Singlet Extension of the SM Higgs Sector

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December 3, 2015

Overview

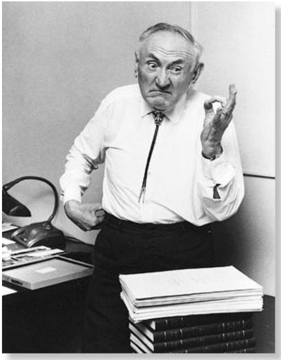
Motivation for an extension of the Higgs sector

Addition of a real singlet scalar (xSM)

Addition of a complex singlet scalar (cxSM)

Cosmological mystery: the 'missing mass' problem

Jan Oort (1900-1992)



Fritz Zwicky (1898-1974)

Motion of galaxies and stars in the universe $\Rightarrow$ dark matter.
Estimated to account for about 85% of the mass in the universe.
But where does it come from?

MACHOs and WIMPs

Massive compact halo object

Weakly interacting massive particle

MACHOs and WIMPs

Properties of WIMPs

- Little interaction with SM particles.
- large mass (for a particle).
- Readily predicted by simple extensions of the SM Higgs sector.

We study the addition of a real (xSM) as well as a complex (cxSM) singlet scalar to the Higgs doublet.

Let H be the SM Higgs doublet and s be a single gauge singlet real scalar field.

Consider the potential

$$\begin{aligned} V =& \mu^2 \left(H^\dagger H\right) + \lambda \left(H^\dagger H\right)^2 \\ &+ a_1 \left(H^\dagger H\right) s + a_2 \left(H^\dagger H\right) s^2 \\ &+ \frac{b_2}{2} s^2 + \frac{b_3}{3} s^3 + \frac{b_4}{4} s^4 . \end{aligned}$$

Note: V is $\mathbb{Z}_2$ symmetric in s for $a_1 = b_3 = 0$ (i.e. symmetric under $s \to -s$).

$$
\begin{aligned}
V = & \mu^2 \left(H^\dagger H\right) + \lambda \left(H^\dagger H\right)^2 \\
& + a_1 \left(H^\dagger H\right) s + a_2 \left(H^\dagger H\right) s^2 \\
& + \frac{b_2}{2} s^2 + \frac{b_3}{3} s^3 + \frac{b_4}{4} s^4 .
\end{aligned}
$$

What are the conditions on V?

- It must be bounded from below (existence of a vacuum).
- It must accomodate electroweak symmetry breaking $\Rightarrow \langle H \rangle \neq 0$.
- It should yield a massive stable scalar s.

The stationary conditions

We write

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h \end{pmatrix}, \text{ with } h \text{ real (unitary gauge)},$$

and denote the vacuum expectation values of h and s with v and v_s.

With this, the stationary conditions of V

$$\left.\frac{\partial V}{\partial h}\right|_{(h,s)=(v,v_s)} = \left.\frac{\partial V}{\partial s}\right|_{(h,s)=(v,v_s)} = 0$$

yield

$$\mu^2 = -\lambda v^2 - v_s(a_1 + a_2 v_s),$$
$$a_1 = -a_2 v_s - \frac{2b_2 v_s}{v^2} - \frac{2b_3 v_s^2}{v^2} - \frac{2b_4 v_s^3}{v^2}.$$

Using the equation for μ^2 we now calculate the mass squared matrix

$$M^2 = \left. \begin{pmatrix} \frac{\partial^2 V}{\partial h^2} & \frac{\partial^2 V}{\partial h \partial s} \\ \frac{\partial^2 V}{\partial s \partial h} & \frac{\partial^2 V}{\partial s^2} \end{pmatrix} \right|_{(h,s)=(v,v_s)}$$

$$= \begin{pmatrix} 2\lambda v^2 & a_1 v + 2a_2 v v_s \\ a_1 v + 2a_2 v v_s & a_2 v^2 + b_2 + 2b_3 v_s + 3b_4 v_s^2 \end{pmatrix}.$$

Note: A $\mathbb{Z}_2$ symmetry ($a_1 = b_3 = 0$) is **not** sufficient to eliminate the mixing terms.

This is because the acquisition of a nonzero vev $v_s \neq 0$ of the scalar s **breaks** the $\mathbb{Z}_2$ symmetry (if imposed) **spontaneously**.
$\Rightarrow$ unwanted mixing terms.
$\Rightarrow$ instability of the mass eigenstates.
$\Rightarrow$ no DM candidate.
So, in order to obtain a viable dark matter candidate, we now assume

$$a_1 = b_3 = \langle s \rangle = 0.$$

Constraints on the potential

After electroweak symmetry breaking, for which we shift $h \equiv v + h$, the potential reads

$$V = -\frac{\mu^4}{4\lambda} - \mu^2 h^2 + \lambda v h^3 + \frac{\lambda}{4} h^4 + \frac{1}{2}(b_2 + a_2 v^2) s^2 + \frac{b_4}{4} s^4 + a_2 v s^2 h + \frac{a_2}{2} s^2 h^2 .$$

Necessary conditions:

- Existence of a vacuum: $\lambda, b_4 \geq 0$ and $\lambda b_4 \geq a_2^2$ for negative a_2.
- The mass squared matrix $M^2 = \mathrm{diag}(2\lambda v, b_2 + a_2 v^2)$ must be positive definite.

Note: The phenomenological properties of this model are completely determined by a_2 and b_2, or a_2 and $m_s^2 = b_2 + a_2 v^2$.

Experimental and theoretical constraints on the parameters

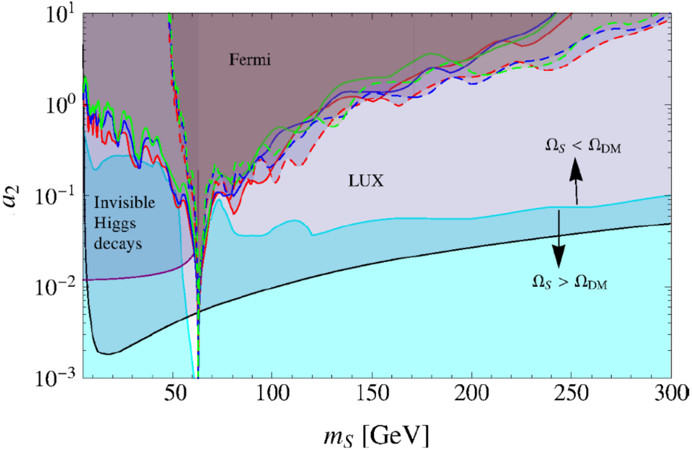


Figure: taken from Lei Feng, S. Profumo, L. Ubaldi, [arXiv:1412.1105]

Highly constrained parameter space for the xSM!

Another cosmological mystery: the baryon asymmetry

Number of baryons $\gg$ number of antibaryons in the **observable** universe.

Possible Explanations:

- There **is** as much antimatter, as there is matter, but its all clunked together far away.
- The universe **began** with a small preference for matter.
- The universe was initially perfectly symmetric, but somehow matter was favoured over time.
  This requires the electroweak symmetry breaking to be a first order phase transition.
  In the context of SM, this requires $m_h \lesssim 70\,\text{GeV}$. In the context of xSM, this requires $\langle S \rangle \neq 0$.

xSM - Conclusive remarks

The xSM Model

- yields either a stable CDM candidate , that **doesn't affect EWPT** ($\langle S \rangle = 0$), or
- generates strong first order EWPT, but only yields **unstable mass eigenstates** ($\langle S \rangle \neq 0$).

So, it is impossible to explain both these mysteries in the context of a single xSM.

Unsatisfactory?

Let $\mathbb{S} = S + iA$ be a single gauge singlet complex scalar field.
Consider the $U(1)$ and $\mathbb{Z}_2$ symmetric Potential

$$V = \frac{m^2}{2}\left(H^\dagger H\right) + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2} H^\dagger H \left|\mathbb{S}\right|^2 + \frac{b_2}{2}\left|\mathbb{S}\right|^2 + \frac{d_2}{4}\left|\mathbb{S}\right|^4$$

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Glodstone's theorem: $\langle \mathbb{S} \rangle \neq 0 \Rightarrow$ massless particle (Spontaneous breaking of the $U(1)$ symmetry).

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$$V = \frac{m^2}{2}\left(H^\dagger H\right) + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2}H^\dagger H\left|\mathbb{S}\right|^2 + \frac{b_2}{2}\left|\mathbb{S}\right|^2 + \frac{d_2}{4}\left|\mathbb{S}\right|^4 + \left(\frac{|b_1|}{4}e^{i\phi_{b_1}}\mathbb{S}^2 + c.c.\right)$$

Glodstone's theorem: $\langle\mathbb{S}\rangle \neq 0 \Rightarrow$ massless particle (Spontaneous breaking of the $U(1)$ symmetry).

We therefore break the $U(1)$ symmetry explicitly.

Let $\mathbb{S} = S + iA$ be a single gauge singlet complex scalar field. Consider the Potential

$$V = \frac{m^2}{2}\left(H^\dagger H\right) + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2} H^\dagger H \left|\mathbb{S}\right|^2 + \frac{b_2}{2}\left|\mathbb{S}\right|^2 + \frac{d_2}{4}\left|\mathbb{S}\right|^4$$
$$+ \left(\frac{|b_1|}{4} e^{i\phi_{b_1}}\mathbb{S}^2 + |a_1|\, e^{i\phi_{a_1}}\mathbb{S} + c.c.\right)$$

In the same fashion, we explicitly break the $\mathbb{Z}_2$ symmetry.

Let $\mathbb{S} = S + iA$ be a single gauge singlet complex scalar field. Consider the Potential

$$V = \frac{m^2}{2}\left(H^\dagger H\right) + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2} H^\dagger H \left|\mathbb{S}\right|^2 + \frac{b_2}{2}\left|\mathbb{S}\right|^2 + \frac{d_2}{4}\left|\mathbb{S}\right|^4$$
$$+ \left(\frac{|b_1|}{4} e^{i\phi_{b_1}} \mathbb{S}^2 + |a_1| \, e^{i\phi_{a_1}} \mathbb{S} + c.c.\right)$$

We study the cases

- A1 $\langle\mathbb{S}\rangle = 0;\ \ a_1 = b_1 = 0.$ (Unbroken $U(1)$)
- A2 $\langle\mathbb{S}\rangle = 0;\ \ a_1 = 0,\ b_1 \neq 0.$ (explicitly broken $U(1)$)
- B1 $\langle\mathbb{S}\rangle \neq 0;\ \ a_1 = b_1 = 0.$ (spontaneously broken $U(1)$)
- B2 $\langle\mathbb{S}\rangle \neq 0;\ \ a_1 \neq 0,\ b_1 \neq 0.$ (explicitly broken $U(1)$ and $\mathbb{Z}_2$)

Constraints on the potential

$$V = \frac{m^2}{2}\left(H^\dagger H\right) + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2} H^\dagger H \left|\mathbb{S}\right|^2 + \frac{b_2}{2}\left|\mathbb{S}\right|^2 + \frac{d_2}{4}\left|\mathbb{S}\right|^4 + \left(\frac{|b_1|}{4} e^{i\phi_{b_1}} \mathbb{S}^2 + |a_1| e^{i\phi_{a_1}} \mathbb{S} + c.c.\right)$$

- Existence of a vacuum (v, v_S):
  We take $\lambda > 0,\ \ d_2 > 0 \ \Rightarrow\ $ if $\delta_2 < 0$ then $\lambda d_2 > \delta_2^2$.
- For simplicity, we take $\phi_{b_1} = \phi_{a_1} = \pi \ \Rightarrow\ \langle A \rangle = 0$.
- The vacuum must be a local minimum, so the mass squared matrix must be positive definite.

Case A: $\langle \mathbb{S} \rangle = 0$.

The mass matrix in $(v, 0)$ is $M^2 = \mathrm{diag}\left(M_h^2, M_S^2, M_A^2\right)$,where

$$M_h^2 = \frac{1}{2}\lambda v^2,$$

$$M_S^2 = -\frac{1}{2}|b_1| + \frac{1}{2}b_2 + \frac{\delta_2 v^2}{4},$$

$$M_A^2 = \frac{1}{2}|b_1| + \frac{1}{2}b_2 + \frac{\delta_2 v^2}{4}.$$

For case A1, that is $b_1 = 0$, we obtain two phenomenologically aquivalent particles.
$\longrightarrow$ xSM.

Case A2: $\langle \mathbb{S} \rangle = 0$; $a_1 = 0$, $b_1 \neq 0$

$$V = \frac{m^2}{2}\left(H^\dagger H\right) + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2} H^\dagger H \left|\mathbb{S}\right|^2 + \frac{b_2}{2}\left|\mathbb{S}\right|^2 + \frac{d_2}{4}\left|\mathbb{S}\right|^4 + \left(-\frac{|b_1|}{4}\mathbb{S}^2 + c.c.\right)$$

$$M^2_{S/A} = \mp\frac{1}{2}\left|b1\right| + \frac{1}{2} b_2 + \frac{\delta_2 v^2}{4}.$$

No mixing of the scalars.
Stable two-component dark matter scenario.

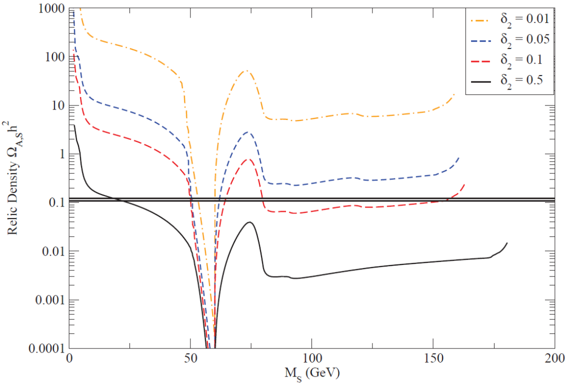

Figure: taken from V. Barger et al., [arXiv:0811.0393]
Contribution to the relic density over the mass of the light scalar
$M_S^2 = -\frac{1}{2}\,|b1| + \frac{1}{2}b_2 + \frac{\delta_2 v^2}{4}$.
$M_H = 120\,\mathrm{GeV}$,
$b_2 = 50000\,\mathrm{GeV}^2$,
$d_2 = 1$.

Case B1: $\langle \mathbb{S} \rangle \neq 0; \;\; a_1 = b_1 = 0.$

$$V = \frac{m^2}{2}\left(H^\dagger H\right) + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2} H^\dagger H \left|\mathbb{S}\right|^2 + \frac{b_2}{2}\left|\mathbb{S}\right|^2 + \frac{d_2}{4}\left|\mathbb{S}\right|^4 .$$

$$M^2 = \begin{pmatrix} \lambda v^2/2 & \delta_2 v v_S/2 & 0 \\ \delta_2 v v_s/2 & d_2 v_S^2/2 & 0 \\ 0 & 0 & 0 \end{pmatrix} .$$

Two unstable mixed scalars.
A is stable but massles.
$\Rightarrow$ no dark matter candidate.

Case B2: $\langle \mathbb{S} \rangle \neq 0$; $a_1 \neq 0$, $b_1 \neq 0$.

$$V = \frac{m^2}{2}\left(H^\dagger H\right) + \frac{\lambda}{4}\left(H^\dagger H\right)^2 + \frac{\delta_2}{2} H^\dagger H \left|\mathbb{S}\right|^2 + \frac{b_2}{2}\left|\mathbb{S}\right|^2 + \frac{d_2}{4}\left|\mathbb{S}\right|^4$$
$$+ \left(-\frac{|b_1|}{4}\mathbb{S}^2 - |a_1|\,\mathbb{S} + c.c.\right).$$

$$M^2 = \begin{pmatrix} \lambda v^2/2 & \delta_2 v v_S/2 & 0 \\ \delta_2 v v_s/2 & d_2 v_S^2/2 + \sqrt{2}\,|a_1|\,/v_S & 0 \\ 0 & 0 & |b_1| + \sqrt{2}\,|a_1|\,/v_S \end{pmatrix}.$$

Two unstable mixed scalars.
A remains stable (no mixing) and $M_A^2 = |b_1| + \frac{\sqrt{2}|a_1|}{v_S} > 0$.
$\Rightarrow$ A candidate for dark matter!

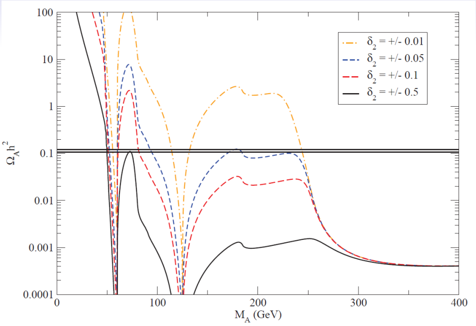

Figure: Contribution to the relic density over the mass M_A. $v_S = 100$ GeV, $M_{h_1} = 120$ GeV, $M_{h_2} = 250$ GeV. (V. Barger et al., [arXiv:0811.0393])

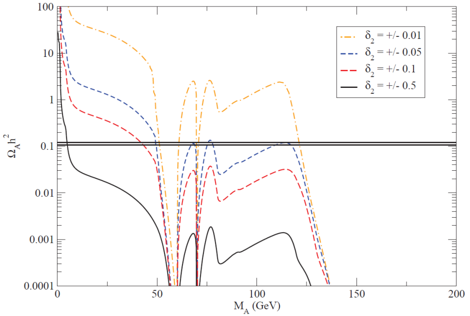

Figure: Contribution to the relic density over the mass M_A. $v_S = 10$ GeV, $M_{h_1} = 120$ GeV, $M_{h_2} = 140$ GeV. (V. Barger et al., [arXiv:0811.0393])

cxSM - Conclusive remarks

The cxSM model

- yields a simple two-component DM scenario, if the $U(1)$ symmetry is explicitly but not spontaneously broken.
- yields a single-component DM scenario **and** allows for first order EWPT, as required for electroweak baryogenesis, if the $U(1)$ symmetry is both explicitly and spontaneously broken.