$$

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$$\mathrm{Disc}_N M(z) = (-2\pi i)^N \int d^D q \frac{N(q,z)\delta^+(q_1^2 - m_1^2)\dots\delta^+(q_N^2 - m_N^2)}{D_{N+1}\dots D_M}$$

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- By considering $\mathrm{Disc}_N M(z)$, *i.e.* making N cuts on both sides, one can read off the coefficients.

OPP method

Working at the integrand level in $D = 4 + \epsilon$ dims: $\bar{q} = q + \tilde{q}$, $\tilde{q} \cdot q = \tilde{q} \cdot p_i = 0$

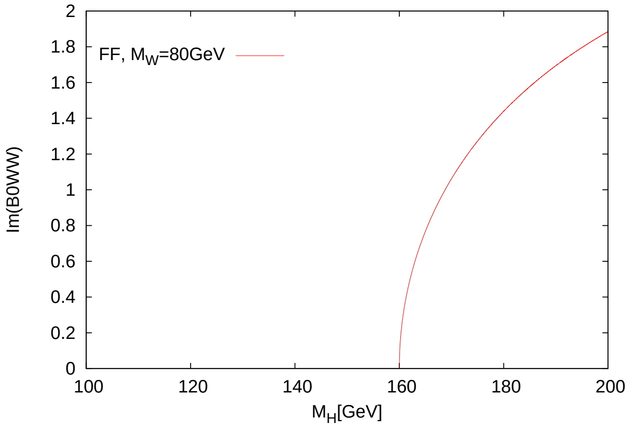
$$M(z) = \int d^D\bar{q} \frac{\bar{N}(\bar{q})}{\bar{D}_1 \ldots \bar{D}_5}, \quad \bar{N}(\bar{q}) = N(q) + \tilde{N}(\tilde{q}, q, \epsilon), \quad \bar{D}_i = D_i + \tilde{q}^2$$

$$N(q) = d(q; i)D_i + c(q; i, j)D_iD_j + b(q; i, j, k)D_iD_jD_k + a(q; i, j, k, l)D_iD_jD_kD_l$$

- Spurious terms appear: $d(q; i) = d(i) + \tilde{d}(q; i)$. These terms vanish upon integration over q. Expressions for all spurious terms are known.
- The mismatch between D_i and $\bar{D}_i$; $\bar{N}(\bar{q})$ and $N(q)$ give rise to rational terms. Indeed, this is a good method to get the rational term.
- One can solve for *all* the coefficients by choosing special values of q. For instance, by requiring q to satisfy $D_1 = D_2 = D_3 = D_4 = 0$ one can remove a lot of terms on the RHS, only $[d(5) + \tilde{d}(q; 5)]D_5$ survives. This can be done numerically.

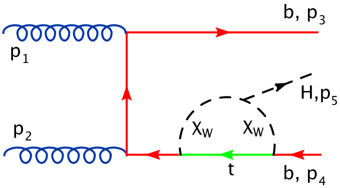
Landau singularities

Introduction: normal threshold (2-point)



$$B_0 = B_0(M_H^2, M_W^2, M_W^2), \quad \underline{M_H^{\text{th}} = 2M_W}.$$

Introduction: anomalous threshold (3-point)



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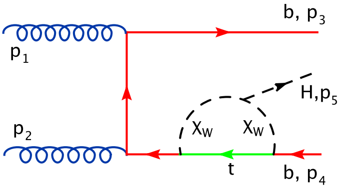

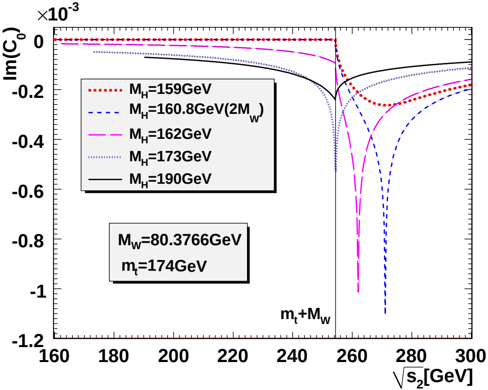

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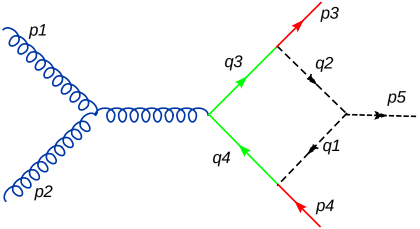

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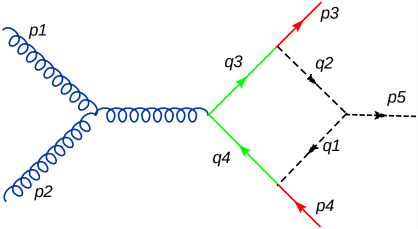

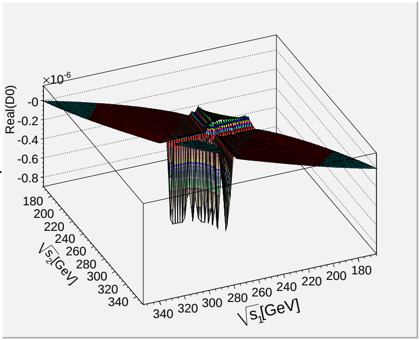

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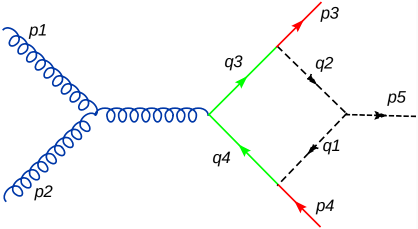

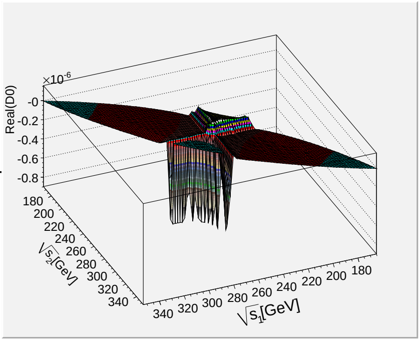

- What happens if $\sqrt{\hat{s}} \geq 2m_t$ and $M_H \geq 2M_W$?
  $\rightsquigarrow$ the loop particles can be *all* on-shell.
- Cross section: NLO: finite, (1 loop)^2($\lambda_{bbH} \rightarrow 0$): divergent.

Singularities: complex integrals

- End-point singularities:

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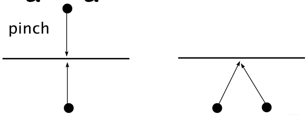

2 Landau equations

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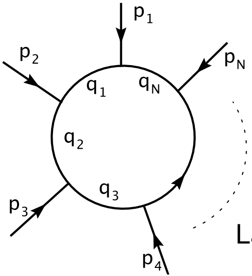

$$T_0^N \propto \int_0^\infty \prod_{i=1}^N dx_i \int \frac{d^Dq}{(2\pi)^D} \frac{\delta(\sum_{i=1}^N x_i - 1)}{[\sum_{i=1}^N x_i(q_i^2 - m_i^2 + i\epsilon)]^N}$$

Physical region: $[x_i = x_i^*, x_i \geq 0, q_i = q_i^*]$

Singular only for: $\epsilon \to 0^+$

Landau: $\begin{cases} \forall i \;\; x_i(q_i^2 - m_i^2) & = 0 \\ \sum_{i=1}^M x_i q_i & = 0 \end{cases}$ $\to$ Landau singularity

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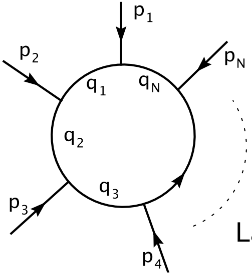

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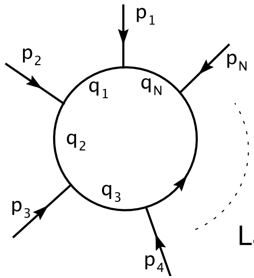

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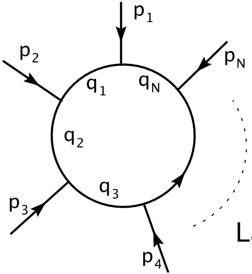

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- How to check those conditions in practice?

2 important conditions

- Landau equations: $Q_{ij} \equiv 2q_i.q_j = m_i^2 + m_j^2 - (q_i - q_j)^2$ (Landau matrix) ,

$$\sum_{i=1}^{M} x_i q_i = 0 \Longleftrightarrow \begin{cases} Q_{11}x_1 + Q_{12}x_2 + \cdots Q_{1M}x_M & = 0, \\ Q_{21}x_1 + Q_{22}x_2 + \cdots Q_{2M}x_M & = 0, \\ \vdots & \\ Q_{M1}x_1 + Q_{M2}x_2 + \cdots Q_{MM}x_M & = 0. \end{cases}$$

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- Sign condition (occurring in the physical region):

$$x_i > 0,\ i = 1, \ldots, M \Longleftrightarrow x_j = \det(\hat{Q}_{jM})/\det(\hat{Q}_{MM}) > 0,\ j = 1, \ldots, M-1$$

$$\det(\hat{Q}_{MM}) = d[\det(Q)]/dQ_{MM},\ \ \det(\hat{Q}_{1j}) = \frac{1}{2}d[\det(Q)]/dQ_{1j}.$$

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- For $N = 2$, $D = 4 - 2\varepsilon$: $(B_0)_{div} \propto [\det(Q_2) - i\epsilon]^{1/2}$ (finite)
- For $N = 3$, $D = 4 - 2\varepsilon$: $(C_0)_{div} \propto \ln[\det(Q_3) - i\epsilon]$ (integrable)
- For $N = 4$, $D = 4$: $(D_0)_{div} \propto \frac{1}{\sqrt{\det(Q_4) - i\epsilon}}$ (integrable, the square is not integrable)
- For $N = 5$, $D = 4$: $(E_0)_{div} \propto \frac{1}{\det(Q_5) - i\epsilon}$ (not integrable)
- For $N \geq 6$: No LLS but several sub-LLSs.

.

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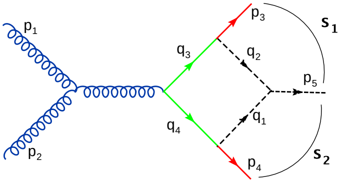
The LLSs are integrable or not?

- For $N = 2$, $D = 4 - 2\varepsilon$: $(B_0)_{div} \propto [\det(Q_2) - i\epsilon]^{1/2}$ (finite)
- For $N = 3$, $D = 4 - 2\varepsilon$: $(C_0)_{div} \propto \ln[\det(Q_3) - i\epsilon]$ (integrable)
- For $N = 4$, $D = 4$: $(D_0)_{div} \propto \frac{1}{\sqrt{\det(Q_4) - i\epsilon}}$ (integrable, the square is not integrable)
- For $N = 5$, $D = 4$: $(E_0)_{div} \propto \frac{1}{\det(Q_5) - i\epsilon}$ (not integrable)
- For $N \geq 6$: No LLS but several sub-LLSs.

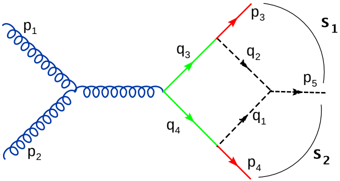
More details in: Boudjema and LDN arXiv: 0806.1498, LDN arXiv:0810.4078.

4-point LLS: $g^* \to b\bar{b}H$ (I)



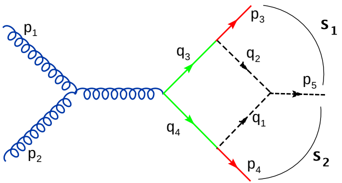
Question: What are the physical conditions to have a LLS?

4-point LLS: $g^* \to b\bar{b}H$ (I)



Question: What are the physical conditions to have a LLS?

$$\begin{cases} q_i^2 & = m_i^2 \\ q_i & = q_i^* \\ x_1 q_1 + x_4 q_4 & = x_2 q_2 + x_3 q_3 \\ x_i > 0 & \end{cases} \xrightarrow{\text{E-p conservation}} \begin{cases} M_H & \geq 2M_W \\ \sqrt{s} & \geq 2m_t \\ s_{1,2} & \geq (m_t + M_W)^2 \\ m_t & > M_W \end{cases}$$



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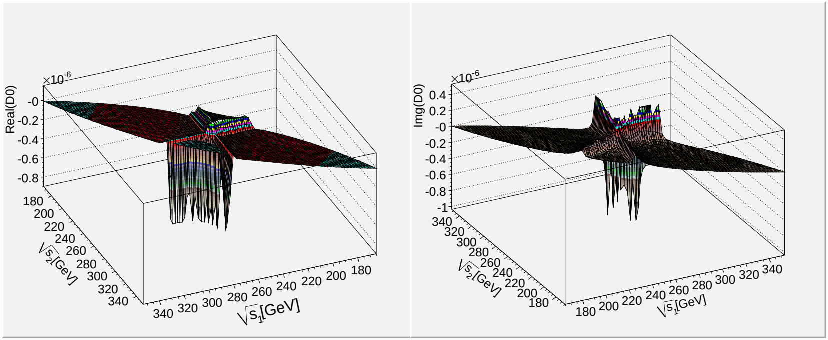
Physical picture: the off-shell gluon splits into two on-shell top quarks, each top quark then decays into a bottom quark and an on-shell W gauge boson. Finally, the W gauge bosons fuse into the Higgs. The problem is related to internal unstable particles.

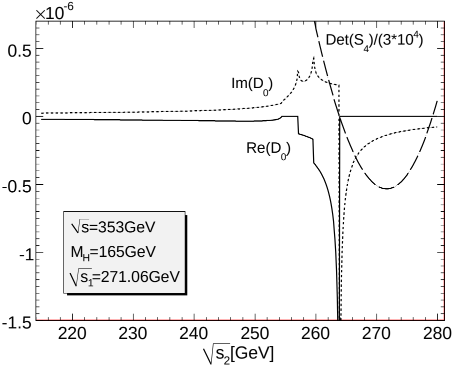
4-point LLS: $g^* \to b\bar{b}H$ (III)



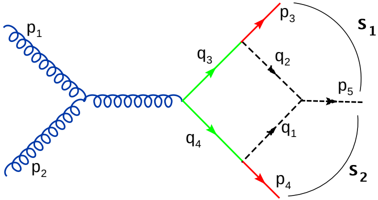
$$D_0 = D_0(M_H^2, 0, s, 0, s_1, s_2, M_W^2, M_W^2, m_t^2, m_t^2).$$

Input parameters: $\sqrt{s} = 353GeV > 2m_t$, $M_H = 165GeV > 2M_W$, $m_b = 0$.
Region of LLS at the center of the phase space.
Take $\sqrt{s_1} = \sqrt{2(m_t^2 + M_W^2)} \approx 271.06GeV \to$

sub-LLSs

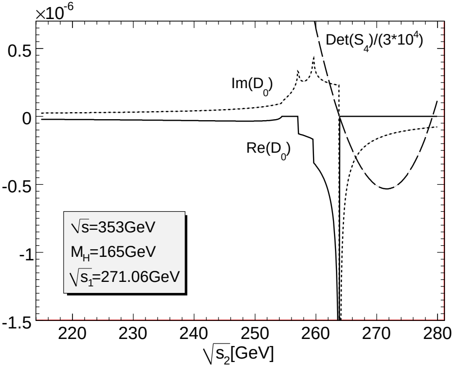


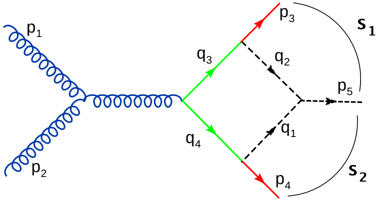
$\times 10^{-6}$
0.5
0
-0.5
-1
-1.5
220
230
240
250
260
270
280
$\sqrt{s_2}$[GeV]
Det(S_4)/(3*10^4)
Im(D_0)
Re(D_0)
$\sqrt{s}$=353GeV
M_H=165GeV
$\sqrt{s_1}$=271.06GeV

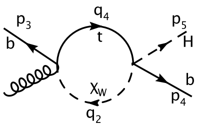


p_1
p_2
p_3
p_4
p_5
q_1
q_2
q_3
q_4
s_1
s_2

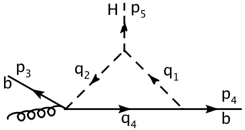
sub-LLSs



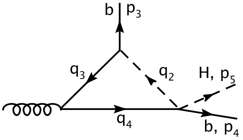




$x_1 = x_3 = 0$



$x_3 = 0$



$x_1 = 0$

Solution: complex masses

$$m_t^2 \to m_t^2 - i m_t \Gamma_t, \quad M_W^2 \to M_W^2 - i M_W \Gamma_W \quad \text{(Dyson summation)}$$

Mathematically, the width effect is to move Landau singularities into the complex plane, so they do not occur in the physical region.

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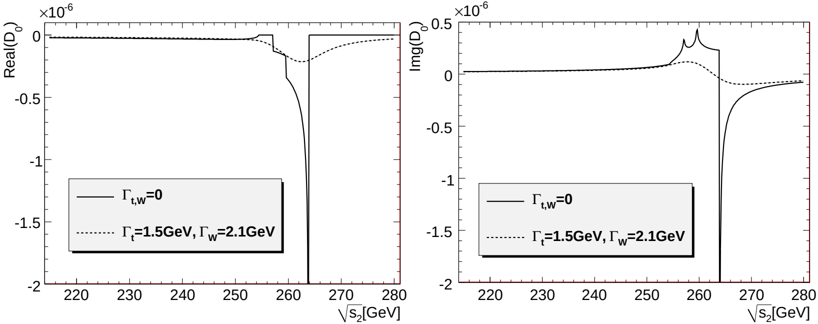

- All Landau singularities are completely regularized.

Special cases

- Soft and collinear divergences are enhanced Landau singularities.

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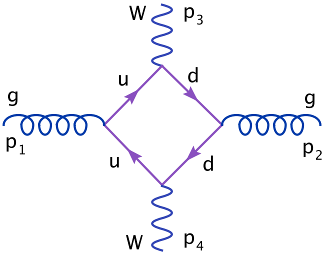
- Double parton scattering singularity: $\vec{p}_1 + \vec{p}_2 = 0$

Condition: $t = (p_1 - p_3)^2$, $u = (p_2 - p_3)^2$



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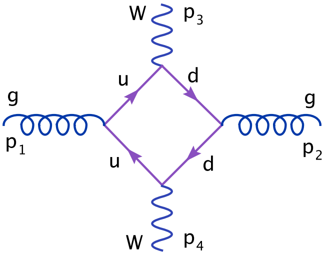

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$$D_0^{div} = \frac{i}{4\pi sp_{T,W}^2}\frac{1}{\varepsilon} + \ldots$$

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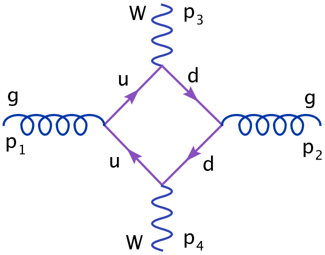

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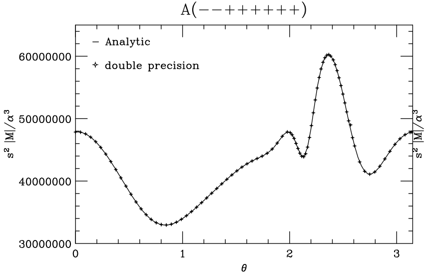
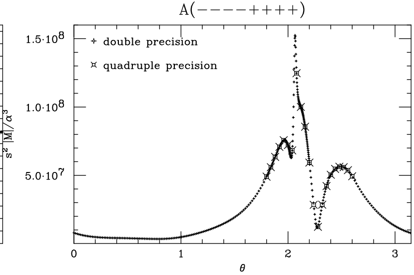
Numerator (QED/QCD): kills the sing. or reduces it to $\log(p_{T,W}^2)$ (integrable), depending on the helicity configuration.

Gaunt and Stirling arXiv: 1103.1888 and refs therein.

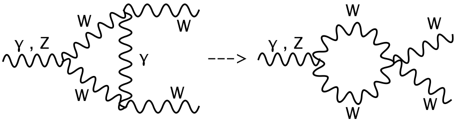
8 photon amplitudes





Mastrolia, Ossola, Reiter and Tramontano, arXiv: 1006.0710 and refs therein.

Coulomb singularity



Landau determinant: $detQ = -s(s - 4M_W^2) = -s^2\beta^2$,
$\beta = \sqrt{1 - 4M_W^2/s}$: $W^\pm$ velocity in the CMS.
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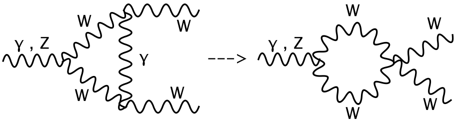

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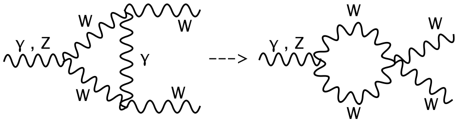

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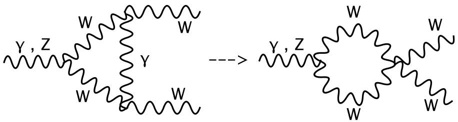

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Solution: resummation!!!

[see e.g. Beneke, Falgari, Klein and Schwinn arXiv:1109.1536]

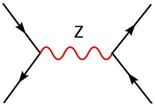
Summary

- Landau singularities: soft, collinear, double-parton scattering and threshold (normal, anomalous, Coulomb).
- Leading Landau singularity (all loop particles are on-shell), sub-leading singularities (some loop particles are pinched).
- Soft and collinear: enhanced sub-leading singularities.
- Double-parton scattering (log): integrable.
- Threshold: some are integrable. Some are not, resummation is the solution.


THANK
YOU
THANK

Backup

Unstable particles



- Free propagator:

$$\Delta_{\text{free}}(p^2) = \frac{i}{p^2 - m^2 + i\epsilon} \tag{1}$$

- Full propagator (with interaction):

$$\begin{aligned}\Delta_{\text{full}}(p^2) &= \int_0^\infty \frac{ds}{2\pi}\rho(s)\frac{i}{p^2 - s + i\epsilon} \\ &= \frac{i\tilde{Z}}{p^2 - m^2 + i\epsilon} + \int_{\text{th}}^\infty \frac{ds}{2\pi}\rho(s)\frac{i}{p^2 - s + i\epsilon}\end{aligned} \tag{2}$$

- Dyson summation:

$$\Delta(p^2) = \frac{i}{p^2 - m^2 + \Sigma(p^2)} \tag{3}$$

- Narrow-width approximation: $\Gamma \ll m$

$$\Delta(p^2) = \frac{i}{p^2 - m^2 + im\Gamma} \tag{4}$$