Parton Shower Monte Carlos

Stefan Gieseke

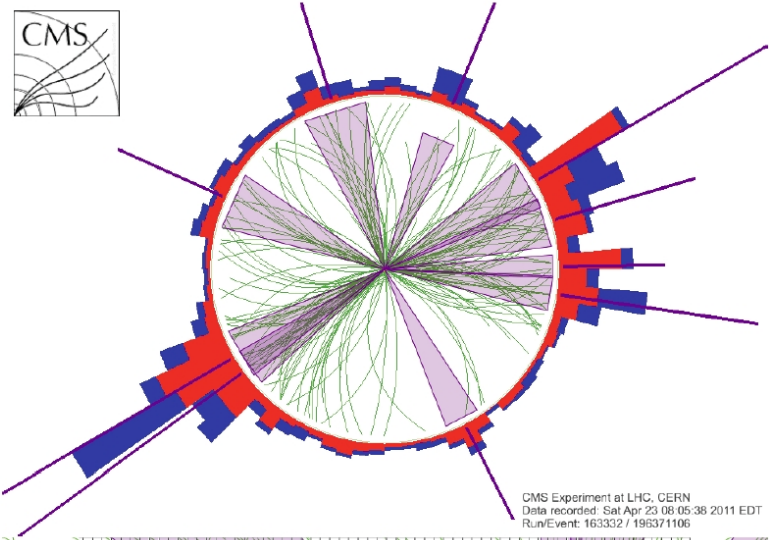
Institut für Theoretische Physik
KIT

UAM PhD course
14–17 March 2016

Motivation: jets

[Google Images]

Motivation: jets (at LHC of course)



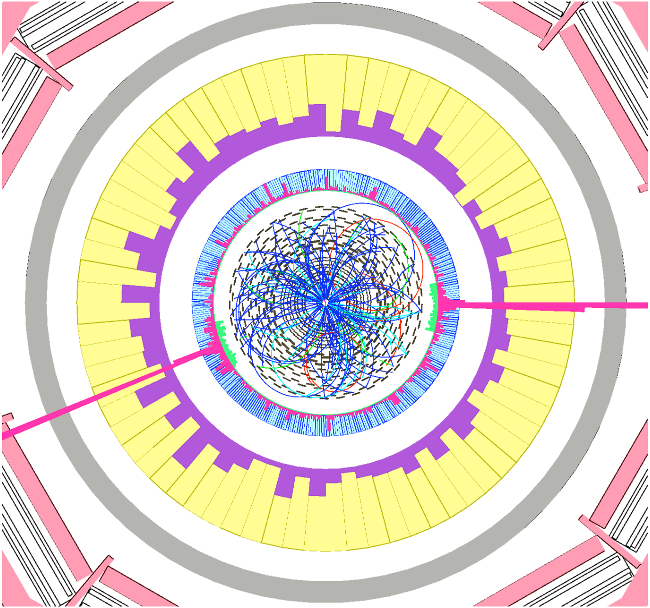
[CMS 2011]

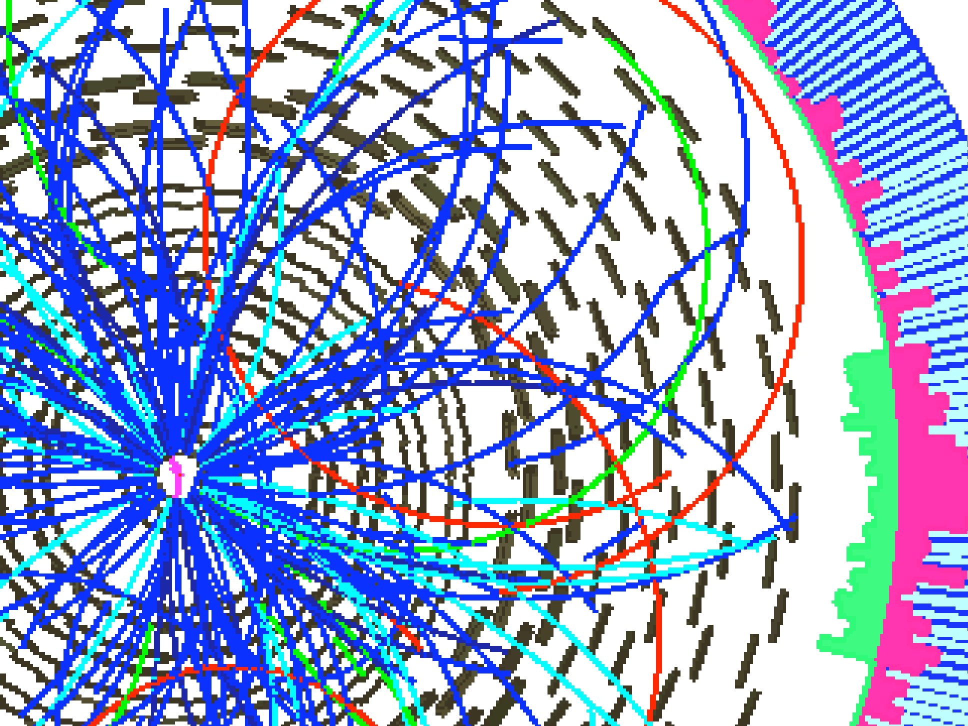
Why Monte Carlos?

We want to understand

$$\mathscr{L}_{\text{int}} \longleftrightarrow \text{Final states}\ .$$

Can you spot the Higgs?





Why Monte Carlos?

LHC experiments require
sound understanding of signals and *backgrounds*.

$\uparrow$

Full detector simulation.

$\uparrow$

Fully exclusive hadronic final state.

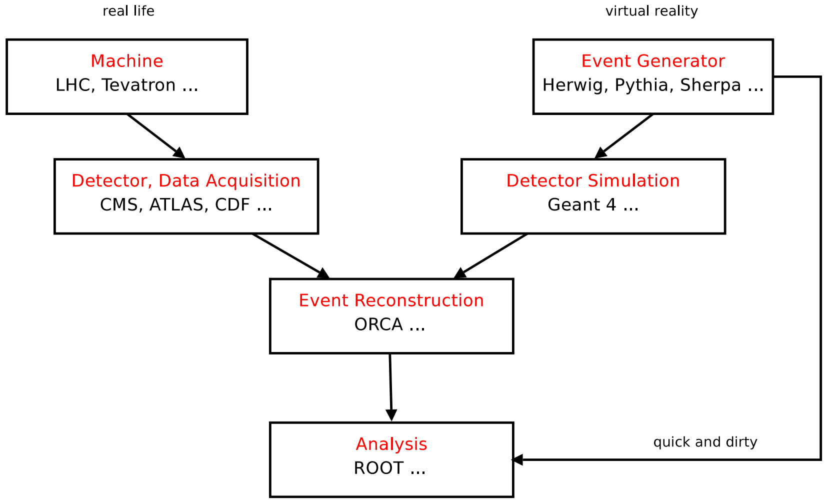
$\uparrow$

Monte Carlo event generator with
parton shower, hadronization model, decays of unstable particles.

$\uparrow$

Parton level computations.

Experiment and Simulation



Monte Carlo Event Generators

- Complex final states in full detail (jets).
- Arbitrary observables and cuts from final states.
- Studies of new physics models.

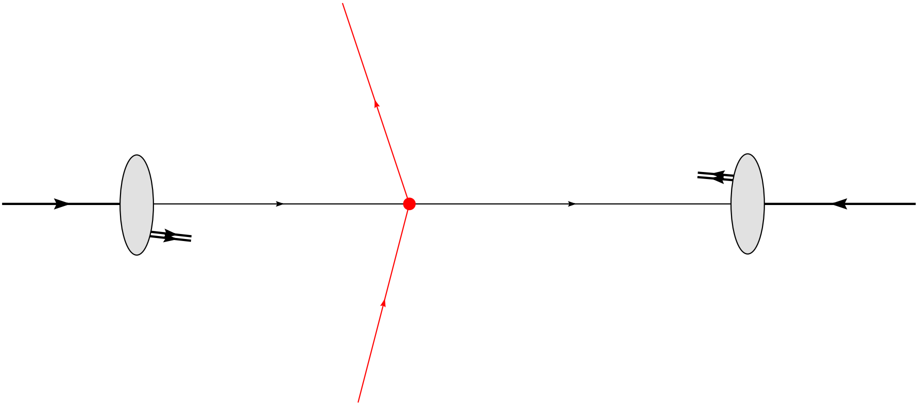
- Rates and topologies of final states.
- Background studies.
- Detector Design.
- Detector Performance Studies (Acceptance).

- *Obvious* for calculation of observables on the quantum level

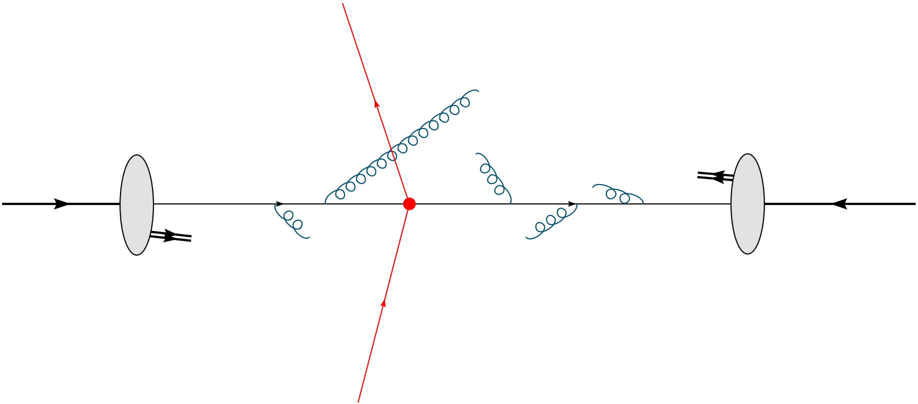
$$|A|^2 \longrightarrow \text{Probability.}$$

pp Event Generator

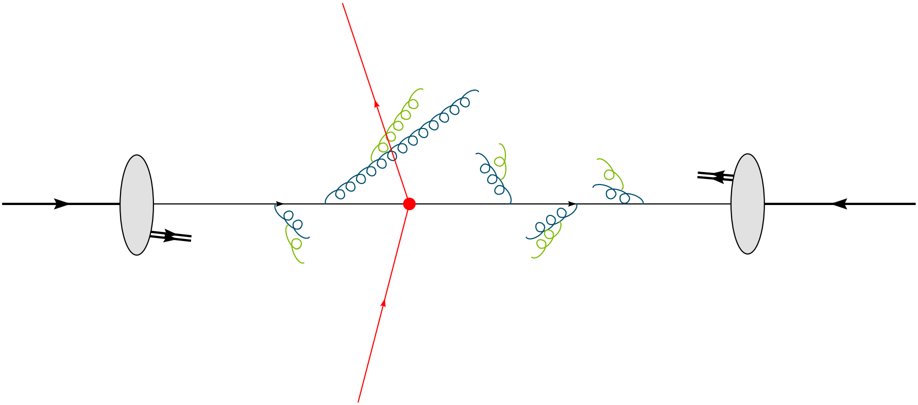
pp Event Generator



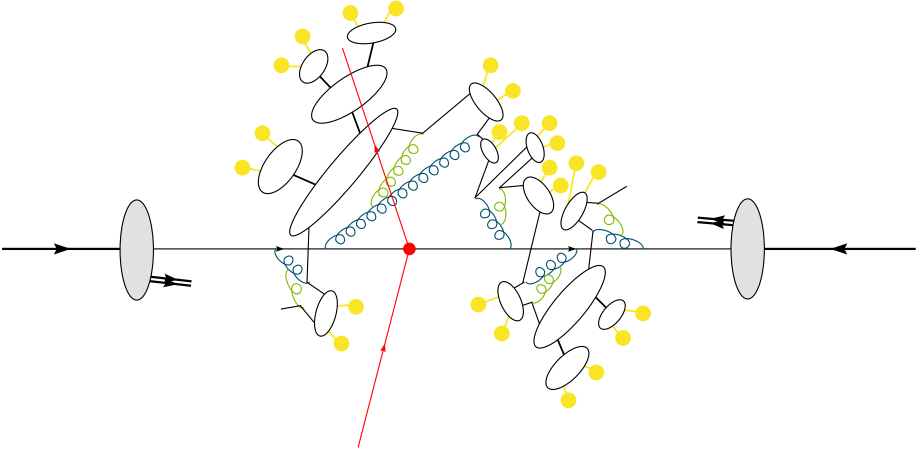
pp Event Generator



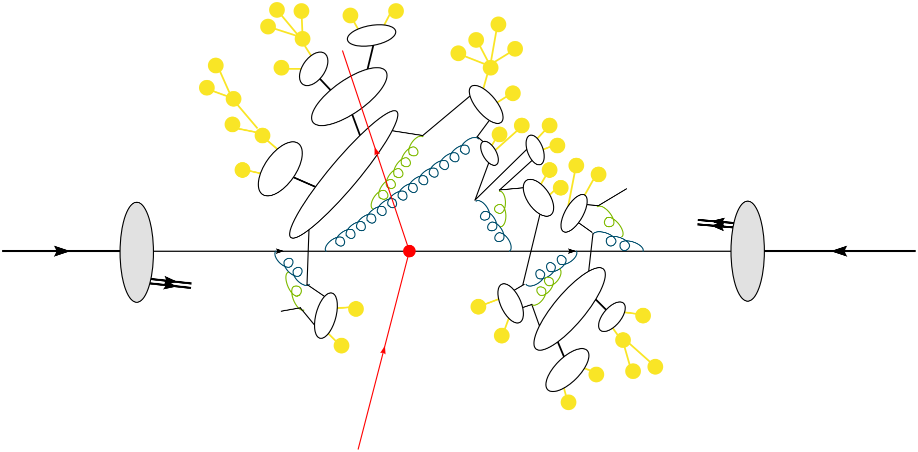
pp Event Generator



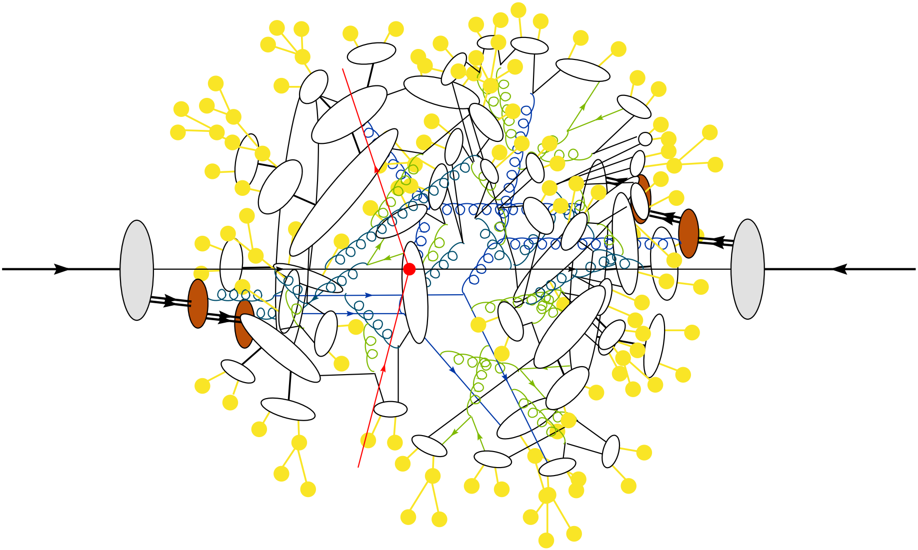
pp Event Generator



pp Event Generator



pp Event Generator



Divide and conquer

Partonic cross section from Feynman diagrams

$$\mathrm{d}\sigma = \mathrm{d}\sigma_{\mathrm{hard}}\mathrm{d}P(\mathrm{partons} \to \mathrm{hadrons})$$

$$\begin{aligned}\mathrm{d}P(\mathrm{partons} \to \mathrm{hadrons}) = \; & \mathrm{d}P(\text{resonance decays}) & [\Gamma > Q_0] \\ & \times \mathrm{d}P(\text{parton shower}) & [\mathrm{TeV} \to Q_0] \\ & \times \mathrm{d}P(\text{hadronisation}) & [\sim Q_0] \\ & \times \mathrm{d}P(\text{hadronic decays}) & [O(\mathrm{MeV})]\end{aligned}$$

Underlying event from multiple partonic interactions

$$\mathrm{d}\sigma \longleftarrow \mathrm{d}\sigma(\mathrm{QCD}\; 2 \to 2)$$

Plan for these lectures

- Monte Carlo Methods
- Hard scattering
- Parton Showers
- Hadronization and Hadronic Decays
- Matching and Merging with Higher Orders
- Underlying Event/Multiple Parton Interactions
  (if time permits)

Monte Carlo Methods

Monte Carlo Methods

Introduction to the most important MC sampling
(= integration) techniques.

1. Hit and miss.
2. Simple MC integration.
3. (Some) methods of variance reduction.
4. Adaptive MC, VEGAS.
5. Multichannel.
6. Mini event generator in particle physics.

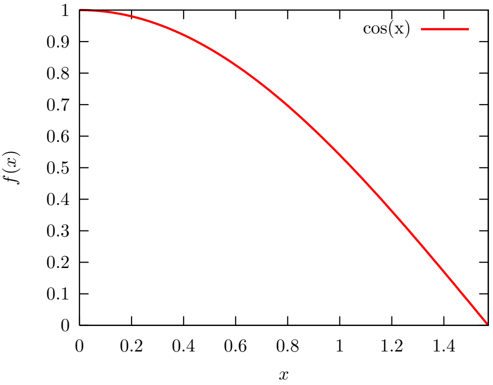
Probability

Example: $f(x) = \cos(x)$.

Probability density:

$$dP = f(x)\,dx$$

is probability to find value x.



Probability

Probability density:

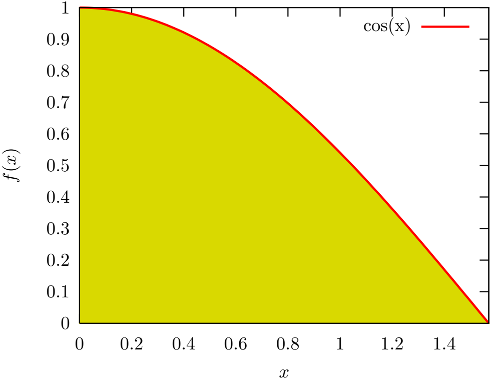
$$dP = f(x)\,dx$$

is probability to find value x.

$$F(x) = \int_{x_0}^{x} f(x)\,dx$$

is called *probability distribution*.

Example: $f(x) = \cos(x)$.



Probability

Example: $f(x) = \cos(x)$.

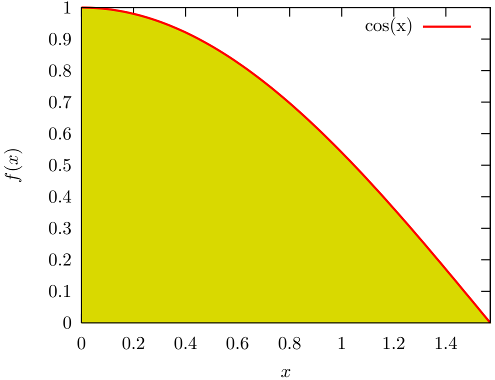
Probability density:

$$dP = f(x)\,dx$$

is probability to find value x.

$$F(x) = \int_{x_0}^{x} f(x)\,dx$$

is called *probability distribution*.



Probability $\sim$ *Area*

Hit and Miss

Hit and miss method:

- throw N random points (x, y) into region.
- Count hits N_{hit}, i.e. whenever $y < f(x)$.

Then
$$I \approx V \frac{N_{\text{hit}}}{N}.$$
approaches 1 again in our example.

Hit and Miss

Hit and miss method:

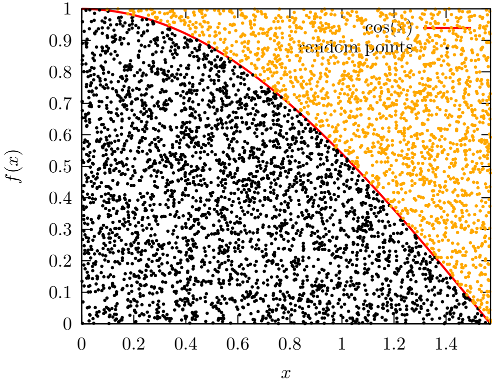
- throw N random points (x, y) into region.
- Count hits N_{hit}, i.e. whenever $y < f(x)$.

Then

$$I \approx V \frac{N_{\text{hit}}}{N}.$$

approaches 1 again in our example.

Example: $f(x) = \cos(x)$.



Hit and Miss

Hit and miss method:

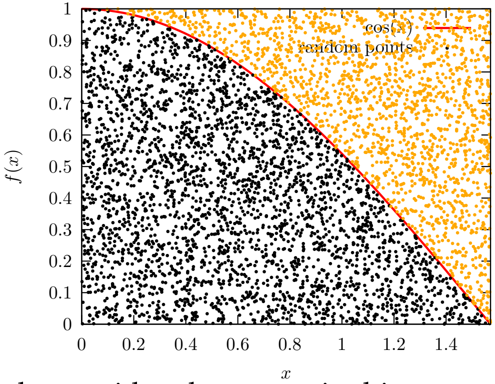
- throw N random points (x, y) into region.
- Count hits $N_{\rm hit}$, i.e. whenever $y < f(x)$.

Then

$$I \approx V \frac{N_{\rm hit}}{N}.$$

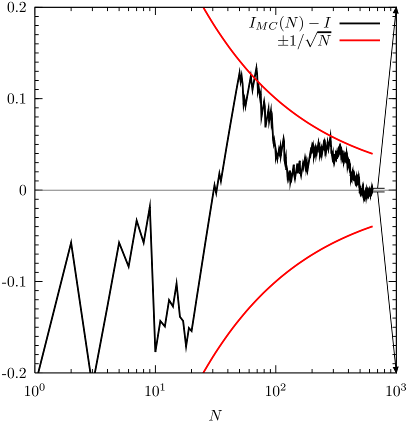
approaches 1 again in our example.

Example: $f(x) = \cos(x)$.



Every accepted value of x can be considered an event in this picture. As $f(x)$ is the 'histogram' of x, it seems obvious that the x values are distributed as $f(x)$ from this picture.

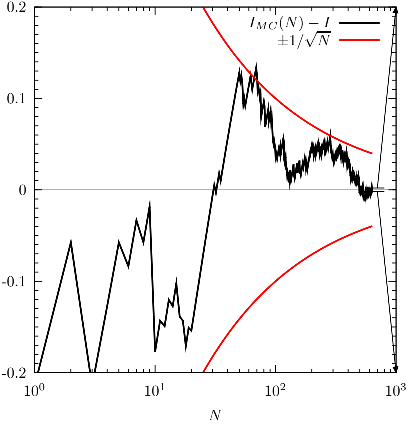
Hit and Miss



How well does it converge?

Error $1/\sqrt{N}$.

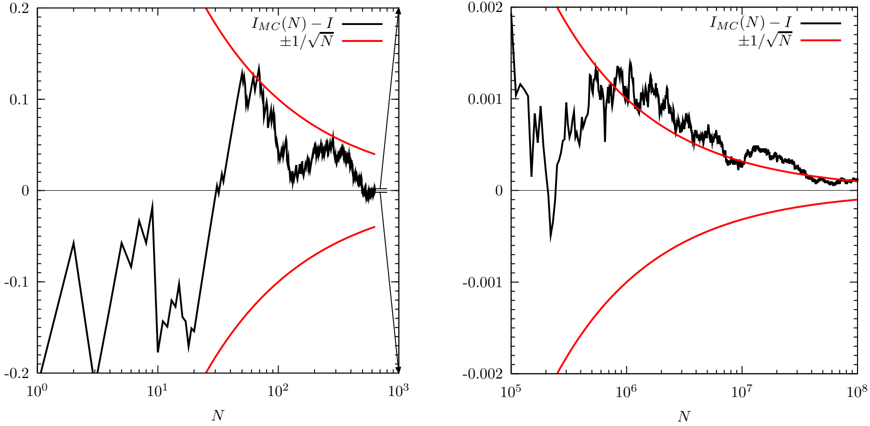
Hit and Miss



More points, zoom in…

Error $1/\sqrt{N}$.

Hit and Miss



Error $1/\sqrt{N}$.

Hit and Miss

This method is used in many event generators. However, it is not sufficient as such.

- Can handle any density $f(x)$,
  however wild and unknown it is.
- $f(x)$ should be bounded from above.
- Sampling will be very *inefficient* whenever $\mathrm{Var}(f)$ is large.

Improvements go under the name variance reduction as they improve the error of the crude MC at the same time.

Simple MC integration

Mean value theorem of integration:

$$\begin{aligned} I &= \int_{x_0}^{x_1} f(x)\,dx \\ &= (x_1 - x_0)\langle f(x)\rangle \end{aligned}$$

(Riemann integral).

Simple MC integration

Mean value theorem of integration:

$$\begin{aligned} I &= \int_{x_0}^{x_1} f(x)\,dx \\ &= (x_1 - x_0)\langle f(x)\rangle \\ &\approx (x_1 - x_0)\frac{1}{N}\sum_{i=1}^{N} f(x_i) \end{aligned}$$

(Riemann integral).

Simple MC integration

Mean value theorem of integration:

$$\begin{aligned} I &= \int_{x_0}^{x_1} f(x)\,dx \\ &= (x_1 - x_0)\langle f(x)\rangle \\ &\approx (x_1 - x_0)\frac{1}{N}\sum_{i=1}^{N} f(x_i) \end{aligned}$$

(Riemann integral).

Sum doesn't depend on ordering
$\longrightarrow$ randomize x_i.

Simple MC integration

Mean value theorem of integration:

$$
\begin{aligned}
I &= \int_{x_0}^{x_1} f(x)\,dx \\
&= (x_1 - x_0)\langle f(x)\rangle \\
&\approx (x_1 - x_0)\frac{1}{N}\sum_{i=1}^{N} f(x_i)
\end{aligned}
$$

(Riemann integral).

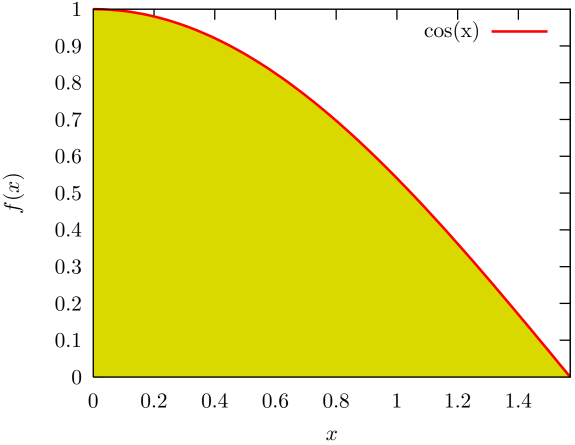
Sum doesn't depend on ordering
$\longrightarrow$ randomize x_i.

Yields a flat distribution of events x_i,
but weighted with *weight* $f(x_i)$ ($\rightarrow$ unweighting).

Simple MC integration

Pictorially:

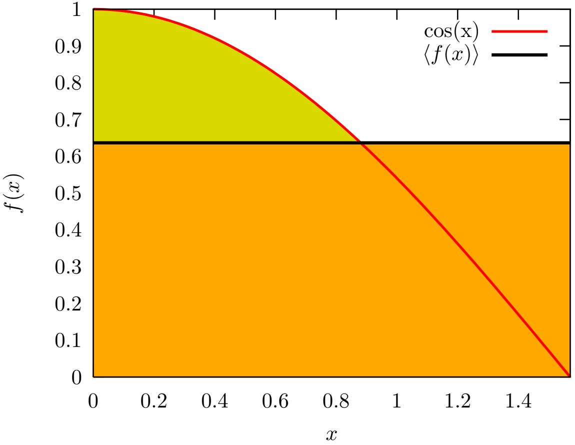
$$I = \int_{x_0}^{x_1} f(x)\,dx$$
$$= (x_1 - x_0)\langle f(x)\rangle$$



Simple MC integration

Pictorially:

$$I = \int_{x_0}^{x_1} f(x)\,dx$$
$$= (x_1 - x_0)\langle f(x)\rangle$$

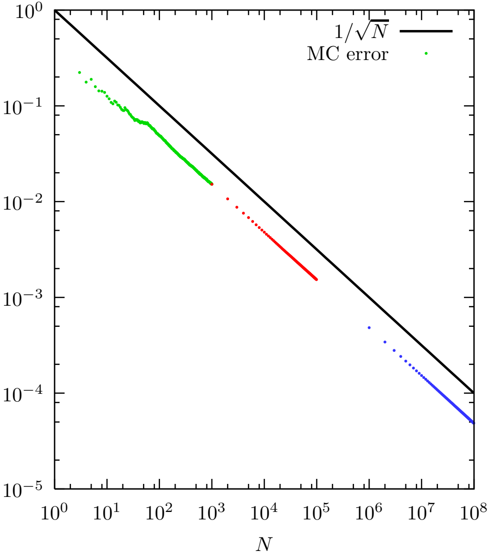


Simple MC integration

What's the error?

Again, looks like

$$\sigma \sim \frac{1}{\sqrt{N}}$$



Simple MC integration

What's the error?

We can calculate it (central limit theorem for the average):

In general: *Crude MC*

$$
\begin{aligned}
I &= \int f dV \\
&\approx V\langle f\rangle \pm V\sqrt{\frac{\langle f\rangle^2 - \langle f^2\rangle}{N}} \\
&\approx V\langle f\rangle \pm V\frac{\sigma}{\sqrt{N}}
\end{aligned}
$$

Simple MC integration

What's the error?

We can calculate it (central limit theorem for the average):

Our example: $\cos(x), 0 \leq x \leq \pi/2$,
compute σ_{MC} from

$$\langle f \rangle = \frac{1}{N}\sum_{i=1}^{N} f(x_i)$$
$$\langle f^2 \rangle = \frac{1}{N}\sum_{i=1}^{N} f^2(x_i).$$

Simple MC integration

What's the error?

We can calculate it (central limit theorem for the average):

Compute σ directly ($V = \pi/2$):

$$V\langle f\rangle = \int_0^{\pi/2} \cos(x)\,dx = 1$$
$$V\langle f^2\rangle = \int_0^{\pi/2} \cos^2(x)\,dx = \frac{1}{2}$$

then

$$V\sigma = \sqrt{1 - \frac{\pi}{2}\frac{1}{2}} \approx 0.4633.$$

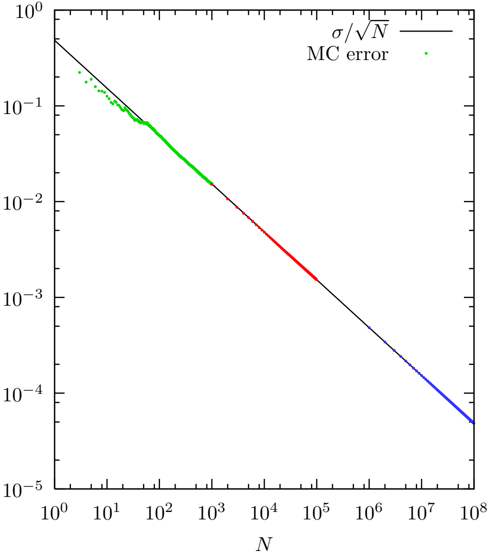
Simple MC integration

What's the error?

Now, compare

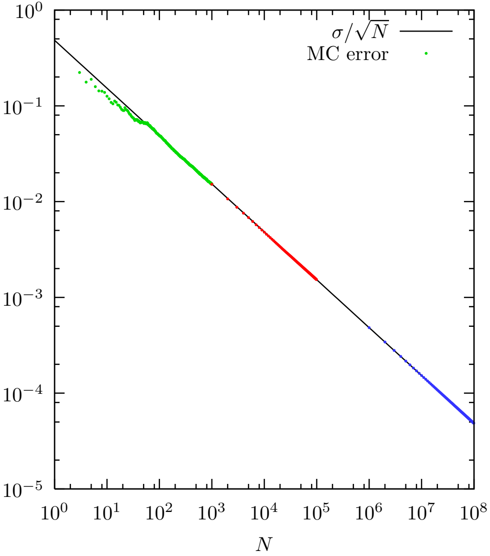
$$\sigma_{MC} = \frac{0.4633}{\sqrt{N}}$$

with error estimate from MC.



Simple MC integration

What's the error?



Now, compare

$$\sigma_{MC} = \frac{0.4633}{\sqrt{N}}$$

with error estimate from MC.

Spot on.

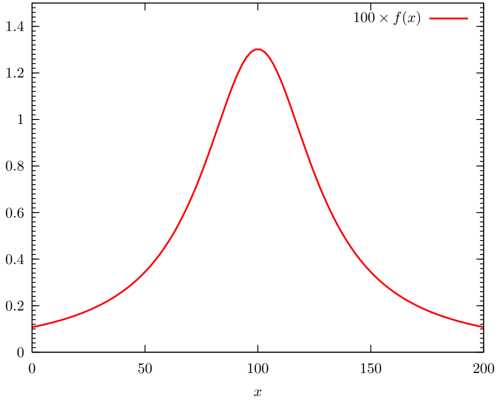
Inverting the Integral

Another basic MC method, based on the observation that

$$\textit{Probability} \sim \textit{Area}$$

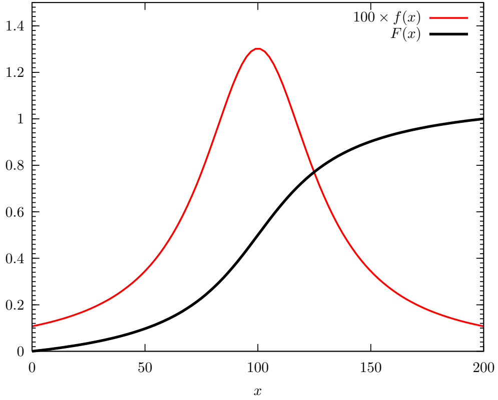
Inverting the Integral

- Probability density $f(x)$. Not necessarily normalized.



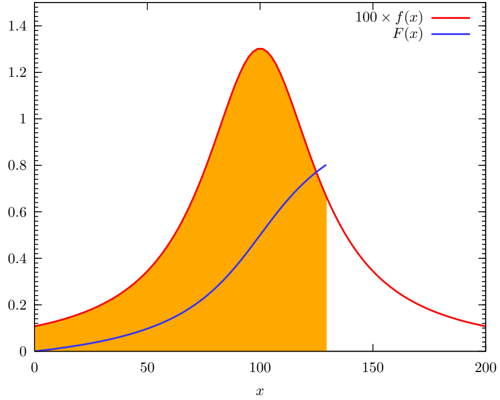
Inverting the Integral

- Probability density $f(x)$. Not necessarily normalized.
- Integral $F(x)$ known,



Inverting the Integral

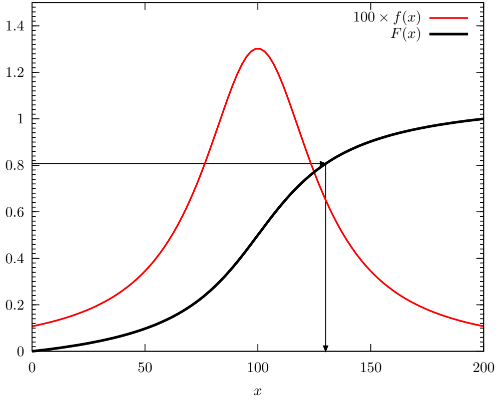
- Probability density $f(x)$. Not necessarily normalized.
- Integral $F(x)$ known,
- $P(x < x_s) = F(x_s)$.



Inverting the Integral

- Probability density $f(x)$. Not necessarily normalized.
- Integral $F(x)$ known,
- $P(x < x_s) = F(x_s)$.
- Probability = 'area', distributed evenly,

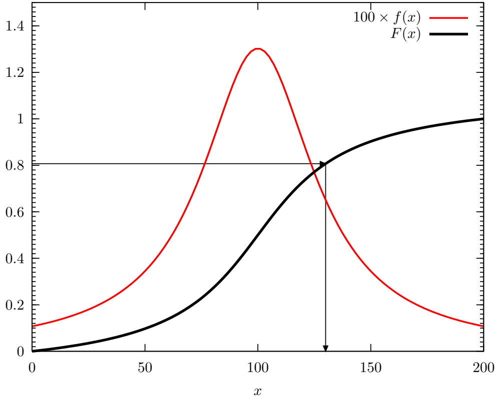
$$\int_{x_0}^{x} dP = r$$



Inverting the Integral

- Probability density $f(x)$. Not necessarily normalized.
- Integral $F(x)$ known,
- $P(x < x_s) = F(x_s)$.
- Probability = 'area', distributed evenly,

$$\int_{x_0}^{x} dP = r$$



Sample x according to $f(x)$ with

$$x = F^{-1}\left[F(x_0) + r\big(F(x_1) - F(x_0)\big)\right] .$$

Inverting the Integral

Another basic MC method, based on the observation that

$$\textit{Probability} \sim \textit{Area}$$

Sample x according to $f(x)$ with

$$x = F^{-1}\left[F(x_0) + r\big(F(x_1) - F(x_0)\big)\right] .$$

Optimal method, but we need to know

- The integral $F(x) = \int f(x)\,\mathrm{d}x$,
- It's inverse $F^{-1}(y)$.

That's rarely the case for real problems.

But very powerful in combination with other techniques.

Importance sampling

Error on Crude MC $\sigma_{MC} = \sigma/\sqrt{N}$.
$\Longrightarrow$ Reduce error by reducing variance of integrand.

Importance sampling

Error on Crude MC $\sigma_{MC} = \sigma/\sqrt{N}$.
$\Longrightarrow$ Reduce error by reducing variance of integrand.

Idea: *Divide out the singular structure.*

$$I = \int f\,\mathrm{d}V = \int \frac{f}{p} p \mathrm{d}V \approx \left\langle \frac{f}{p} \right\rangle \pm \sqrt{\frac{\langle f^2/p^2\rangle - \langle f/p\rangle^2}{N}} \,.$$

where we have chosen $\int p\,\mathrm{d}V = 1$ for convenience.

Note: need to sample flat in $p\,\mathrm{d}V$, so we better know $\int p\mathrm{d}V$ and it's inverse.

Importance sampling

Consider error term:

$$
\begin{aligned}
E &= \left\langle \frac{f^2}{p^2} \right\rangle - \left\langle \frac{f}{p} \right\rangle^2 = \int \frac{f^2}{p^2} p \mathrm{d}V - \left[\int \frac{f}{p} p \mathrm{d}V \right]^2 \\
&= \int \frac{f^2}{p} \, \mathrm{d}V - \left[\int f \, \mathrm{d}V \right]^2 .
\end{aligned}
$$

Importance sampling

Consider error term:

$$E = \int \frac{f^2}{p} \, \mathrm{d}V - \left[\int f \, \mathrm{d}V \right]^2 .$$

Best choice of p? Minimises $E \to$ functional variation of error term with (normalized) p:

$$0 = \delta E = \delta \left(\int \frac{f^2}{p} \, \mathrm{d}V - \left[\int f \, \mathrm{d}V \right]^2 + \lambda \int p \, \mathrm{d}V \right)$$

$$= \int \left(-\frac{f^2}{p^2} + \lambda \right) \mathrm{d}V \, \delta p \, ,$$

Importance sampling

Consider error term:

$$E = \int \frac{f^2}{p}\,\mathrm{d}V - \left[\int f\,\mathrm{d}V\right]^2 .$$

Best choice of p? Minimises E $\to$ functional variation of error term with (normalized) p:

$$0 = \delta E = \int \left(-\frac{f^2}{p^2} + \lambda\right) \mathrm{d}V\,\delta p ,$$

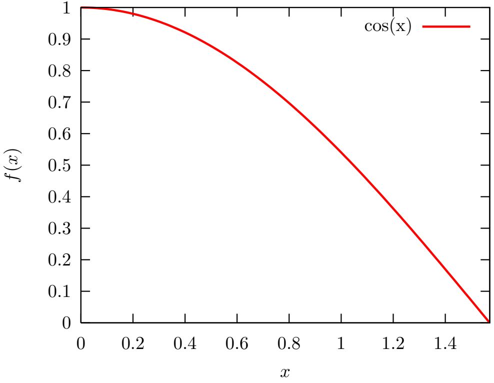
hence

$$p = \frac{|f|}{\sqrt{\lambda}} = \frac{|f|}{\int |f|\,\mathrm{d}V} .$$

Choose p as close to f as possible.

Importance sampling — example

Improving $\cos(x)$ sampling,



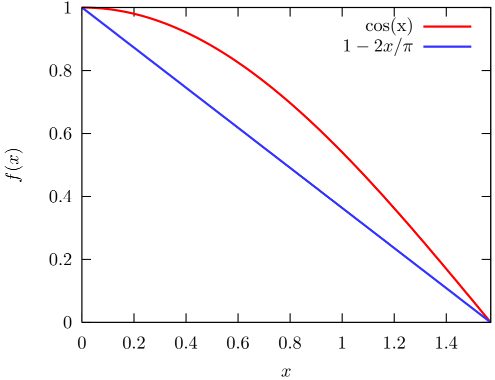
Importance sampling — example

Improving $\cos(x)$ sampling,

$$I = \int_0^{\pi/2} \cos(x)\, dx$$

$$= \int_0^{\pi/2} \frac{\cos(x)}{1-\frac{2}{\pi}x} \left(1 - \frac{2}{\pi}x\right) dx$$

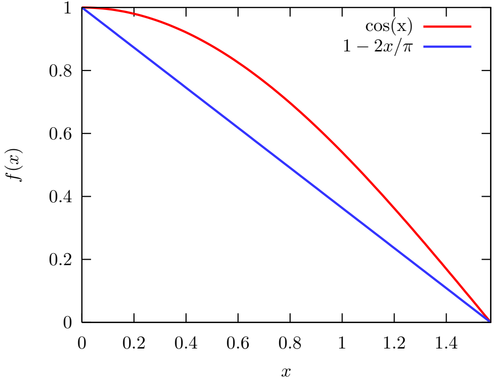
$$= \int_0^1 \left.\frac{\cos(x)}{1-\frac{2}{\pi}x}\right|_{x=x(\rho)} d\rho \ .$$



Importance sampling — example

Improving $\cos(x)$ sampling,

$$I = \int_0^{\pi/2} \cos(x)\, dx$$
$$= \int_0^{\pi/2} \frac{\cos(x)}{1 - \frac{2}{\pi}x} \left(1 - \frac{2}{\pi}x\right) dx$$
$$= \int_0^1 \left.\frac{\cos(x)}{1 - \frac{2}{\pi}x}\right|_{x=x(\rho)} d\rho \ .$$



Sample x with *inverting the integral* technique (flat random number ρ),

$$x = \frac{\pi}{2}\left(1 - \sqrt{1-\rho}\right) \mathrel{\hat{=}} \frac{\pi}{2}\left(1 - \sqrt{\rho}\right) \quad \left(I = \int_0^1 \frac{\cos\left(\frac{\pi}{2}\left(1-\sqrt{\rho}\right)\right)}{\sqrt{\rho}}\, d\rho.\right)$$

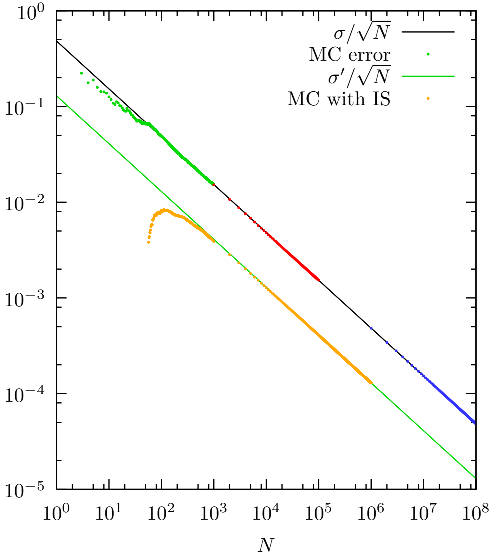
Importance sampling — example

Improving cos(x) sampling,

much better convergence,

about 80% “accepted events”.

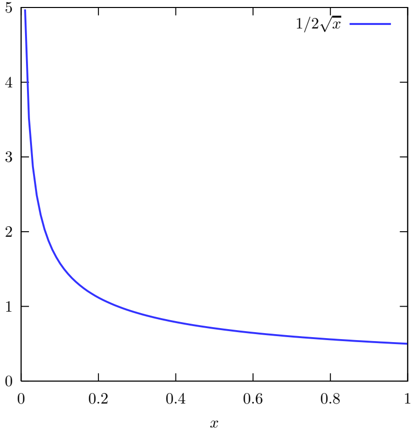
Reduced variance ($\sigma' = 0.027$)
$\Rightarrow$ better efficiency.



Importance sampling — better example

More interesting for divergent integrands, eg

$$\frac{1}{2\sqrt{x}} \, ,$$



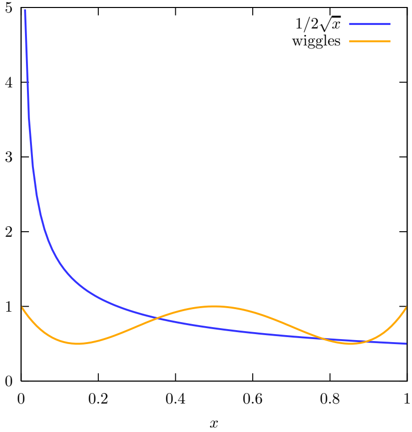
Importance sampling — better example

More interesting for divergent integrands, eg

$$\frac{1}{2\sqrt{x}} \, ,$$

with some wiggles,

$$p(x) = 1 - 8x + 40x^2 - 64x^3 + 32x^4 \, .$$



Importance sampling — better example

More interesting for divergent integrands, eg

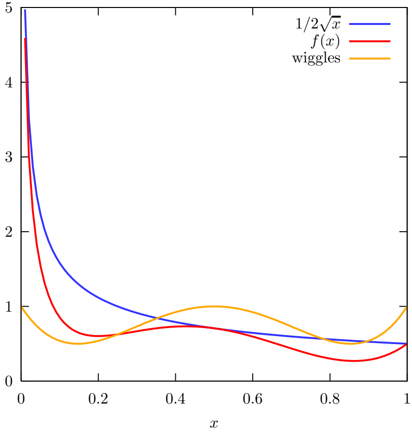
$$\frac{1}{2\sqrt{x}}\ ,$$

with some wiggles,

$$p(x) = 1 - 8x + 40x^2 - 64x^3 + 32x^4\ .$$

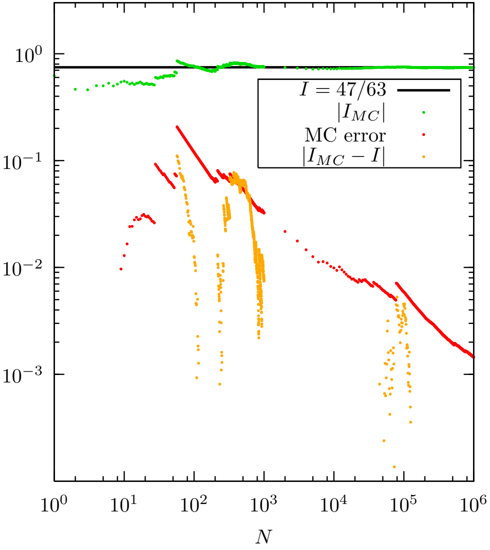
i.e. we want to integrate

$$f(x) = \frac{p(x)}{2\sqrt{x}}\ .$$



Importance sampling — better example

- Crude MC gives result in reasonable 'time'.
- Error a bit unstable.
- Event generation with maximum weight $w_{\max} = 20$. (that's arbitrary.)
- hit/miss/events with $(w > w_{\max}) =$ 36566/963434/617 with 1M generated events.

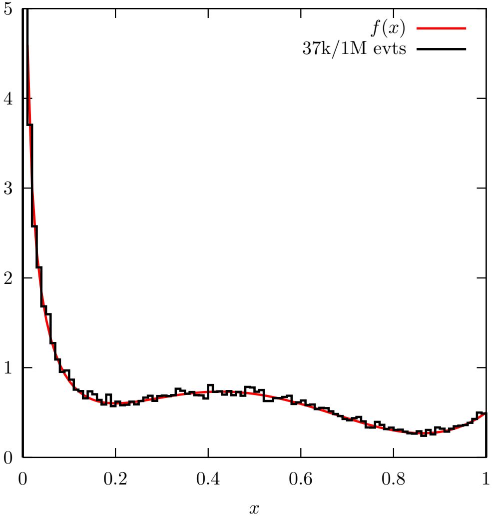


Importance sampling — better example

Want events:

use hit+mass variant here:

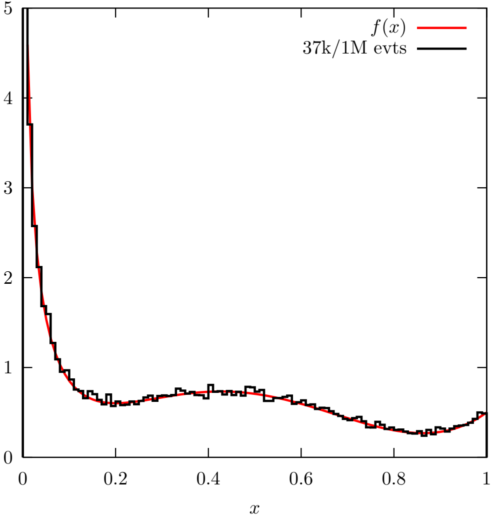
- Choose new random number r
- $w = f(x)$ in this case.
- if $r < w/w_{\max}$ then "hit".
- MC efficiency = hit$/N$.



Importance sampling — better example

Want events:
use hit+mass variant here:

- Choose new random number r
- $w = f(x)$ in this case.
- if $r < w/w_{\max}$ then "hit".
- MC efficiency = hit$/N$.
- Efficiency for MC events only 3.7%.
- Note the wiggly histogram.



Importance sampling — better example

Now importance sampling, i.e. divide out $1/2\sqrt{x}$.

$$\int_0^1 \frac{p(x)}{2\sqrt{x}}\,dx = \int_0^1 \left(\frac{p(x)}{2\sqrt{x}} \bigg/ \frac{1}{2\sqrt{x}} \right) \frac{dx}{2\sqrt{x}}$$
$$= \int_0^1 p(x)\,d\sqrt{x}$$
$$= \int_0^1 p(x(\rho))\,d\rho$$
$$= \int_0^1 1 - 8\rho^2 + 40\rho^4 - 64\rho^6 + 32\rho^8\,d\rho$$

so,

$$\rho = \sqrt{x}, \qquad d\rho = \frac{dx}{2\sqrt{x}}$$

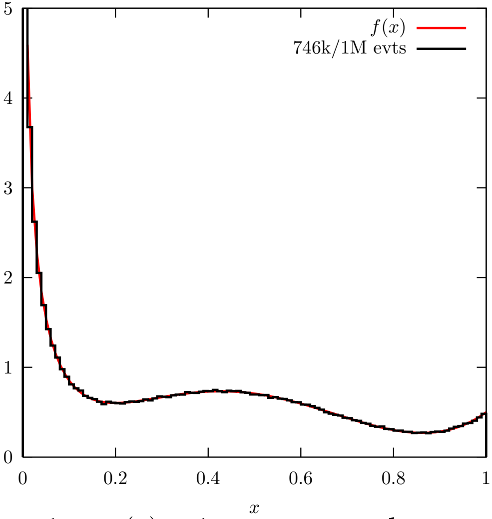
x sampled with *inverting the integral* from flat random numbers ρ, $x = \rho^2$.

Importance sampling — better example

$$\int_0^1 \frac{p(x)}{2\sqrt{x}}\, dx = \int_0^1 p(x(\rho))\, d\rho$$

with

$$\rho = \sqrt{x}, \qquad d\rho = \frac{dx}{2\sqrt{x}}$$



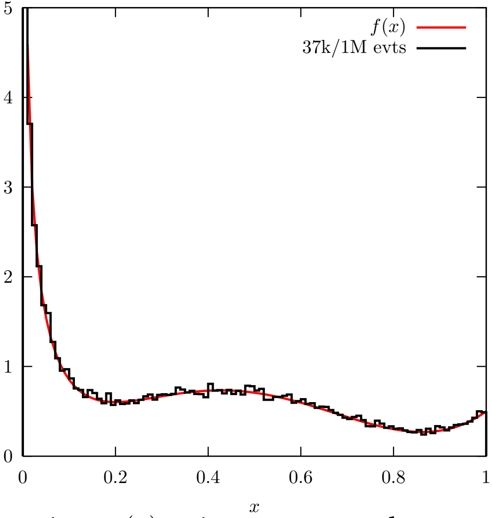
Events generated with $w_{\max} = 1$, as $p(x) \le 1$, no guesswork needed here! Now, we get 74.6% MC efficiency.

Importance sampling — better example

$$\int_0^1 \frac{p(x)}{2\sqrt{x}}\,dx = \int_0^1 p(x(\rho))\,d\rho$$

with

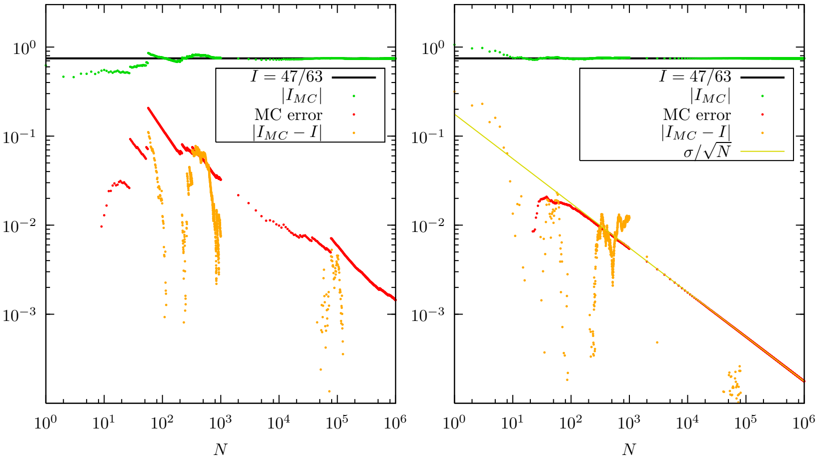
$$\rho = \sqrt{x}, \qquad d\rho = \frac{dx}{2\sqrt{x}}$$



Events generated with $w_{\max} = 1$, as $p(x) \leq 1$, no guesswork needed here! Now, we get 74.6% MC efficiency.
…as opposed to 3.7%.

Importance sampling — better example

Crude MC vs Importance sampling.



100× more events needed to reach same accuracy.

Importance sampling — another useful example

Breit–Wigner peaks appear in many realistic MEs for cross sections and decays.

$$I = \int_{s_0}^{s_1} \frac{ds}{(s-m^2)^2 + m^2\Gamma^2}$$

Importance sampling — another useful example

Breit–Wigner peaks appear in many realistic MEs for cross sections and decays.

$$I = \int_{s_0}^{s_1} \frac{ds}{(s-m^2)^2 + m^2\Gamma^2} = \frac{1}{m\Gamma}\int_{y_0}^{y_1} \frac{dy}{y^2+1} \qquad (y = \frac{s-m^2}{m\Gamma})$$

$$= \frac{1}{m\Gamma} \arctan \frac{s-m^2}{m\Gamma}\Big|_{s_0}^{s_1}$$

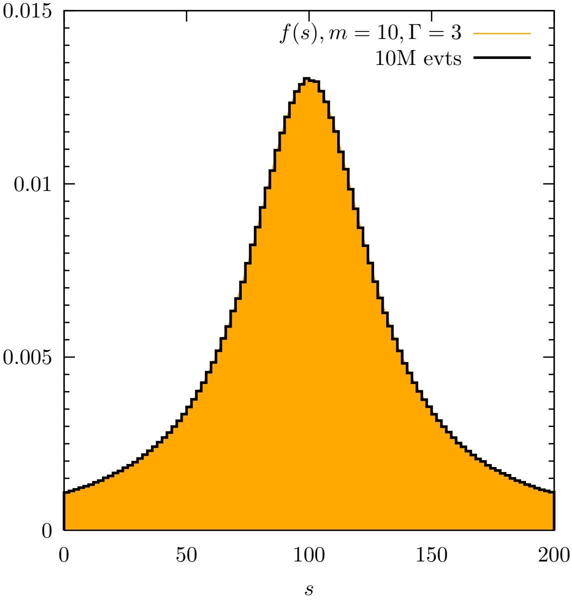
Inverting the integral gives ("tan mapping").

$$f(s) = \frac{m\Gamma}{(s-m^2)^2 + m^2\Gamma^2} \; ,$$

$$F(s) = \arctan \frac{s-m^2}{m\Gamma} = \rho \; ,$$

$$F^{-1}(\rho) = m^2 + m\Gamma \tan \rho \; .$$

Importance sampling — another useful example



VEGAS

- Classic algorithm.
- Automatic impotance sampling.
- Adopt grid size.
- Often used for multidimensional integration.
- Very robust.

VEGAS

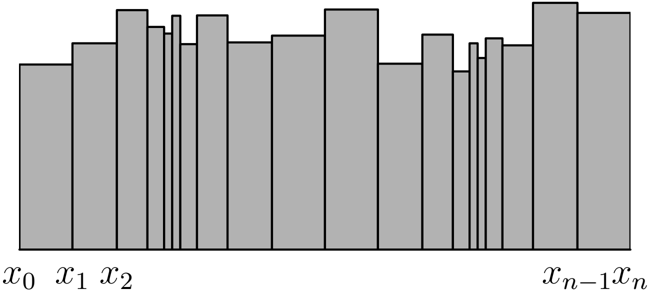
- start with equidistant grid $x_0, x_1, \ldots, x_N$.
- Sample a number of points $\big(x_{s,i}, f(x_{s,i})\big)$, compute first estimate of integral as $\langle f \rangle$.
- Resize grid:
  choose x_i' such that contribution from partial areas inside $x_i < x < x_{i+1}$ to integral is $\langle f \rangle / N$.
- Remember, optimal $p(x) \sim |f(x)|$.
- Sample again with same number of points into every bin $x_i < x < x_{i+1}$. Results in step weight function with steps

$$p_i = \frac{1}{N(x_i - x_{i-1})}\,, \qquad x_i < x < x_{i+1}\;.$$

- $\Rightarrow$ Sample often where density is high.

VEGAS

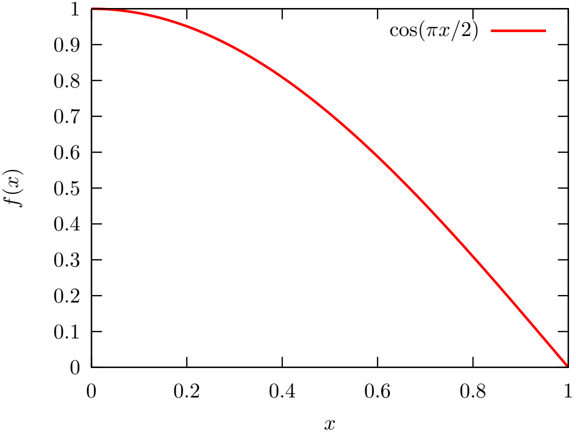
Rebinning:



[from T. Ohl, VAMP]

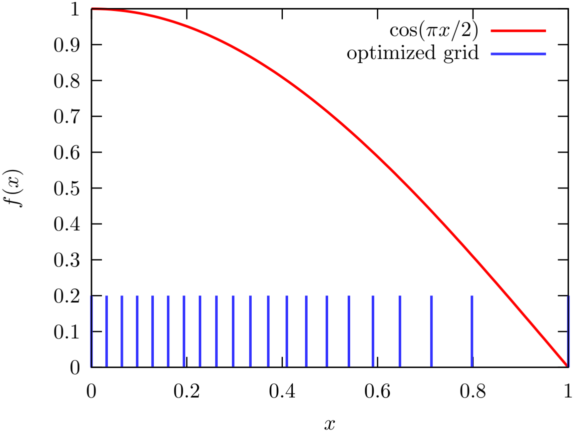
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20,100$
Convergence improved.



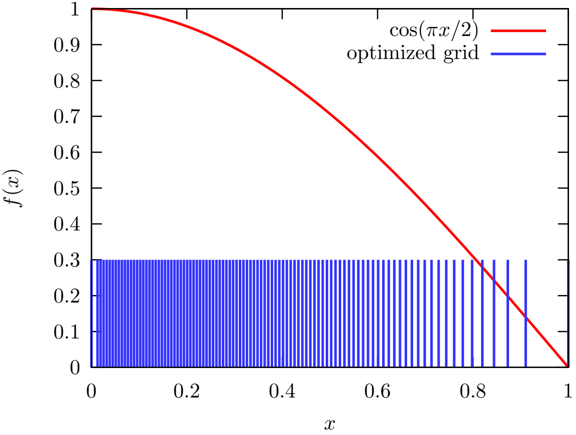
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20,100$
Convergence improved.



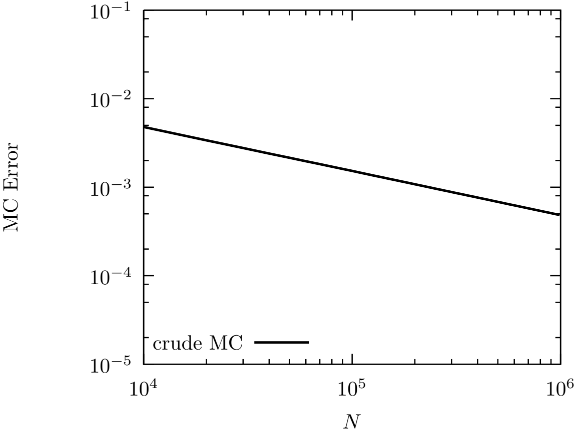
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20, 100$
Convergence improved.



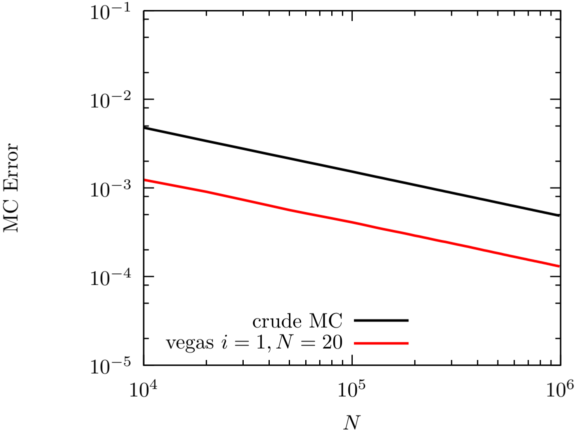
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20,100$
Convergence improved.



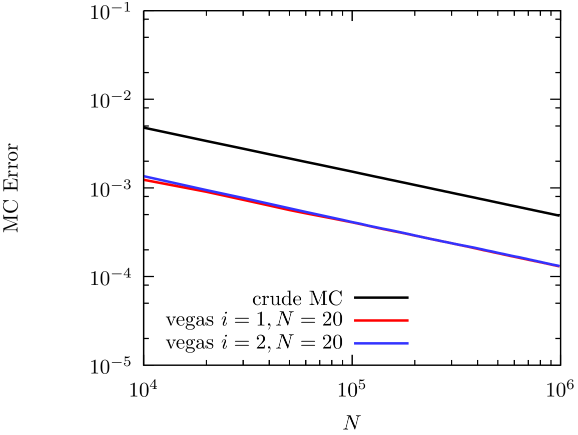
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20, 100$
Convergence improved.



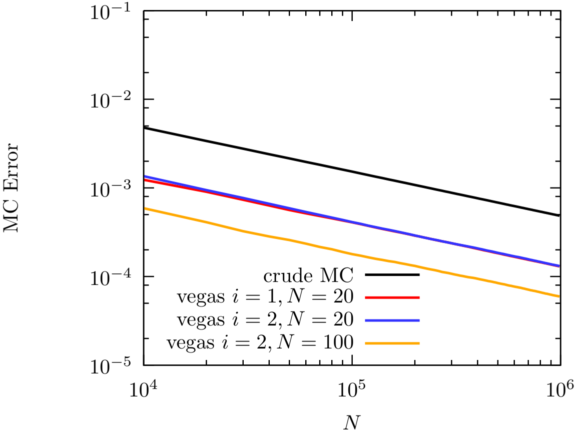
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20, 100$
Convergence improved.



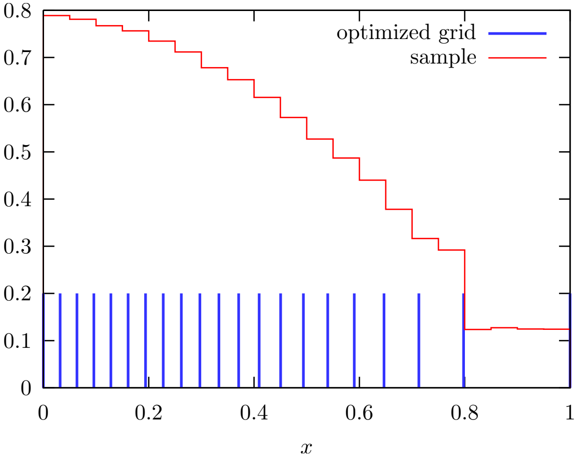
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20, 100$
Convergence improved.



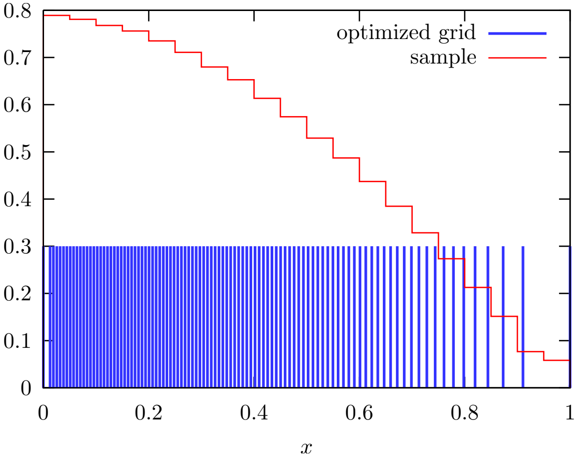
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20,100$
Convergence improved.



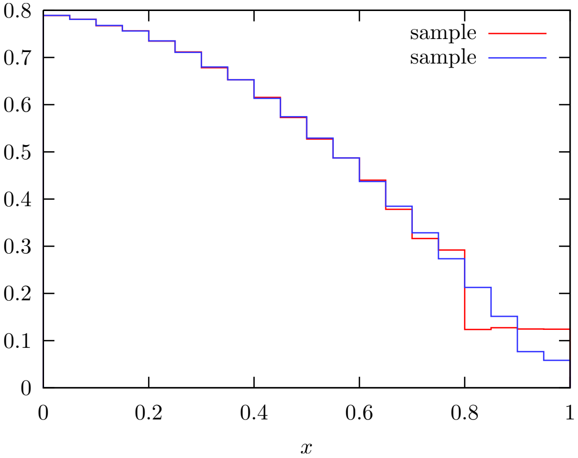
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20,100$
Convergence improved.



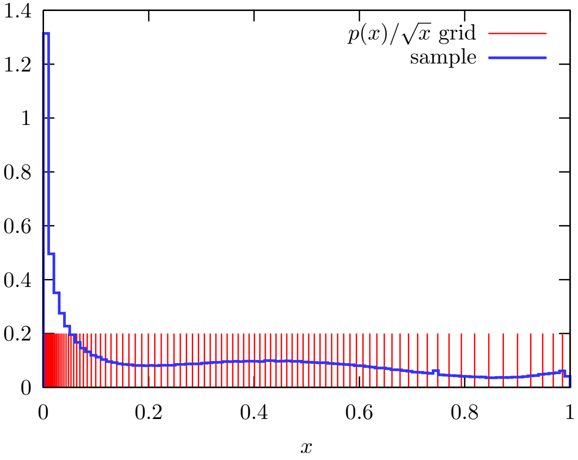
VEGAS

Example: $\cos(\frac{\pi x}{2})$
$N = 20, 100$
Convergence improved.



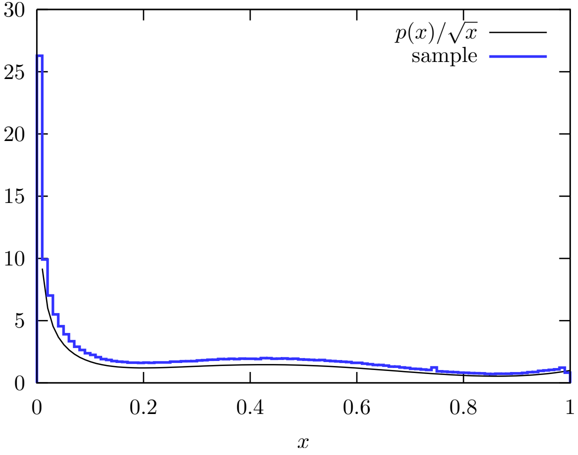
VEGAS

Second example: $p(x)/\sqrt{x}$ (divergence with wiggles)



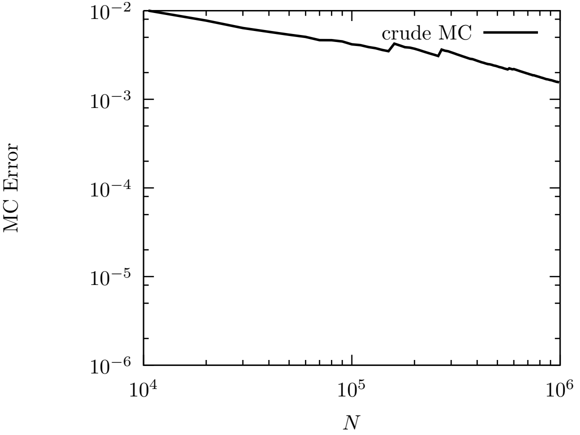
VEGAS

Second example:
$p(x)/\sqrt{x}$
(divergence with wiggles)



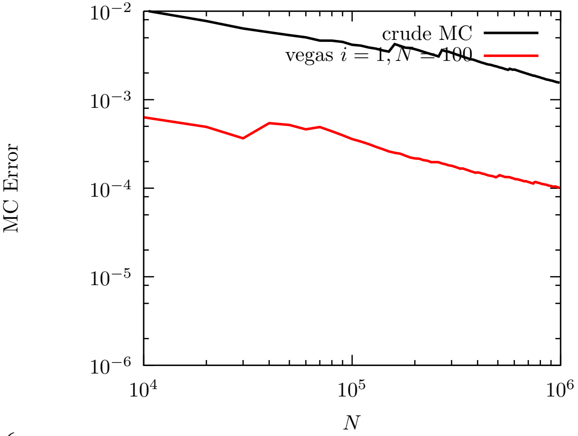
VEGAS

Second example:
$p(x)/\sqrt{x}$
(divergence with wiggles)



VEGAS

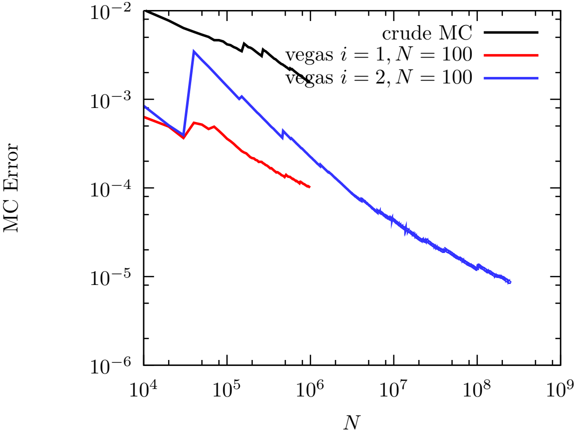
Second example:
$p(x)/\sqrt{x}$
(divergence with wiggles)



Acc 10^{-4} after $N = 10^6$ comparable with ’inverting the integral’.

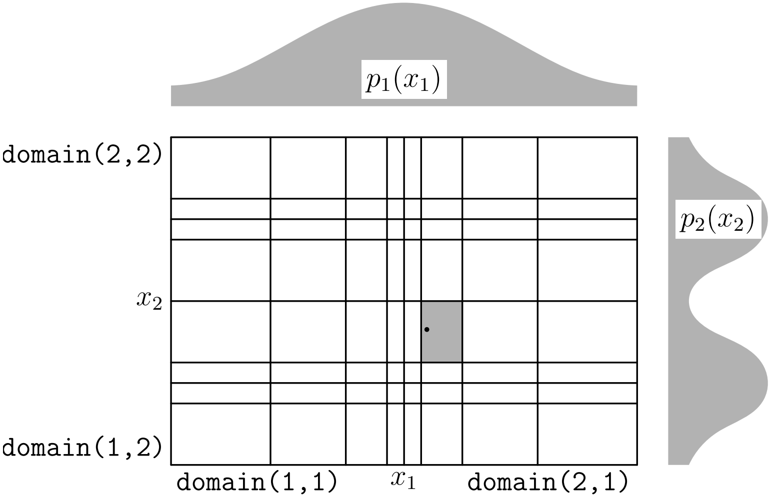
VEGAS

Second example:
$p(x)/\sqrt{x}$
(divergence with wiggles)



VEGAS

Problem to adapt in multiple dimensions:

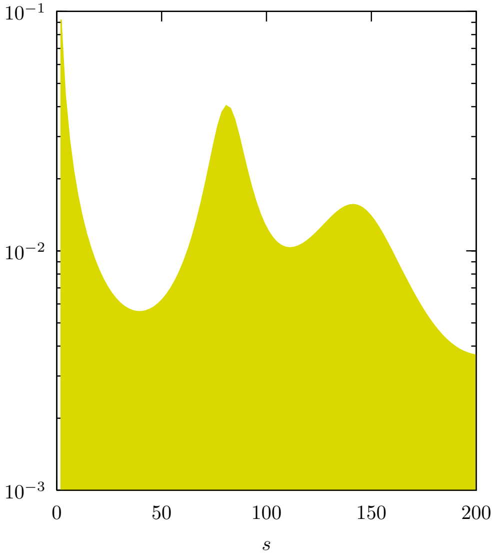


[from T. Ohl, VAMP]

Multichannel MC

Typical problem:

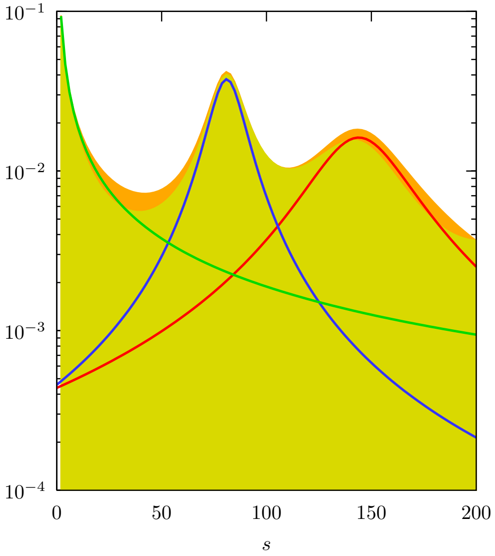
- $f(s)$ has multiple peaks ($\times$ wiggles from ME).



Multichannel MC

Typical problem:

- $f(s)$ has multiple peaks ($\times$ wiggles from ME).
- Usually have some idea of the peak structure.

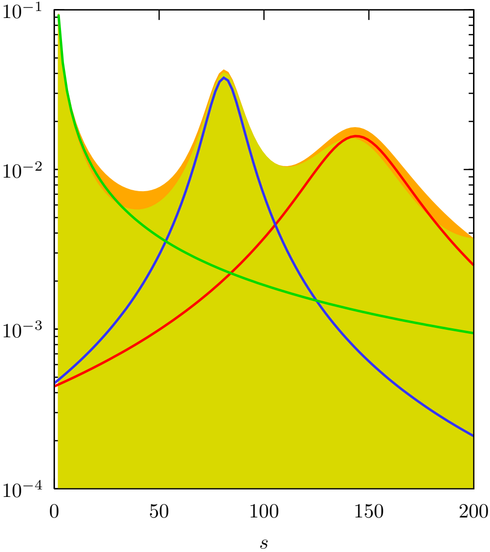


Multichannel MC

Typical problem:

- $f(s)$ has multiple peaks ($\times$ wiggles from ME).
- Usually have some idea of the peak structure.
- Encode this in sum of sample functions $g_i(s)$ with weights $\alpha_i, \sum_i \alpha_i = 1$.

$$g(s) = \sum_i \alpha_i g_i(s) \; .$$



Multichannel MC

Now rewrite

$$\int_{s_0}^{s_1} f(s)\,ds = \int_{s_0}^{s_1} \frac{f(s)}{g(s)} g(s)\,ds$$
$$= \int_{s_0}^{s_1} \frac{f(s)}{g(s)} \sum_i \alpha_i g_i(s)\,ds$$
$$= \sum_i \alpha_i \int_{s_0}^{s_1} \frac{f(s)}{g(s)} g_i(s)\,ds$$

Now $g_i(s)\,ds = d\rho_i$ (inverting the integral).

Multichannel MC

Now rewrite

$$\int_{s_0}^{s_1} f(s)\,ds = \int_{s_0}^{s_1} \frac{f(s)}{g(s)} g(s)\,ds$$
$$= \int_{s_0}^{s_1} \frac{f(s)}{g(s)} \sum_i \alpha_i g_i(s)\,ds$$
$$= \sum_i \alpha_i \int_{s_0}^{s_1} \frac{f(s)}{g(s)} g_i(s)\,ds$$

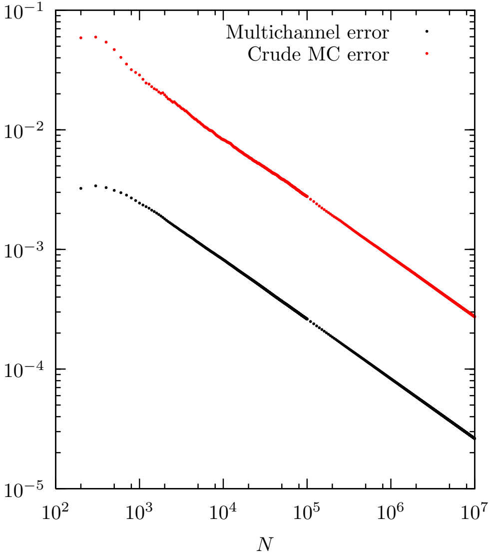
Now $g_i(s)\,ds = d\rho_i$ (inverting the integral).

Select the distribution $g_i(s)$ you'd like to sample next event from acc to weights α_i.

α_i can be optimized after a number of trials.

Multichannel MC

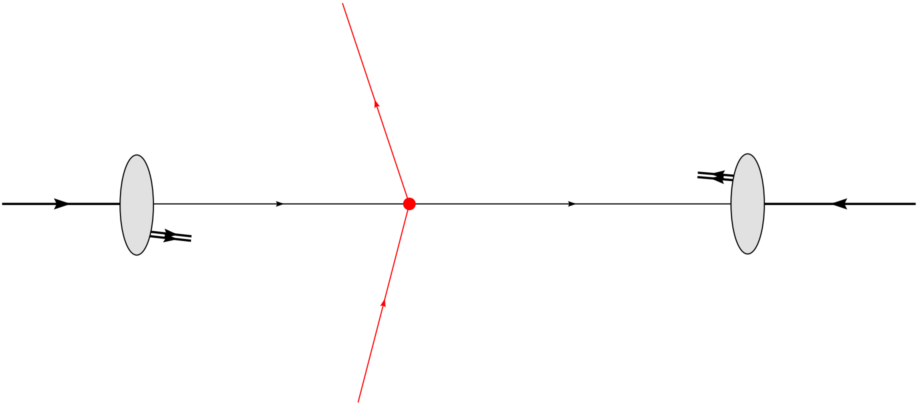
Works quite well:



Hard Scattering

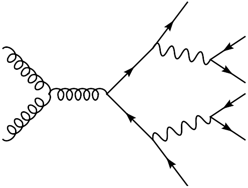
Hard scattering

Hard scattering



Matrix elements

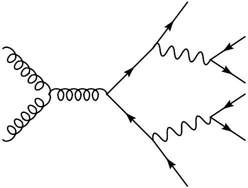
- Perturbation theory/Feynman diagrams give us (fairly accurate) final states for a few number of legs ($O(1)$).



- OK for very inclusive observables.

Matrix elements

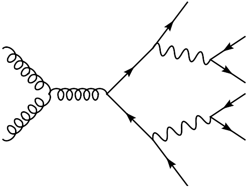
- Perturbation theory/Feynman diagrams give us (fairly accurate) final states for a few number of legs ($O(1)$).



- OK for very inclusive observables.
- Starting point for further simulation.
- Want exclusive final state at the LHC ($O(100)$).

Matrix elements

- Perturbation theory/Feynman diagrams give us (fairly accurate) final states for a few number of legs ($O(1)$).



- OK for very inclusive observables.
- Starting point for further simulation.
- Want exclusive final state at the LHC ($O(100)$).
- Want arbitrary cuts.
- $\rightarrow$ use Monte Carlo methods.

Matrix elements

Where do we get (LO) $|M|^2$ from?

- Most/important simple processes (SM) are 'built in'.
- Calculate yourself (≤ 3 particles in final state).
- Matrix element generators:
  - MadGraph/MadEvent.
  - Comix/AMEGIC (part of Sherpa).
  - HELAC/PHEGAS.
  - Whizard.
  - CalcHEP/CompHEP.

  generate code or event files that can be further processed.
- $\rightarrow$ FeynRules interface to ME generators.

Also NLO mostly automatically available.
See "Matching and Merging".

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1,\mu^2) f_j(x_2,\mu^2) \frac{1}{F} \overline{\sum} |M|^2 \qquad\qquad \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n \ ,$$

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1, \mu^2) f_j(x_2, \mu^2) \frac{1}{F} \overline{\sum} |M|^2 \, \Theta(\text{cuts}) \, \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n \; ,$$

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1,\mu^2) f_j(x_2,\mu^2) \frac{1}{F} \overline{\sum} |M|^2 \, \Theta(\text{cuts}) \, \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n \ ,$$

now,

$$\frac{1}{F} \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n = J(\vec{x}) \prod_{i=1}^{3n-2} \mathrm{d}x_i \qquad \left(\mathrm{d}\Phi_n = (2\pi)^4 \delta^{(4)}(\ldots) \prod_{i=1}^{n} \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 2E_i} \right)$$

such that

$$\sigma = \int g(\vec{x}) \, \mathrm{d}^{3n-2}\vec{x} \ , \qquad \left(g(\vec{x}) = J(\vec{x}) f_i f_j \overline{\sum} |M|^2 \Theta(\text{cuts}) \right)$$

$$= \frac{1}{N} \sum_{i=1}^{N} \frac{g(\vec{x}_i)}{p(\vec{x}_i)} = \frac{1}{N} \sum_{i=1}^{N} w_i \ .$$

Cross section formula

From Matrix element, we calculate

$$\sigma = \int f_i(x_1,\mu^2) f_j(x_2,\mu^2) \frac{1}{F} \overline{\sum} |M|^2 \, \Theta(\text{cuts}) \, \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n \ ,$$

now,

$$\frac{1}{F} \mathrm{d}x_1 \mathrm{d}x_2 \mathrm{d}\Phi_n = J(\vec{x}) \prod_{i=1}^{3n-2} \mathrm{d}x_i \qquad \left(\mathrm{d}\Phi_n = (2\pi)^4 \delta^{(4)}(\ldots) \prod_{i=1}^{n} \frac{\mathrm{d}^3\vec{p}}{(2\pi)^3 2E_i} \right)$$

such that

$$\begin{aligned} \sigma &= \int g(\vec{x}) \, \mathrm{d}^{3n-2}\vec{x} \ , \qquad \left(g(\vec{x}) = J(\vec{x}) f_i f_j \overline{\sum} |M|^2 \Theta(\text{cuts}) \right) \\ &= \frac{1}{N} \sum_{i=1}^{N} \frac{g(\vec{x}_i)}{p(\vec{x}_i)} = \frac{1}{N} \sum_{i=1}^{N} w_i \ . \end{aligned}$$

We generate events $\vec{x}_i$ with weights w_i.

Mini event generator

- We generate pairs $(\vec{x}_i, w_i)$.

Mini event generator

- We generate pairs $(\vec{x}_i, w_i)$.
- Use immediately to book weighted histogram of arbitrary observable (possibly with additional cuts!)

Mini event generator

- We generate pairs $(\vec{x}_i, w_i)$.
- Use immediately to book weighted histogram of arbitrary observable (possibly with additional cuts!)
- Keep event $\vec{x}_i$ with probability

$$P_i = \frac{w_i}{w_{\max}} \ .$$

Generate events with same frequency as in nature!

Mini event generator

- We generate pairs $(\vec{x}_i, w_i)$.
- Use immediately to book weighted histogram of arbitrary observable (possibly with additional cuts!)
- Keep event $\vec{x}_i$ with probability

$$P_i = \frac{w_i}{w_{\max}} \ ,$$

where $w_{\max}$ has to be chosen sensibly.
$\rightarrow$ reweighting, when $\max(w_i) = \bar{w}_{\max} > w_{\max}$, as

$$P_i = \frac{w_i}{\bar{w}_{\max}} = \frac{w_i}{w_{\max}} \cdot \frac{w_{\max}}{\bar{w}_{\max}} \ ,$$

i.e. reject events with probability $(w_{\max}/\bar{w}_{\max})$ afterwards.
(can be ignored when #(events with $w_i > \bar{w}_{\max}$) small.)

Mini event generator

- We generate pairs $(\vec{x}_i, w_i)$.
- Use immediately to book weighted histogram of arbitrary observable (possibly with additional cuts!)
- Keep event $\vec{x}_i$ with probability

$$P_i = \frac{w_i}{w_{\max}} \ .$$

Generate events with same frequency as in nature!

Matrix elements

Some comments:

- Use common Monte Carlo techniques to generate events efficiently. Goal: small variance in w_i distribution!

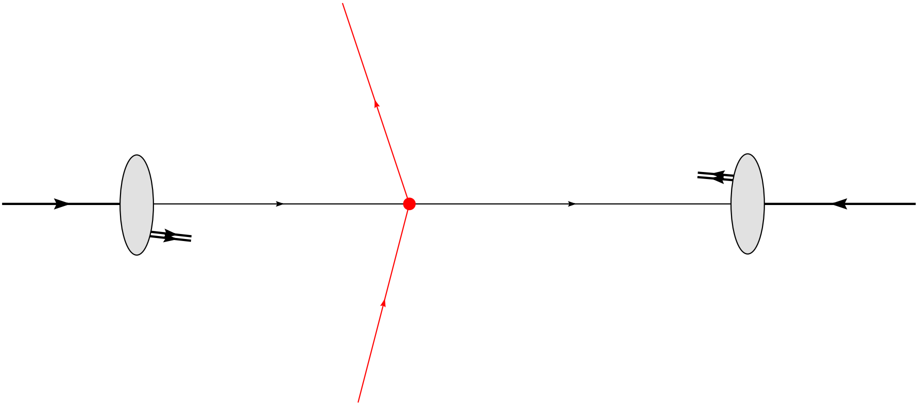
Matrix elements

Some comments:

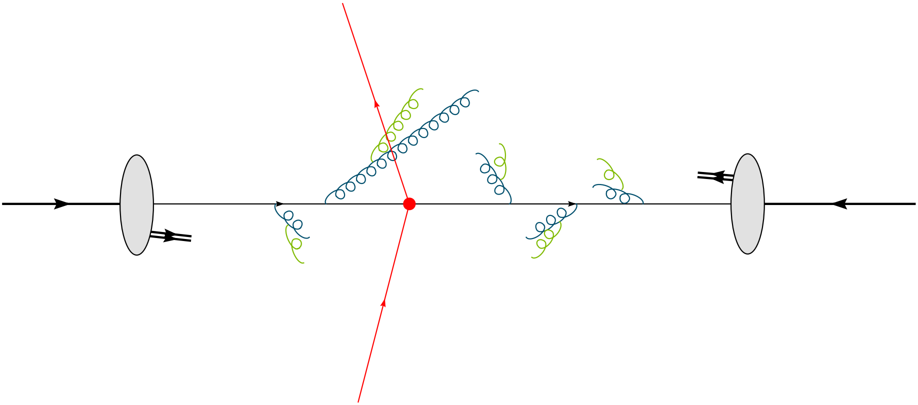
- Use common Monte Carlo techniques to generate events efficiently. Goal: small variance in w_i distribution!
- Efficient generation closely tied to knowledge of $f(\vec{x}_i)$, *i.e.* the matrix element's propagator structure.
  $\rightarrow$ build phase space generator already while generating ME's automatically.

Parton Showers

Hard matrix element



Hard matrix element → parton showers



Parton showers

Quarks and gluons in final state, pointlike.

Parton showers

Quarks and gluons in final state, pointlike.

- Know short distance (short time) fluctuations from matrix element/Feynman diagrams: $Q \sim$ few GeV to O(TeV).

- Measure hadronic final states, long distance effects, $Q_0 \sim 1\,\mathrm{GeV}$.

Parton showers

Quarks and gluons in final state, pointlike.

- Know short distance (short time) fluctuations from matrix element/Feynman diagrams: $Q \sim$ few GeV to O(TeV).
- Parton shower evolution, multiple gluon emissions become resolvable at smaller scales. TeV $\rightarrow 1\,$GeV.
- Measure hadronic final states, long distance effects, $Q_0 \sim 1\,$GeV.

Parton showers

Quarks and gluons in final state, pointlike.

- Know short distance (short time) fluctuations from matrix element/Feynman diagrams: $Q \sim$ few GeV to O(TeV).
- Parton shower evolution, multiple gluon emissions become resolvable at smaller scales. TeV $\to 1\,\mathrm{GeV}$.
- Measure hadronic final states, long distance effects, $Q_0 \sim 1\,\mathrm{GeV}$.

Dominated by large logs, terms

$$\alpha_S^n \log^{2n} \frac{Q}{Q_0} \sim 1\,.$$

Generated from emissions *ordered* in Q.

Quarks and gluons in final state, pointlike.

- Know short distance (short time) fluctuations from matrix element/Feynman diagrams: $Q \sim$ few GeV to O(TeV).
- Parton shower evolution, multiple gluon emissions become resolvable at smaller scales. TeV $\to$ 1 GeV.
- Measure hadronic final states, long distance effects, $Q_0 \sim 1\,\mathrm{GeV}$.

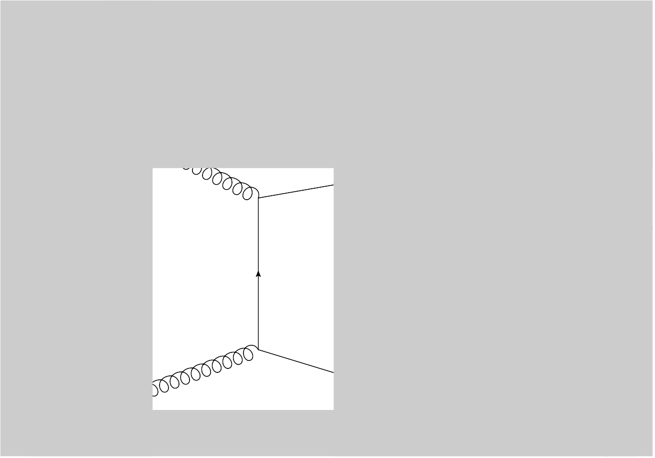
Dominated by large logs, terms

$$\alpha_S^n \log^{2n} \frac{Q}{Q_0} \sim 1\,.$$

Generated from emissions *ordered* in Q.
Soft and/or collinear emissions.

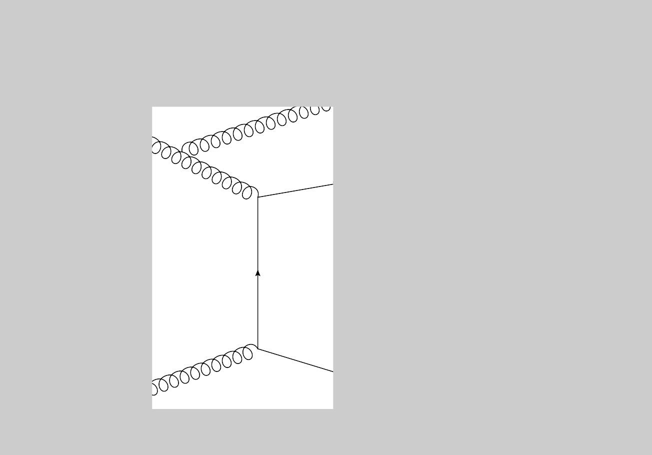
ME approximated by parton cascade

Evolution in scale, typically $Q \sim 1\,\mathrm{TeV}$ down to $Q \sim 1\,\mathrm{GeV}$.



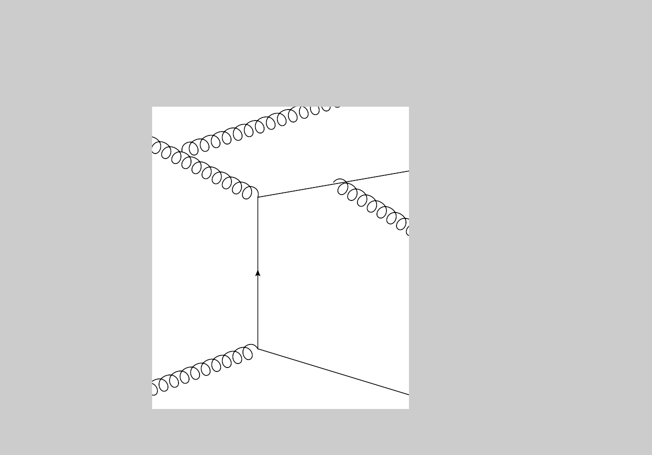
ME approximated by parton cascade

Evolution in scale, typically $Q \sim 1\,\mathrm{TeV}$ down to $Q \sim 1\,\mathrm{GeV}$.



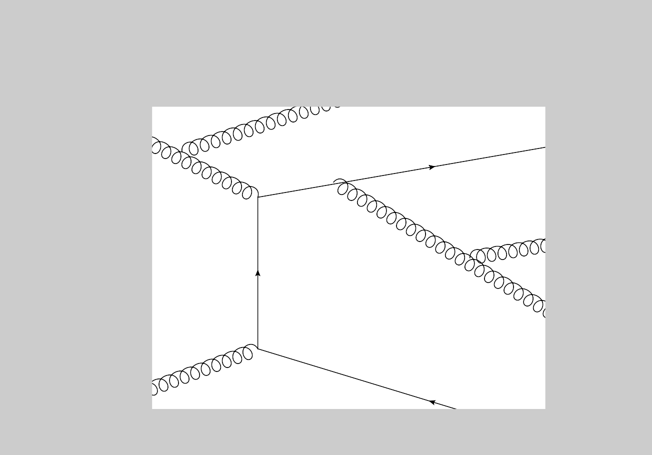
ME approximated by parton cascade

Evolution in scale, typically $Q \sim 1\,\mathrm{TeV}$ down to $Q \sim 1\,\mathrm{GeV}$.



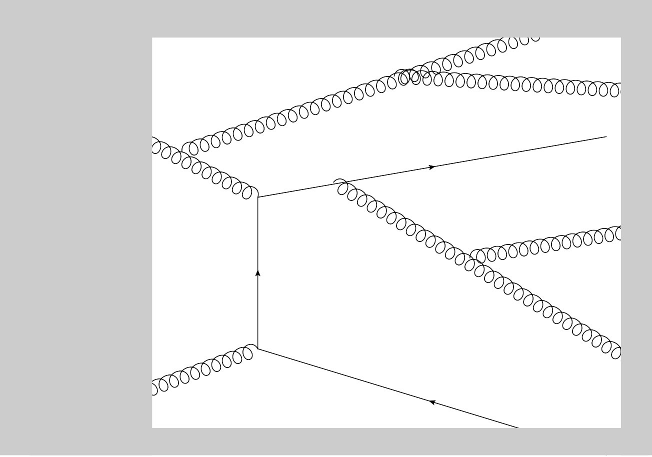
ME approximated by parton cascade

Evolution in scale, typically $Q \sim 1\,\text{TeV}$ down to $Q \sim 1\,\text{GeV}$.



ME approximated by parton cascade

Evolution in scale, typically $Q \sim 1\,\text{TeV}$ down to $Q \sim 1\,\text{GeV}$.



e^+e^- annihilation

Good starting point: $e^+e^- \to q\bar{q}g$:

Final state momenta in one plane (orientation usually averaged).
Write momenta in terms of

$$x_i = \frac{2p_i \cdot q}{Q^2} \quad (i = 1,2,3) \ ,$$

$$0 \le x_i \le 1 \ , x_1 + x_2 + x_3 = 2 \ ,$$

$$q = (Q,0,0,0) \ ,$$

$$Q \equiv E_{cm} \ .$$

$(x_1, x_2) = (x_q, x_{\bar{q}})$ –plane:

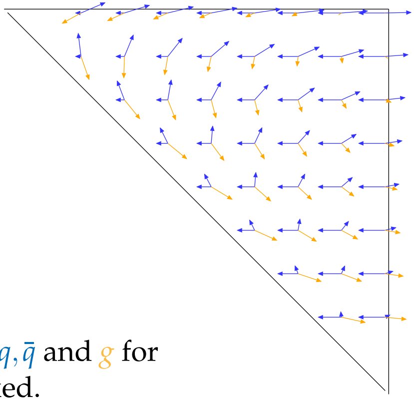


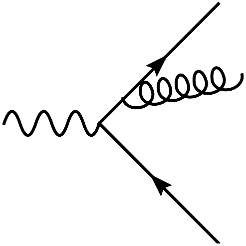
Fig: momentum configuration of $q, \bar{q}$ and g for given point (x_1, x_2), $\bar{q}$ direction fixed.

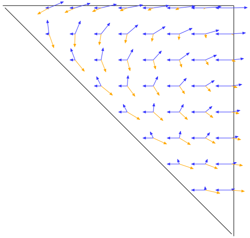
e^+e^- annihilation

Differential cross section:

$$\frac{\mathrm{d}\sigma}{\mathrm{d}x_1\mathrm{d}x_2} = \sigma_0 \frac{C_F\alpha_S}{2\pi} \frac{x_1 + x_2}{(1-x_1)(1-x_2)}$$

Collinear singularities: $x_1 \to 1$ or $x_2 \to 1$. Soft singularity: $x_1, x_2 \to 1$.

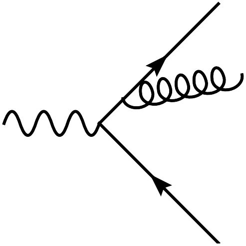




e^+e^- annihilation

Differential cross section:

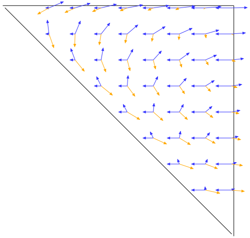
$$\frac{d\sigma}{dx_1 dx_2} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \frac{x_1 + x_2}{(1-x_1)(1-x_2)}$$



Collinear singularities: $x_1 \to 1$ or $x_2 \to 1$. Soft singularity: $x_1, x_2 \to 1$.

Rewrite in terms of x_3 and $\theta = \angle(q, g)$:

$$\frac{d\sigma}{d\cos\theta \, dx_3} = \sigma_0 \frac{C_F \alpha_S}{2\pi} \left[\frac{2}{\sin^2\theta} \frac{1 + (1-x_3)^2}{x_3} - x_3 \right]$$



Singular as $\theta \to 0$ and $x_3 \to 0$.

e^+e^- annihilation

Can separate into two jets as

$$\begin{aligned}\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} &= \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\theta}{1+\cos\theta}\\ &= \frac{\mathrm{d}\cos\theta}{1-\cos\theta}+\frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}}\\ &\approx \frac{\mathrm{d}\theta^2}{\theta^2}+\frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}\end{aligned}$$

e^+e^- annihilation

Can separate into two jets as

$$\begin{aligned} \frac{2\mathrm{d}\cos\theta}{\sin^2\theta} &= \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta} \\ &= \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar\theta}{1-\cos\bar\theta} \\ &\approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar\theta^2}{\bar\theta^2} \end{aligned}$$

So, we rewrite $\mathrm{d}\sigma$ in collinear limit as

$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z^2} \mathrm{d}z$$

Can separate into two jets as

$$\frac{2\mathrm{d}\cos\theta}{\sin^2\theta} = \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\theta}{1+\cos\theta}$$
$$= \frac{\mathrm{d}\cos\theta}{1-\cos\theta} + \frac{\mathrm{d}\cos\bar{\theta}}{1-\cos\bar{\theta}}$$
$$\approx \frac{\mathrm{d}\theta^2}{\theta^2} + \frac{\mathrm{d}\bar{\theta}^2}{\bar{\theta}^2}$$

So, we rewrite $\mathrm{d}\sigma$ in collinear limit as

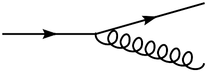
$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} C_F \frac{1+(1-z)^2}{z^2} \mathrm{d}z$$
$$= \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) \mathrm{d}z$$

with DGLAP splitting function $P(z)$.

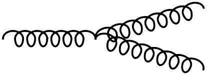
Collinear limit

Universal DGLAP splitting kernels for collinear limit:

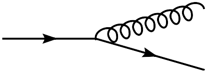
$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z)\mathrm{d}z$$



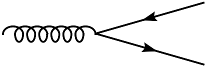
$$P_{q\to qg}(z) = C_F \frac{1+z^2}{1-z}$$



$$P_{g\to gg}(z) = C_A \frac{(1-z(1-z))^2}{z(1-z)}$$



$$P_{q\to gq}(z) = C_F \frac{1+(1-z)^2}{z}$$



$$P_{g\to qq}(z) = T_R(1-2z(1-z))$$

Collinear limit

Universal DGLAP splitting kernels for collinear limit:

$$d\sigma = \sigma_0 \sum_{\text{jets}} \frac{d\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z) dz$$

Note: Other variables may equally well characterize the collinear limit:

$$\frac{d\theta^2}{\theta^2} \sim \frac{dQ^2}{Q^2} \sim \frac{dp_\perp^2}{p_\perp^2} \sim \frac{d\tilde{q}^2}{\tilde{q}^2} \sim \frac{dt}{t}$$

whenever $Q^2, p_\perp^2, t \to 0$ means "collinear".

Collinear limit

Universal DGLAP splitting kernels for collinear limit:

$$\mathrm{d}\sigma = \sigma_0 \sum_{\text{jets}} \frac{\mathrm{d}\theta^2}{\theta^2} \frac{\alpha_S}{2\pi} P(z)\mathrm{d}z$$

Note: Other variables may equally well characterize the colline-ar limit:

$$\frac{\mathrm{d}\theta^2}{\theta^2} \sim \frac{\mathrm{d}Q^2}{Q^2} \sim \frac{\mathrm{d}p_\perp^2}{p_\perp^2} \sim \frac{\mathrm{d}\tilde{q}^2}{\tilde{q}^2} \sim \frac{\mathrm{d}t}{t}$$

whenever $Q^2, p_\perp^2, t \to 0$ means "collinear".

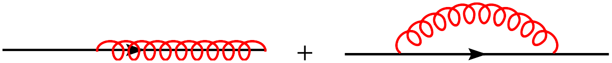
- θ: HERWIG
- Q^2: PYTHIA ≤ 6.3, SHERPA.
- $p_\perp$: PYTHIA ≥ 6.4, ARIADNE, Catani–Seymour showers.
- $\tilde{q}$: Herwig++.

Resolution

Need to introduce resolution t_0, e.g. a cutoff in $p_\perp$. Prevent us from the singularity at $\theta \to 0$.

Emissions below t_0 are unresolvable.

Finite result due to virtual corrections:



= finite.

unresolvable + virtual emissions are included in Sudakov form factor via unitarity (see below!).

Towards multiple emissions

Starting point: factorisation in collinear limit, single emission.

$$\sigma_{2+1}(t_0) = \sigma_2(t_0) \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \int_{z_-}^{z_+} \mathrm{d}z \, \frac{\alpha_S}{2\pi} \hat{P}(z) \; = \sigma_2(t_0) \int_{t_0}^{t} \mathrm{d}t \, W(t) \; .$$

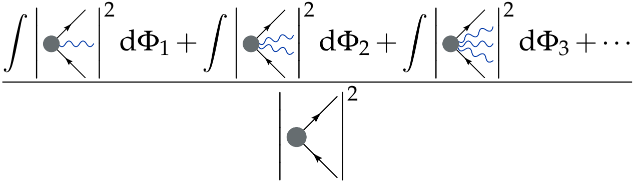
Towards multiple emissions

Starting point: factorisation in collinear limit, single emission.

$$\sigma_{2+1}(t_0) = \sigma_2(t_0) \int_{t_0}^{t} \frac{\mathrm{d}t'}{t'} \int_{z_-}^{z_+} \mathrm{d}z \, \frac{\alpha_S}{2\pi} \hat{P}(z) \; = \sigma_2(t_0) \int_{t_0}^{t} \mathrm{d}t \, W(t) \; .$$

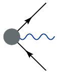
Simple example:
Multiple photon emissions, strongly ordered in t.
We want

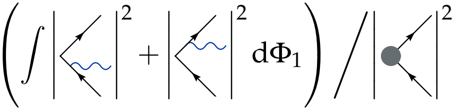
$$W_{\rm sum} = \sum_{n=1} W_{2+n} =$$



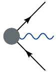
for any number of emissions.

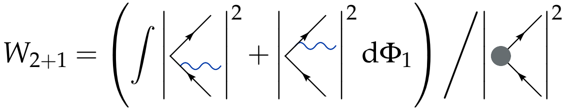
Towards multiple emissions

$(n=1)$ 

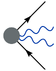
$W_{2+1} =$  $= \frac{2}{1!}\int_{t_0}^{t} \mathrm{d}t\, W(t)\ .$

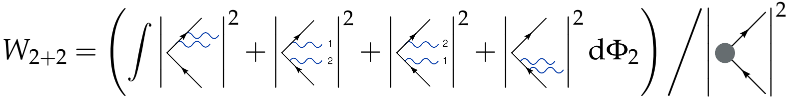
Towards multiple emissions

$(n=1)$ 



$$= \frac{2}{1!}\int_{t_0}^{t} \mathrm{d}t\, W(t)\ .$$

$(n=2)$ 

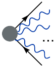


$$= 2^2 \int_{t_0}^{t} \mathrm{d}t' \int_{t_0}^{t'} \mathrm{d}t''\, W(t')W(t'') = \frac{2^2}{2!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^2\ .$$

We used

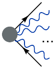
$$\int_{t_0}^{t} \mathrm{d}t_1 \ldots \int_{t_0}^{t_{n-1}} \mathrm{d}t_n\ W(t_1)\ldots W(t_n) = \frac{1}{n!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^n\ .$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

$$W_{2+n} = \frac{2^n}{n!} \left(\int_{t_0}^{t} \mathrm{d}t \, W(t) \right)^n$$

Towards multiple emissions

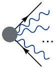
Easily generalized to n emissions  by induction. *i.e.*

$$W_{2+n} = \frac{2^n}{n!}\left(\int_{t_0}^{t} \mathrm{d}t\, W(t)\right)^n$$

So, in total we get

$$\sigma_{>2}(t_0) = \sigma_2(t_0)\sum_{k=1}^{\infty}\frac{2^k}{k!}\left(\int_{t_0}^{t}\mathrm{d}t\, W(t)\right)^k = \sigma_2(t_0)\left(\mathrm{e}^{2\int_{t_0}^{t}\mathrm{d}t\, W(t)} - 1\right)$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

$$W_{2+n} = \frac{2^n}{n!} \left(\int_{t_0}^{t} \mathrm{d}t\, W(t) \right)^n$$

So, in total we get

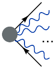
$$\sigma_{>2}(t_0) = \sigma_2(t_0) \sum_{k=1}^{\infty} \frac{2^k}{k!} \left(\int_{t_0}^{t} \mathrm{d}t\, W(t) \right)^k = \sigma_2(t_0) \left(\mathrm{e}^{2 \int_{t_0}^{t} \mathrm{d}t\, W(t)} - 1 \right)$$

$$= \sigma_2(t_0) \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right)$$

Sudakov Form Factor

$$\Delta(t_0, t) = \exp\left[-\int_{t_0}^{t} \mathrm{d}t\, W(t) \right]$$

Towards multiple emissions

Easily generalized to n emissions  by induction. *i.e.*

$$W_{2+n} = \frac{2^n}{n!} \left(\int_{t_0}^{t} \mathrm{d}t \, W(t) \right)^n$$

So, in total we get

$$\sigma_{>2}(t_0) = \sigma_2(t_0) \sum_{k=1}^{\infty} \frac{2^k}{k!} \left(\int_{t_0}^{t} \mathrm{d}t \, W(t) \right)^k = \sigma_2(t_0) \left(\mathrm{e}^{2 \int_{t_0}^{t} \mathrm{d}t \, W(t)} - 1 \right)$$

$$= \sigma_2(t_0) \left(\frac{1}{\Delta^2(t_0, t)} - 1 \right)$$

Sudakov Form Factor in QCD

$$\Delta(t_0, t) = \exp\left[- \int_{t_0}^{t} \mathrm{d}t \, W(t) \right] = \exp\left[- \int_{t_0}^{t} \frac{\mathrm{d}t}{t} \int_{z_-}^{z_+} \frac{\alpha_S(z,t)}{2\pi} \hat{P}(z,t) \mathrm{d}z \right]$$

Sudakov form factor

Note that

$$\sigma_{\rm all} = \sigma_2 + \sigma_{>2} = \sigma_2 + \sigma_2 \left(\frac{1}{\Delta^2(t_0,t)} - 1 \right) ,$$
$$\Rightarrow \Delta^2(t_0,t) = \frac{\sigma_2}{\sigma_{\rm all}} .$$

$$\text{Two jet rate} = \Delta^2 = P^2(\text{No emission in the range } t \to t_0) .$$

$$\text{Sudakov form factor} = \text{No emission probability} .$$

Often $\Delta(t_0,t) \equiv \Delta(t)$.

- Hard scale t, typically CM energy or $p_\perp$ of hard process.
- Resolution t_0, two partons are resolved as two entities if inv mass or relative $p_\perp$ above t_0.
- P^2 (not P), as we have two legs that evolve independently.

Sudakov form factor from Markov property

Unitarity

$$P(\text{“some emission”}) + P(\text{“no emission”}) = P(0 < t \leq T) + \bar{P}(0 < t \leq T) = 1 \,.$$

Multiplication law (no memory)

$$\bar{P}(0 < t \leq T) = \bar{P}(0 < t \leq t_1)\bar{P}(t_1 < t \leq T)$$

Sudakov form factor from Markov property

Unitarity

$$P(\text{"some emission"}) + P(\text{"no emission"}) \\ = P(0 < t \leq T) + \bar{P}(0 < t \leq T) = 1 \; .$$

Multiplication law (no memory)

$$\bar{P}(0 < t \leq T) = \bar{P}(0 < t \leq t_1)\bar{P}(t_1 < t \leq T)$$

Then subdivide into n pieces: $t_i = \frac{i}{n}T, 0 \leq i \leq n$.

$$\begin{aligned}\bar{P}(0 < t \leq T) &= \lim_{n\to\infty} \prod_{i=0}^{n-1} \bar{P}(t_i < t \leq t_{i+1}) = \lim_{n\to\infty} \prod_{i=0}^{n-1} \big(1 - P(t_i < t \leq t_{i+1})\big) \\ &= \exp\left(-\lim_{n\to\infty} \sum_{i=0}^{n-1} P(t_i < t \leq t_{i+1})\right) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right) \; .\end{aligned}$$

Sudakov form factor

Again, no–emission probability!

$$\bar{P}(0 < t \leq T) = \exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)$$

So,

$$\begin{aligned}\mathrm{d}P(\text{first emission at } T) &= \mathrm{d}P(T)\bar{P}(0 < t \leq T) \\ &= \mathrm{d}P(T)\exp\left(-\int_0^T \frac{\mathrm{d}P(t)}{\mathrm{d}t}\mathrm{d}t\right)\end{aligned}$$

That's what we need for our parton shower! Probability density for next emission at t:

$$\mathrm{d}P(\text{next emission at } t) =$$
$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Parton shower Monte Carlo

Probability density:

$$
\mathrm{d}P(\text{next emission at } t) =
\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]
$$

Conveniently, the probability distribution is $\Delta(t)$ itself.

Parton shower Monte Carlo

Probability density:

$$\mathrm{d}P(\text{next emission at } t) =$$

$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\ \exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Conveniently, the probability distribution is $\Delta(t)$ itself.
Hence, parton shower very roughly from (HERWIG):

1. Choose flat random number $0 \le \rho \le 1$.
2. If $\rho < \Delta(t_{\mathrm{max}})$: no resolbable emission, stop this branch.
3. Else solve $\rho = \Delta(t_{\mathrm{max}})/\Delta(t)$
   (= no emission between t_{max} and t) for t.
   Reset $t_{\mathrm{max}} = t$ and goto 1.

Determine z essentially according to integrand in front of exp.

Parton shower Monte Carlo

Probability density:

$$\mathrm{d}P(\text{next emission at } t) =$$
$$\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\,\exp\left[-\int_{t_0}^{t}\frac{\mathrm{d}t}{t}\int_{z_-}^{z_+}\frac{\alpha_S(z,t)}{2\pi}\hat{P}(z,t)\mathrm{d}z\right]$$

Conveniently, the probability distribution is $\Delta(t)$ itself.

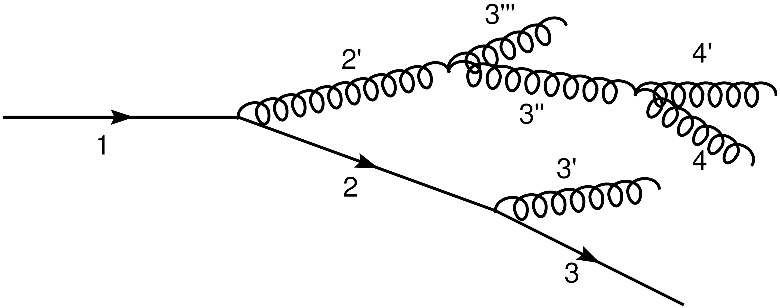
- That was old HERWIG variant. Relies on (numerical) integration/tabulation for $\Delta(t)$.
- Pythia, now also Herwig++, use the Veto Algorithm.
- Method to sample x from distribution of the type

$$\mathrm{d}P = F(x)\exp\left[-\int^{x}\mathrm{d}x'\,F(x')\right]\mathrm{d}x\ .$$

Simpler, more flexible, but slightly slower.

Parton cascade

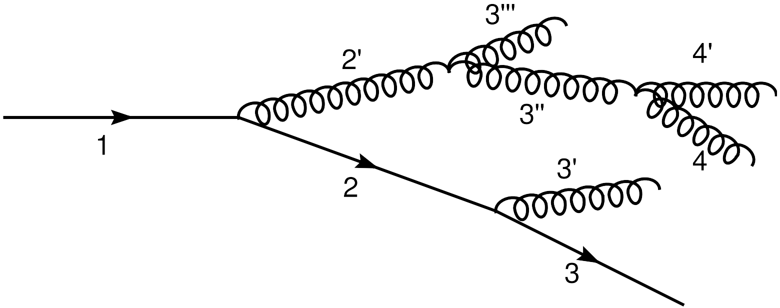
Get tree structure, ordered in evolution variable t:



Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

Parton cascade

Get tree structure, ordered in evolution variable t:

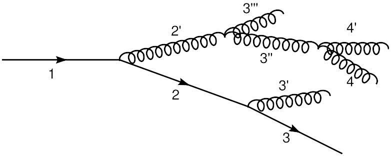


Here: $t_1 > t_2 > t_3$; $t_2 > t_{3'}$ etc.
Construct four momenta from (t_i, z_i) and (random) azimuth ϕ.

Not at all unique!
Many (more or less clever) choices still to be made.

Parton cascade

Get tree structure, ordered in evolution variable t:



- t can be θ, Q^2, $p_\perp$, …
- Choice of hard scale $t_{\max}$ not fixed. "Some hard scale".
- z can be light cone momentum fraction, energy fraction, …
- Available parton shower phase space.
- Integration limits.
- Regularisation of soft singularities.
- …

Good choices needed here to describe wealth of data!

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emissions

- Only *collinear* emissions so far.
- Including *collinear+soft*.
- *Large angle+soft* also important.

Soft emission: consider *eikonal factors*,
here for $q(p+q) \to q(p)g(q)$, soft g:

$$u(p)\ \not{\varepsilon}\frac{\not{p}+\not{q}+m}{(p+q)^2-m^2} \longrightarrow u(p)\frac{p\cdot\varepsilon}{p\cdot q}$$

soft factorisation. Universal, *i.e.* independent of emitter.
In general:

$$d\sigma_{n+1} = d\sigma_n \frac{d\omega}{\omega}\frac{d\Omega}{2\pi}\frac{\alpha_S}{2\pi}\sum_{ij} C_{ij}W_{ij} \quad \text{(“QCD–Antenna”)}$$

with

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})}\ .$$

Soft emissions

We define

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})} \equiv W_{ij}^{(i)} + W_{ij}^{(j)}$$

with

$$W_{ij}^{(i)} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{qj}}\right) .$$

$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .

Soft emissions

We define

$$W_{ij} = \frac{1-\cos\theta_{ij}}{(1-\cos\theta_{iq})(1-\cos\theta_{qj})} \equiv W_{ij}^{(i)} + W_{ij}^{(j)}$$

with

$$W_{ij}^{(i)} = \frac{1}{2}\left(W_{ij} + \frac{1}{1-\cos\theta_{iq}} - \frac{1}{1-\cos\theta_{qj}}\right) .$$

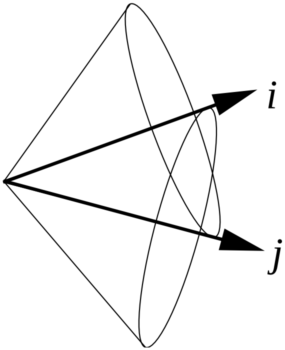
$W_{ij}^{(i)}$ is only collinear divergent if $q \| i$ etc .
After integrating out the azimuthal angles, we find

$$\int \frac{d\phi_{iq}}{2\pi} W_{ij}^{(i)} = \begin{cases} \dfrac{1}{1-\cos\theta_{iq}} & (\theta_{iq} < \theta_{ij}) \\ 0 & \text{otherwise} \end{cases}$$

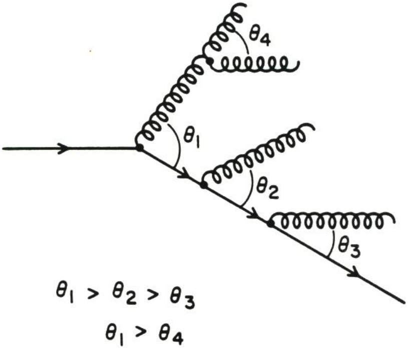
That's angular ordering.

Angular ordering

Radiation from parton *i* is bound to a cone, given by the colour partner parton *j*.



Results in angular ordered parton shower and suppresses soft gluons viz. hadrons in a jet.



Colour coherence from CDF

Events with 2 hard ($> 100\,\mathrm{GeV}$) jets and a soft 3rd jet ($\sim 10\,\mathrm{GeV}$)

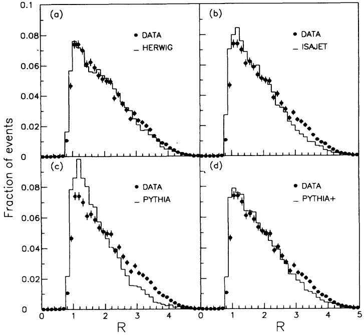


FIG. 14. Observed R distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

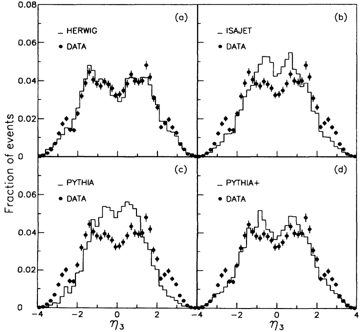


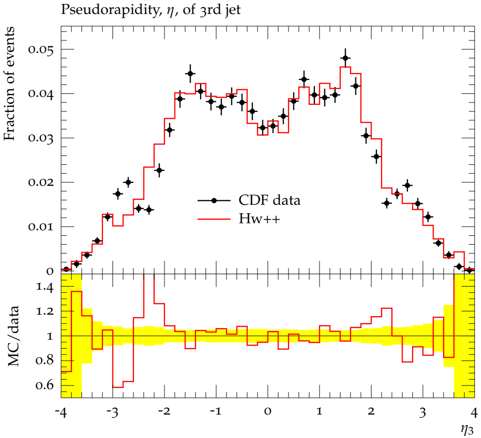
FIG. 13. Observed η_3 distribution compared to the predictions of (a) HERWIG; (b) ISAJET; (c) PYTHIA; (d) PYTHIA+.

F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

Colour coherence from CDF

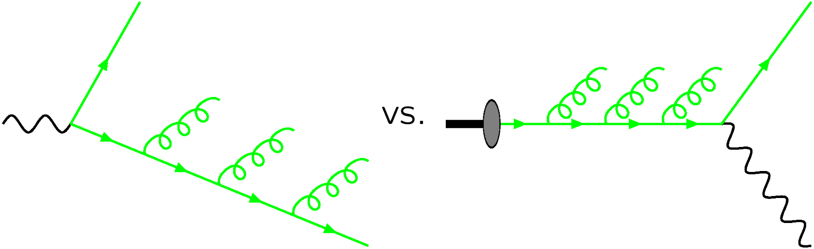
Events with 2 hard ($> 100\,\text{GeV}$) jets and a soft 3rd jet ($\sim 10\,\text{GeV}$)



F. Abe *et al.* [CDF Collaboration], Phys. Rev. D **50** (1994) 5562.

Best description with angular ordering.

Initial state radiation



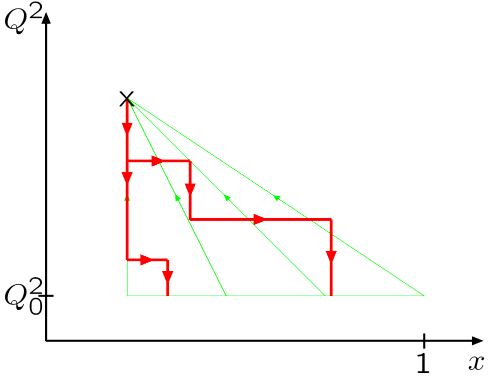
Similar to final state radiation. Sudakov form factor ($x' = x/z$)

$$\Delta(t, t_{\max}) = \exp\left[-\sum_b \int_t^{t_{\max}} \frac{\mathrm{d}t}{t} \int_{z_-}^{z_+} \mathrm{d}z \frac{\alpha_S(z,t)}{2\pi} \frac{x' f_b(x',t)}{x f_a(x,t)} \hat{P}_{ba}(z,t)\right]$$

Have to divide out the pdfs.

Initial state radiation

Evolve backwards from hard scale Q^2 *down* towards cutoff scale Q_0^2. Thereby increase x.



With parton shower we *undo* the DGLAP evolution of the pdfs.

Reconstruction of Kinematics

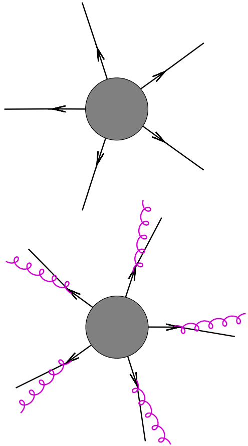
After shower: original partons acquire virtualities q_i^2
$\rightarrow$ boost/rescale jets:
Started with

$$\sqrt{s} = \sum_{i=1}^{n} \sqrt{m_i^2 + \vec{p}_i^{\,2}}$$

we *rescale* momenta with common factor k,

$$\sqrt{s} = \sum_{i=1}^{n} \sqrt{q_i^2 + k\vec{p}_i^{\,2}}$$

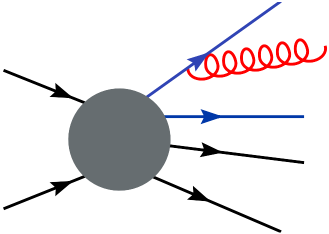
to preserve overall energy/momentum.
$\rightarrow$ resulting jets are boosted accordingly.



Dipoles

Exact kinematics when recoil is taken by spectator(s).

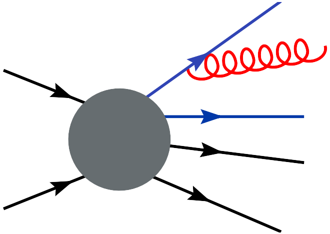
- Dipole showers.
- Ariadne.
- Recoils in Pythia.



Dipoles

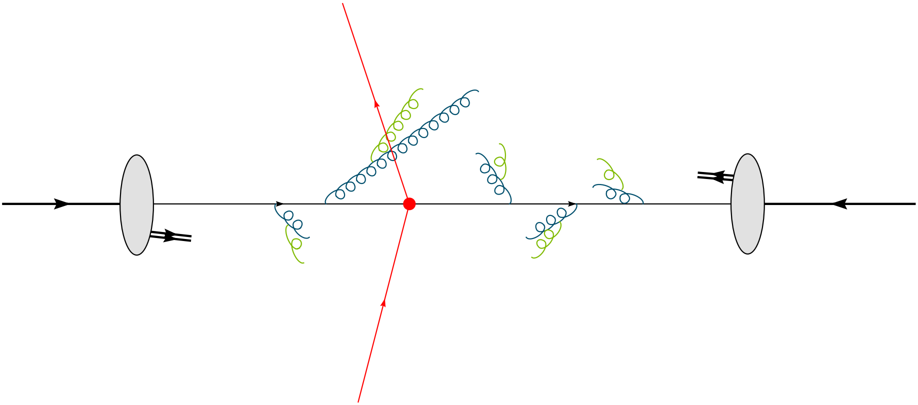
Exact kinematics when recoil is taken by spectator(s).

- Dipole showers.
- Ariadne.
- Recoils in Pythia.
- New dipole showers, based on
  - Catani Seymour dipoles.
  - QCD Antennae.
  - Goal: matching with NLO.
- Generalized to IS–IS, IS–FS.

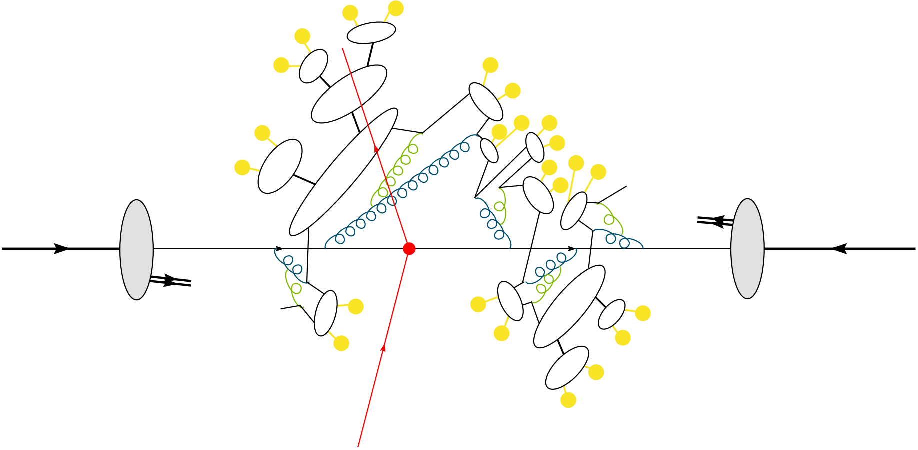


Hadronization

Parton shower



Parton shower ⟶ hadrons

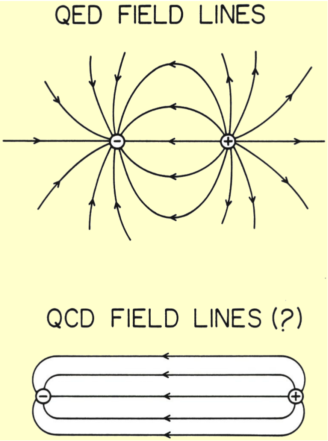


Parton shower $\longrightarrow$ hadrons

- Parton shower terminated at t_0 = lower end of PT.
- Can't measure quarks and gluons.
- Degrees of freedom in the detector are hadrons.
- Need a description of confinement.

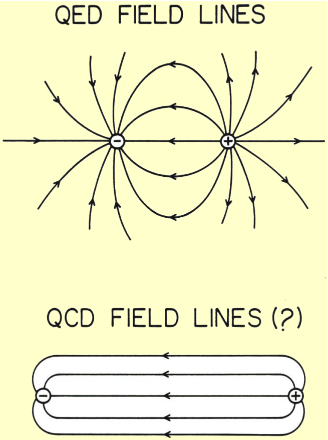
Physical input

Self coupling of gluons
$\leftrightarrow$ "attractive field lines"

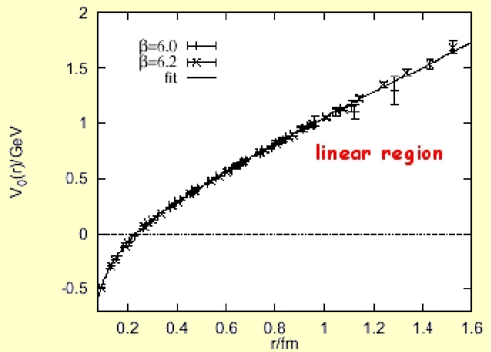


Physical input

Self coupling of gluons
$\leftrightarrow$ "attractive field lines"



Linear static potential $V(r) \approx \kappa r$.



Supported by lattice QCD,
hadron spectroscopy.

Hadronization models

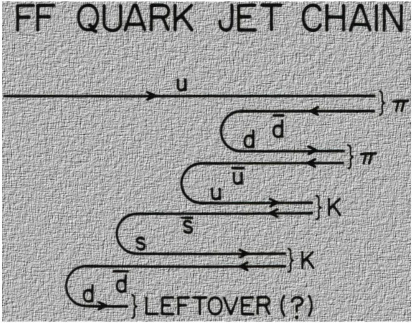
Older models:

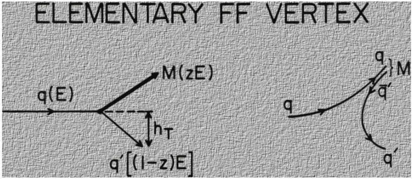
- Flux tube model.
- Independent fragmentation.

Today's models.

- Lund string model (Pythia).
- Cluster model (Herwig).

Independent fragmentation

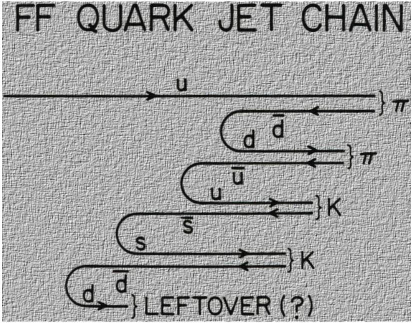


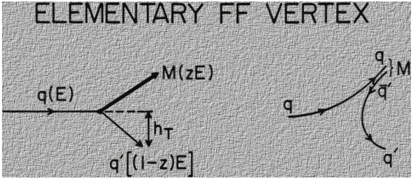


Feynman–Field fragmentation ('78).

- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q\to h}(z)$ = density of momentum fraction z carried away by hadron h from quark q.
- Gaussian $p_\perp$ distribution.

Independent fragmentation



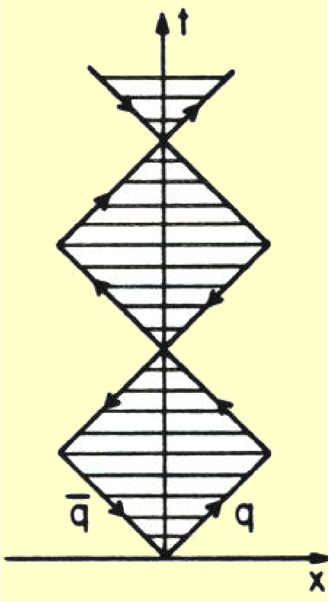


Feynman–Field fragmentation ('78).

- $q\bar{q}$ pairs created from vacuum to dress bare quarks.
- Fragmentation function $f_{q\to h}(z)$ = density of momentum fraction z carried away by hadron h from quark q.
- Gaussian $p_\perp$ distribution.
- Problems:
  - "last quark".
  - not Lorentz invariant.
  - infrared safety.
  - …
- Good at that time.
- Still usefull for inclusive descriptions.

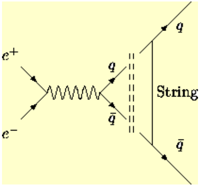
Lund string model

String model of mesons.
$L = 0$ mesons move in yoyo modes.
Area law: $m^2 \sim$ area.

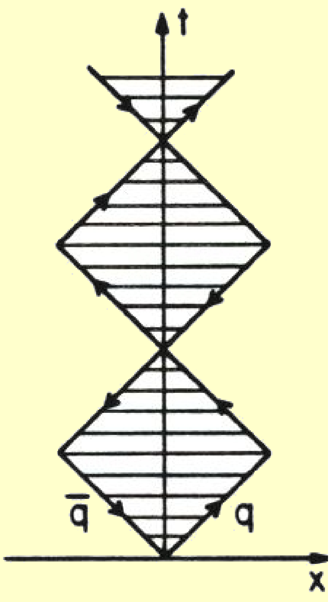


Lund string model

String model of mesons.
$L = 0$ mesons move in yoyo modes.
Area law: $m^2 \sim$ area.
Simple model for particle production in e^+e^- annihilation:



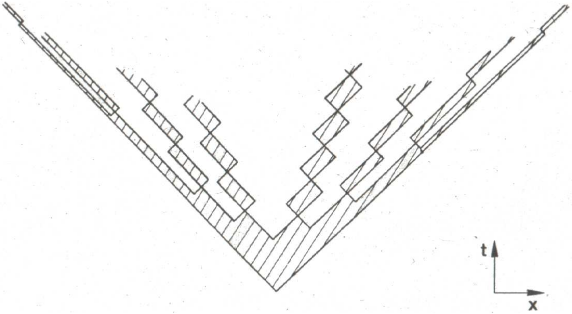
$q\bar{q}$ pair as pointlike source of string.



Lund string model

String energy $\sim$ intense chromomagnetic field.
$\longrightarrow$ Additional $q\bar{q}$ pairs created by QM tunneling.

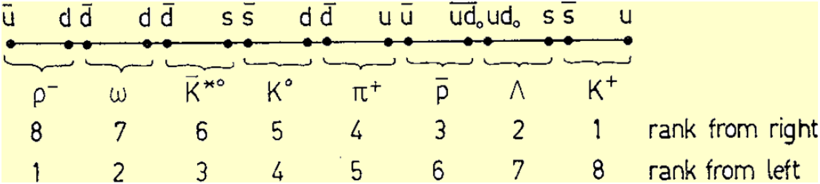
$$\frac{\mathrm{dProb}}{\mathrm{d}x\mathrm{d}t} \sim \exp\left(-\pi m_q^2/\kappa\right) \qquad \kappa \sim 1\,\mathrm{GeV}\,.$$



String breaking expected long before yoyo point.

Lund string model

Ajacent breaks form hadrons.



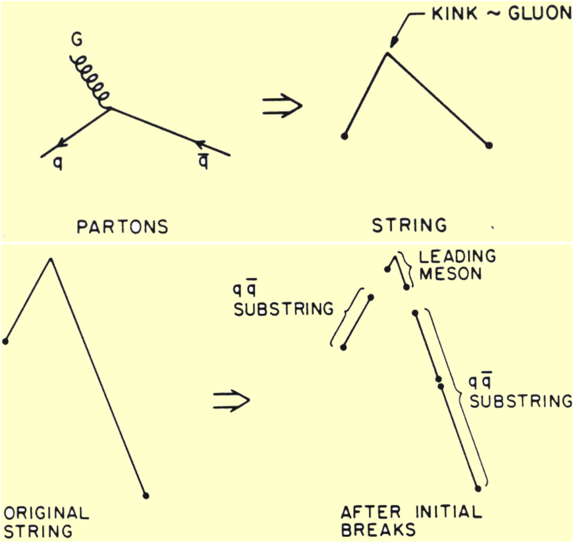
Works in both directions (symmetry).
Lund symmetric fragmentation function

$$f(z,p_\perp) \sim \frac{1}{z}(1-z)^a \exp\left(-\frac{b(m_h^2+p_\perp^2)}{z}\right)$$

a, b, m_h^2 main adjustable parameters.
Note: diquarks $\to$ baryons.

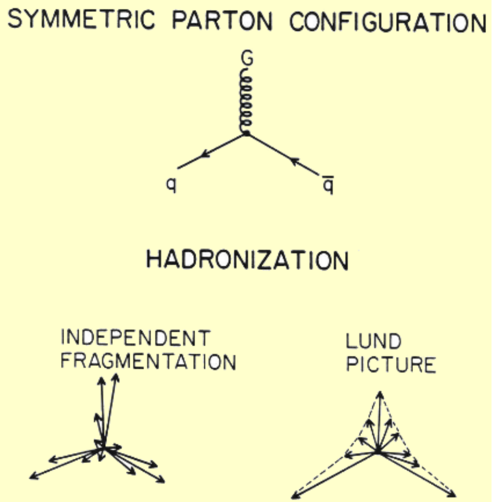
Lund string model

gluon = kink on string = motion pushed into the $q\bar{q}$ system.



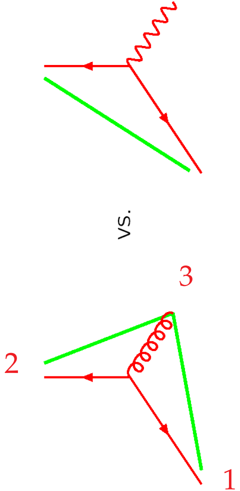
Lund string model

gluon = kink on string = motion pushed into the $q\bar{q}$ system.

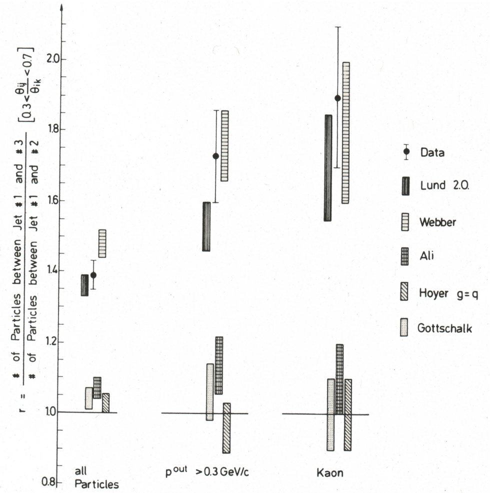


Lund string model

gluon = kink on string = motion pushed into the $q\bar{q}$ system.



"String effect"



Some remarks:

- Originally invented without parton showers in mind.

Lund string model

Some remarks:

- Originally invented without parton showers in mind.
- Stong physical motivation.
- Very successful desription of data.
- Universal desription of data
  (fit at e^+e^-, transfer to hadron-hadron).
- Many parameters, ~ 1 per hadron.
- Too easy to hide errors in perturbative description?

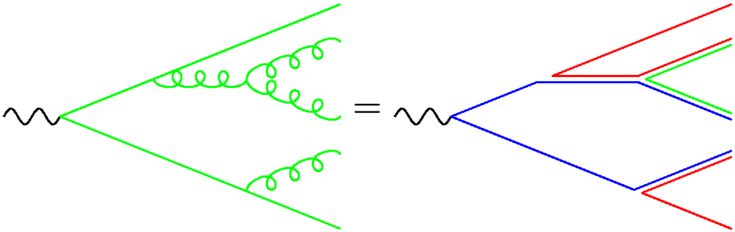
Some remarks:

- Originally invented without parton showers in mind.
- Stong physical motivation.
- Very successful desription of data.
- Universal desription of data
  (fit at e^+e^-, transfer to hadron-hadron).
- Many parameters, ~ 1 per hadron.
- Too easy to hide errors in perturbative description?

$\longrightarrow$ try to use more QCD information/intuition.

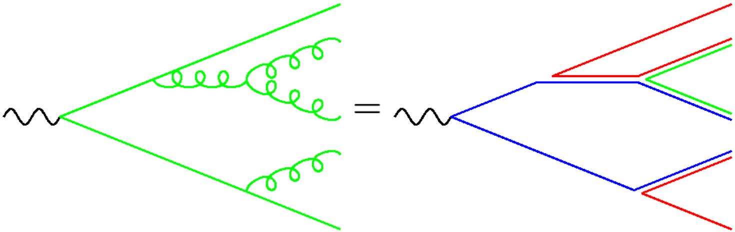
Colour preconfinement

Large N_C limit $\longrightarrow$ planar graphs dominate.
Gluon = colour — anticolourpair



Colour preconfinement

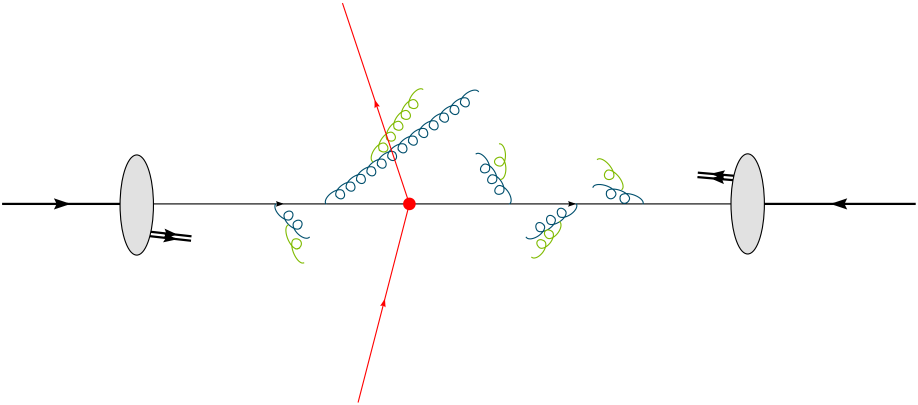
Large N_C limit $\longrightarrow$ planar graphs dominate.
Gluon = colour — anticolourpair



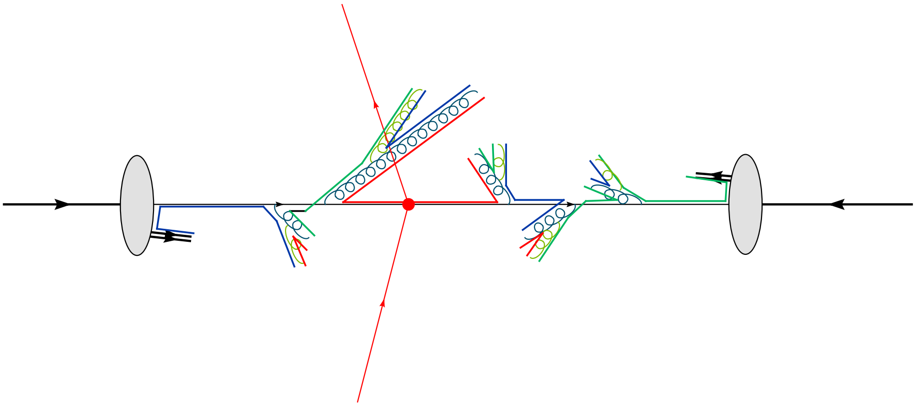
Parton shower organises partons in colour space. Colour partners (=colour singlet pairs) end up close in phase space.

$\longrightarrow$ Cluster hadronization model

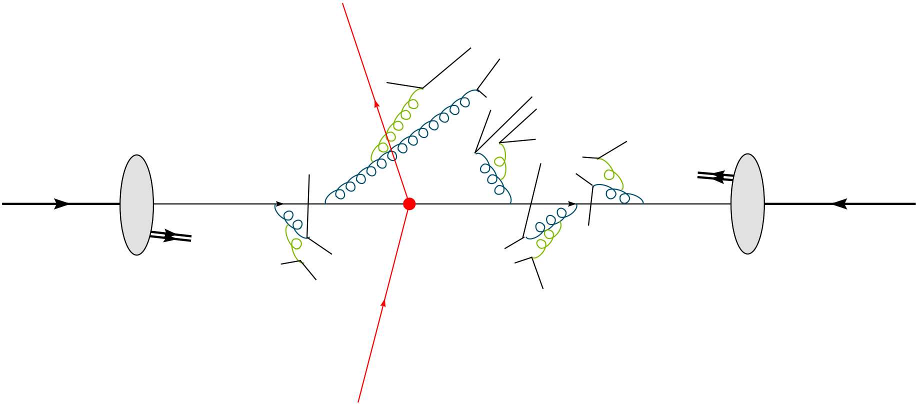
Cluster hadronization



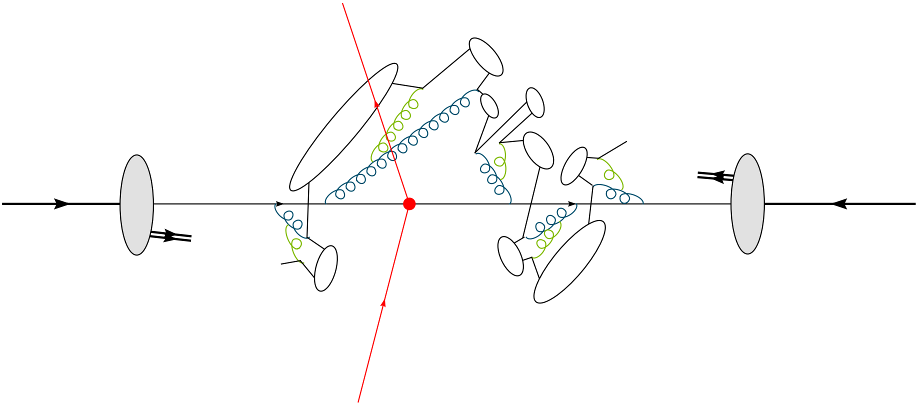
Cluster hadronization



Cluster hadronization

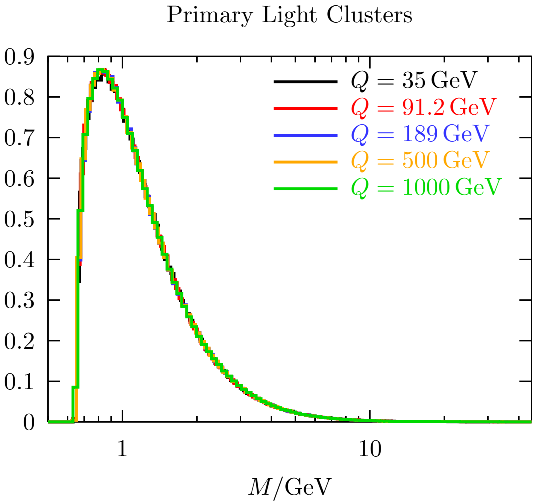


Cluster hadronization



Cluster hadronization

Primary cluster mass spectrum independent of production mechanism. Peaked at some low mass.



Cluster hadronization

Primary cluster mass spectrum independent of production mechanism. Peaked at some low mass.

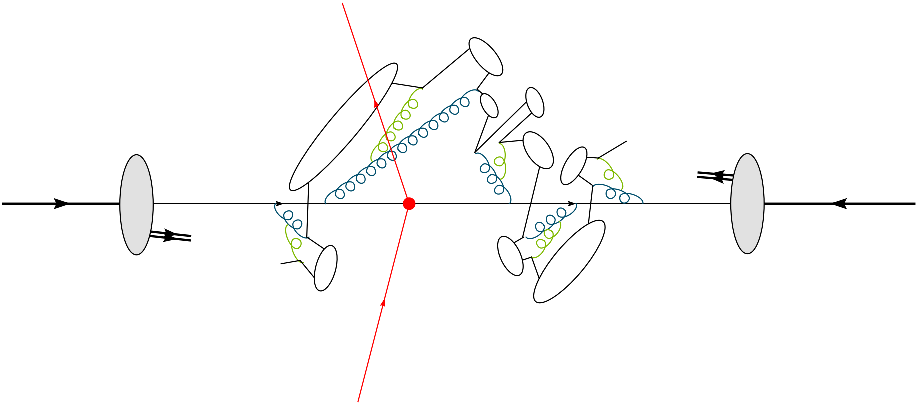
Cluster = continuum of high mass resonances.
Decay into well–known lighter mass resonances
= discrete spectrum of hadrons.

No spin information carried over, i.e. only phase space.

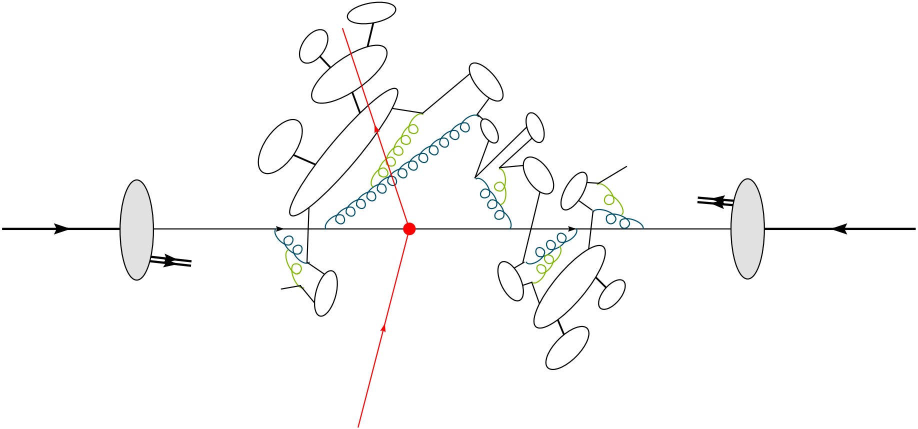
Suppression of heavier particles
(particularly baryons, can be problematic).

Cluster spectrum determined entirely by parton shower, i.e. perturbation theory. Hence, t_0 crucial parameter.

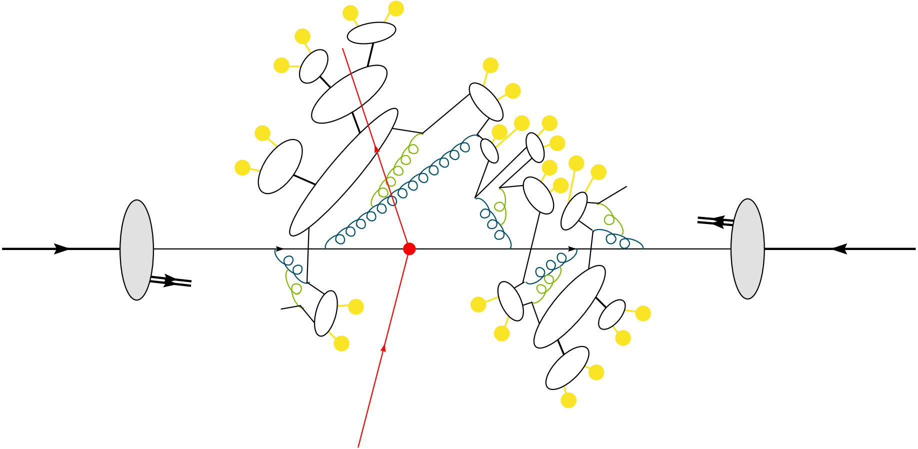
Cluster hadronization



Cluster hadronization



Cluster hadronization



Cluster hadronization in a nutshell

- Nonperturbative $g \to q\bar{q}$ splitting ($q = uds$) isotropically. Here, $m_g \approx 750\,\mathrm{MeV} > 2m_q$.
- Cluster formation, universal spectrum (see below)
- Cluster fission, until

  $$M^p < M^p_{\max} + (m_1 + m_2)^p$$

  where masses are chosen from

  $$M_i = \left[\left(M^P - (m_i + m_3)^P\right) r_i + (m_i + m_3)^P\right]^{1/P},$$

  with additional phase space contraints. Constituents keep moving in their original direction.
- Cluster Decay

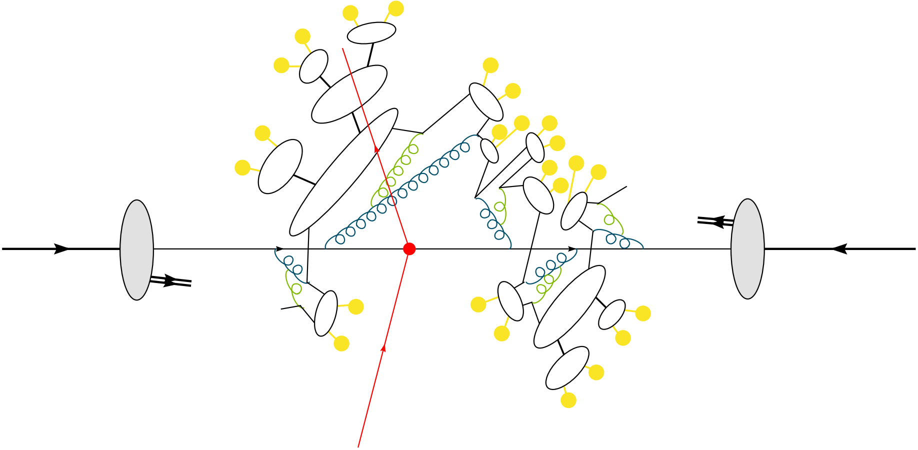
  $$P(a_{i,q}, b_{q,j}|i,j) = \frac{W(a_{i,q}, b_{q,j}|i,j)}{\sum_{M/B} W(c_{i,q'}, d_{q',j}|i,j)}.$$

Hadronization

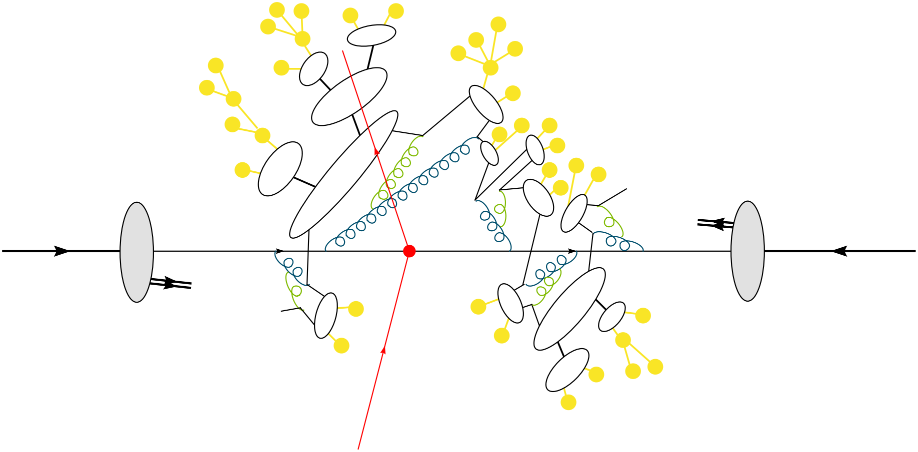
- Only string and cluster models used in recent MC programs.
  Independent fragmentation only for inclusive observables.
- Strings started non–perturbatively,
  improved by parton shower.
- Cluster model started mostly on perturbative side,
  improved by string like cluster fission.

Hadronic Decays

Hadronic decays



Hadronic decays



Hadronic decays

Many aspects:

$$B^{*0} \to \gamma B^0$$
$$\hookrightarrow \bar{B}^0$$
$$\hookrightarrow e^- \bar{\nu}_e D^{*+}$$
$$\hookrightarrow \pi^+ D^0$$
$$\hookrightarrow K^- \rho^+$$
$$\hookrightarrow \pi^+ \pi^0$$
$$\hookrightarrow e^+ e^- \gamma$$

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \quad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad\qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\qquad\qquad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad\qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\qquad\qquad\qquad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

EM decay.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \qquad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad\qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\qquad\qquad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad\qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\qquad\qquad\qquad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Weak mixing.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \qquad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad\qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\qquad\qquad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad\qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\qquad\qquad\qquad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Weak decay.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \to \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \qquad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad\qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\qquad\qquad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad\qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\qquad\qquad\qquad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Strong decay.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} \rightarrow \gamma B^0 & \\
& \hookrightarrow \bar{B}^0 \\
& \quad \hookrightarrow e^- \bar{\nu}_e D^{*+} \\
& \qquad \hookrightarrow \pi^+ D^0 \\
& \qquad\quad \hookrightarrow K^- \rho^+ \\
& \qquad\qquad \hookrightarrow \pi^+ \pi^0 \\
& \qquad\qquad\quad \hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Weak decay, ρ^+ mass smeared.

Hadronic decays

Many aspects:

$$
\begin{aligned}
B^{*0} &\to \gamma B^0 \\
&\quad\hookrightarrow \bar{B}^0 \\
&\quad\quad\hookrightarrow e^- \bar{\nu}_e D^{*+} \\
&\quad\quad\quad\hookrightarrow \pi^+ D^0 \\
&\quad\quad\quad\quad\hookrightarrow K^- \rho^+ \\
&\quad\quad\quad\quad\quad\hookrightarrow \pi^+ \pi^0 \\
&\quad\quad\quad\quad\quad\quad\hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

ρ^+ polarized, angular correlations.

Hadronic decays

Many aspects:

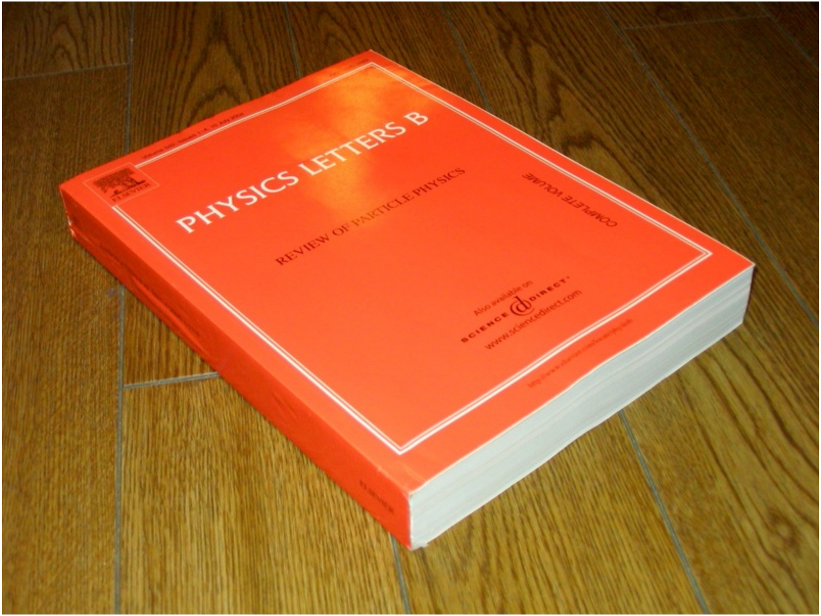
$$
\begin{aligned}
B^{*0} \to \gamma B^0 \\
\hookrightarrow \bar{B}^0 \\
\hookrightarrow e^- \bar{\nu}_e D^{*+} \\
\hookrightarrow \pi^+ D^0 \\
\hookrightarrow K^- \rho^+ \\
\hookrightarrow \pi^+ \pi^0 \\
\hookrightarrow e^+ e^- \gamma
\end{aligned}
$$

Dalitz decay, m_{ee} peaked.

Hadronic decays

Tedious.
100s of different particles, 1000s of decay modes, phenomenological matrix elements with parametrized form factors…

Hadronic decays



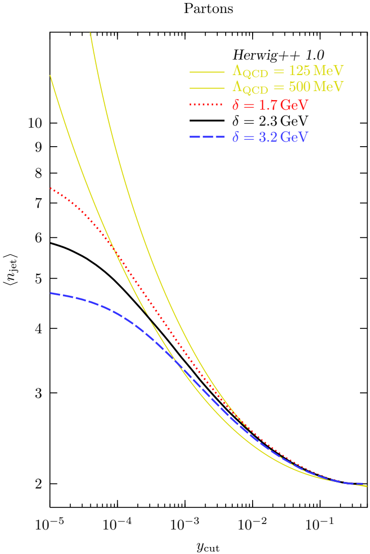
A few plots

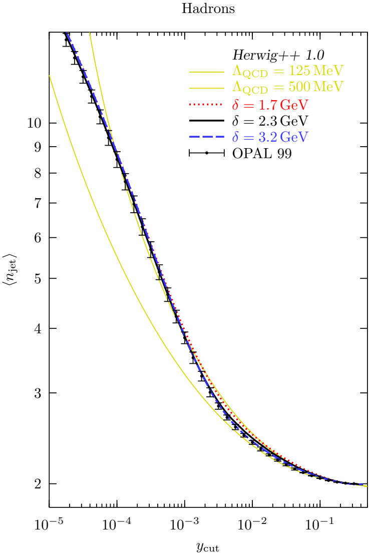
How well does it work?

- $e^+e^- \to$ hadrons, mostly at LEP.
- Jet shapes, jet rates, event shapes, identified particles…
- ‘Tuning’ of parameters.
- Want to get *everything* right with *one* parameter set.
- Compare to literally 100s of plots.

How well does it work?

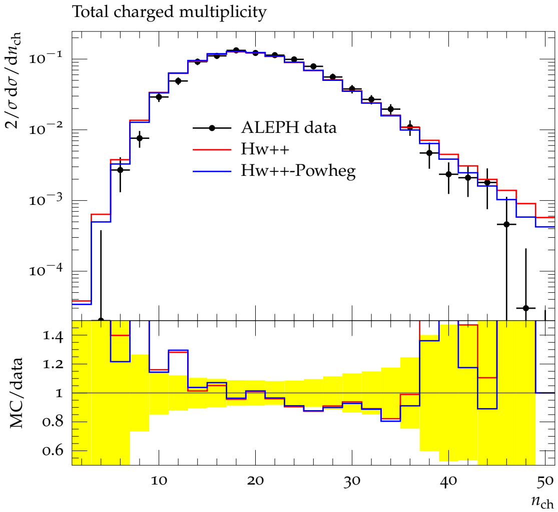
Smooth interplay between shower and hadronization.





How well does it work?

N_{ch} at LEP. Crucial for t_0 (Herwig++ 2.5.2)



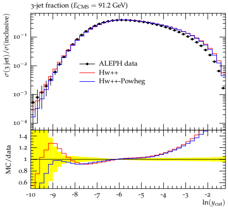
How well does it work?

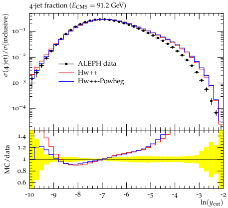
Jet rates at LEP.

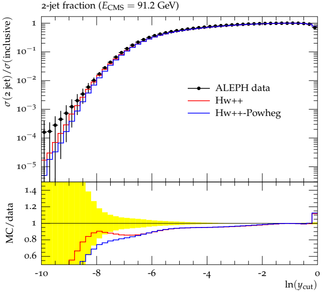
$$R_n = \sigma(n\text{–jets})/\sigma(\text{jets})$$

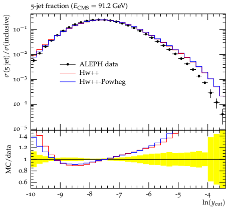
$$R_6 = \sigma(>5\text{–jets})/\sigma(\text{jets})$$

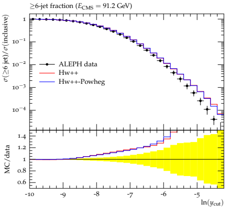
(Herwig++ 2.5.2)





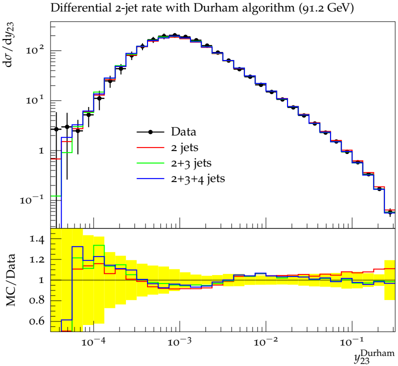


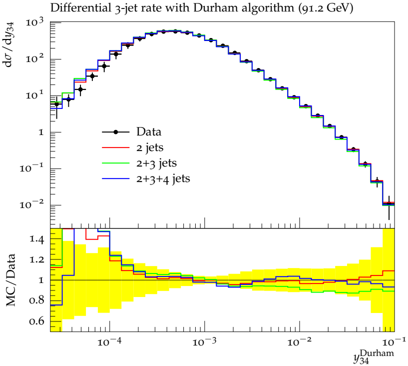




How well does it work?

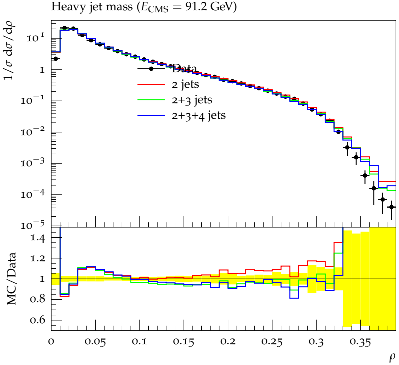
Differential Jet Rates at LEP (Herwig++ pre-3.0).
Dipole shower + some merging

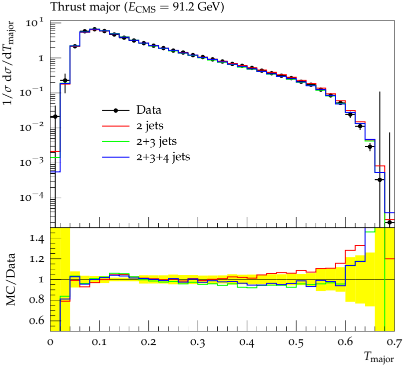




How well does it work?

Event Shapes at LEP (Herwig++ pre-3.0).
Dipole shower + some merging

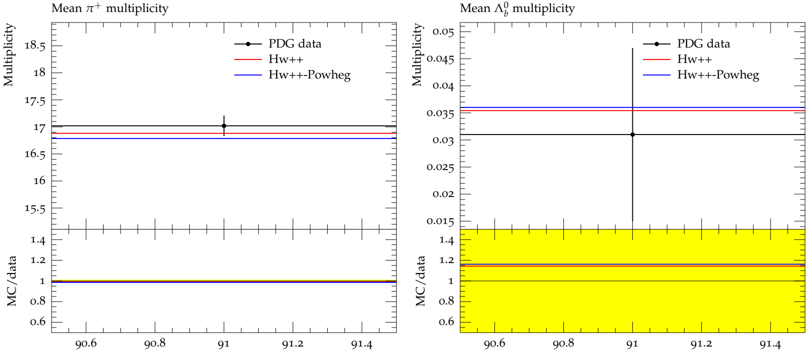




Parton showers do very well, today!

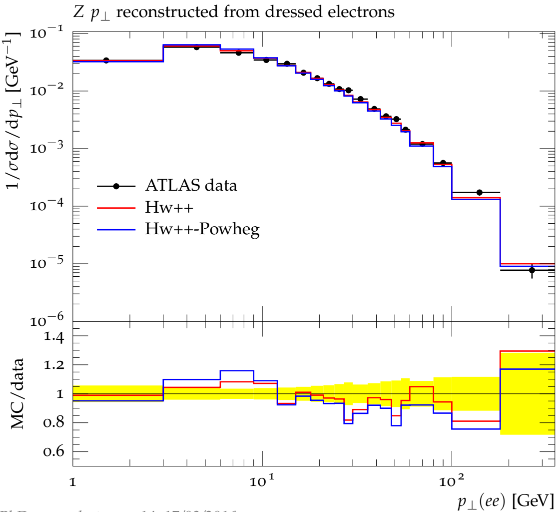
How well does it work?

Hadron Multiplicities at LEP (e.g. π^+, Λ_b^0).

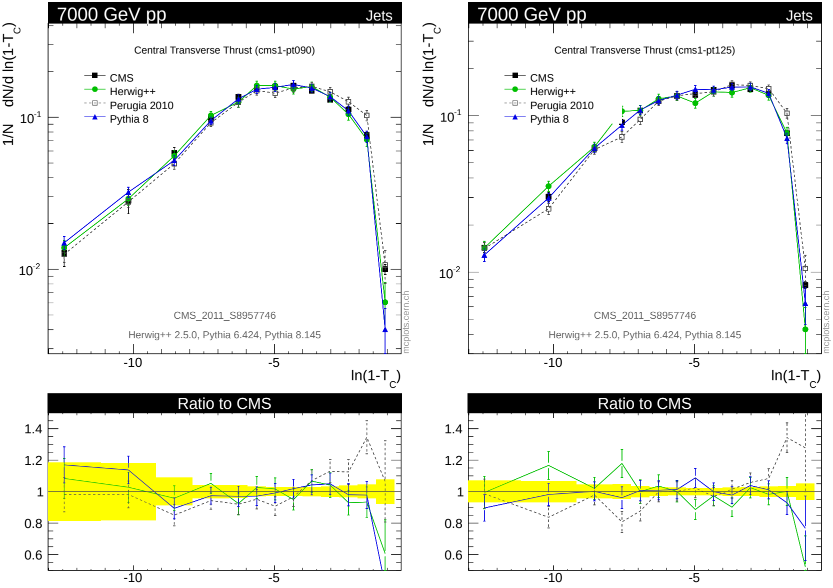


How well does it work?

$p_\perp(Z^0) \to$ intrinsic $k_\perp$ (LHC 7 TeV).
See also in context of matching/marging.

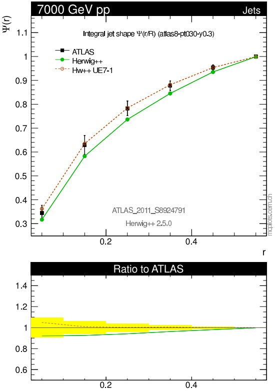


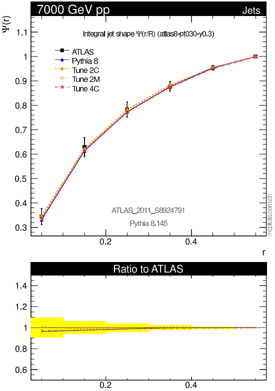
Transverse thrust

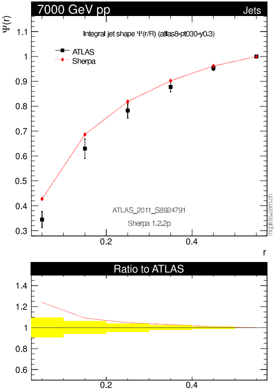


Integral jet shapes

not too hard, central ($30 < p_T/\text{GeV} < 40; 0 < |y| < 0.3$)

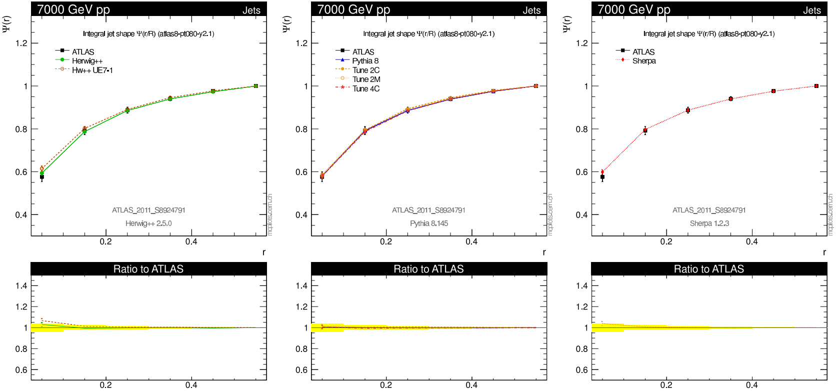






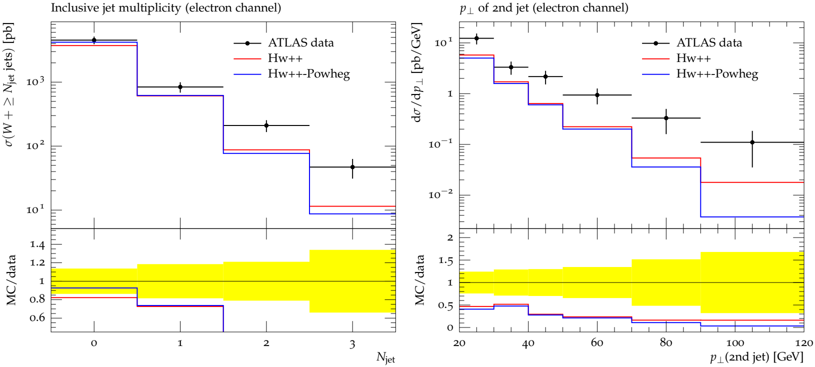
Integral jet shapes

harder, more forward ($80 < p_T/\mathrm{GeV} < 110; 1.2 < |y| < 2.1$)



Limits of parton shower

$W +$ jets, LHC 7 TeV.



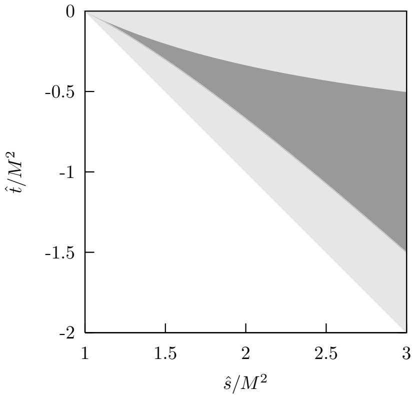
Higher jets not covered by parton shower only → matching.

Matching and Merging

Matrix element corrections

Hard ME correction (e.g. in DY)

- Light: collinear/soft regions.
- Dark: Dead region, filled with extra hard emissions — not accessable by parton shower.
- To be complemented by soft matrix element corrections.

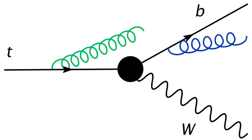


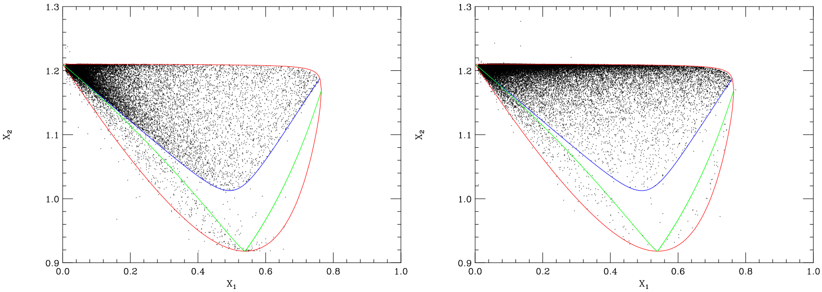
Also for $V^* \to q\bar{q}$, t–decay (2.0)
$gg \to h^0$ (2.2),

Simplest matching.

Soft ME Corrections in t Decays

Smooth coverage of soft gluon region from both parton showers.

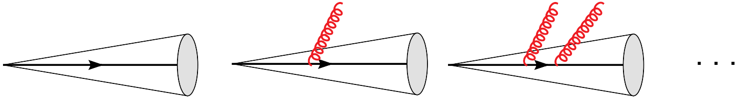




w/o and with soft ME correction

Matching tree level ME and parton showers

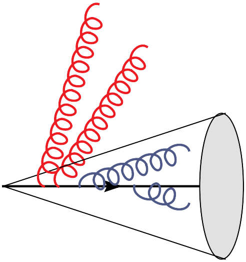
- Problem: have multiple tree level MEs for $X + 0, 1, \dots n$ jets.



- Jets well separated and *inclusive*.
- Merge this into one exclusive multijet sample.
- Idea: use Sudakov form factors to disallow "+ anything softer" (which is normally inside an inclusive ME).
- That's done in the CKKW(-L) approach. [Catani, Krauss, Kuhn, Webber, JHEP 0111:063,2001, Krauss JHEP 0208:015,2002, L. Lönnblad, JHEP 0205:046,2002, Gleisberg, Höche, Winter, Schälicke, Schumann.]
- Alternative: MLM matching. [M.L. Mangano]
- Systematic study and comparison of implementations. [J. Alwall, S. Höche, F. Krauss, N. Lavesson, L. Lönnblad, F. Maltoni, M.L. Mangano, M. Moretti, C.G. Papadopoulos, F. Piccinini, S. Schumann, M. Treccani, J. Winter, M. Worek, EPJC53:473-500,2008.]

Matching tree level ME and parton showers

- Separates ME and parton shower at intermediate scale Q_{ini}.
- Parton shower fills region below Q_{ini}.
- All emissions resolvable above Q_0.



Merges ME and parton shower at scale Q_{ini}.

CKKW weights

Starting point: Sudakov form factors

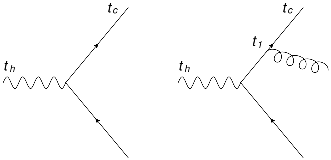
$$\Delta_q(T,t) = \exp\left\{-\int_t^T \frac{\mathrm{d}t'}{t'} \int_{z_-(t')}^{z_+(t')} \frac{\alpha_S(t,z)}{2\pi} P_{qq}(z)\right\}$$
$$= \exp\left\{-\int_t^T \mathrm{d}t\, \Gamma_q(T,t)\right\} \ ,$$
$$\Delta_g(T,t) = \exp\left\{-\int_t^T \mathrm{d}t\, \Gamma_g(T,t) + \Gamma_f(t)\right\}$$

and integrated splitting functions

$$\Gamma_q(t_h,t) = \frac{C_F}{\pi}\frac{\alpha_S(t)}{t}\left[\frac{1}{2}\ln\frac{t_h}{t} - \frac{3}{4}\right] \ ,$$
$$\Gamma_g(t_h,t) = \frac{C_A}{\pi}\frac{\alpha_S(t)}{t}\left[\frac{1}{2}\ln\frac{t_h}{t} - \frac{11}{12}\right] \ ,$$
$$\Gamma_f(t_h,t) = \frac{T_R\, n_F}{3\pi}\frac{\alpha_S(t)}{t} \ .$$

CKKW weights

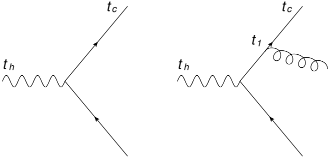
Sudakov FF = no branching probability.
Get probability for a jet configuration to survive exclusivly:
e.g.



$$w_2 = \left[\Delta_q(t_h, t_c)\right]^2 \ .$$

CKKW weights

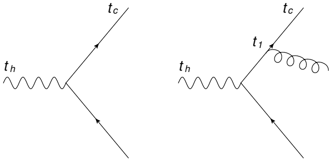
Sudakov FF = no branching probability.
Get probability for a jet configuration to survive exclusivly:
e.g.



$$w_3 = \Delta_q(t_h, t_c)\Delta_q(t_h, t_1)\Gamma_q(t_h, t_1)\Delta_q(t_1, t_c)\Delta_g(t_1', t_c)$$

CKKW weights

Sudakov FF = no branching probability.
Get probability for a jet configuration to survive exclusivly:
e.g.



$$
\begin{aligned}
w_3 &= \Delta_q(t_h, t_c)\Delta_q(t_h, t_1)\Gamma_q(t_h, t_1)\Delta_q(t_1, t_c)\Delta_g(t_1', t_c) \\
&= \left[\Delta_q(t_h, t_c)\right]^2 \Gamma_q(t_h, t_1)\Delta_g(t_1', t_c)\ .
\end{aligned}
$$

CKKW algorithm

Results in CKKW algorithm:

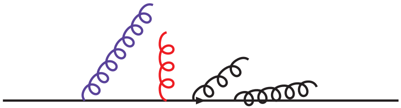
1. Pick multiplicity according to $\sigma_n / \sum_k \sigma_k$.
2. Pick phase space point $\rightarrow w_h$.
3. Jet algorithm $\rightarrow$ shower history $\rightarrow$ nodal scales t_i.
4. Reweighting factor $w_\alpha = \frac{\alpha_S(t_1)}{\alpha_S(t_{\rm ini})} \cdots \frac{\alpha_S(t_n)}{\alpha_S(t_{\rm ini})}$.
5. Assign Sudakov weight $\rightarrow w_S$.
6. Final event weight $w = w_h w_S w_\alpha$.

Parton shower $t_c \rightarrow t_0$.

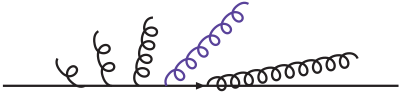
-L variant: Apply trial showers in step 5.

Matching tree level ME and PS — trouble?

Hard emission, to be complemented by parton shower.



$p_\perp$ ordered shower. Angular ordering from additional vetos.



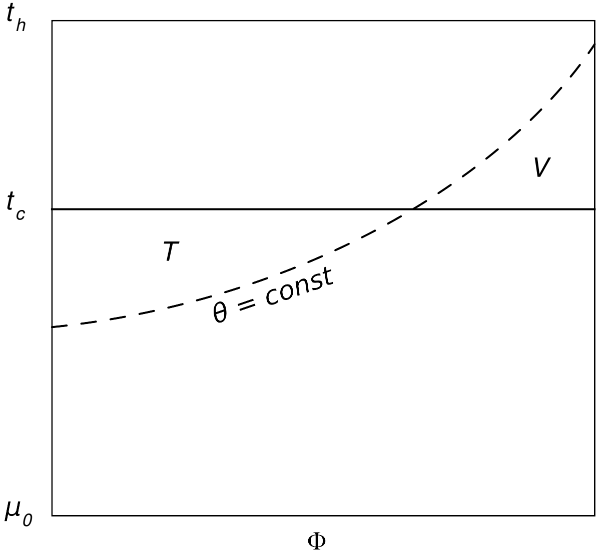
Angular ordered shower. Some softer emissions before hardest one.

Potential holes in phase space $\longrightarrow$ *truncated showers*.

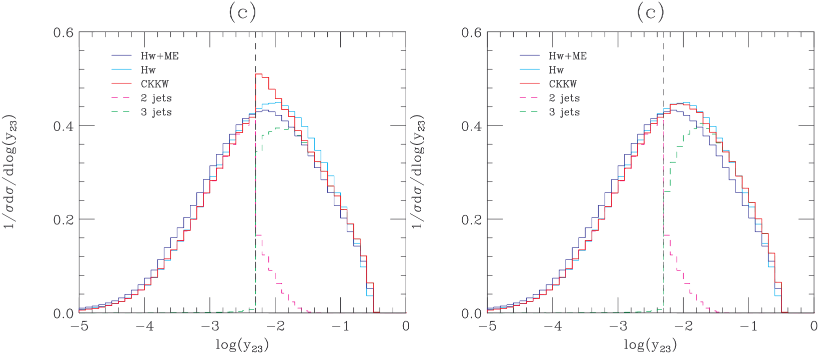
[S. Höche, F. Krauss, S. Schumann, F. Siegert, JHEP 0905:053,2009.]

[K. Hamilton, P. Richardson, J. Tully, JHEP 0911:038,2009.]

Matching tree level ME and PS — trouble?



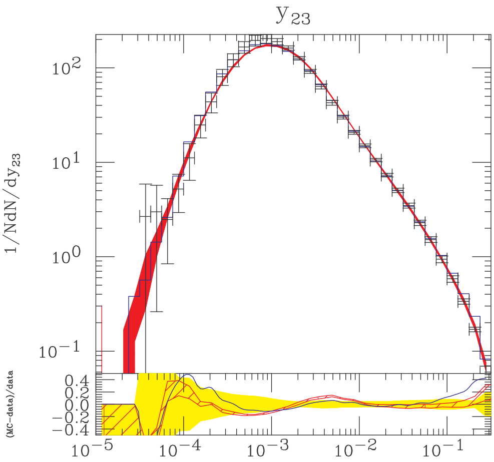
Matching tree level ME and parton showers



[K. Hamilton, P. Richardson, J. Tully, JHEP 0911:038,2009.]

Parton level merging for illustration.
Instabilities at $Q_{\rm ini}$ removed.

Matching tree level ME and parton showers

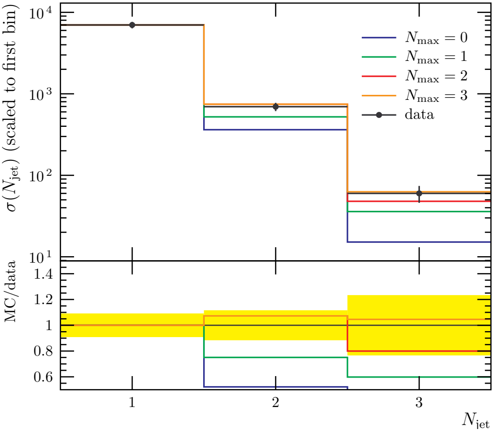


[K. Hamilton, P. Richardson, J. Tully, JHEP 0911:038,2009.]

Hadron level with matching uncertainty band vs OPAL.

Matching tree level ME and parton showers

Sherpa CS shower, matched with $Z^0 + N_{\rm jet}$ jets vs CDF data.



[S. Höche, F. Krauss, S. Schumann, F. Siegert, JHEP 0905:053,2009.]

Reached remarkable stability wrt $Q_{\rm ini}$ variation.

Matching NLO computations and parton showers

The problem:
Consider n and $n+1$ body ME

$$|M_n^{(0)}|^2 \qquad 2\mathrm{Re}M_n^{(0)}M_n^{(1)} \qquad |M_{n+1}^{(0)}|^2 \ .$$

- Both present in NLO as Born+Virtual and Real ME.
- Parton shower adds $n+1$ st emission as well (accurate to leading log accuracy).

$\Rightarrow$ Potential double counting!

Matching NLO computations and parton showers

The problem:
Consider n and $n+1$ body ME

$$|M_n^{(0)}|^2 \qquad 2\mathrm{Re}M_n^{(0)}M_n^{(1)} \qquad |M_{n+1}^{(0)}|^2\ .$$

- Both present in NLO as Born+Virtual and Real ME.
- Parton shower adds $n+1$ st emission as well (accurate to leading log accuracy).

$\Rightarrow$ Potential double counting!

Two popular approaches:

- MC@NLO
- POWHEG

NLO with subtraction method

Toy model: NLO calculation with subtraction method,
$x =$ real emission phase space, *B*orn, *O*bservable, *R*eal, *V*irtual.

$$\langle O \rangle_{\text{NLO}} = BO(0) + VO(0) + \int_0^1 dx \, \frac{O(x)R(x)}{x} ,$$

NLO with subtraction method

Toy model: NLO calculation with subtraction method,
x = real emission phase space, *B*orn, *O*bservable, *R*eal, *V*irtual.

$$\langle O \rangle_{\mathrm{NLO}} = BO(0) + VO(0) + \int_0^1 dx\, \frac{O(x)R(x)}{x} ,$$

Add/subtract soft/collinear piece $A(x)$ ($\lim_{x\to 0} A(x) = R(x)$):

$$\langle O \rangle_{\mathrm{NLO}} = BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x} ,$$

where

$$\bar{V} = V + \int_0^1 dx\, \frac{A(x)}{x} = \text{IR finite} .$$

PS@NLO

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{ -\int_\mu \frac{dx}{x} P(x) \right\} \qquad .$$

From Born $\otimes$ zero/one parton shower emission:

$$\langle O \rangle_{\mathrm{PS}} = \int dx\, O(x) \left[B\Delta\delta(x) + B\frac{P(x)}{x}\Delta\Theta(x-\mu) \right]$$

PS@NLO

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{-\int_\mu \frac{dx}{x}P(x)\right\} \approx 1 - \int dx\,\frac{P(x)}{x}\ .$$

From Born $\otimes$ zero/one parton shower emission:

$$\begin{aligned}\langle O\rangle_{\mathrm{PS}} &= \int dx\,O(x)\left[B\Delta\delta(x) + B\frac{P(x)}{x}\Delta\Theta(x-\mu)\right] \\ &= BO(0)\left[1 - \int_\mu \frac{dx}{x}P(x)\right] + \int_\mu dx\,O(x)B\frac{P(x)}{x}\ .\end{aligned}$$

PS@NLO

Calculate parton shower contribution with Sudakov FF,

$$\Delta = \exp\left\{ -\int_\mu \frac{dx}{x} P(x) \right\} \approx 1 - \int dx \, \frac{P(x)}{x} \; .$$

From Born $\otimes$ zero/one parton shower emission:

$$\begin{aligned} \langle O \rangle_{\rm PS} &= \int dx \, O(x) \left[B \Delta \delta(x) + B \frac{P(x)}{x} \Delta \Theta(x-\mu) \right] \\ &= B O(0) \left[1 - \int_\mu \frac{dx}{x} P(x) \right] + \int_\mu dx \, O(x) B \frac{P(x)}{x} \; . \end{aligned}$$

Terms that contribute at $O(\alpha_S)$/NLO $\Rightarrow$ double counting.

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O\rangle_{\text{NLO}} = BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x}$$

$$\langle O\rangle_{\text{PS}} = BO(0)\left[1 - \int_\mu \frac{dx}{x}P(x)\right] + \int_\mu dx\, O(x)B\frac{P(x)}{x}$$

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\begin{aligned}\langle O\rangle_{\mathrm{NLO}}{}' =& BO(0)+\bar{V}O(0)+\int_0^1 dx\,\frac{O(x)R(x)-O(0)A(x)}{x}\\ &+\int_\mu \frac{dx}{x}P(x)-\int_\mu dx\,O(x)B\frac{P(x)}{x}\end{aligned}$$

Matching MC and NLO

Solution: subtract doubly counted terms.

$$\langle O \rangle_{\mathrm{NLO}}{}' = BO(0) + \bar{V}O(0) + \int_0^1 dx\, \frac{O(x)R(x) - O(0)A(x)}{x} + \int_\mu \frac{dx}{x} P(x) - \int_\mu dx\, O(x) B \frac{P(x)}{x}$$

Result ("MC@NLO master formula")

$$\langle O \rangle_{\mathrm{MC@NLO}} = O(0) \left[B + \bar{V} + \int_0^1 dx\, \frac{BP(x) - A(x)}{x} \right] + \int dx\, O(x) \frac{R(x) - BP(x)}{x} \; .$$

Note: ($O(0)B \otimes$ parton shower) adds back subtracted terms
$\Rightarrow$ NLO result is exactly reproduced after parton shower.

Matching MC and NLO

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B + \bar{V} + \int_0^1 dx\,\frac{BP(x) - A(x)}{x}\right] + \int dx\,O(x)\,\frac{R(x) - BP(x)}{x}\ .$$

Observations/remarks:

- Events with n and $n+1$ legs are seperately finite. No cancellation of large weights.
- NLO result can be recovered strictly upon expansion in powers of α (with parton shower emission).
- Interface to MC program very well defined.
- Dropping $\mu \to 0$ is only a power correction.

Matching MC and NLO

$$\begin{aligned}\langle O\rangle_{\text{MC@NLO}} =& O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{BP(x)-A(x)}{x}\right]\\ &+\int dx\,O(x)\frac{R(x)-BP(x)}{x}\ .\end{aligned}$$

Three types of matching

1. MC@NLO (classic, Frixione and Webber).
2. Simpler: parton shower with $P(x) = A(x)/B$.
3. Or, also simpler, $P(x) = R(x)/B$.

Matching MC and NLO

$$\langle O \rangle_{\mathrm{MC@NLO}} = O(0) \left[B + \bar{V} + \int_0^1 dx \, \frac{BP(x) - A(x)}{x} \right] + \int dx \, O(x) \frac{R(x) - BP(x)}{x} \, .$$

1. Classic MC@NLO (Frixione and Webber)
   - $A(x)$ = FKS subtraction terms
   - $P(x)$ and phase space specific for HERWIG.
   - Generic, calculate once and for all.
   - New for every process.

Matching MC and NLO

$$\langle O\rangle_{\text{MC@NLO}} = O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{BP(x)-A(x)}{x}\right] + \int dx\,O(x)\frac{R(x)-BP(x)}{x}\,.$$

2. 'Custom' parton shower
e.g. with Catani–Seymour subtraction kernels

- CS subtraction already used in many NLO calculations.
- $P(x) = A(x)/B$, so terms vanish.
- $R(x) - A(x)$ already in NLO parton level program.

$\Rightarrow$ (almost) no need to modify NLO calculation!

Matching MC and NLO

$$\langle O\rangle_{\mathrm{MC@NLO}} = O(0)\left[B+\bar{V}+\int_0^1 dx\,\frac{BP(x)-A(x)}{x}\right] + \int dx\,O(x)\,\frac{R(x)-BP(x)}{x}\,.$$

3. Simpler in a different way, $P(x) = R(x)/B$
   - $R(x) - A(x)$ now only needed as integral available in NLO parton level program.
   - No $n+1$ body events.
   - ≥ 1 PS emission from $R(x)/B$ as splitting kernel $\rightarrow$`POWHEG`.
   - Positive weights (terms $\neq 0$ are $\sigma_{\mathrm{NLO}}^{\mathrm{incl}}$).
   - Further emissions from (truncated) standard PS.

MC@NLO

- Introduced 2002 Frixione, Webber, JHEP 0206:029,2002 [hep-ph/0204244].
- Extended to heavy quarks

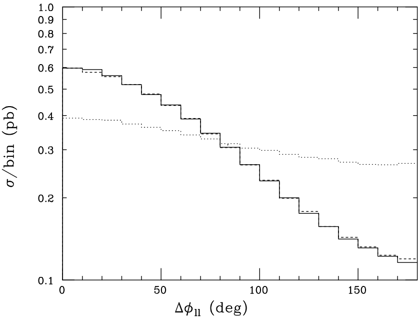
  Frixione, Nason, Webber, JHEP 0308:007,2003 [hep-ph/0305252].
- further extensions to many processes (single top etc.)
- MC@NLO customised to use with HERWIG.
- Some processes in Herwig++ as well
  $e^+e^- \to$ jets, DY, W', h^0 decay

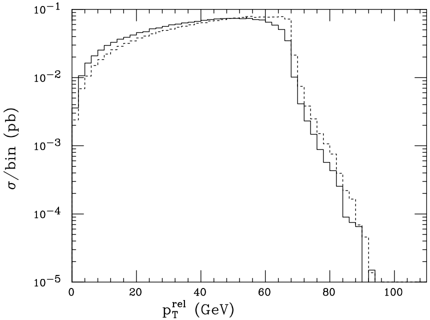
  Latunde–Dada 0708.4390, 0903.4135, Latunde–Dada, Papaefstatiou, 0901.3685.
- MC@NLO package adopted to Herwig++ as well.

  S. Frixione, F. Stoeckli P. Torrielli and B.R. Webber, 1010.0568.

MC@NLO

Examples with Herwig++ (solid) Herwig6 (dash)





$h^0 \to WW \to l\nu l\nu$,
(no spin corr dotted)

$t\bar{t}$, $p_t(b)$ rel to t (right).

S. Frixione, F. Stoeckli P. Torrielli and B.R. Webber, 1010.0568.

POWHEG

- Alternative proposed by P. Nason.
- Modified Sudakov FF for first emission.
- Angular ordered Parton Shower tricky (see below).
- *Truncated Shower* adds in missing radiation afterwards.
- Finally evolution with 'ordinary' Parton Shower.

[Nason, hep-ph/0409146; Nason, Ridolfi hep-ph/0606275]

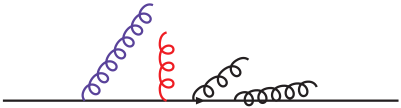
Recently systematically extended.

- `POWHEG` formulation independent of the event generator implementation.
- Worked out for different subtraction schemes.

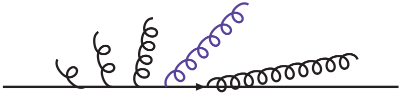
[Frixione, Nason, Ridolfi, 0707.3081, 0707.3088; Frixione, Nason, Oleari, 0709.2092]

POWHEG

Angular ordered showers and POWHEG



$p_\perp$ ordered shower. Angular ordering from additional vetos.



Angular ordered shower. Some softer emissions before hardest one.

Need truncated showers.

POWHEG in Herwig++

- First implementation of method for e^+e^- annihilation

[O. Latunde–Dada, SG, B. Webber, hep-ph/0612281]

- Many more processes now available with release:
$\mathrm{DY}\ (\gamma^*/Z^0/W^\pm), h^0, h^0Z^0, h^0W^\pm, W^+W^-, W^\pm Z^0, Z^0Z^0$

[K. Hamilton, P. Richardson and J. Tully, 0806.0290, 0903.4345, Hamilton, JHEP 1101:009]

- and with contributed code:
$e^+e^- \to \mathrm{jets}, t\bar{t}, t - \mathrm{decay}, W', h^0 - \mathrm{decay}$

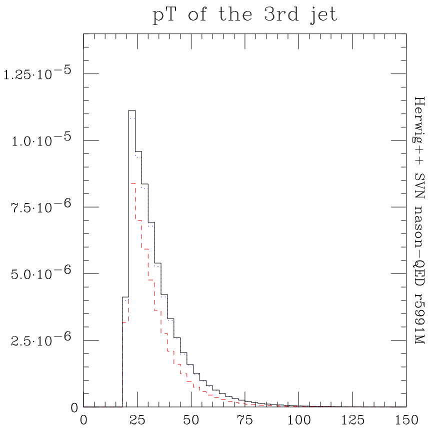
[O. Latunde-Dada, 0812.3297, Eur. Phys. J. C **58**, 543 (2008)]

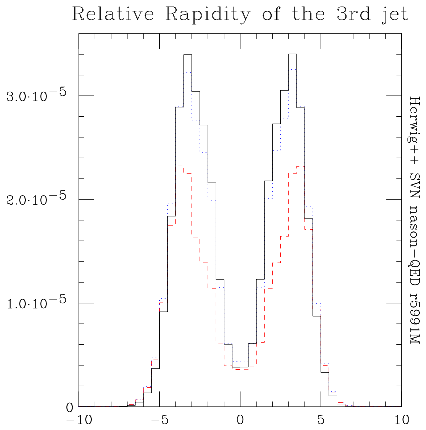
[A. Papaefstathiou and O. Latunde-Dada, JHEP **0907**, 044]

- includes full truncated showers.
- Interface to PowhegBox straightforward.
- More processes underway ($\gamma\gamma$, VBF, SUSY pair prod…).

POWHEG in Herwig++

Higgs production in VBF. (POWHEG, MEC, LO+PS)





[L. D'Errico, P. Richardson in preparation]

Matchbox in Herwig++

- Upcoming Herwig++ 3.0 with Matchbox working horse.
  $\rightarrow$ NLO as default.
- Interfaces to various programs.
- Formalism and code to generate matched/merged events.

What’s in Matchbox?

- Matching/merging formalism completely genereric.
- Two showers
  - Angular ordered shower.
  - Catani–Seymour dipoles.
- Two matching formalisms
  - MC@NLO like.
  - POWHEG like.
- Many interfaces to (automatic) NLO programs.
- Automatic CS subtraction terms.
- Improved phase space.

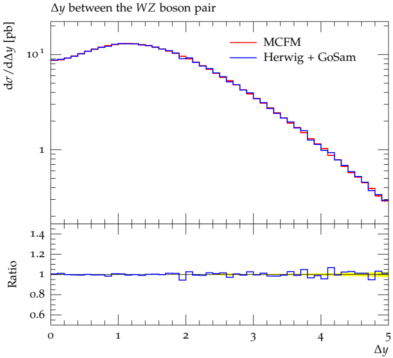
Interfaces to Matchbox

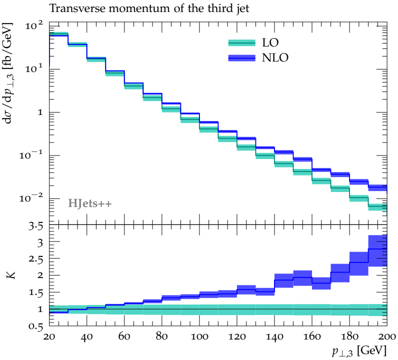
- Amplitude level
  - Hand-coded MEs
  - Hjet++ [F. Campanario, T. Figy, S. Plätzer, M. Sjödahl]
  - MadGraph5 [MadGraph, SG, S. Plätzer, J. Bellm]
  - Colour correlations with ColourFull [S. Plätzer, M. Sjödahl]
- Squared amplitude level
  - GoSam [GoSam & J. Bellm, SG, S. Plätzer, C. Reuschle]
  - OpenLoops [OpenLoops & J. Bellm, SG, S. Plätzer]
  - NJet [NJet & S. Plätzer]
  - VBFNLO [VBFNLO & J. Bellm, SG, S. Plätzer]

Many details validated, see e.g. below.

Processes at the parton level

E.g. WZ production, $H + 2$ jets (EW) as more complicated example. Many processes tested.



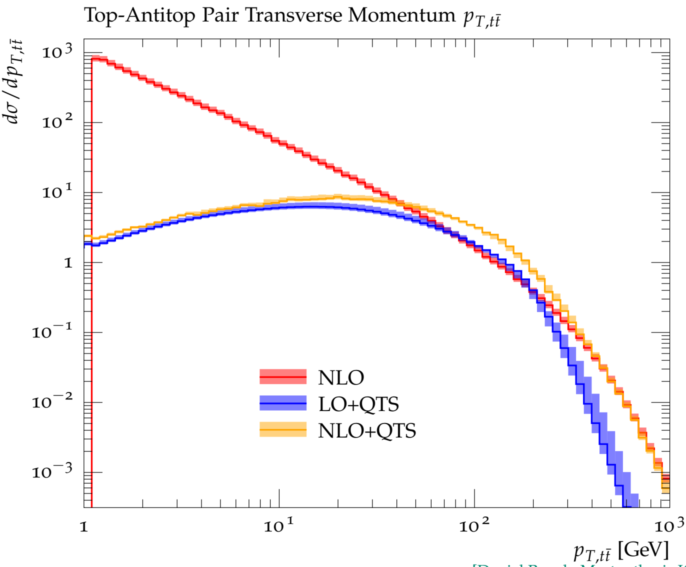


[N. Fischer, KIT 2013]

[F. Campanario, T. Figy, S. Plätzer, M. Sjödahl, PRL 111 (2013) 211802]

All SM $2 \to 2$ processes validated in detail. Plus more.

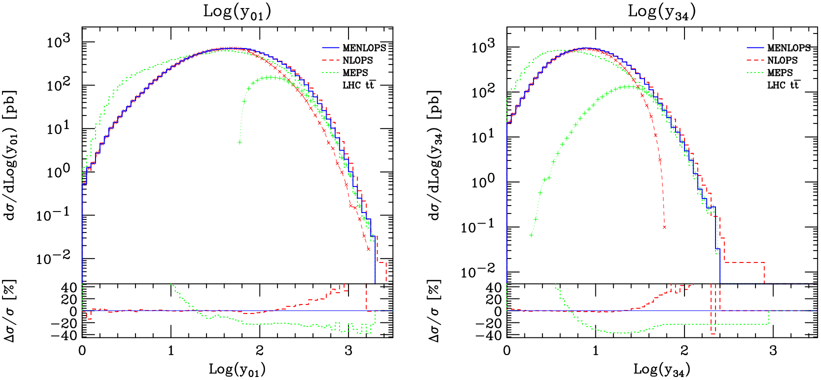
$t\bar{t}$ Matched with parton shower



[Daniel Rauch, Master thesis KIT 2014]

MENLOPS

ME+PS merging with lowest multiplicity at NLO.

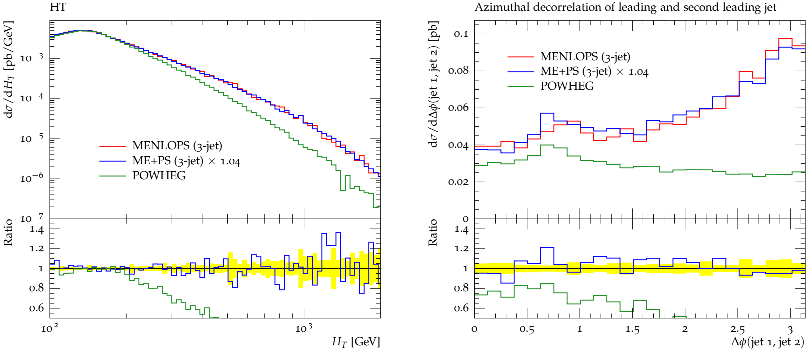


Test generic method with Pythia. y_{nm} in $t\bar{t}$+jets

[Hamilton, Nason, JHEP 1006:039]

MENLOPS

ME+PS merging with lowest multiplicity at NLO.



WW+jets implementation in Sherpa.

[Hoeche, Krauss, Schönherr, Siegert, 1009.1127]

Outlook: unitarized Matching/Merging

New approach in Herwig++/Matchbox. [S. Plätzer, 1211.5467]

Idea: Approximation of Sudakov "$\Delta \approx 1 - \int BP$" violates parton shower unitarity. Replace BP by full LO matrix element also in reweighting of events.

Leads to unified NLO matching and (LO/NLO)-merging prescription. [J. Bellm, SG, S. Plätzer]

Outlook: unitarized Matching/Merging

Consider parton shower acting on Born ME,

$$PS[B_0] = \Delta_\mu^0 B_0 + PS[P_1\,\Delta_0^1\,B_0]\ ,$$

iterate once,

$$PS[B_0] = \Delta_\mu^0 B_0 + \Delta_\mu^1\,P_1\,\Delta_1^0 B_0 + PS[P_2\,\Delta_2^1\,P_1\,\Delta_1^0\,B_0]\ ,$$

replace

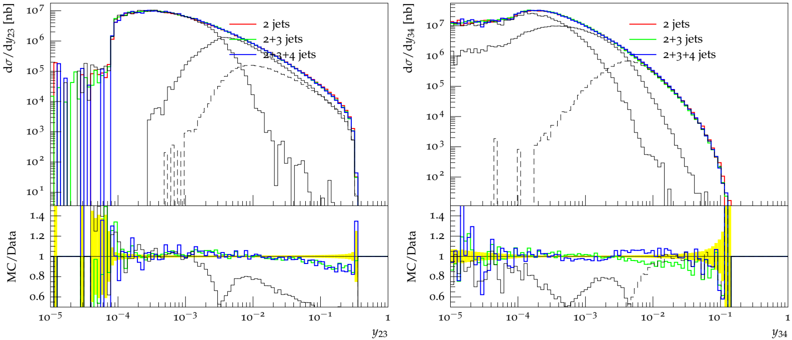
$$P_1\,B_0 \to \frac{\alpha_S(q_1)}{\alpha_S(q_0)} B_1\ ,$$

etc., but induces unitarity violation in Sudakov weights, so

$$\Delta_\mu^1 \approx 1 - P_1 B_0 \to 1 - \frac{\alpha_S(q_1)}{\alpha_S(q_0)} B_1\ .$$

Outlook: unitarized Matching/Merging

Preliminary example: LEP with merging contributions

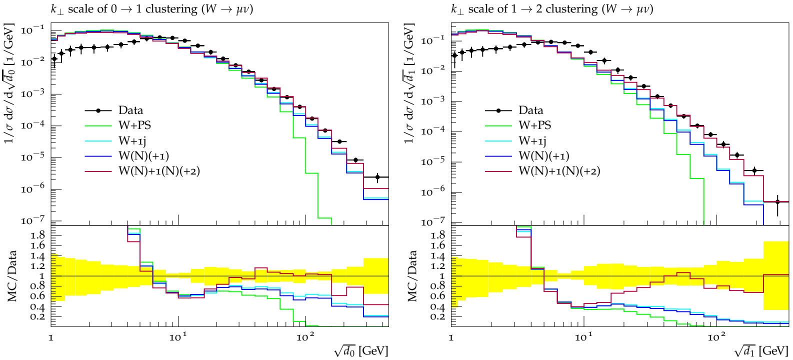


[J. Bellm, KIT]

Note: no hadronization in small y_{ij} region.

Outlook: unitarized Matching/Merging

W+jets. Note residual hadronization dependance.

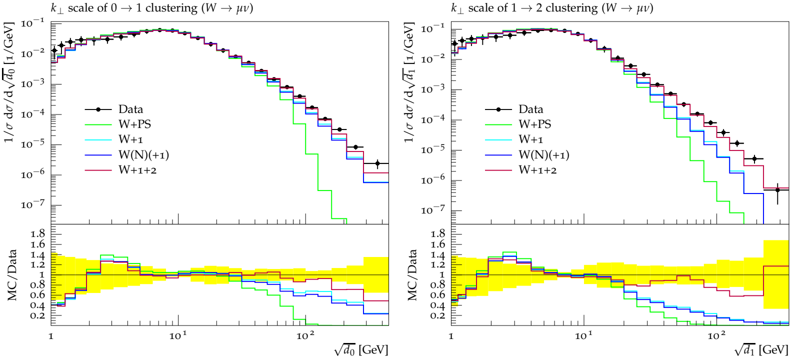


[J. Bellm, KIT]

MPI/Hadronization off.
$W+1, W+1+2$: LO merging with 1(2) jets.
$W(N)+1$: 0j NLO with 0j+1j LO ("matching through merging").

Outlook: unitarized Matching/Merging

W+jets. Note residual MPI/hadronization dependance.

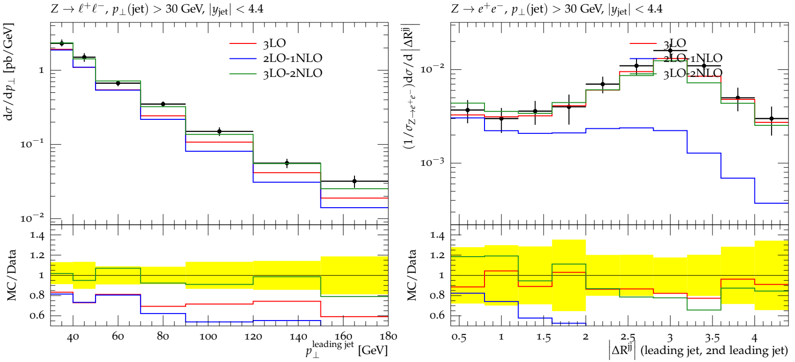


[J. Bellm, KIT]

MPI/Hadronization on.

Outlook: unitarized Matching/Merging

Preliminary example: Z production, jet-jet correlation.

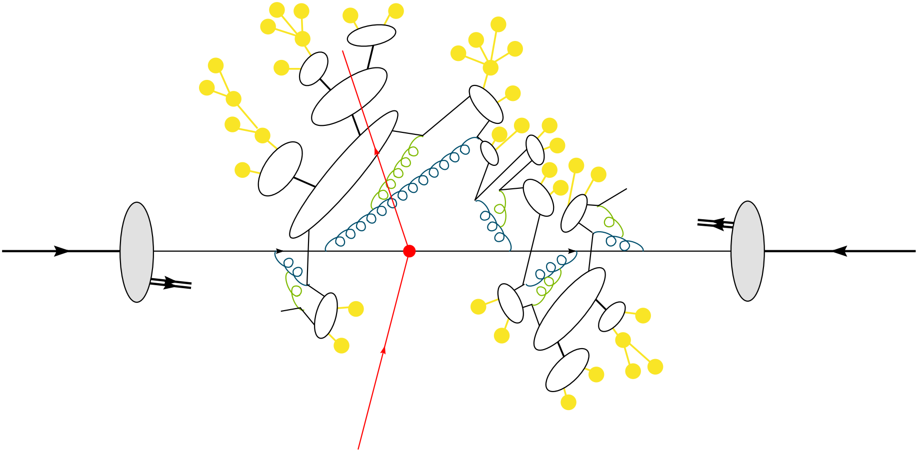


[J. Bellm, KIT]

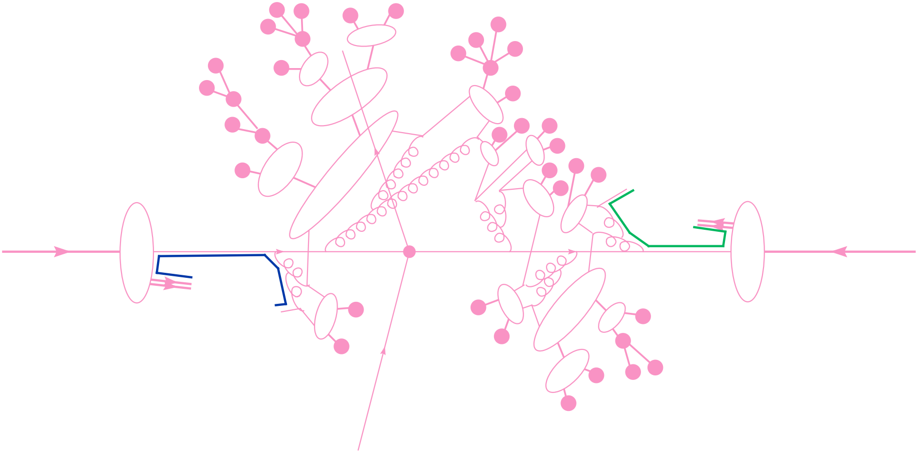
3LO-2NLO = Z+0, 1, 2 (tree) and Z+0,1 NLO (virtual).

Min Bias/Underlying event in data

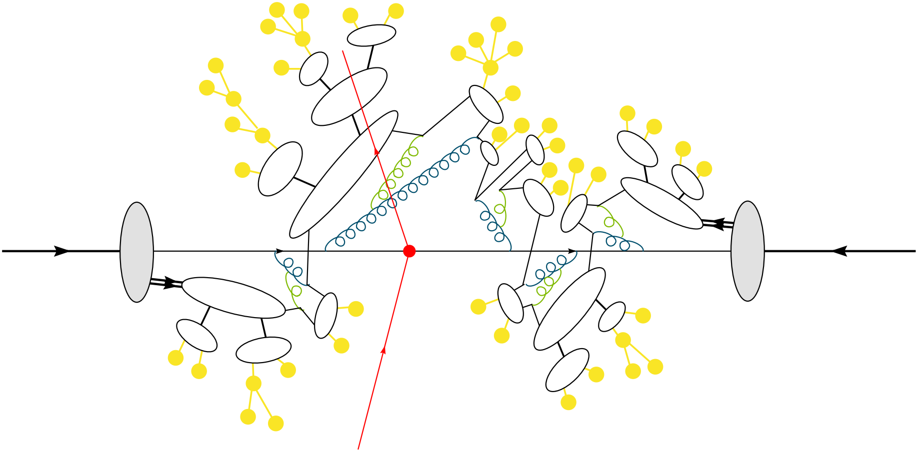
pp Event Generator



pp Event Generator



pp Event Generator

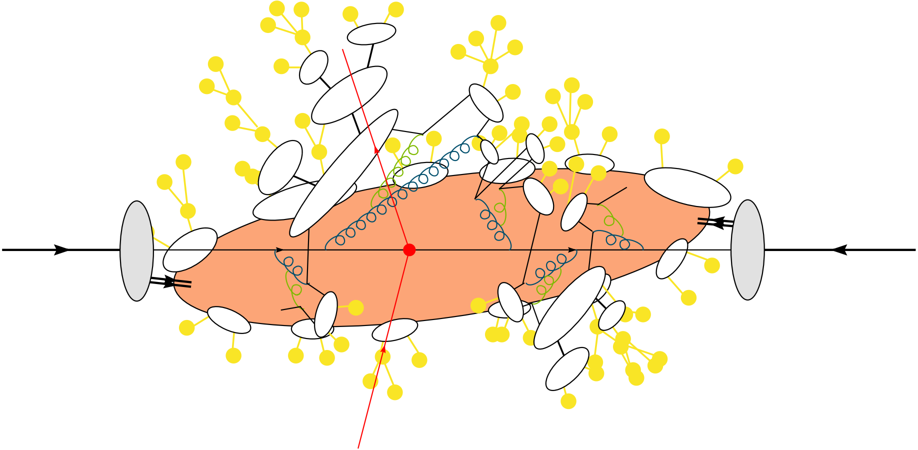


No UE model

Just remnant clusters

- Simplest model?
- Connects loose colour ends and produces some N_{ch}.
- No extra transverse energy.
- Fails.

pp Event Generator



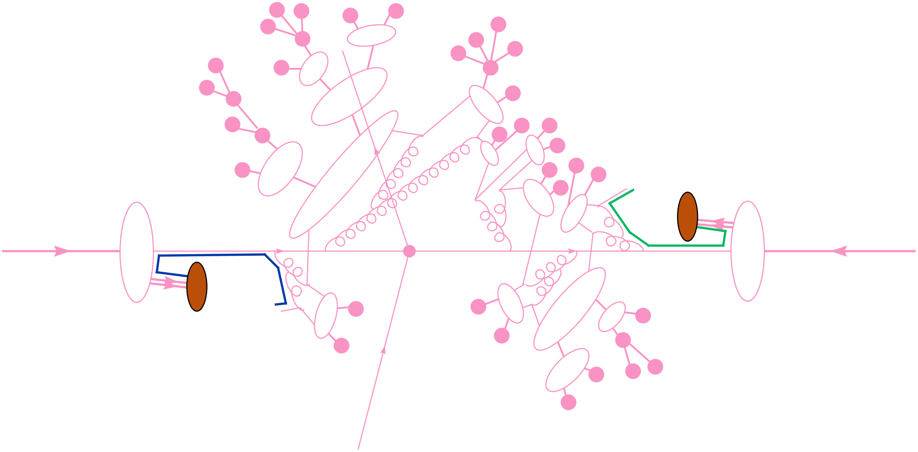
UA5 model

UA5 model

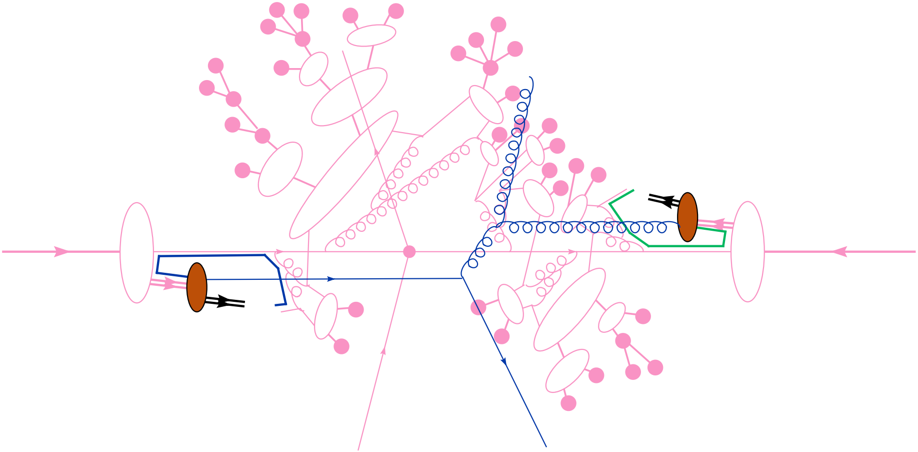
- Produce $\langle n \rangle$ extra clusters, flat in y, with soft $p_\perp$ sprectrum.
- Included from Herwig++ 2.0. [Herwig++, hep-ph/0609306]
- Little predictive power.
- Only gets averages right, not large (and interesting!) fluctiations $\to$ mini jets.
- Was default in fHerwig. Superseded by `JIMMY`.

[JM Butterworth, JR Forshaw, MH Seymour, ZP C72 637 (1996)]

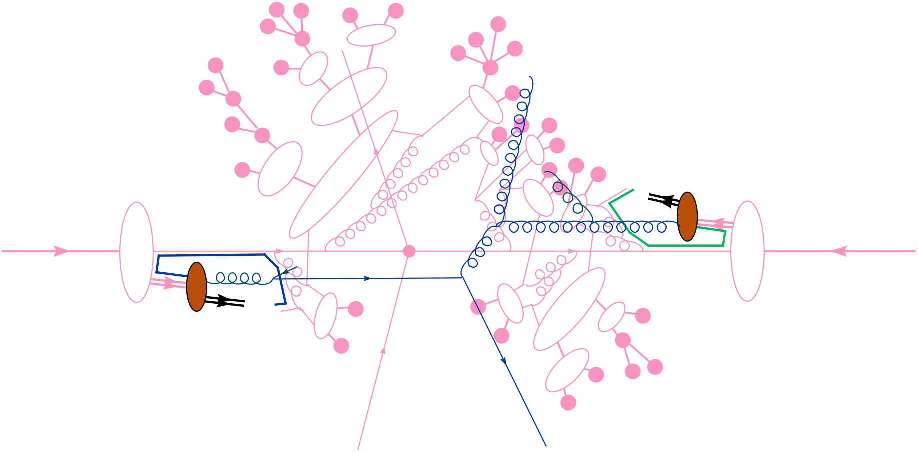
pp Event Generator



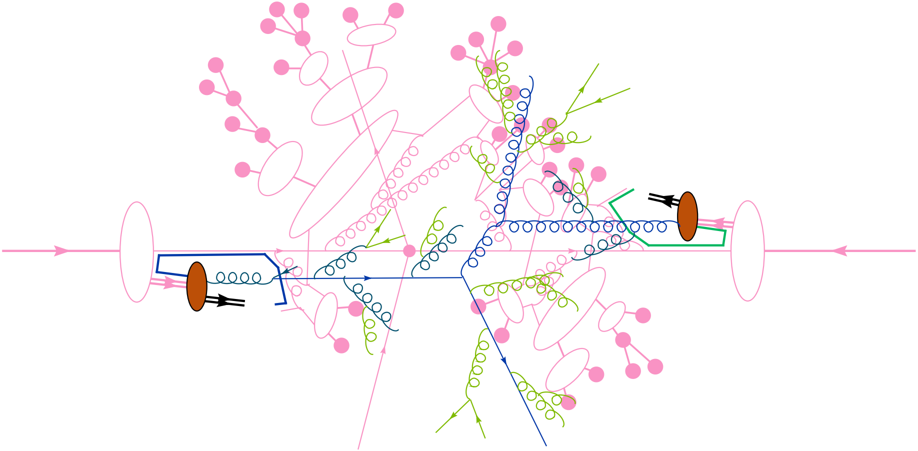
pp Event Generator



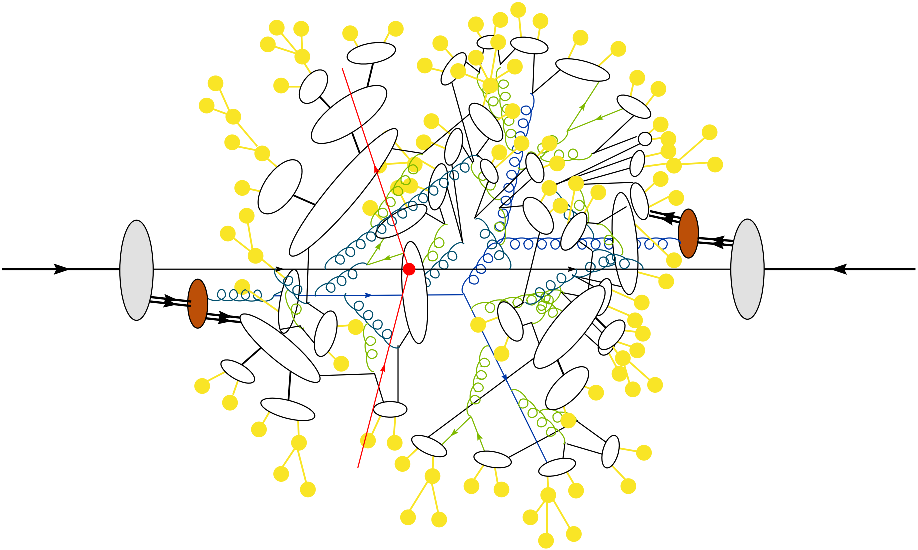
pp Event Generator



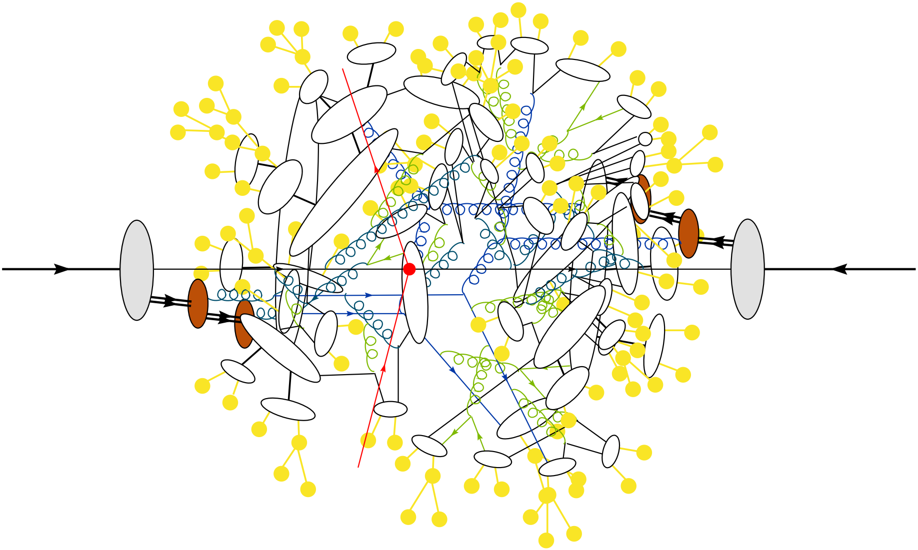
pp Event Generator



pp Event Generator



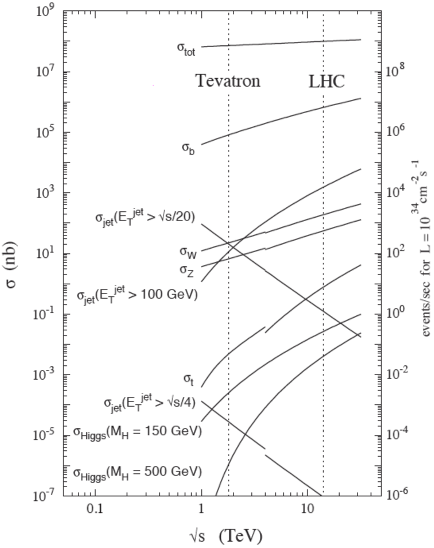
pp Event Generator



Collider cross sections

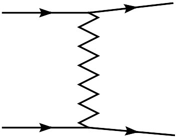
$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \overbrace{\sigma_{\text{DD}} + \underbrace{\sigma_{\text{soft}} + \sigma_{\text{hard}}}_{\sigma_{\text{ND}}}}^{\sigma_{\text{NSD}}}$$

Collider cross sections



What is the Underlying event?

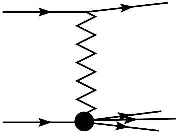
$$\sigma_{\rm tot} = \sigma_{\rm el} + \sigma_{\rm SD} + \overbrace{\sigma_{\rm DD} + \underbrace{(\sigma_{\rm soft} + \sigma_{\rm hard})}_{\sigma_{\rm ND}}}^{\sigma_{\rm NSD}}$$

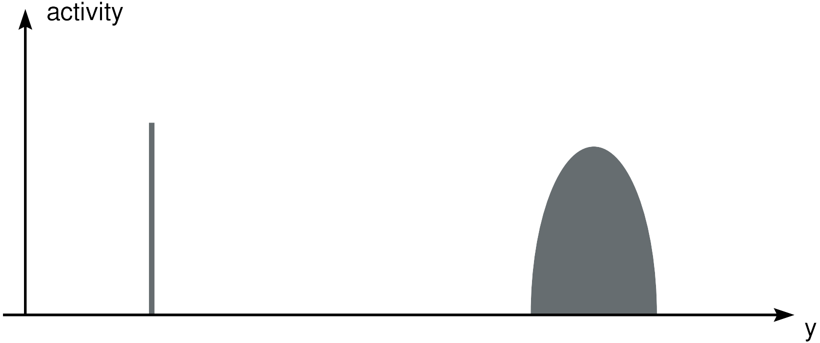


elastic

What is the Underlying event?

$$\sigma_{\rm tot} = \sigma_{\rm el} + \sigma_{\rm SD} + \overbrace{\sigma_{\rm DD} + \underbrace{(\sigma_{\rm soft} + \sigma_{\rm hard})}_{\sigma_{\rm ND}}}^{\sigma_{\rm NSD}}$$

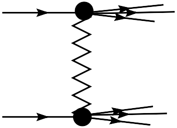


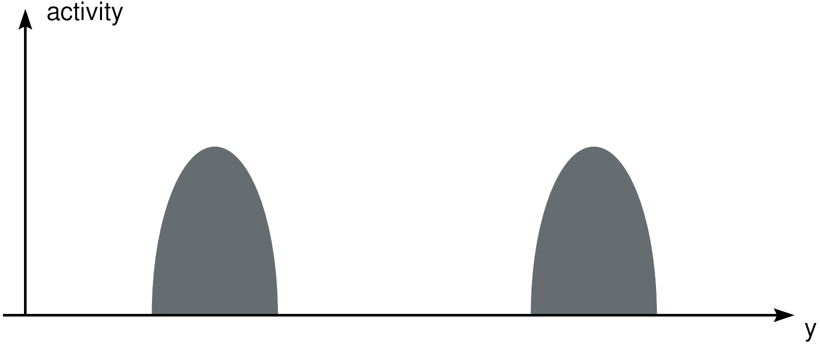


single diffractive

What is the Underlying event?

$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{el}} + \sigma_{\mathrm{SD}} + \overbrace{\color{red}{\sigma_{\mathrm{DD}}} + \underbrace{(\sigma_{\mathrm{soft}} + \sigma_{\mathrm{hard}})}_{\sigma_{\mathrm{ND}}}}^{\sigma_{\mathrm{NSD}}}$$

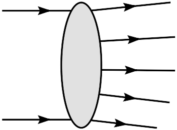




double diffractive

What is the Underlying event?

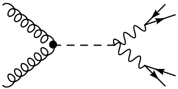
$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \overbrace{\sigma_{\text{DD}} + \underbrace{(\sigma_{\text{soft}} + \sigma_{\text{hard}})}_{\sigma_{\text{ND}}}}^{\sigma_{\text{NSD}}}$$

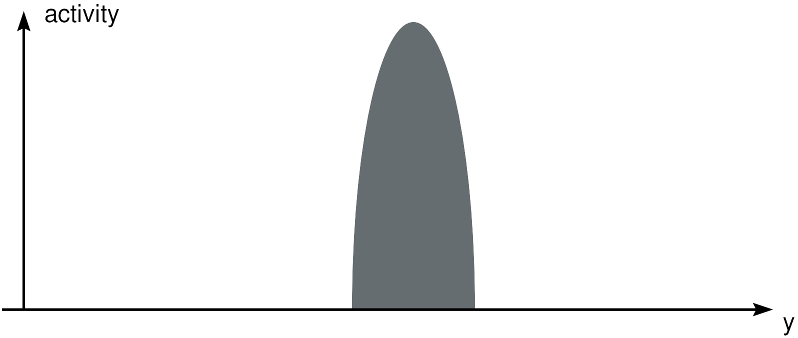


(multiple/soft) interactions

What is the Underlying event?

$$\sigma_{\mathrm{tot}} = \sigma_{\mathrm{el}} + \sigma_{\mathrm{SD}} + \overbrace{\sigma_{\mathrm{DD}} + \underbrace{\left(\sigma_{\mathrm{soft}} + \sigma_{\mathrm{hard}}\right)}_{\sigma_{\mathrm{ND}}}}^{\sigma_{\mathrm{NSD}}}$$

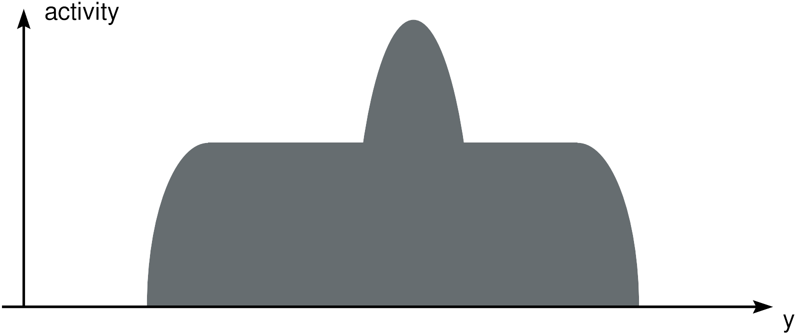




hard scattering

What is the Underlying event?

$$\sigma_{\text{tot}} = \sigma_{\text{el}} + \sigma_{\text{SD}} + \overbrace{\sigma_{\text{DD}} + \underbrace{(\sigma_{\text{soft}} + \sigma_{\text{hard}})}_{\sigma_{\text{ND}}}}^{\sigma_{\text{NSD}}}$$



hard scattering + underlying event

What is the Underlying event?

"Everything except the process of interest."

- Experimentalist: "includes parton showers etc."
- MC author: "everything on top of primary hard process."

The Underlying event (UE) is everywhere in the detector.

- Cannot select UE
- May spoil measurements.
- What characteristics?
- Hard?
- Soft?

Why should I learn about it?

- UE comes with every event.
- Can't trigger/select it away.
- Gives additional tracks and calorimeter hits, in the same cells as your signal.
- Jet energy scale determination.
- Important systematic error.
- Jets where your signal shouldn't give any (VBF).

Triggers

- Zero bias
  - *Every* event in a perfect 4π detector.

Triggers

- Zero bias
  - *Every* event in a perfect 4π detector.
- Minimum bias (MB)
  - Require "some activity"
  - At least have to distinguish from noise/cosmics.
  - small number of tracks of charged tracks (e.g. 1, 2, 6),
  - forward calorimeter hits,
  - $\rightarrow$ with some minimum $p_\perp$.
  - Often want non–single–diffractive

Triggers

- Zero bias
  - *Every* event in a perfect 4π detector.
- Minimum bias (MB)
  - Require "some activity"
  - At least have to distinguish from noise/cosmics.
  - small number of tracks of charged tracks (e.g. 1, 2, 6),
  - forward calorimeter hits,
  - $\to$ with some minimum $p_\perp$.
  - Often want non–single–diffractive
- Hard scattering
  - Very selective trigger
  - BUT accompanied by soft stuff $\to$ underlying event.

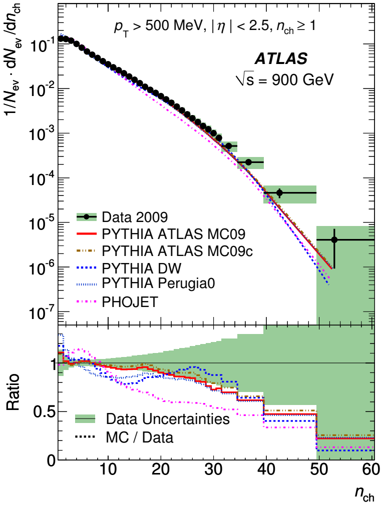
Triggers

- Zero bias
  - *Every* event in a perfect 4π detector.
- Minimum bias (MB)
  - Require "some activity"
  - At least have to distinguish from noise/cosmics.
  - small number of tracks of charged tracks (e.g. 1, 2, 6),
  - forward calorimeter hits,
  - $\rightarrow$ with some minimum $p_\perp$.
  - Often want non–single–diffractive
- Hard scattering
  - Very selective trigger
  - BUT accompanied by soft stuff $\rightarrow$ underlying event.

Physics in MB and UE very similar.

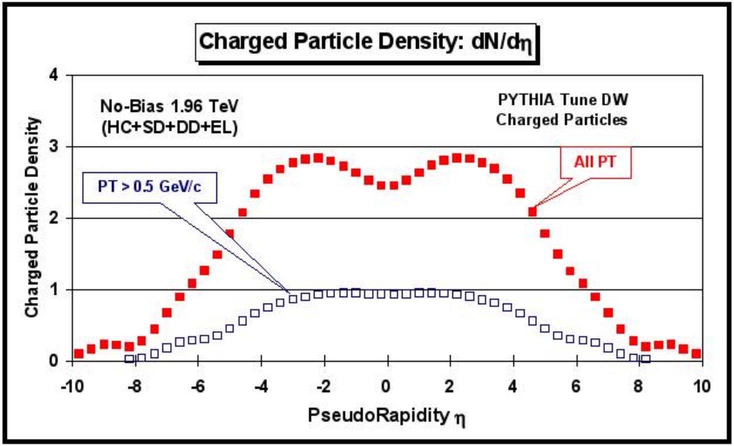
Charakteristics of MB events

N_{ch}



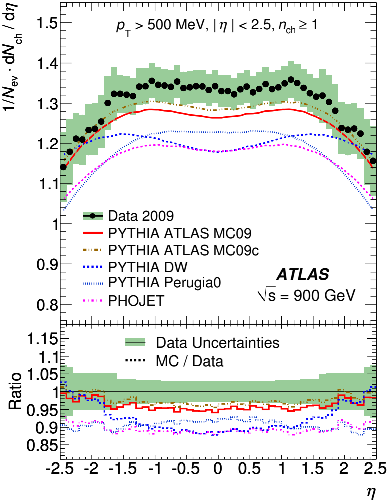
Charakteristics of MB events

$dN/d\eta$ Zero bias vs min bias (Tevatron)



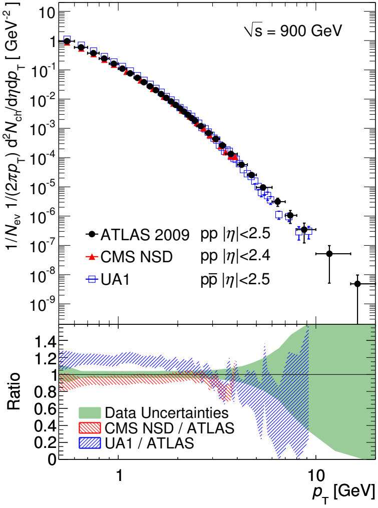
Charakteristics of MB events

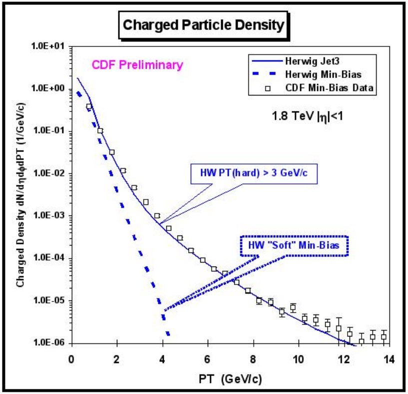
$dN/d\eta$ ATLAS



Charakteristics of MB events

$p_\perp$ spectra of all particles





Charakteristics of MB events

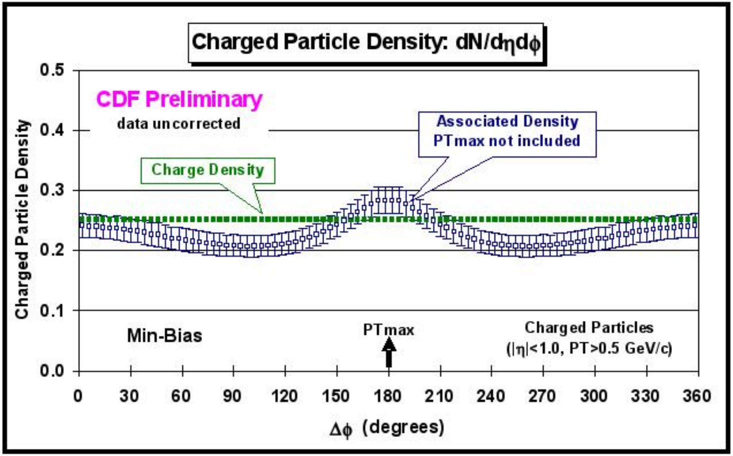
- Inclusive quantities have to be correct, of course.
- Already show, that soft component is important in modelling.

Charakteristics of MB events

- Inclusive quantities have to be correct, of course.
- Already show, that soft component is important in modelling.
- Don't tell much about morphology of event.
- $\rightarrow$ look at distributions inside detector.
- $\rightarrow$ leading particles.

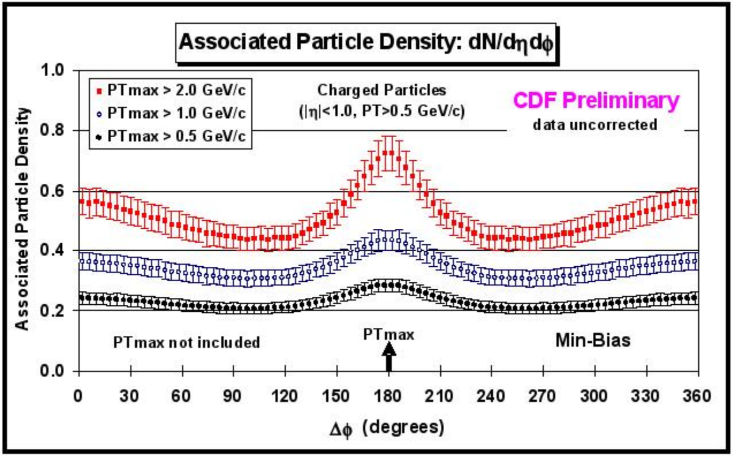
Azimuthal distributions

Measure $\Delta\phi$ relative to leading particle/jet/track.



Azimuthal distributions

Measure $\Delta\phi$ relative to leading particle/jet/track.



Azimuthal distributions

Observation:

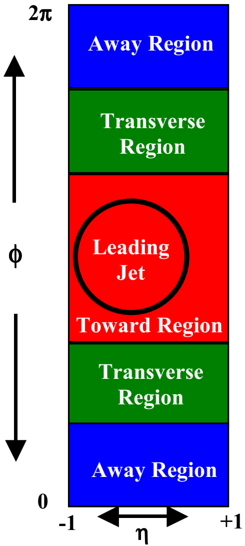
- Events not flat. Have 'leading object'.
- Harder leading object:
  - $\rightarrow$ harder recoil.
  - $\rightarrow$ more activity everywhere, also transverse.

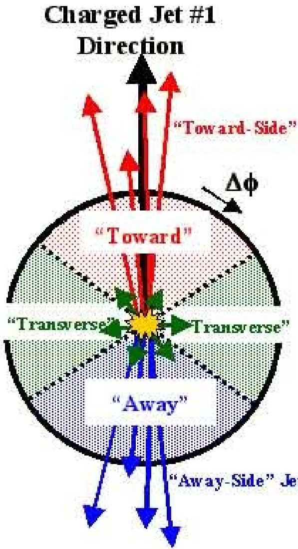
Trigger: The harder leading object, the more jets are inclusively just below this threshold (pedestal effect).

Closer look at transverse region!

"Rick Field analysis".

Towards, away, transverse

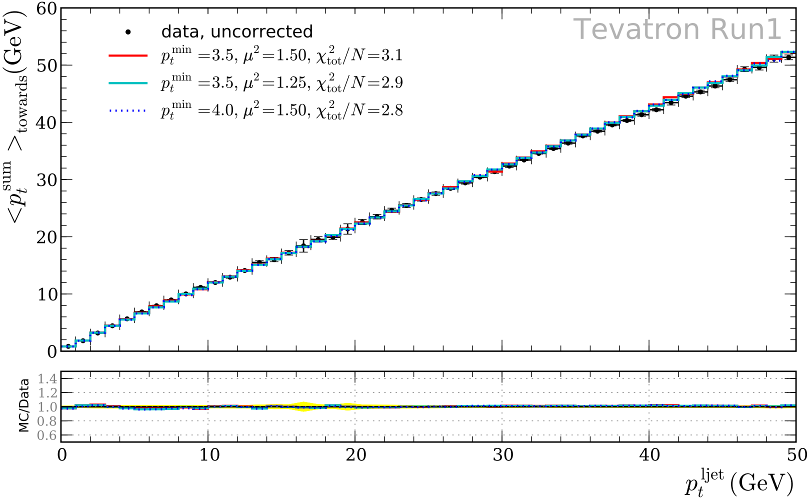




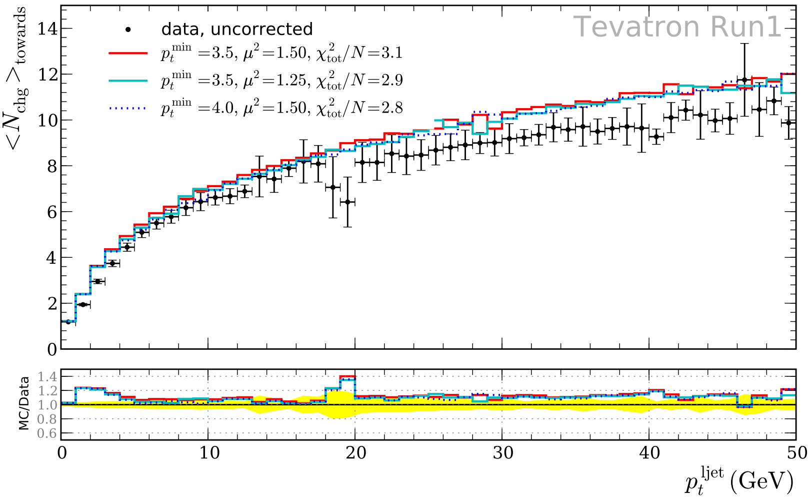
Measurements of the UE: separate from hard bit of event.

- How big is the 'activity' in the different regions?
- How does it depend on the leading object?
- If UE is really *underlying*,
  should decouple from leading event.

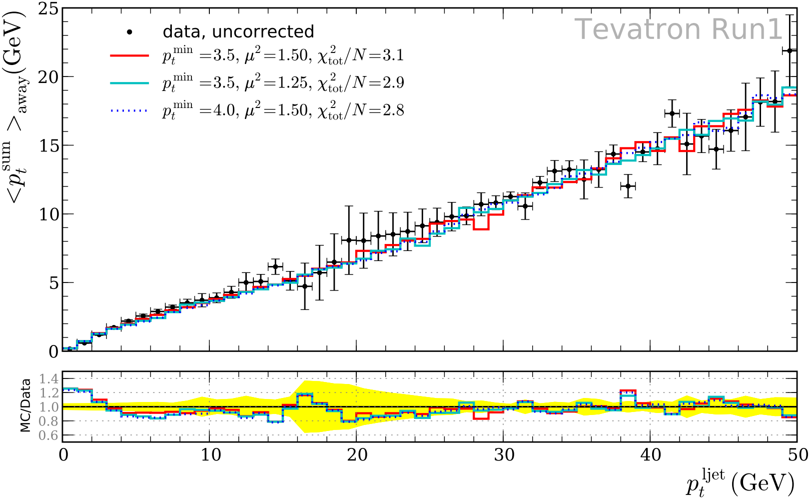
Detailed look at observables: Towards Region



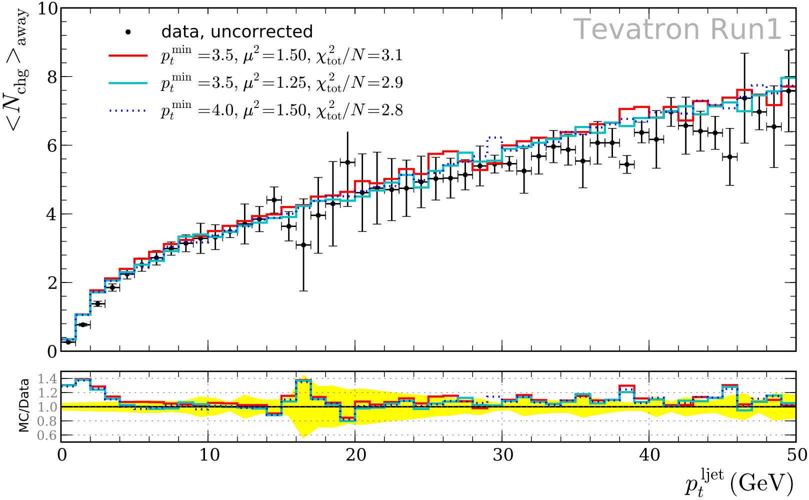
Detailed look at observables: Towards Region



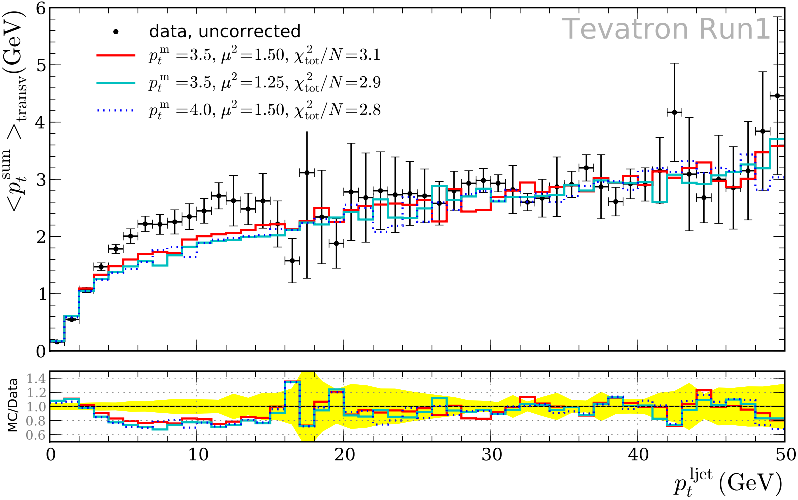
Detailed look at observables: Away Region



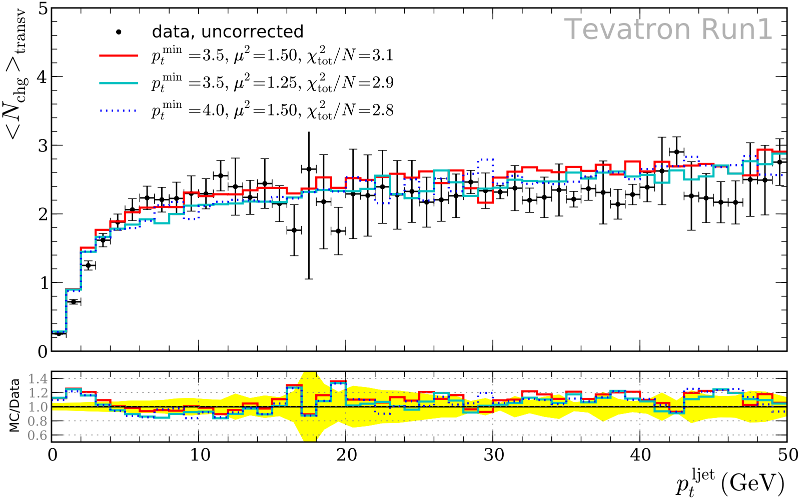
Detailed look at observables: Away Region



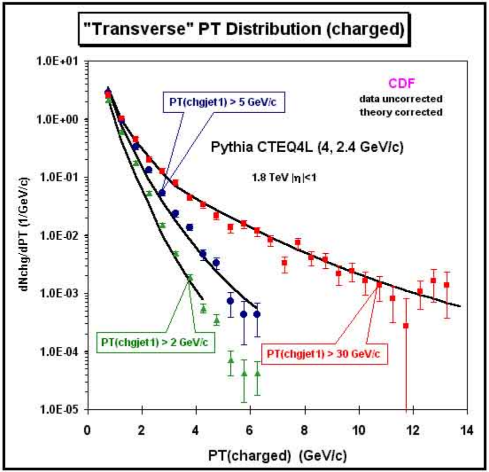
Detailed look at observables: Transverse Region



Detailed look at observables: Transverse Region



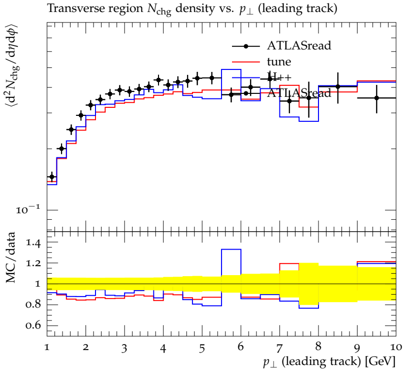
Spectrum in transverse region

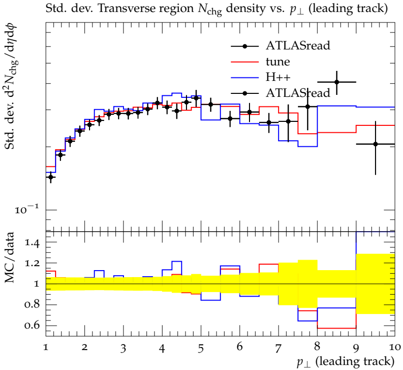


Not only average important. The UE has a jetty substructure!

Underlying Event (ATLAS 900 GeV)

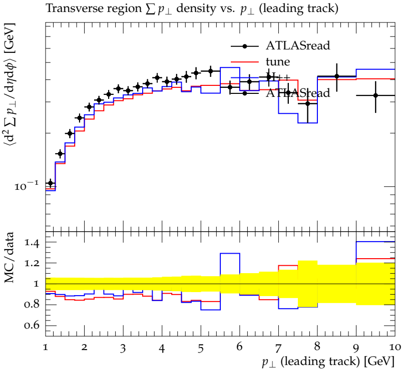
Also include Std deviation!

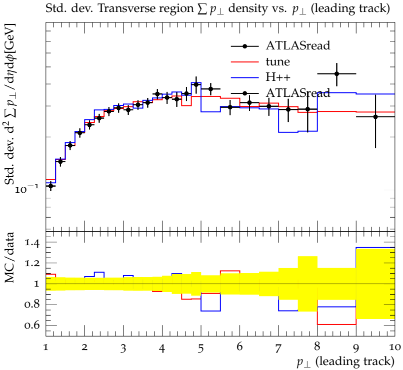




Underlying Event (ATLAS 900 GeV)

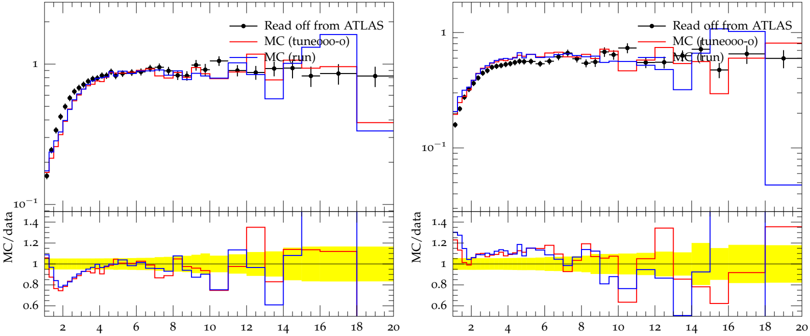
Also include Std deviation!





Underlying Event (ATLAS 7 TeV)

N_{ch}/StdDev transverse vs p_t^{lead}/GeV.

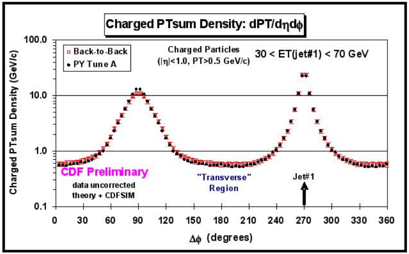


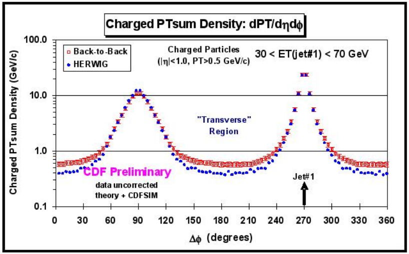
So far

- Idea of decoupling UE from hard event seems to hold.
- UE has jetty structure.
- Must contain hard physics as well.

More azimuthal distributions

Require at least two nearly b2b jets.
Dominated by hard physics.

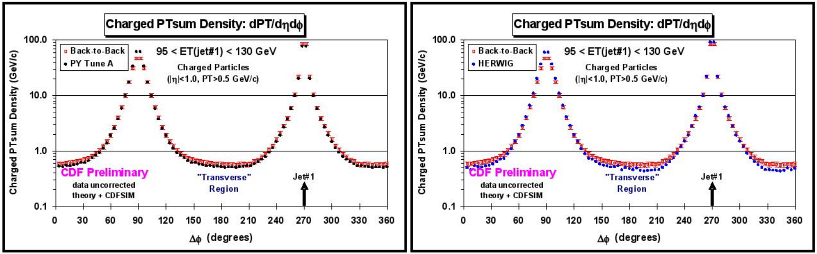




Old Herwig soft model not sufficient.

More azimuthal distributions

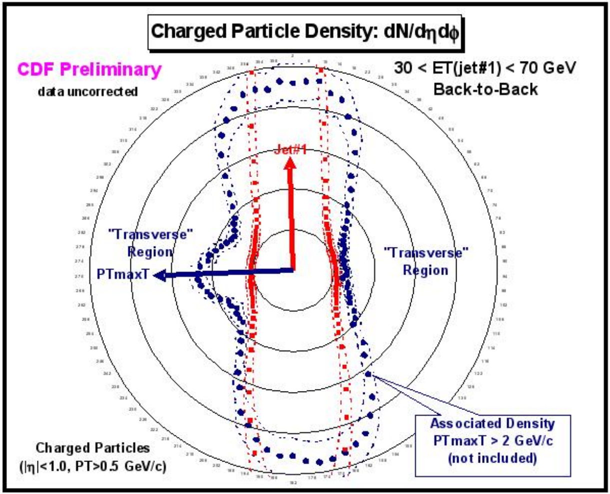
Require at least two nearly b2b jets.
Dominated by hard physics.



Better with harder jets.

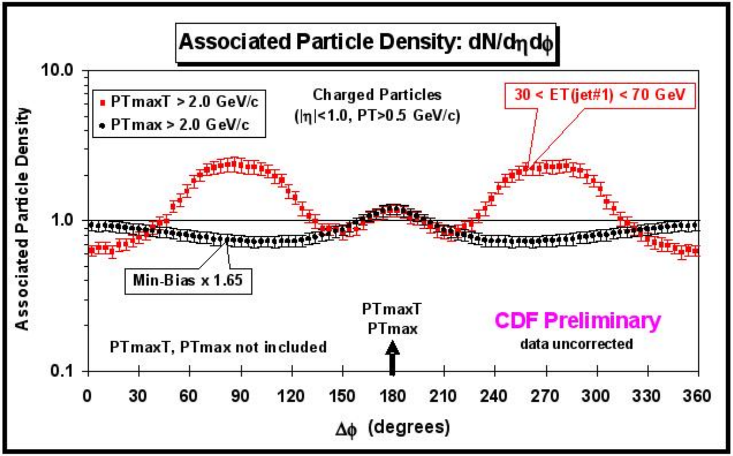
More azimuthal distributions

Now select the hardest of the two transverse regions only (TransMAX): associated distribution:



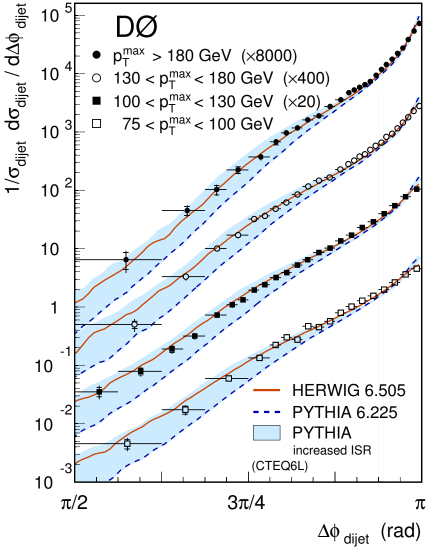
More azimuthal distributions

Now select the hardest of the two transverse regions only (TransMAX): associated distribution:



Birth of 3rd jet ∼ leading jet in MinBias

Hard dijets



Angles between hard jets modeled by parton showers.

Towards modelling

- Leading jet in Minimum bias $\sim$ 3rd jet in back–to–back sample.
- UE and MB really seem to reflect the same physics.
- Hard component important.
- Hard jets not sufficient
  (but well described $\to$ D0 dijet angular decorrelation).

Hard jets in the UE via multiple interactions?

- Additional Partonic $2 \to 2$ interactions (MPI).
- No correlation with hard event.

Indirect evidence for MPI

N_{ch} distribution (vs UA5; Sjöstrand, van Zijl (1987))

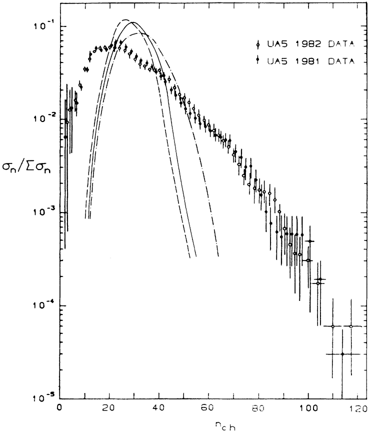


FIG. 3. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs simple models: dashed low p_T only, full including hard scatterings, dash-dotted also including initial- and final-state radiation.

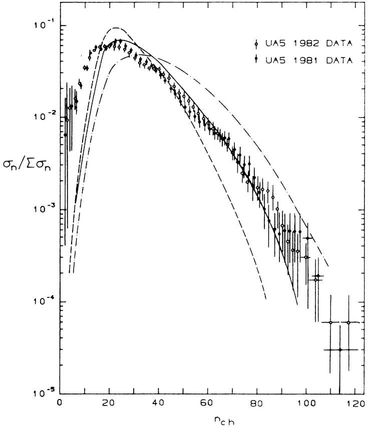


FIG. 5. Charged-multiplicity distribution at 540 GeV, UA5 results (Ref. 32) vs impact-parameter-independent multiple-interaction model: dashed line, $p_{T\min}=2.0$ GeV; solid line, $p_{T\min}=1.6$ GeV; dashed-dotted line, $p_{T\min}=1.2$ GeV.

no MPI (left)/MPI (right).

Indirect evidence for MPI

FB correlation in η bins (vs UA5; Sjöstrand, van Zijl (1987))

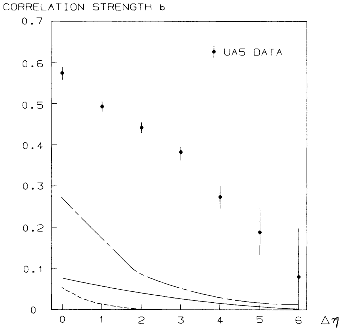


FIG. 4. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs simple models; the latter models with notation as in Fig. 3.

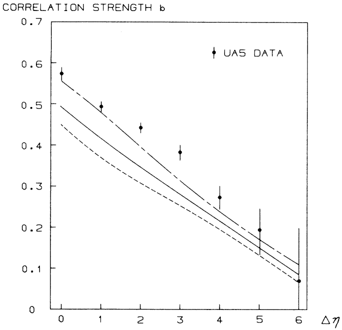
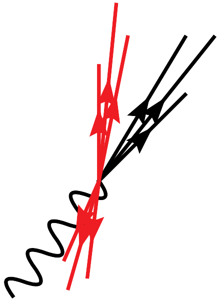


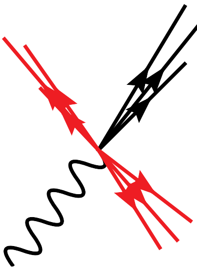
FIG. 6. Forward-backward multiplicity correlation at 540 GeV, UA5 results (Ref. 33) vs impact-parameter-independent multiple-interaction model; the latter with notation as in Fig. 5.

no MPI (left)/MPI (right).

Evidence for MPI

Angle ϕ from 4 final state objects (jets, γ).

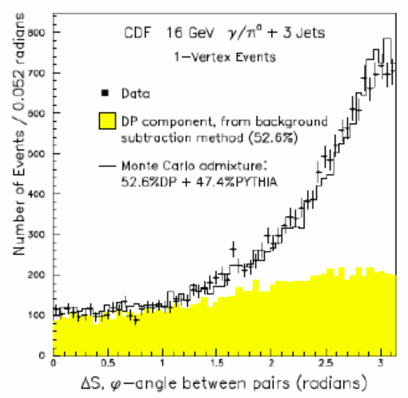




Evidence for MPI

Angle ϕ from 4 final state objects (jets, γ). Latest: CDF ('97).

$$\phi = \angle(\vec{p}_1 \pm \vec{p}_2, \vec{p}_3 \pm p_4)$$



53% double parton scattering needed!

Modelling MPI (in Herwig++)

Semihard UE

- Default from Herwig++ 2.1. [Herwig++, 0711.3137]
- Multiple hard interactions, $p_t \leq p_t^{\mathrm{min}}$. [Bähr, SG, Seymour, JHEP 0807:076]
- Similar to `JIMMY`.
- Good description of harder Run I UE data (Jet20).

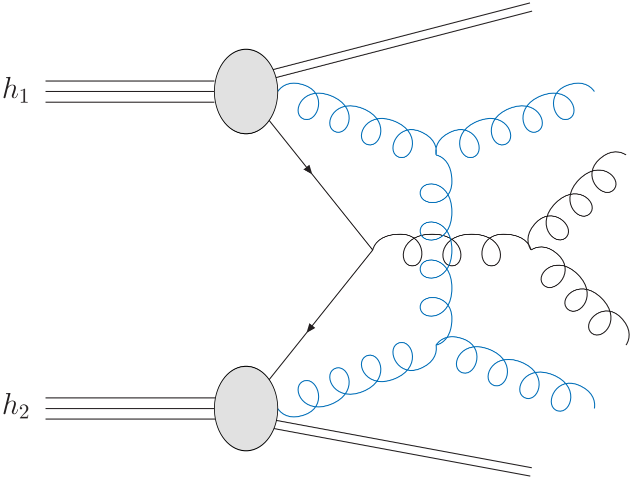
Underlying event in Herwig++

Soft UE

- Default from Herwig++ 2.3. [Herwig++, 0812.0529]
- Extension to soft interactions $p_t < p_t^{\min}$.
- Relation to total cross section, Exploration of parameter space.
- Extrapolation to LHC?
- Theoretical work with simplest possible extension. [Bähr, Butterworth, Seymour, JHEP 0901:065]
- “Hot Spot” model. [Bähr, Butterworth, SG, Seymour, 0905.4671]

Eikonal model basics

Mulitple hard interactions



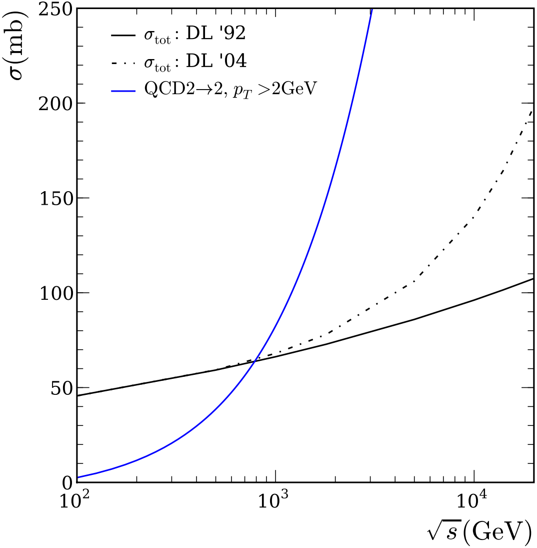
Eikonal model basics

Starting point: hard inclusive jet cross section.

$$\sigma^{\rm inc}(s;p_t^{\min}) = \sum_{i,j}\int_{p_t^{\min 2}} {\rm d}p_t^2 f_{i/h_1}(x_1,\mu^2)\otimes\frac{{\rm d}\hat\sigma_{i,j}}{{\rm d}p_t^2}\otimes f_{j/h_2}(x_2,\mu^2)\,,$$

$\sigma^{\rm inc} > \sigma_{\rm tot}$ eventually (for moderately small $p_t^{\min}$).

Eikonal model basics



Eikonal model basics

Starting point: hard inclusive jet cross section.

$$\sigma^{\mathrm{inc}}(s;p_t^{\mathrm{min}}) = \sum_{i,j}\int_{p_t^{\mathrm{min}\,2}} \mathrm{d}p_t^2 f_{i/h_1}(x_1,\mu^2)\otimes\frac{\mathrm{d}\hat{\sigma}_{i,j}}{\mathrm{d}p_t^2}\otimes f_{j/h_2}(x_2,\mu^2)\,,$$

$\sigma^{\mathrm{inc}} > \sigma_{\mathrm{tot}}$ eventually (for moderately small p_t^{min}).

Interpretation: σ^{inc} counts *all* partonic scatters that happen during a single *pp* collision $\Rightarrow$ more than a single interaction.

$$\sigma^{\mathrm{inc}} = \bar{n}\sigma_{\mathrm{inel}}.$$

Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number m of additional scatters,

$$P_m(\vec{b}, s) = \frac{\bar{n}(\vec{b}, s)^m}{m!} \mathrm{e}^{-\bar{n}(\vec{b}, s)} \ .$$

Then we get σ_{inel}:

$$\sigma_{\mathrm{inel}} = \int \mathrm{d}^2\vec{b} \sum_{m=1}^{\infty} P_m(\vec{b}, s) = \int \mathrm{d}^2\vec{b} \left(1 - \mathrm{e}^{-\bar{n}(\vec{b}, s)}\right) \ .$$

Eikonal model basics

Use eikonal approximation (= independent scatters). Leads to Poisson distribution of number m of additional scatters,

$$P_m(\vec{b},s) = \frac{\bar{n}(\vec{b},s)^m}{m!} \mathrm{e}^{-\bar{n}(\vec{b},s)} \, .$$

Then we get σ_{inel}:

$$\sigma_{\mathrm{inel}} = \int \mathrm{d}^2\vec{b} \sum_{m=1}^{\infty} P_m(\vec{b},s) = \int \mathrm{d}^2\vec{b} \left(1 - \mathrm{e}^{-\bar{n}(\vec{b},s)}\right) \, .$$

Cf. σ_{inel} from scattering theory in eikonal approx. with scattering amplitude $a(\vec{b},s) = \frac{1}{2i}(\mathrm{e}^{-\chi(\vec{b},s)} - 1)$

$$\sigma_{\mathrm{inel}} = \int \mathrm{d}^2\vec{b} \left(1 - \mathrm{e}^{-2\chi(\vec{b},s)}\right) \qquad \Rightarrow \qquad \chi(\vec{b},s) = \frac{1}{2}\bar{n}(\vec{b},s) \, .$$

$\chi(\vec{b},s)$ is called *eikonal* function.

Eikonal model basics

Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$
\begin{aligned}
\bar{n}(\vec{b},s) &= L_{\text{partons}}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&\qquad \times D_{i/A}(x_1,p_t^2,|\vec{b}'|) D_{j/B}(x_2,p_t^2,|\vec{b}-\vec{b}'|)
\end{aligned}
$$

Eikonal model basics

Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$\bar{n}(\vec{b},s) = L_{\text{partons}}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

$$= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

$$\times D_{i/A}(x_1,p_t^2,|\vec{b}'|) D_{j/B}(x_2,p_t^2,|\vec{b}-\vec{b}'|)$$

$$= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2}$$

$$\times f_{i/A}(x_1,p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2,p_t^2) G_B(|\vec{b}-\vec{b}'|)$$

$$= A(\vec{b})\sigma^{\text{inc}}(s;p_t^{\text{min}}) \ .$$

Eikonal model basics

Calculation of $\bar{n}(\vec{b},s)$ from parton model assumptions:

$$
\begin{aligned}
\bar{n}(\vec{b},s) &= L_{\text{partons}}(x_1,x_2,\vec{b}) \otimes \sum_{ij} \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&\qquad \times D_{i/A}(x_1,p_t^2,|\vec{b}'|) D_{j/B}(x_2,p_t^2,|\vec{b}-\vec{b}'|) \\
&= \sum_{ij} \frac{1}{1+\delta_{ij}} \int \mathrm{d}x_1 \mathrm{d}x_2 \int \mathrm{d}^2\vec{b}' \int \mathrm{d}p_t^2 \frac{\mathrm{d}\hat{\sigma}_{ij}}{\mathrm{d}p_t^2} \\
&\qquad \times f_{i/A}(x_1,p_t^2) G_A(|\vec{b}'|) f_{j/B}(x_2,p_t^2) G_B(|\vec{b}-\vec{b}'|) \\
&= A(\vec{b}) \sigma^{\text{inc}}(s;p_t^{\min}) \ .
\end{aligned}
$$

$$
\Rightarrow \quad \chi(\vec{b},s) = \tfrac{1}{2}\bar{n}(\vec{b},s) = \tfrac{1}{2}A(\vec{b})\sigma^{\text{inc}}(s;p_t^{\min}) \ .
$$

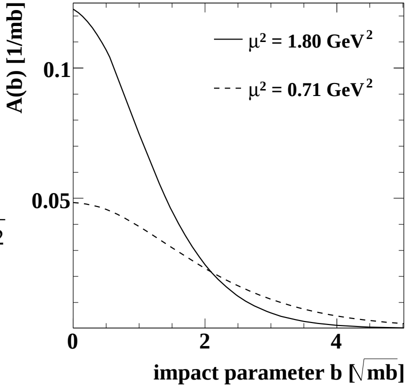
Overlap function

$$A(b) = \int \mathrm{d}^2\vec{b}' G_A(|\vec{b}'|) G_B(|\vec{b} - \vec{b}'|)$$

$G(\vec{b})$ from electromagnetic FF:

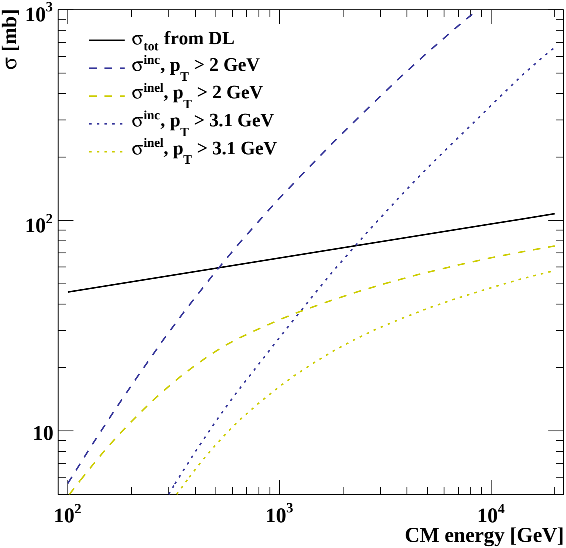
$$G_p(\vec{b}) = G_{\bar{p}}(\vec{b}) = \int \frac{\mathrm{d}^2\vec{k}}{(2\pi)^2} \frac{e^{i\vec{k}\cdot\vec{b}}}{(1+\vec{k}^2/\mu^2)^2}$$

But μ^2 *not fixed* to the *electromagnetic* $0.71\,\mathrm{GeV}^2$.
Free for colour charges.



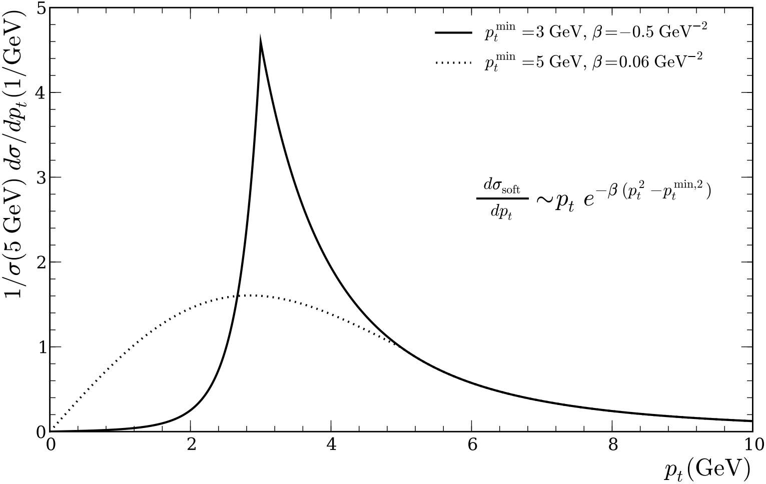
$\Rightarrow$ Two main parameters: $\mu^2, p_t^{\min}$.

Unitarized cross sections



Extending into the soft region

Continuation of the differential cross section into the soft region $p_t < p_t^{\mathrm{min}}$ (here: p_t integral kept fixed)



Hot Spot model

Fix the two parameters μ_{soft} and $\sigma^{\text{inc}}_{\text{soft}}$ in

$$\chi_{\text{tot}}(\vec{b},s) = \frac{1}{2}\left(A(\vec{b};\mu)\sigma^{\text{inc}}\text{hard}(s;p_t^{\text{min}}) + A(\vec{b};\mu_{\text{soft}})\sigma^{\text{inc}}_{\text{soft}}\right)$$

from two constraints. Require simultaneous description of σ_{tot} and b_{el} (measured/well predicted),

$$\sigma_{\text{tot}}(s) \stackrel{!}{=} 2\int \mathrm{d}^2\vec{b}\left(1-\mathrm{e}^{-\chi_{\text{tot}}(\vec{b},s)}\right) ,$$

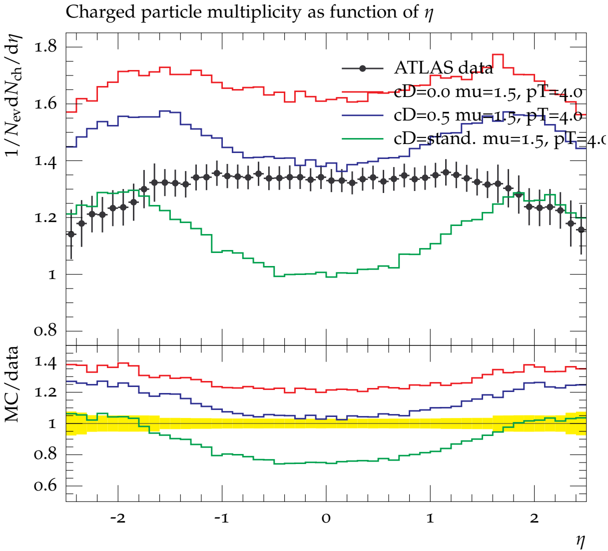
$$b_{\text{el}}(s) \stackrel{!}{=} \int \mathrm{d}^2\vec{b}\,\frac{b^2}{\sigma_{\text{tot}}}\left(1-\mathrm{e}^{-\chi_{\text{tot}}(\vec{b},s)}\right) .$$

First plots against LHC data

- Not all aspects well described.
- Despite very good agreement with Rick Field's CDF UE analysis.
- Observe sensitivity to colour structure.

First plots against LHC data

Colour structure of soft events.

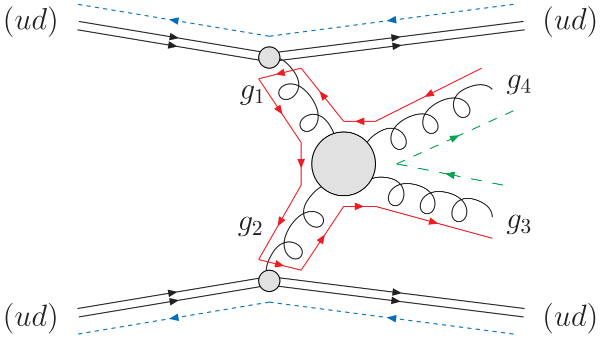


Colour structure

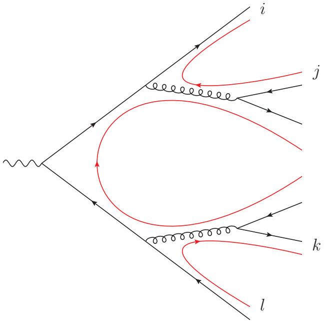
Sensitivity to parameter

$$\texttt{colourDisrupt} = P(\text{disrupt colour lines})$$

(as opposed to hard QCD).



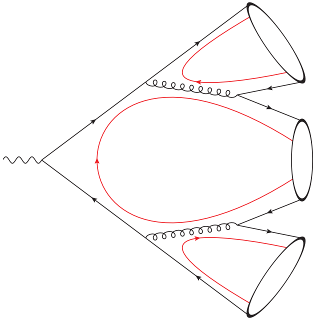
Colour reconnection (CR) in Herwig++



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* ⇒ colour-anticolour pairs

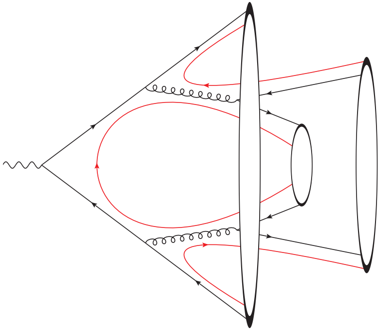
Colour reconnection (CR) in Herwig++



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* $\Rightarrow$ colour-anticolour pairs
- $\rightarrow$ *clusters*

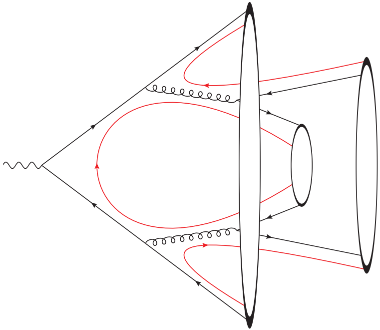
Colour reconnection (CR) in Herwig++



Extend cluster hadronization:

- QCD parton showers provide *pre-confinement* $\Rightarrow$ colour-anticolour pairs
- $\rightarrow$ *clusters*
- CR in the cluster hadronization model: allow *reformation* of clusters, *e.g.* $(il) + (jk)$

Colour reconnection (CR) in Herwig++



Extend cluster hadronization:

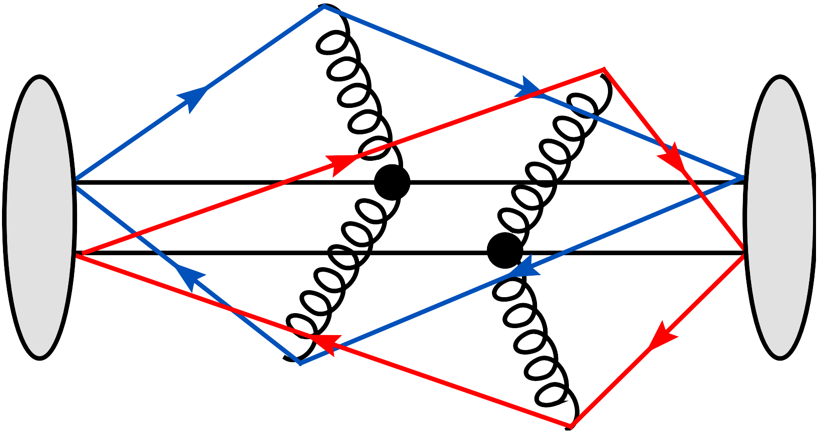
- QCD parton showers provide *pre-confinement* $\Rightarrow$ colour-anticolour pairs
- $\rightarrow$ *clusters*
- CR in the cluster hadronization model: allow *reformation* of clusters, *e.g.* $(il) + (jk)$

- Allow CR if the cluster mass decreases,

$$M_{il} + M_{kj} < M_{ij} + M_{kl},$$

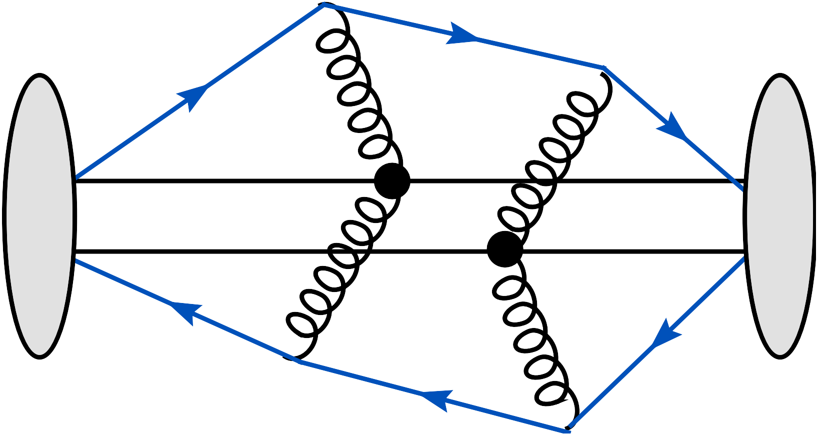
- Accept alternative clustering with probability p_{reco} (model parameter) $\Rightarrow$ this allows to switch on CR smoothly

Colour reconnection at hadron colliders



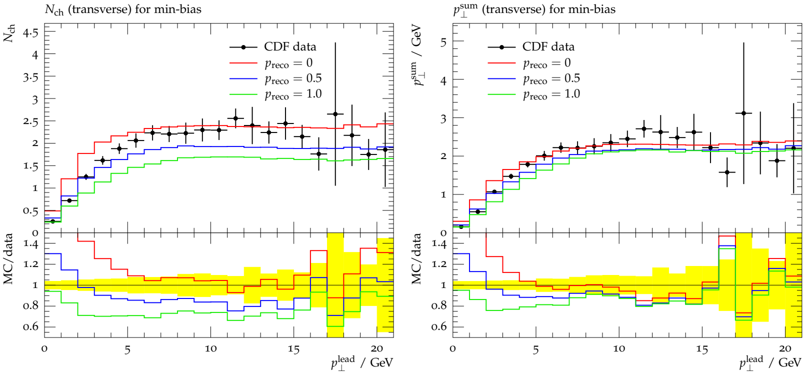
- Colour preconfinement
- Shorten colour string/lower mass clusters.

Colour reconnection at hadron colliders



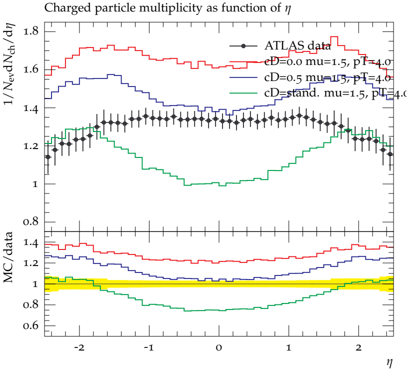
- Colour preconfinement
- Shorten colour string/lower mass clusters.

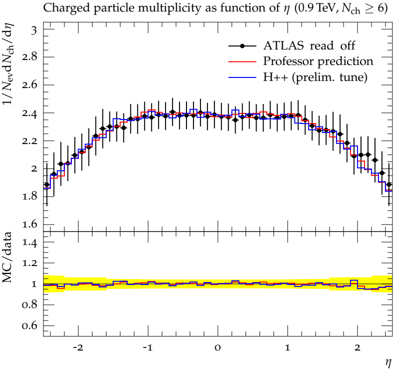
A quick look at CDF data



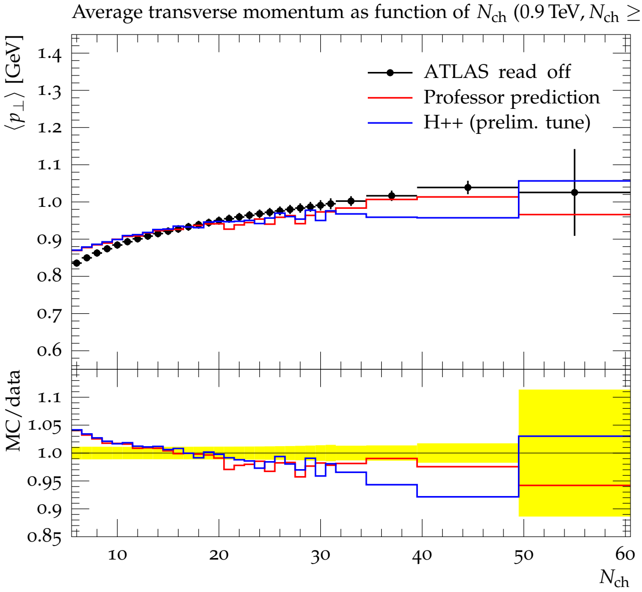
Sensitivity different for the two observables.

Comparison with MinBias ATLAS data (900 GeV)

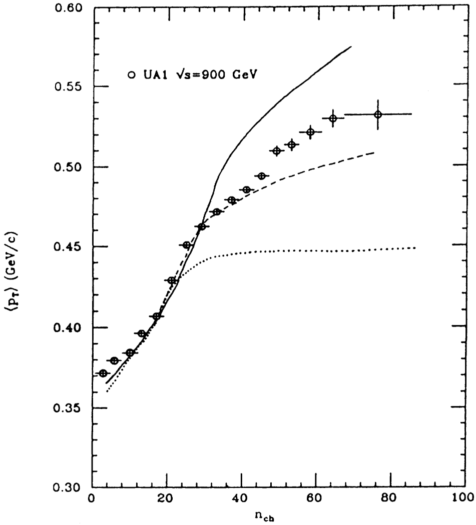




Comparison with MinBias ATLAS data (900 GeV)



Colour reconnections

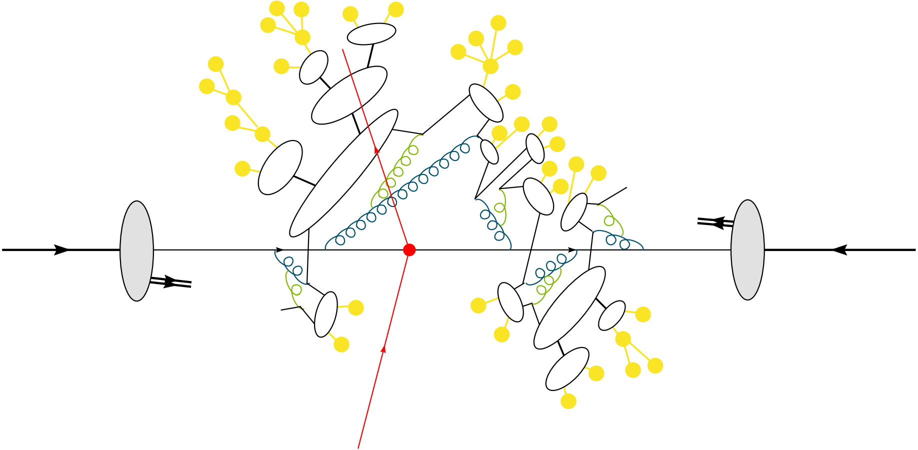


- Sensitivity to CR already known from UA5.
- (From Sjöstrand / van Zijl)

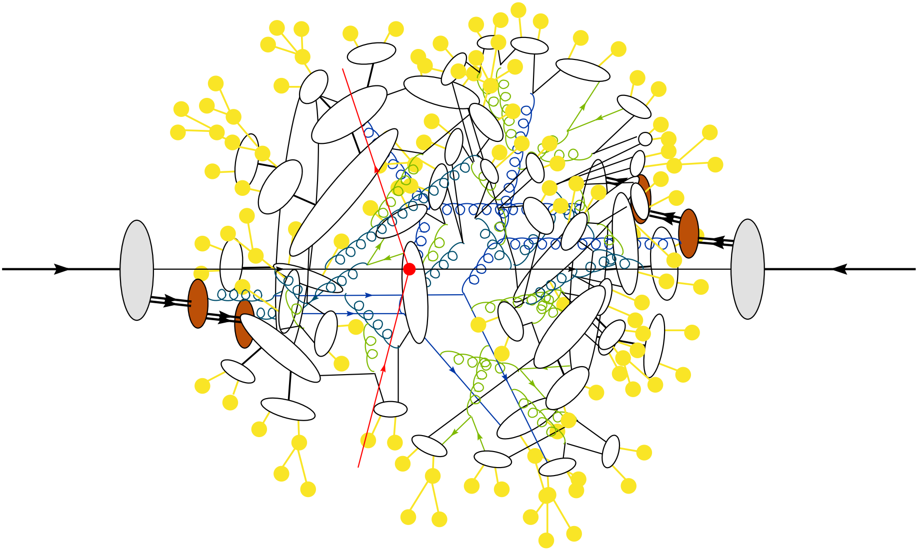
MPI Summary

- MPI (with colour reconnections) currently model of choice.
- Describes averages *and* fluctuations.
- Not always universal, but all models tunable.
- soft component needed for MB modelling.
- Constraints from inclusive cross sections.
- Different emphasis on hard/soft modelling between generators.
- Many details still only models.

Brief graphical summary



Brief graphical summary



Monte Carlo

training studentships

3-6 month fully funded studentships for current PhD students at one of the MCnet nodes. An excellent opportunity to really understand the Monte Carlos you use!

Application rounds every 3 months.

MCnet

funded by:

MARIE CURIE ACTIONS

for details go to:
www.montecarlonet.org