MSSM Higgs sector with CP violation

- Introduction of two complex Higgs doublets

- Explicit SUSY breaking $\Rightarrow$ many new (complex) parameters

- Complex parameters can lead to CP- or T-violation:

  T-operator: antiunitary:
  complex conjugation of complex parameters

- MSSM: parameters are in general complex:

  they are not forbidden by a symmetry as

  CP-symmetry is no fundamental symmetry in nature:
  - ▷ observation of CP-violation in K- and B-systems
  - ▷ CP-violation is needed for explanation of baryogenesis

MSSM Higgs potenial at Born level

$$
\begin{aligned}
V_{Higgs} &= \frac{g^2 + g'^2}{8}(H_1^\dagger H_1 - H_2^\dagger H_2)^2 + \frac{g^2}{2}|H_1^\dagger H_2|^2 + |\mu|^2(H_1^\dagger H_1 + H_2^\dagger H_2) \\
&\quad +(m_1^2 H_1^\dagger H_1 + m_2^2 H_2^\dagger H_2) + (\epsilon_{ij}|m_{12}^2|e^{i\varphi_{m_{12}^2}} H_1^i H_2^j + h.c.)
\end{aligned}
$$

g, g' :	gauge couplings
μ :	coupling between Higgs superfields
m_1^2, m_2^2, m_{12}^2:	soft breaking parameters

- one complex parameter: $m_{12}^2 e^{i\varphi_{m_{12}^2}}$

◇ If $m_{12} = 0$ and $\mu = 0$:

MSSM: further U(1) symmetry: Peccei-Quinn symmetry

◇ If $m_{12} \neq 0$ and $\mu \neq 0$:

perform a Peccei-Quinn transformation with angle φ_{PQ}, redefine

$$
\begin{aligned}
(m_{12}^2)' &= |m_{12}|^2 e^{i(\varphi_{m_{12}^2} - \varphi_{PQ})} \\
(\mu)' &= |\mu| e^{i(\varphi_\mu - \varphi_{PQ})}
\end{aligned}
$$

physical content of Lagrangian does not change

$\Rightarrow$ m_{12}^2 can always be chosen to be real

Higgs vacuum expectation values

Scalar Higgs doublets in the vacuum state:

$$H_1|_{vac} = \begin{pmatrix} v_1 \\ 0 \end{pmatrix} \qquad H_2|_{vac} = e^{i\xi} \begin{pmatrix} 0 \\ v_2 \end{pmatrix}$$

$\Rightarrow$ vacuum expection values can differ by a phase ξ

Expansion about the vacuum

$$H_1|_{vac} = \begin{pmatrix} v_1 + \frac{1}{\sqrt{2}}(\phi_1^0 - i\zeta_1^0) \\ -\phi_1^- \end{pmatrix} \qquad H_2|_{vac} = e^{i\xi} \begin{pmatrix} \phi_2^+ \\ v_2 + \frac{1}{\sqrt{2}}(\phi_2^0 + i\zeta_2^0) \end{pmatrix}$$

- four real scalar fields: $\phi_1^0, \phi_2^0, \zeta_1^0, \zeta_2^0$
- two complex scalar fields: $\phi_1^\pm, \phi_2^\pm$

} no mass eigenstates

Higgs sector at Born level

Mass terms: bilinear terms in the Higgs potential

$$V_{\text{Higgs}}|_{\text{bil}} = \frac{1}{2}(\phi_1^0,\phi_2^0)\mathcal{M}_{\phi^0}\begin{pmatrix}\phi_1^0\\ \phi_2^0\end{pmatrix} + \ldots \qquad \text{mixing between}(\phi_1^0,\phi_2^0),(\zeta_1^0,\zeta_2^0)$$

$$+ \frac{1}{2}(\phi_1^0,\phi_2^0)\mathcal{M}_{\phi^0\zeta^0}\begin{pmatrix}\zeta_1^0\\ \zeta_2^0\end{pmatrix} + \ldots \qquad \text{mixing between}(\phi_1^0,\phi_2^0,\zeta_1^0,\zeta_2^0)$$

Matrix $\mathcal{M}_{\phi^0\zeta^0}$ generates mixing between the ϕ^0- and ζ^0- fields

$$\mathcal{M}_{\phi^0\zeta^0} = \begin{pmatrix} 0 & m_{12}^2\sin(\xi) \\ -m_{12}^2\sin(\xi) & 0 \end{pmatrix} \tag{1}$$

⇒ no such mixing in case of real parameters

Minimum condition for the vacuum:

$$\frac{\partial V_{\text{Higgs}}}{\partial H_j^i}|_{vac} = 0 \quad , \qquad i,j = 1,2$$

Fulfilled if terms linear in the Higgs fields vanish:

$$V_{\text{Higgs}}|_{lin} = -t_{\phi_1^0}\phi_1^0 - t_{\phi_2^0}\phi_2^0 - t_{\zeta_1^0}\zeta_1^0 - t_{\zeta_2^0}\zeta_2^0$$

In particular

$$t_{\zeta_1^0} = -\frac{v_1}{v_2} t_{\zeta_2^0} = \sqrt{2} m_{12}^2 v_2 \sin(\xi) \stackrel{!}{=} 0$$

$\Rightarrow$ phase ξ must vanish: $\xi = 0$
at Born level: no CP-violating phases

Higgs sector at Born level

Physical mass eigenstates:

- $\triangleright$ 2 CP-even Higgs bosons: H^0, h^0 $\xleftarrow{\text{mixing}}$ ϕ_1^0, ϕ_2^0
- $\triangleright$ 1 CP-odd Higgs boson: A^0 $\xleftarrow{\text{mixing}}$ ζ_1^0, ζ_2^0
- $\triangleright$ 2 charged Higgs bosons: $H^\pm$ $\xleftarrow{\text{mixing}}$ $\phi_1^\pm, \phi_2^\pm$

Masses of the Higgs bosons:

- $\triangleright$ not all independent
- $\triangleright$ lightest Higgs boson: h^0

- Real parameters:

  upper theoretical Born mass limit: $M_{h^0} \leq M_Z = 91$ GeV

  with quantum corrections of higher orders: $M_{h^0} \lesssim 140$ GeV

  quantum corrections are dependent on the MSSM parameters

- Complex parameters:

  quantum corrections will also depend on parameter phases

M_{h^0} as precision observable

- Experiment:

  exclusion limits, accurate measurement $\rightsquigarrow$ mass of the Higgs boson

- Theory:

  precise prediction $\rightsquigarrow$ mass of the Higgs boson

◇ $\Rightarrow$ precision calculation necessary!

i.e. taking into account higher order corrections
(truncation of perturbation series $\Rightarrow$ theoretical uncertainty)

◇ Exclusion of parts of the parameter space $\rightsquigarrow$ strong constraints on the MSSM parameters

CP violating phases in other sectors

- Soft SUSY breaking parameters

▷ Sfermion sector: phases φ_{A_f} of the trilinear couplings A_f

▷ Gaugino sector: phases of gaugino mass parameters M_1, M_2, M_3

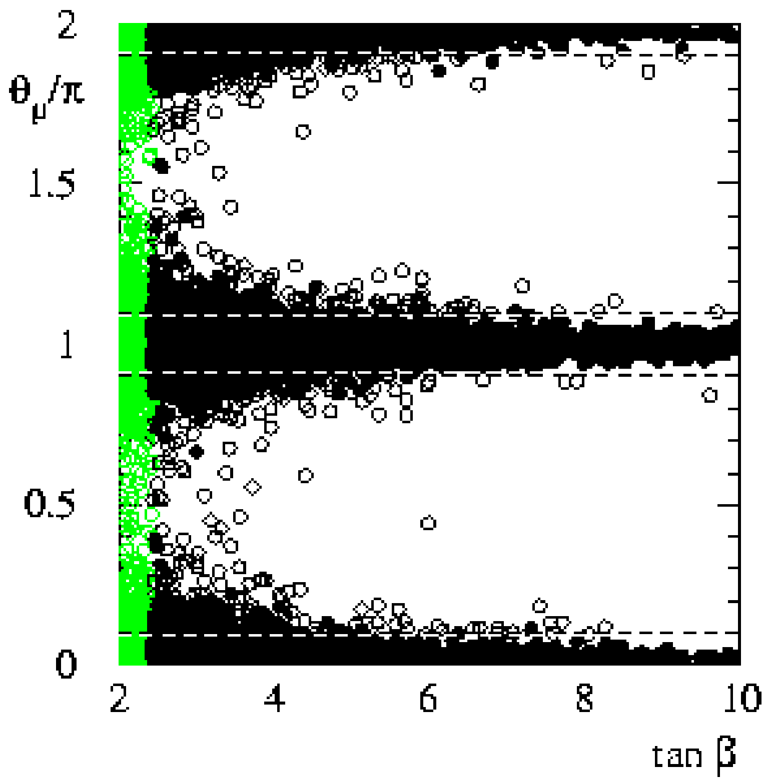
- one phase can be eliminated (R-transformation), often φ_{M_2}
- φ_{M_3} is the phase of the gluino mass parameter

$\Rightarrow$ contributes to the phase dependence of the Higgs sector from the 2-loop level on

▷ Higgsino sector: phase of μ

Experimental constraints: Phase of μ

Constrained by measurements of electric dipole moments:



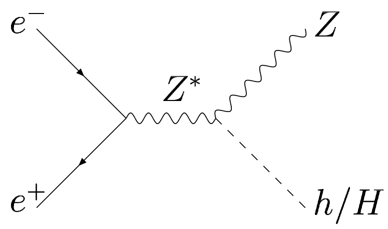
▷ 15 parameter scan: allowed values: black dots

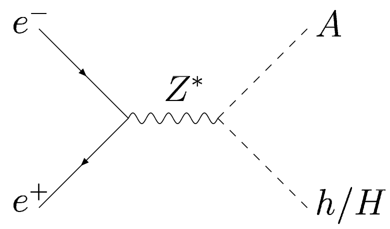
Barger et al.

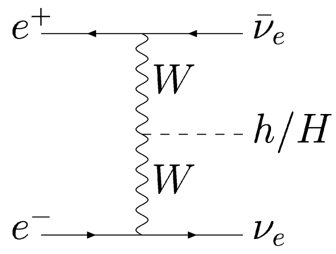
⇒ In large areas: phase of μ is small

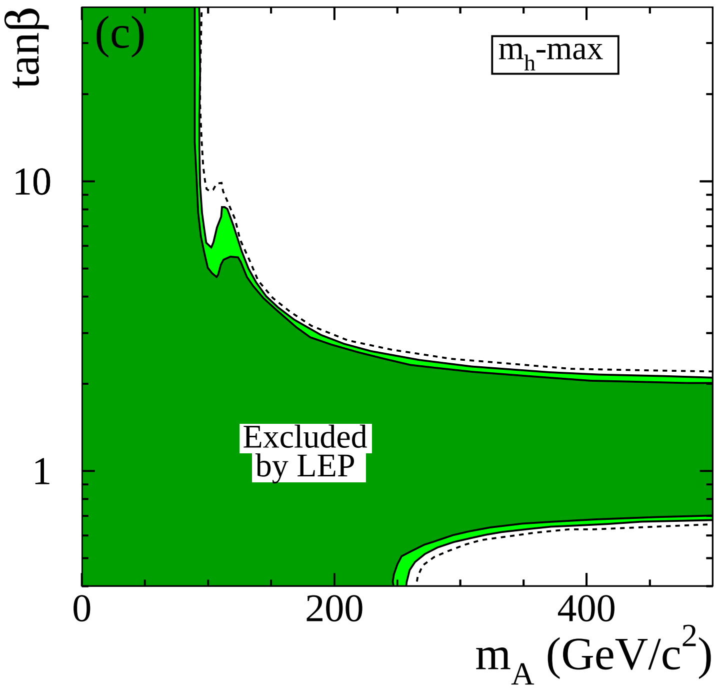
Higgs boson search: MSSM Higgs mass limits

▷ Direct Search at LEP: $e^+e^- \to Z + h/H, A + h/H, \nu_e\bar{\nu}_e + h/H$









$M_{h/H} \gtrsim 91$ GeV

$M_A \gtrsim 91.9$ GeV

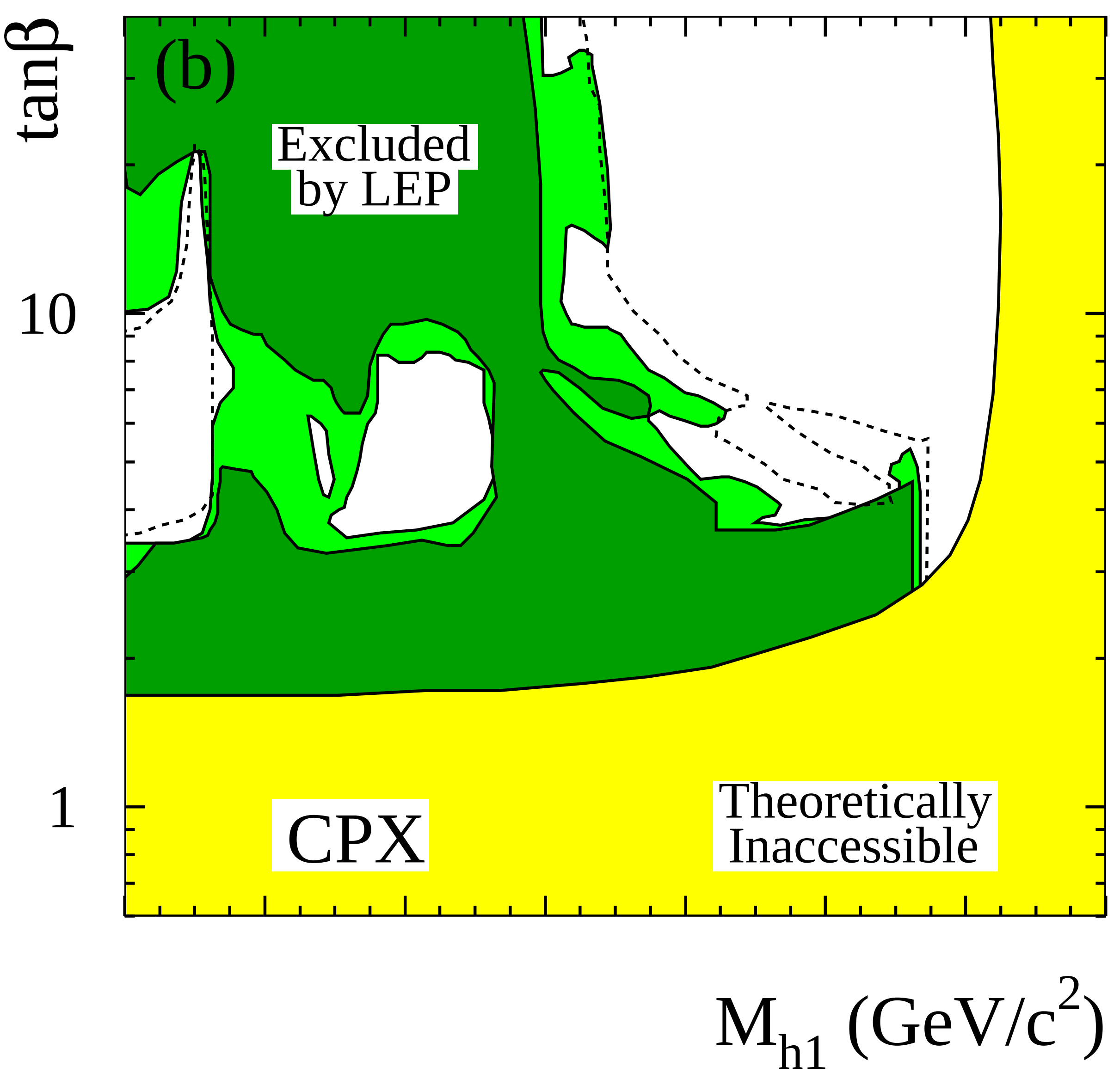
$M_{H^\pm} > 78.6$ GeV

$0.5 < \tan\beta < 2.4$ excluded

(only in this scenario, $m_t = 174.3$ GeV!)

A hole in the exclusion region

CPX scenario: large mixing in the Higgs sector ($\phi_{A_f} = \frac{\pi}{2}$, $\phi_{M_3} = \frac{\pi}{2}$)



Higgs masses and mixings as inputs

In the hole region:

- Coupling of h_1 to the Z-bosons is suppressed

  $\Rightarrow$ Production channel $e^+e^- \to h_1 Z$ is suppressed

- Other production channels:

$$\triangleright \quad e^+e^- \to h_2 Z$$
$$\triangleright \quad e^+e^- \to h_1 h_2$$

with the decay channels

$$\triangleright \quad h_2 \to h_1 h_1$$
$$\triangleright \quad h_2 \to b\bar{b}$$

$\Rightarrow$ Higgs masses and mixings have to be known

Status: Higgs masses at higher orders (incl CP-phases)

Status:

Without CP-phases

- Higgs masses at higher order w/o CP phases $\Rightarrow$ good shape
  Ellis eal; Okada eal; Haber, Hempfling, Hoang eal; Carena eal; Heinemeyer eal; Zhang eal; Brignole eal; Harlander eal; Slavich eal; ...
  (up to leading 3-loop) Martin; Harlander et al.)

Including CP-phases

- Effective potential approach, up to 2-loop leading log contributions CPsuperH
  - up to leading-log contributions at 2-loop level ($\tilde{f}, f$ contributions) Pilaftsis,Wagner;Demir;Choi eal; Carena eal
  - Gaugino contributions Ibrahim, Nath
  - Effects of imaginary parts at one-loop Ellis eal; Choi eal; Bernabeu eal
- Feynman diagrammatic approach: full 1-lopp + 2-loop $\mathcal{O}(\alpha_t \alpha_s)$ FeynHiggs
  Heinemeyer, Hollik, Rzehak, Weiglein

- **Determination of the Higgs masses**

Two-point function

$$-i\hat{\Gamma}(p^2) = p^2 - M(p^2)$$

with the matrix

$$M(p^2) = \begin{pmatrix} M^2_{H^0_{Born}} - \hat{\Sigma}_{H^0H^0}(p^2) & -\hat{\Sigma}_{H^0h^0}(p^2) & -\hat{\Sigma}_{H^0A^0}(p^2) \\ -\hat{\Sigma}_{H^0h^0}(p^2) & M^2_{h^0_{Born}} - \hat{\Sigma}_{h^0h^0}(p^2) & -\hat{\Sigma}_{h^0A^0} \\ -\hat{\Sigma}_{H^0A^0}(p^2) & -\hat{\Sigma}_{h^0A^0}(p^2) & M^2_{A^0_{Born}} - \hat{\Sigma}_{A^0A^0}(p^2) \end{pmatrix}$$

Real parameters:

$$\hat{\Sigma}_{H^0A^0}(p^2) = \hat{\Sigma}_{h^0A^0}(p^2) = 0$$

no mixing between CP-even and CP-odd states

- Calculate the zeros of the determinant of $\hat{\Gamma}$:

$$det[p^2 - M(p^2)] = 0$$

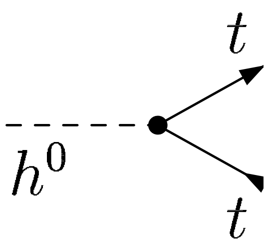
or calculate the eigenvalues $\lambda(p^2)$ of $M(p^2)$:

$$det[\lambda(p^2) - M(p^2)] = 0$$

and solve iteratively

$$p^2 - \lambda(p^2) = 0 \qquad \Rightarrow M_{h_1} \leq M_{h_2} \leq M_{h_3}$$

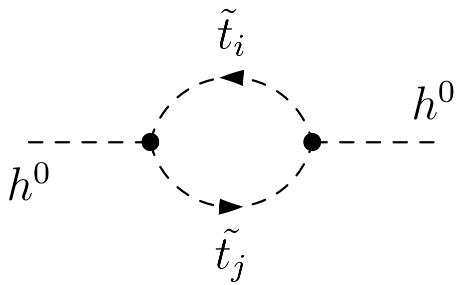
Why large corrections?



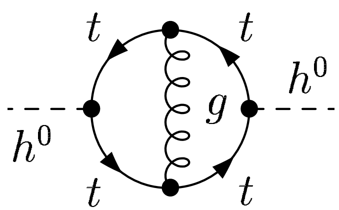
Yukawa coupling:

$\sim \lambda_t \sim \frac{m_t}{M_W \sin\beta}$ Large contribution from the top sector.

◇ One-loop level $\mathcal{O}(\alpha_t)$, $\alpha_t = \frac{\lambda_t^2}{4\pi}$:



◇ Two-loop level $\mathcal{O}(\alpha_t \alpha_s)$



QCD corrections to the dominant one-loop contribution.

For large $\tan\beta$: Contribution from bottom sector becomes also large.

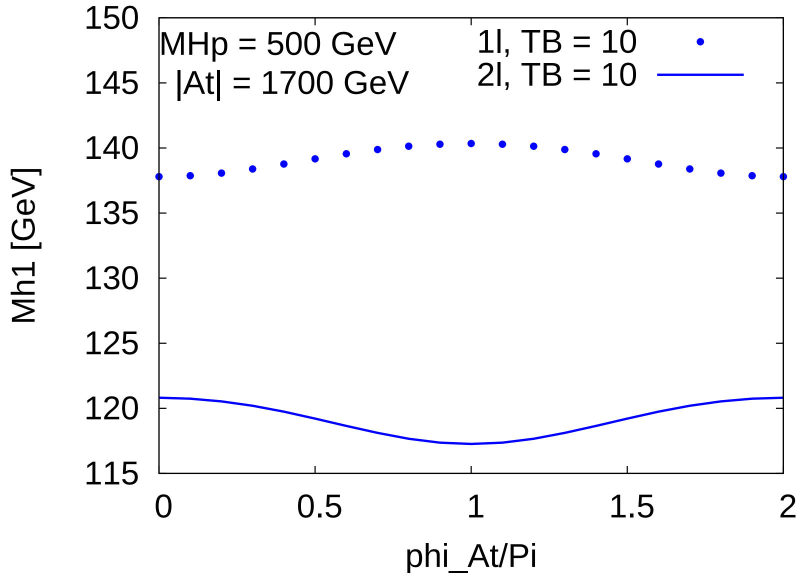
Heinemeyer,Hollik,Rzehak,Weiglein

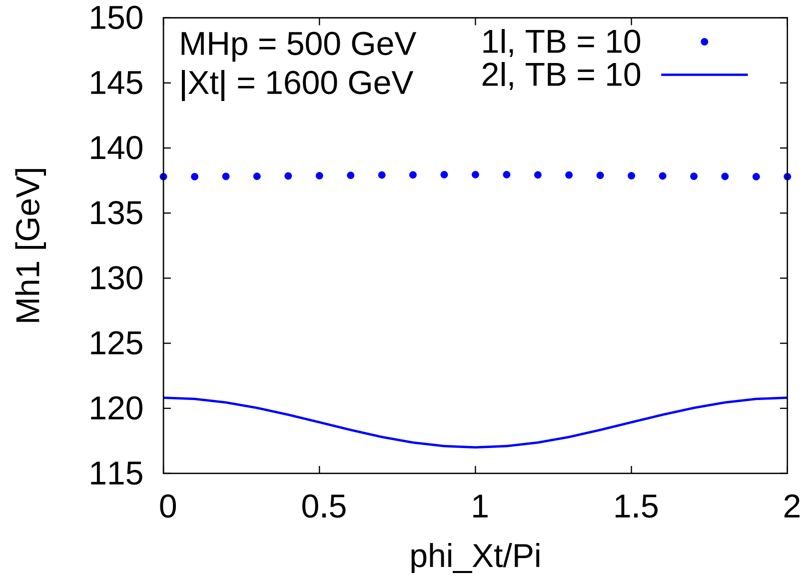
φ_{A_t} versus φ_{X_t} dependence (large $M_{H^\pm}$)

size of the squark mixing:

$X_t := A_t - \mu^* \cot\beta$

$|X_t| = const. \Rightarrow$ squark masses const.

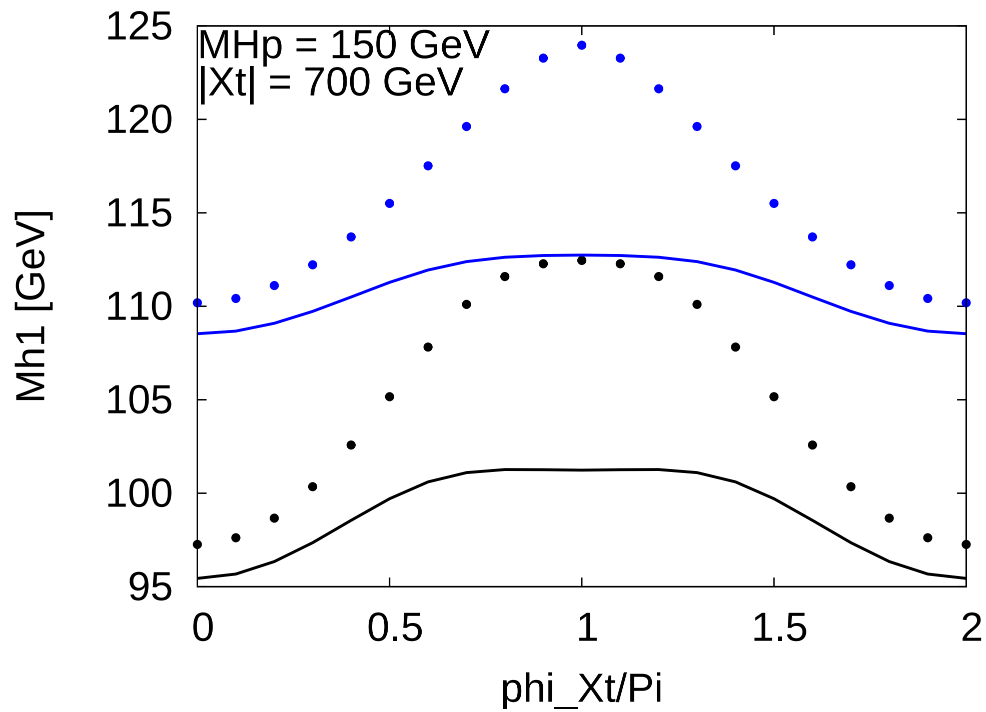




- Qualitative behaviour of M_{h_1} can change with inclusion of quantum corrections of $\mathcal{O}(\alpha_t\alpha_S)$
- Quantum corrections tend to be smaller for constant absolute values of the squark mixing $|X_t| = const.$

Results - φ_{X_t} dependence

φ_{X_t} dependence (small $M_{H^\pm}$)

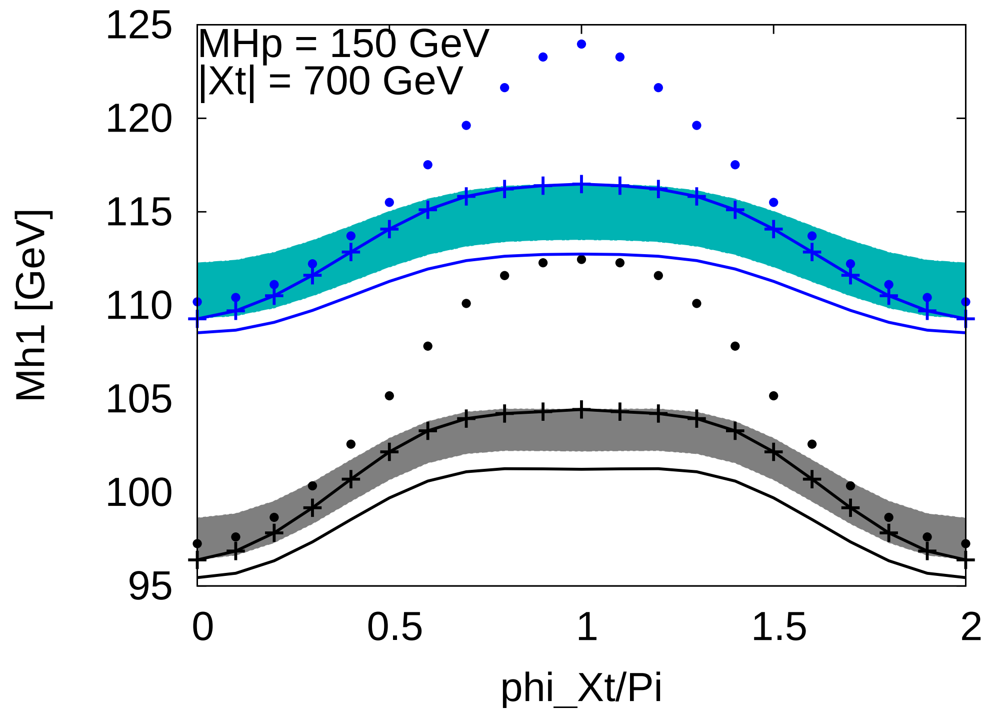


dotted:	$\mathcal{O}(\alpha)$:	$\tan\beta = 5$	$\tan\beta = 15$
full:	$\mathcal{O}(\alpha + \alpha_t\alpha_S)$:	$\tan\beta = 5$	$\tan\beta = 15$

- Higgs mass M_{h_1} depends on φ_{X_t}, $|X_t| = 700$ GeV
- One-loop corrections more sensitive to φ_{X_t} for small $M_{H^\pm}$

Results

φ_{X_t} dependence (small $M_{H^\pm}$)



dotted:	$\mathcal{O}(\alpha)$:	$\tan\beta = 5$	$\tan\beta = 15$
full:	$\mathcal{O}(\alpha + \alpha_t\alpha_S)$:	$\tan\beta = 5$	$\tan\beta = 15$
full w/ bars:	$\mathcal{O}(\alpha + \alpha_t\alpha_S) +$ interpol :	$\tan\beta = 5$	$\tan\beta = 15$

- **Bands:** Estimate of size of corrections of $\mathcal{O}(\alpha_b\alpha_S + \alpha_t^2 + \alpha_t\alpha_b + \alpha_b^2)$ Slavich eal
- **Interpolation:** Size of the above corrections, known for MSSM w/ real parameters: evaluate for $\varphi_{X_t} = 0$ and $\varphi_{X_t} = \pi$ and interpolate

Couplings

One example:

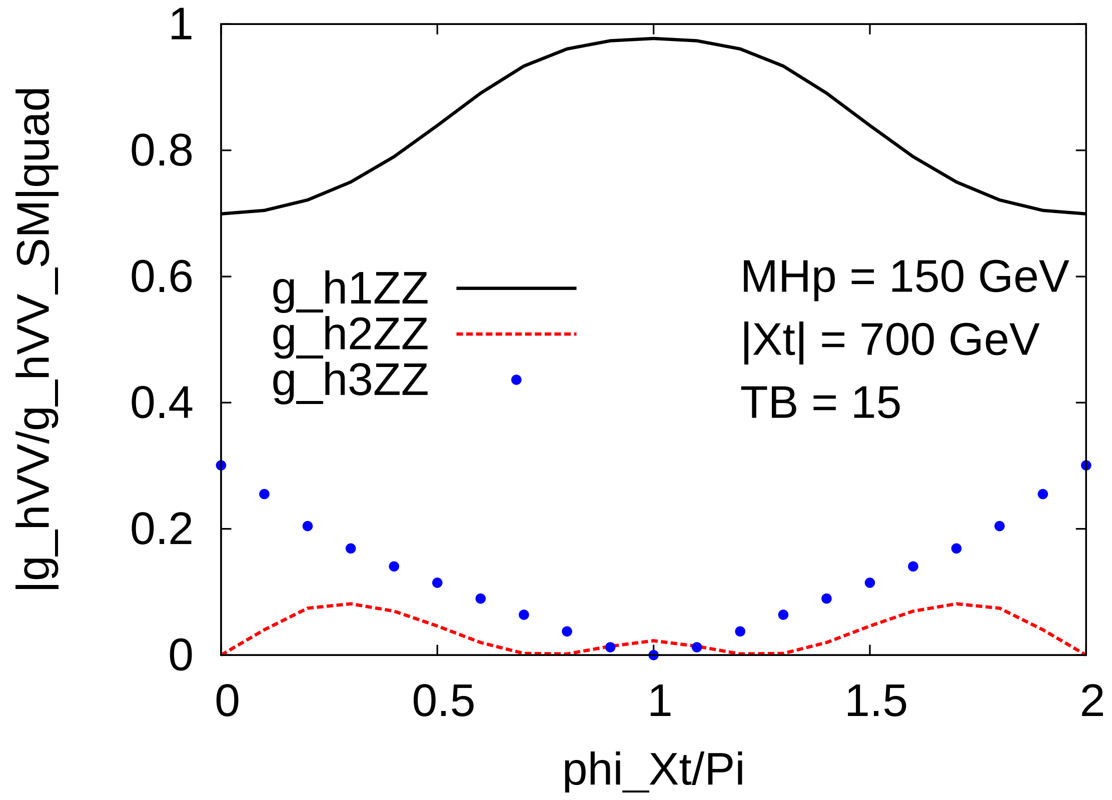
Coupling of two gauge bosons ($V = W, Z$) and one Higgs boson:

$$g_{h_i VV} = [U_{i1} \cos(\beta - \alpha) + U_{i2} \sin(\beta - \alpha)] \, g_{H_{SM} VV}$$

standard model coupling

- only CP-even components of the Higgs boson couple to V
- all three Higgs bosons can have a CP-even component

Results: φ_{X_t} dependence of couplings



- Here: g_{h_iVV} normalized to the SM couplings
- $|g_{h_iVV}|$ do depend on the phase φ_{X_t}, $|X_t| = 700$ GeV
- For $\varphi_{X_t} = 0$ h_2 is the CP-odd Higgs boson, for $\varphi_{X_t} = \pi$ it is h_3

Summary

- At **Born** level: no CP-violation in the Higgs sector

- **Quantum corrections** can **induce** CP-violation

- **Quantum corrections** have to be taken into account:
  - ◇ Prediction of Higgs boson masses
  - ◇ Amplitudes with Higgs bosons

- Programs **FeynHiggs** and **CPsuperH** are available for the evaluation

Higgs boson quantum numbers

Quantum numbers of the Higgs boson: J^{PC}

- J spin
- P parity
- C charge conjugation

◇ $\gamma\gamma \to H$ or $H \to \gamma\gamma \rightsquigarrow J \neq 1$.

Spin and CP quantum numbers: angular correlations

- angular correlations in production: Hjj in vector boson fusion, gluon gluon fusion

  Plehn,Rainwater,Zeppenfeld; Hankele,Klämke,Zeppenfeld

- angular correlations in Higgs decays, e.g. $H \to ZZ \to l^+l^-l^+l^-$

  Dell'Aquila,Nelson; Kramer,Kühn,Stong,Zerwas;

  Choi,Miller,MMM,Zerwas;Bluj; Buszello,Fleck,Marquard,van der Bij

  observables sensitive to CP-violation

  Godbole,Miller,MMM

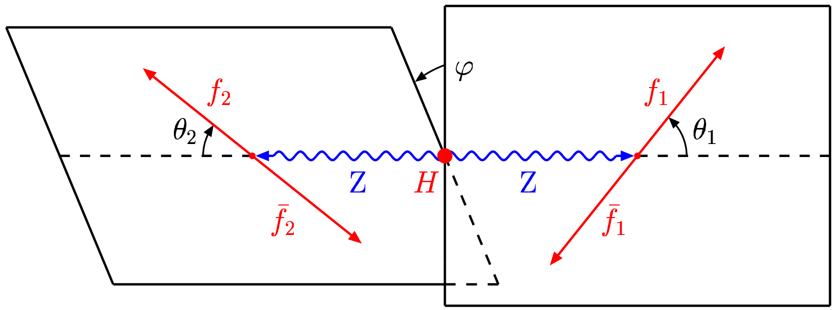
- below ZZ threshold: angular correlations, threshold effects

  Choi,Miller,MMM,Zerwas Buszello,Marquard

Higgs boson quantum numbers

◇ Determination of spin and parity in

$$gg \to H \to ZZ^{(*)} \to (f_1\bar{f}_1)(f_2\bar{f}_2)$$



◇ Helicity methods: general HZZ coupling for arbitrary spin and parity

Differential distributions

◇ Double polar angular distribution ($\mathcal{CP}$ invariant theory)

$$\frac{d\Gamma_H}{d\cos\theta_1 d\cos\theta_2} \sim \sin^2\theta_1 \sin^2\theta_2 \, |\mathcal{T}_{00}|^2 + \frac{1}{2}(1+\cos^2\theta_1)(1+\cos^2\theta_2)\left[|\mathcal{T}_{11}|^2 + |\mathcal{T}_{1,-1}|^2\right]$$
$$+(1+\cos^2\theta_1)\sin^2\theta_2 \, |\mathcal{T}_{10}|^2 + \sin^2\theta_1 \, (1+\cos^2\theta_2) \, |\mathcal{T}_{01}|^2$$
$$+2\,\eta_1\eta_2 \cos\theta_1 \cos\theta_2 \left[|\mathcal{T}_{11}|^2 - |\mathcal{T}_{1,-1}|^2\right]$$

SM: $\mathcal{T}_{00} = M_H^2/(2M_Z^2) - 1, \; \mathcal{T}_{11} = -1, \; \mathcal{T}_{10} = \mathcal{T}_{01} = \mathcal{T}_{1,-1} = 0$

◇ Azimuthal angular distribution ($\mathcal{CP}$ invariant theory)

$$\frac{d\Gamma_H}{d\varphi} \sim |\mathcal{T}_{11}|^2 + |\mathcal{T}_{10}|^2 + |\mathcal{T}_{1,-1}|^2 + |\mathcal{T}_{01}|^2 + |\mathcal{T}_{00}|^2/2$$
$$+\eta_1\eta_2\left(\frac{3\pi}{8}\right)^2 \Re\mathrm{e}(\mathcal{T}_{11}\mathcal{T}_{00}^* + \mathcal{T}_{10}\mathcal{T}_{0,-1}^*)\cos\varphi + \frac{1}{4}\Re\mathrm{e}(\mathcal{T}_{11}\mathcal{T}_{-1,-1}^*)\cos 2\varphi$$

Determination of spin and parity

- **$M_H < 2M_Z$:** $d\Gamma/dM_*^2 \sim \beta$ for $\mathcal{J}^{\mathcal{P}} = 0^+$

  ◇ $d\Gamma/dM_*^2$ rules out $\mathcal{J}^{\mathcal{P}} = 0^-, 1^-, 2^-, 3^\pm, 4^\pm$

  ◇ $d\Gamma/dM_*^2$ and no $[1+\cos^2\theta_1]\sin^2\theta_2$

  $[1+\cos^2\theta_2]\sin^2\theta_1$ rules out $\mathcal{J}^{\mathcal{P}} = 1^+, 2^+$

- **$M_H > 2M_Z$:**

  ◇ odd normality: $\mathcal{J}^{\mathcal{P}} = 0^-, 1^+, 2^-, 3^+, \ldots$ excluded by non-zero $\sin^2\theta_1 \sin^2\theta_2$

  ◇ even normality: $\mathcal{J}^{\mathcal{P}} = 1^-, 3^-, \ldots$ excluded by non-zero $\sin^2\theta_1 \sin^2\theta_2$

  ◇ rule out $\mathcal{J}^{\mathcal{P}} = 2^+, 4^+$ with:

  $\frac{d\sigma}{d\cos\theta}[gg/\gamma\gamma \to H \to ZZ]$ only isotropic for spin 0

$\mathcal{CP}$ Violation

- **$\mathcal{CP}$ Violation:** Examine behaviour with

  most general vertex =
  sum of even and odd normality tensors

- **Case spin 0:** $p = p_{Z_1} + p_{Z_2}$, $k = p_{Z_1} - p_{Z_2}$

$$\text{Vertex } HZZ: \quad \frac{igM_Z}{\cos\theta_W}\left[a\, g_{\mu\nu} + \frac{b}{M_Z^2}\, p_\mu p_\nu + i\, \frac{c}{M_Z^2}\, \epsilon_{\mu\nu\alpha\beta}\, p^\alpha k^\beta\right]$$

  ◇ $a = 1,\ b = c = 0$: SM

  ◇ $(a \neq 0 \wedge c \neq 0) \quad \vee \quad (b \neq 0 \wedge c \neq 0)$: $\mathcal{CP}$-violation

- **Observables sensitive to $\mathcal{CP}$**

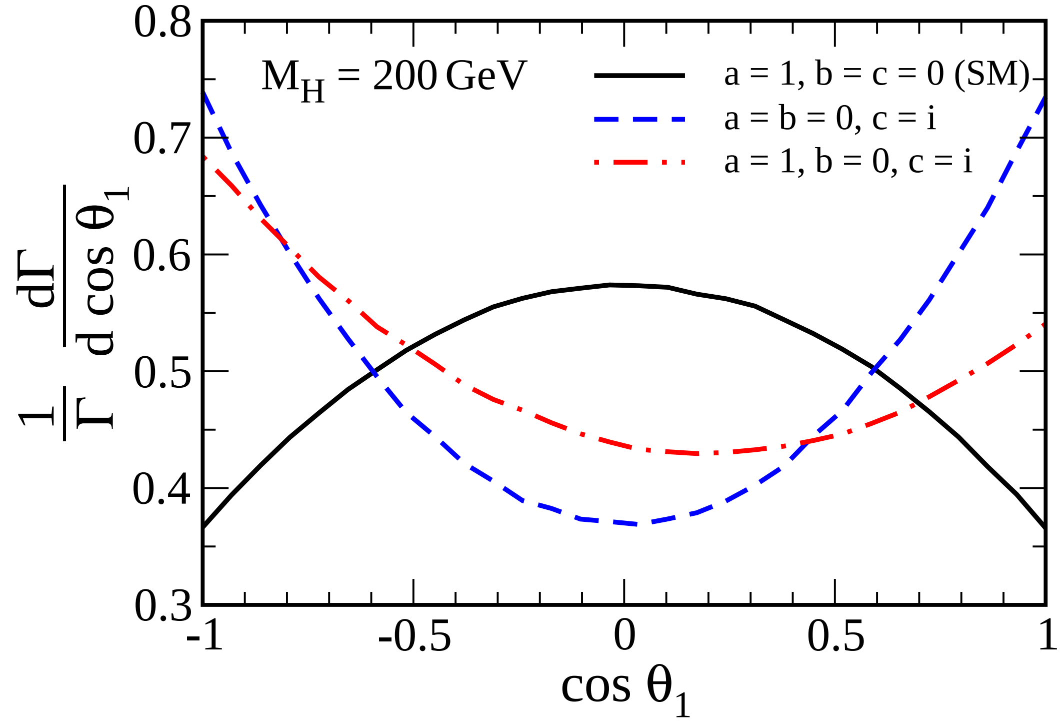
  ◇ angle ϕ between oriented Z decay planes in the Higgs rest frame

  ◇ cos of the fermion polar angle θ in the Z rest frame

Higgs boson quantum numbers

angular distribution in $\cos\theta$

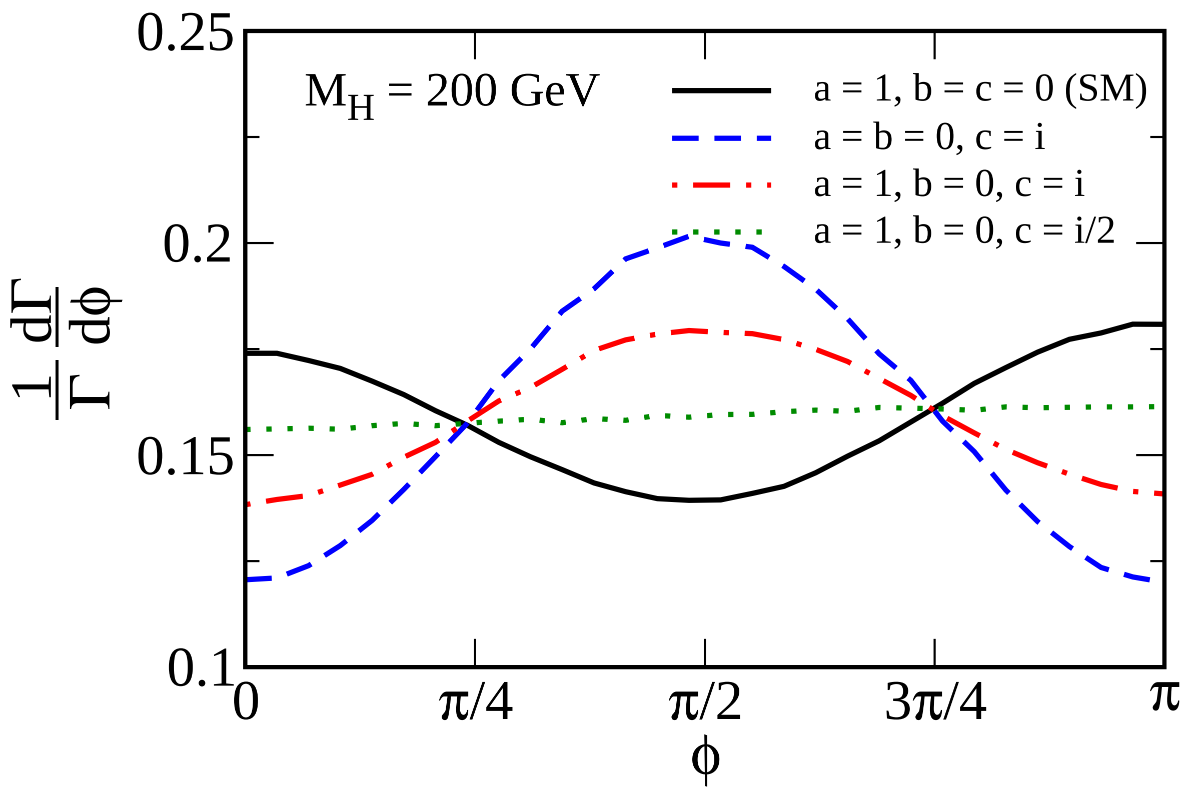
Godbole,Miller,MMM



Higgs boson quantum numbers

angular distribution in ϕ

Godbole,Miller,MMM



Asymmetries sensitive to $\mathcal{CP}$

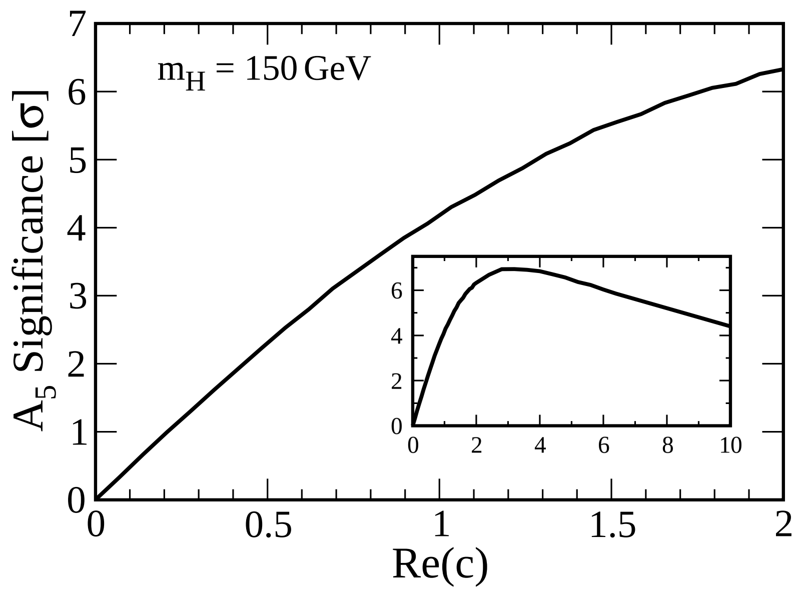
Godbole,Miller,MMM

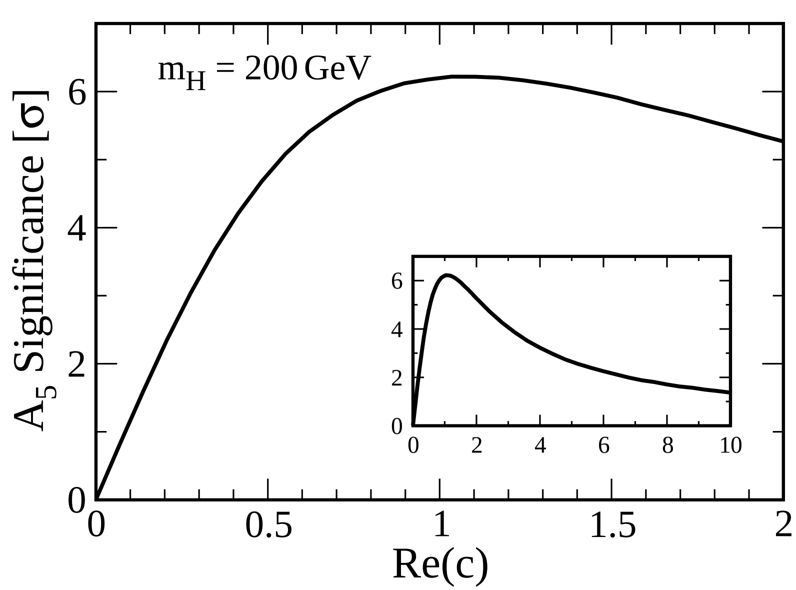
◇ Example:

$$O_5 = \sin\theta_1 \sin\theta_2 \sin\phi[\sin\theta_1 \sin\theta_2 \cos\phi - \cos\theta_1 \cos\theta_2]$$

$$O_5 = \frac{[(\vec{p}_{4H} \times \vec{p}_{3H}) \cdot \vec{p}_{1H}][(\vec{p}_{1Z} - \vec{p}_{2Z}) \cdot \vec{p}_{3Z}]}{|\vec{p}_{3H} + \vec{p}_{4H}||\vec{p}_{3Z} - \vec{p}_{4Z}|^2|\vec{p}_{1Z} - \vec{p}_{2Z}|^2/8}$$

$$\mathcal{A}_5 = \frac{\Gamma(O_5 > 0) - \Gamma(O_5 > 0)}{\Gamma(O_5 > 0) + \Gamma(O_5 > 0)}$$





Higgs boson quantum numbers

gluon gluon fusion	CP-even Htt can be distinguished from CP-odd at $> 5\sigma$ ($M_H = 160$ GeV)	Klämke,Zeppenfeld
$H \to ZZ \to 4l$	consistency with SM; $0^-, 1^\pm$ excluded ($\int \mathcal{L} = 100\text{fb}^{-1}$, $M_H = 200$ GeV)	Buszello,Fleck, Marquard,van der Bij
CMS: $H \to ZZ \to 4l$	scalar, pseudoscalar can be distinguished at 3σ ($\int \mathcal{L} = 60\text{fb}^{-1}$, $M_H = 300$ GeV)	CMS
ALTAS: $H \to ZZ \to 4l$	CP-odd excluded at 8.7σ (2.9σ) $M_H = 200$ GeV (130 GeV), $\int \mathcal{L} = 100\text{fb}^{-1}$ strong limits to anomalous couplings	Buszello,Fleck,Marquard, van der Bij;Strässner
CP-sensitive observable	significance $\sim 5\ \sigma$ ($M_H = 200$ GeV)	Godbole,Miller,MMM