Higgs masses in the complex NMSSM

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Outline

- Introduction & Motivation
- Renormalization
- How to get the Higgs mass from the renormalized self energy
- Results

Higgs sector of the NMSSM

Superpotential:

$$W = \hat{u}_R^* y_u \left(\hat{Q}^\intercal \epsilon \hat{H}_u\right) + \hat{d}_R^* y_d \left(\hat{Q}^\intercal \epsilon \hat{H}_d\right) - \hat{e}_R^* y_e \left(\hat{L}^\intercal \epsilon \hat{H}_d\right) + \lambda \hat{S} \left(\hat{H}_u^\intercal \epsilon \hat{H}_d\right) + \frac{1}{3}\kappa \hat{S}^3$$

- 3+2 neutral Higgs bosons, 2 charged Higgs bosons, 4+1 neutralinos
- Solution to the "μ-problem" $\rightarrow$ an effective μ-term is generated dynamically through the vev of an singlet field

$$\mu_{eff} = \lambda v_s$$

- Higgs sector is very restricted in MSSM:
  Upper bound on tree-level: $M_H \leq M_Z$
  $\rightarrow$ Large quantum corrections are needed to get M_H above LEP-limit

Why complex NMSSM?

Complex parameters can lead to CP-violation
$\rightarrow$ CP-violation in the kaon system can be explained by the phases of the CKM-matrix
$\rightarrow$ But: To explain observed baryon asymmetry in the universe the Shakarov-conditions must be fulfilled:

- Baryon number violation
- C and CP-violation
- Departure from thermal equilibrum

e.g. in GUT-baryogenesis the SM CP-violating phases are not enough!

Complex parameters on tree-level

Higgs scalar potential:

$$
\begin{aligned}
V =& |\lambda|^2 |S|^2 \left(H_u^\dagger H_u + H_d^\dagger H_d \right) + |\lambda \left(H_u^\intercal \epsilon H_d \right) + \kappa S^2|^2 + \frac{1}{2} g_2^2 |H_u^\dagger H_d|^2 \\
&+ \frac{1}{8}(g_1^2 + g_2^2) \left(H_u^\dagger H_u - H_d^\dagger H_d \right)^2 + m_{H_u}^2 H_u^\dagger H_u + m_{H_d}^2 H_d^\dagger H_d + m_S^2 |S|^2 \\
&+ \left(\lambda A_\lambda \left(H_u^\intercal \epsilon H_d \right) S + \frac{1}{3} \kappa A_\kappa S^3 + c.c \right)
\end{aligned}
$$

with

$$
H_d = \begin{pmatrix} \frac{1}{\sqrt{2}} \left(v_d + h_d + i a_d \right) \\ H_d^- \end{pmatrix} \qquad H_u = e^{i\phi_u} \begin{pmatrix} H_u^+ \\ \frac{1}{\sqrt{2}} \left(v_u + h_u + i a_u \right) \end{pmatrix}
$$

$$
S = e^{i\phi_s} \frac{1}{\sqrt{2}} \left(v_s + h_s + i a_s \right)
$$

Complex parameter

In the Higgs sector:

$$\phi_\lambda, \phi_\kappa, \phi_s, \phi_u, \phi_{A_\kappa}, \phi_{A_\lambda}$$

But only in the combinations:

$$\begin{aligned} \Psi &= \phi_\lambda - \phi_\kappa + \phi_u - 2\phi_s \\ \Psi_\lambda &= \phi_\lambda + \phi_{A_\lambda} + \phi_u + \phi_s \\ \Psi_\kappa &= \phi_\kappa + \phi_{A_\kappa} + 3\phi_s \end{aligned}$$

On tree-level:
Only one phase remains due to the tadpole conditions

$$\frac{1}{\sqrt{2}}|A_\lambda| \sin\Psi_\lambda = -\frac{|\kappa| v_s}{2} \sin\Psi, \qquad \frac{1}{\sqrt{2}}|A_\kappa| \sin\Psi_\kappa = -\frac{3|\lambda| v_d v_u}{2 v_s} \sin\Psi$$

In other sectors:

$$\phi_{M_1}, \phi_{M_2}, \phi_{A_t}, \phi_{A_b}, \phi_{A_e}, \dots.$$

Renormalization: A short reminder on Kathrin's talk last semester

Parameter of the Higgs sector:

g_1	g_2	v_d	v_u	v_s	$\lvert\lambda\rvert$	$\lvert\kappa\rvert$	$\lvert A_\kappa\rvert$	$\lvert A_\lambda\rvert$	$m^2_{H_u}$	$m^2_{H_d}$	m^2_s
$\tan\beta$	e	M_W	M_Z	v_s	$\lvert\lambda\rvert$	$\lvert\kappa\rvert$	$\lvert A_\kappa\rvert$	$M^2_{H^\pm}$	t_{h_u}	t_{h_d}	t_{h_s}

On-Shell-renormalization:

$$e, M_W, M_Z, M_{H^\pm}, t_{H_u}, t_{H_d}, t_{H_s}$$

$\overline{\text{DR}}$-renormalization:

$$\tan\beta, \quad \underbrace{v_s, |\lambda|, |\kappa|,}_{\text{neutralino/chargino sector}} \quad \underbrace{|A_\kappa|}_{\text{Higgs sector}}$$

What is new for complex parameters?

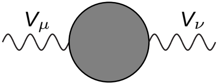
Introduction of the phases:

$$\phi_\lambda,\ \phi_\kappa,\ \phi_s,\ \phi_u,\quad \underbrace{\phi_{A_\lambda},\ \phi_{A_\kappa},}_{\text{replaced by tadpoles}}\quad \underbrace{\phi_{M_1},\ \phi_{M_2},\ \phi_{A_t},\ \phi_{A_b},\ \phi_{A_e}}_{\text{Influences only on loop-level}}$$

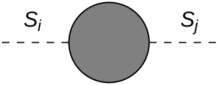
- The mixing matrix is now a 5×5-matrix.
- We introduced for every phase a counterterm.
- Two can be replaced by the two additional tadpole conditions (t_a and t_{a_s}).
- The phases are renormalized in the $\overline{\text{DR}}$-scheme. Of course the tadpoles also get a finite part!

But:
Only the new tadpole conditions get a divergent part. All other counterterms related to phases vanish.
Why?
We don‘t know yet. Maybe they can be interpreted as part of the rotation matrix....

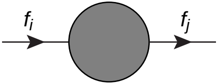
Renormalization



$$\Gamma^V_{ij}(p^2) = -ig_{\mu\nu}(p^2 - M_V^2) - i\left(g_{\mu\nu} - \frac{p_\mu p_\nu}{p^2}\right)\hat{\Sigma}^V_T(p^2) - i\frac{p_\mu p_\nu}{p^2}\hat{\Sigma}^V_L(p^2)$$



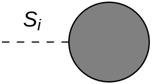
$$\Gamma^S_{ij}(p^2) = i\left[\delta_{ij}\left(p^2 - M_S^2\right) + \hat{\Sigma}_{ij}(p^2)\right]$$



$$\Gamma^F_{ij}(p^2) = i\delta_{ij}\left(\not{p} - m\right) + i\left(\not{p}\,\omega_L\hat{\Sigma}^L_{ij} + \not{p}\,\omega_R\hat{\Sigma}^R_{ij} + m_i\,\omega_L\hat{\Sigma}^{SL}_{ij} + m_j\,\omega_R\hat{\Sigma}^{SR}_{ij}\right)$$

Renormalization conditions for OnShell-parameters

- Tadpoles:



$$\hat{T} = T + \delta t = 0$$

- Vector boson masses: $\widetilde{\mathrm{Re}}\, \hat{\Sigma}_T^V(M_V^2) = 0$

$$M_{W,Z}^2 \to M_{W,Z}^2 + \delta M_{W,Z}^2 \qquad \text{with} \qquad \delta M_{W,Z}^2 = \widetilde{\mathrm{Re}}\, \Sigma_T^{W,Z}(M_{W,Z}^2)$$

- Mass of charged Higgs: $\widetilde{\mathrm{Re}}\, \hat{\Sigma}(M_{H^\pm}^2) = 0$

$$M_{H\pm}^2 \to M_{H\pm}^2 + \delta M_{H\pm}^2 \qquad \text{with} \qquad \delta M_{H\pm}^2 = \widetilde{\mathrm{Re}}\, \Sigma(M_{H\pm}^2)$$

- Electric charge:

$$e \to e\,(1 + \delta Z_e) \qquad \text{with} \qquad \delta Z_e = \frac{1}{2} \frac{\partial \Sigma_T^{AA}(k^2)}{\partial k^2} \Big|_{k^2=0} + \frac{s_W}{c_W} \frac{\Sigma_T^{AZ}(0)}{M_Z^2}$$

Renormalization of the $\overline{\text{DR}}$ parameters For $\tan\beta$ see [Freitas, Stoeckinger]

$$\delta \tan\beta = \delta\left(\frac{v_u}{v_d}\right) = \frac{1}{2}\tan\beta\left(\delta Z_{H_u} - \delta Z_{H_d}\right)|_{div}$$

And the field strenght renormalization condition is

$$\widetilde{\text{Re}}\,\frac{\partial\hat{\Sigma}_{ii}}{\partial k^2}\,|_{k^2=M^2_{H_i}} = 0$$

So

$$\widetilde{\text{Re}}\,\frac{\partial\Sigma_{ii}}{\partial k^2}\,|_{k^2=M^2_{H_i}} = |R_{i1}|^2\delta Z_{H_d} + |R_{i2}|^2\delta Z_{H_u} + |R_{i3}|^2\delta Z_{H_s} + |R_{i4}|^2\left((\sin\beta)^2\delta Z_{H_d} + (\cos\beta)^2\delta Z_{H_u}\right) + |R_{i5}|^2\delta Z_{H_s}$$

Renormalization of the $\overline{\text{DR}}$ parameters

- $\delta|\lambda|$, $\delta|\kappa|$, δv_s, $\delta\phi_\kappa$, $\delta\phi_\lambda$, $\delta\phi_u$, $\delta\phi_s$ are taken from the chargino/neutralino mass matrix $\rightarrow$ we need more conditions as in rNMSSM but we also can impose conditions on the real and imaginary parts

$$M_{Cha} = \begin{pmatrix} M_2 e^{i\phi_{M_2}} & \frac{g_2\, v_u}{\sqrt{2}} e^{i\phi_u} \\ \frac{g_2\, v_d}{\sqrt{2}} & |\lambda|\, v_s\, e^{i\phi_\lambda + i\phi_s} \end{pmatrix}$$

$$M_{Neut} = \begin{pmatrix} M_1 e^{i\phi_{M_1}} & 0 & -M_Z s_W c_B & M_Z s_W s_B e^{-i\phi_u} & 0 \\ 0 & M_2 e^{i\phi_{M_2}} & M_Z c_W c_B & -M_Z c_W s_B e^{-i\phi_u} & 0 \\ -M_Z s_W c_B & M_Z c_W c_B & 0 & -\frac{v_s}{\sqrt{2}}\, \lambda e^{i\phi_s} & -v_u \lambda e^{i\phi_u} \\ M_Z s_W s_B e^{-i\phi_u} & -M_Z c_W s_B e^{-i\phi_u} & -\frac{v_s}{\sqrt{2}} e^{i\phi_s} & 0 & -\frac{v_d}{\sqrt{2}} \\ 0 & 0 & -\frac{v_u}{\sqrt{2}} e^{i\phi_u} & -\frac{v_d}{\sqrt{2}} & \sqrt{2} v_s\, \kappa e^{i\phi_s} \end{pmatrix}$$

- $\delta|A_\kappa|$ is taken from the Higgs mass matrix

Determination of the Higgs mass

Renormalized self-energy:

$$\hat{\Sigma}(p^2)_{ij} = \Sigma(p^2)_{ij} + \frac{1}{2}p^2\left(\delta\tilde{Z}^*_{ji} + \delta\tilde{Z}_{ij}\right) - \frac{1}{2}\left(\delta\tilde{Z}^*_{ki}D_{kj} + D_{ik}\delta\tilde{Z}_{kj}\right) - R_{ik}\delta M_{kl}R^*_{jl}$$

with

$$\delta\tilde{Z} = R\,\delta Z\,R^{\mathsf{T}} \qquad \text{and} \qquad \delta Z = \begin{pmatrix} \delta Z_{H_u} & 0 & \dots \\ 0 & \delta Z_{H_d} & \dots \\ \vdots & \vdots & \ddots \end{pmatrix}$$

and D is the diagonal tree-level mass matrix $D = R\,M\,R^{\mathsf{T}}$

Determination of the Higgs mass: On-Shell and $p^2 = 0$ approximation

On-Shell-approximation:

$$\hat{\Sigma}_{ij}(p^2) \to \mathrm{Re}\,\hat{\Sigma}_{ij}\left(\frac{1}{2}(m_i^2 + m_j^2)\right)$$

Mass determination by the eigenvalues of the matrix $M + \hat{\Sigma}$
Determination of the rotation matrix at one-loop by the eigenvectors of $M + \hat{\Sigma}$

$p^2 = 0$ approximation:

$$\hat{\Sigma}_{ij}(p^2) \to \hat{\Sigma}_{ij}\,(0)$$

Mass determination by the eigenvalues of the matrix $M + \hat{\Sigma}$
Determination of the rotation matrix at one-loop by the eigenvectors of $M + \hat{\Sigma}$

Determination of the Higgs mass: Iterative procedure

- Beginning:
  Calculate the Higgs mass i as the ith eigenvalue of the matrix $M + \hat{\Sigma}(p^2)$ with $p^2 = m^2_{i,treelevel}$

- Iteration steps:
  Take the new Higgs mass (the ith eigenvalue) as new p^2.
  LoopTools can only treat real momenta $\rightarrow$ expansion in $\text{Im}\,(p^2)$.

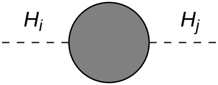
- Iteration end:
  $|\text{assumed } p^2 - \text{newest eigenvalue}| < \epsilon$

Treatment of imaginary parts:
To calculate $\hat{\Sigma}$ at $p^2 = \mathcal{M}^2$

$$\hat{\Sigma}(\mathcal{M}^2) \approx \hat{\Sigma}(\text{Re}\,\mathcal{M}^2) + i\,\text{Im}\,\mathcal{M}^2\ \hat{\Sigma}'(\text{Re}\,\mathcal{M}^2)$$

Problem:
How to determine the rotation matrix at One-Loop

Determination of the Higgs mass: Pole of the propagator [Frank, Hahn, Heinemeyer, Hollik, Rzehak, Weiglein; Williams]



$$\Gamma^H_{ij}(p^2) = i\left[\delta_{ij}\left(p^2 - m_i^2\right) + \hat{\Sigma}_{ij}(p^2)\right]$$

Determine the Higgs mass as pole of the propagator

$$\Delta^H = -\left[\Gamma^H\right]^{-1}$$

$$\Longrightarrow \Delta_{ii} = \frac{i}{p^2 - m_i^2 + \hat{\Sigma}^{eff}_{ii}(p^2)}$$

The complex pole $\mathcal{M}$ of the propagator can be obtained by solving

$$\mathcal{M}^2 - m_i^2 + \hat{\Sigma}^{eff}_{ii}(\mathcal{M}^2) = 0$$

Determination of the Higgs mass: Pole of the propagator

Expansion of $\mathcal{M}^2 = M^2 - iM\Gamma$ in Γ around M^2 leads to

$$M_i^2 - m_i^2 + \mathrm{Re}\,\Sigma_{ii}^{eff}(M_i^2) + \frac{\mathrm{Im}\,\Sigma_{ii}^{eff}(M_i^2)\left(\mathrm{Im}\,\Sigma_{ii}^{eff}\right)'(M_i^2)}{1 + \left(\mathrm{Re}\,\Sigma_{ii}^{eff}\right)'(M_i^2)} = 0$$

The solution can be obtained by an iterative procedure.
Disadvantage:
Very slow! A lot of iteration steps are needed.

Amplitudes for external Higgs bosons

To guarantee the right behaviour for external Higgs bosons mixing must be taken into account

$$\sqrt{\hat{Z}_i}\,(\Gamma_i + \sum_{j=1, j\neq i}^{5} Z_{ij}\Gamma_j)$$

For internal Higgs $\rightarrow$ interpretation as mixing matrix (with $Z_{ii} = 1$):

$$\tilde{\mathbf{Z}}_{ij} = \sqrt{\hat{Z}_i} Z_{ij}$$

with

$$\hat{Z}_i = \frac{1}{1 + \left(\mathrm{Re}\,\Sigma_{ii}^{eff}\right)'(M_i^2)} \qquad \text{and} \qquad Z_{ij} = \frac{\Delta_{ij}(M_i^2)}{\Delta_{ii}(M_i^2)}$$

But:
Mixing matrix is not unitary.

Results

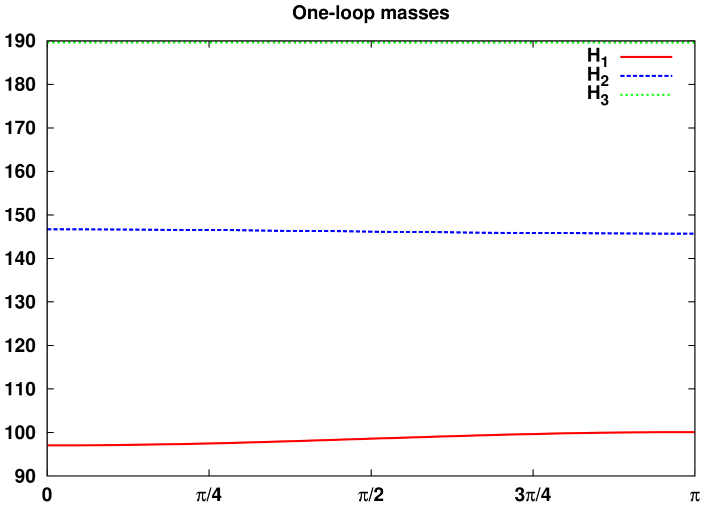
Parameters:

$$\lambda = 0.7, \qquad \kappa = 0.11, \qquad vs = 442.8 \text{ GeV}, \qquad A_\kappa = -230 \text{ GeV}, \qquad A_\lambda = 928 \text{ GeV},$$

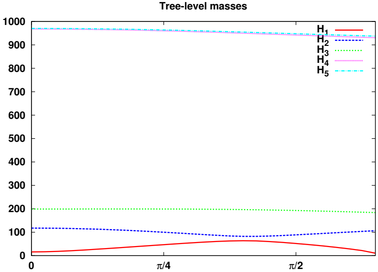
$$M_{SUSY} = 1 \text{ TeV}, \qquad M_2 = 1 \text{ TeV}, \qquad M_1 = \frac{5\sin^2\theta_W}{3\cos^2\theta_W} M_2, \qquad Q = 300 \text{ GeV},$$

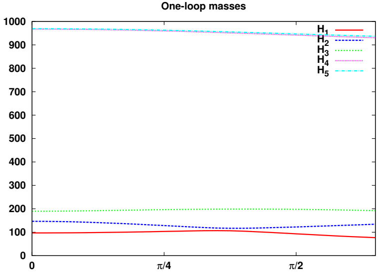
$$A_t = A_b = A_l = -750 \text{ GeV}$$

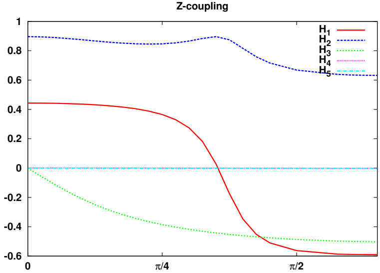
Results: Variation in ϕ_{M_2}

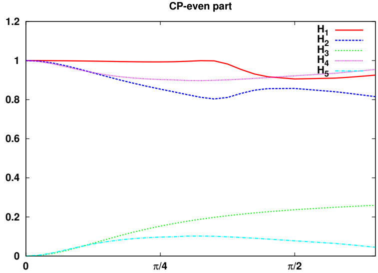


Results: Variation in ϕ_u









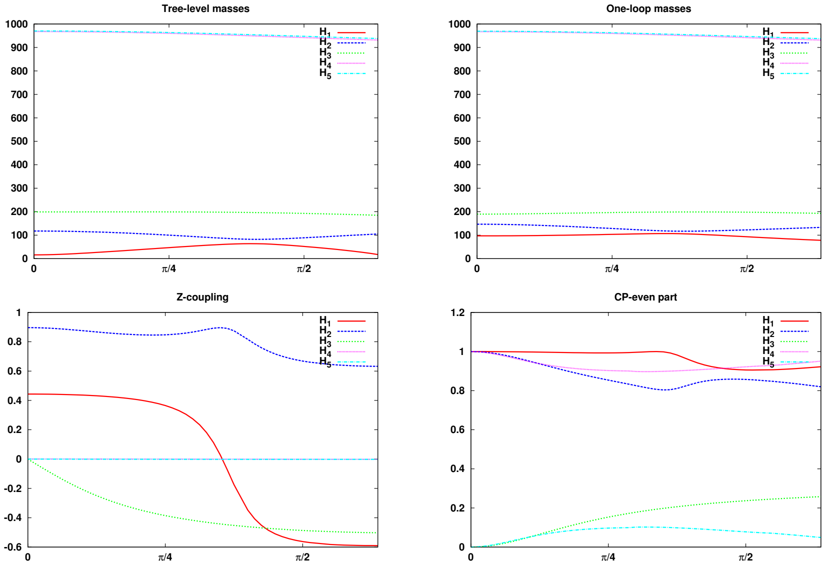
Conclusion and Outlook

- The one-loop corrections are very important.
- The phase dependence can, however, already be found in the tree-level contributions.
- The HZZ-coupling depends strongly on the (Higgs sector) phases $\rightarrow$ light Higgs can have probably escaped detection at LEP

What has still to be done?

- Understand why only two new divergent counterterms are generated.
- Investigate mixing with Goldstone bosons.
- Look at exclusion limits from LEP, Tevatron and LHC

Results: Variation in ϕ_λ



Results: Variation in ϕ_S