Dark Matter

Literature Gianfranco Bertone, Dan Hooper and Joseph Silk,
Particle dark matter: Evidence, candidates and constraints.
Phys. Rept. **405** (2005) 279; arXiv: hep-ph/0404175. And references therein.

Standard Cosmology

∗ Fundamental picture *Big Bang* scenario: Universe is a system evolving from a highly compressed state existing $\sim 10^{10}$ y ago.

∗ Cosmological model: 3 fundamental ingredients

▷ *Einstein equations* relate geometry of the Universe with its matter and energy content

▷ *metrics* describe the symmetries of the problem

▷ *Equation of state* specify the physical properties of the matter and energy content

∗ Einstein field equation:

1) equation invariant under general coordinate transformations
2) equation tends to Newton's law in the limit of weak fields
3) equation is of second differential order and linear in second derivatives

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\frac{8\pi G_N}{c^4} T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$R_{\mu\nu}, R$ - Ricci tensor and Ricci scalar ($\leftarrow$ contraction of the Riemann curvature tensor);
$g_{\mu\nu}$ - metric tensor; G_N - Newton's constant; $T_{\mu\nu}$ - energy-momentum tensor;
Λ - cosmological constant

$*$ Meaning w/o Λ, *geometry of the Universe* (lhs) *determined by its energy content* (energy-momentum tensor rhs) $\leftarrow$ key concept of general relativity

Cosmological constant: (Einstein: stationary solution of the Universe)
"vacuum energy" associated w/ space-time itself
source of gravitational field even w/o matter; contribution to total energy can be important

$*$ Solution: specify symmetries of the problem
- *isotropy* $\leftarrow$ Cosmic Microwave Background (CMB)
- *homogeneity* $\leftarrow$ galaxy surveys $\rightsquigarrow$ homogeneous distribution at scales $\gtrsim$ 100 Mpc

$\Rightarrow$ specific form of the metric

$$ds^2 = -c^2 dt^2 + a(t)^2 \left(\frac{dr^2}{1-kr^2} + r^2 d\Omega^2 \right)$$

$a(t)$ - *scale factor*;
k - spatial curvature, $k = -1, 0, +1$; $k = 0 \rightsquigarrow$ metric of flat Euclidean space $\rightsquigarrow$ Friedman equation

$$\left(\frac{\dot{a}}{a} \right)^2 + \frac{k}{a^2} = \frac{8\pi G_N}{3} \rho_{tot}$$

ρ_{tot} - total average energy density of the universe
Hubble parameter

$$H(t) = \frac{\dot{a}(t)}{a(t)}$$

present value (May 2009) $H_0 = 74.2 \pm 3.6$ km s^{-1} Mpc^{-1}.

$*$ Flat Universe ($k = 0$): energy density = *critical density* ρ_c

$$\rho_c \equiv \frac{3H^2}{8\pi G_N}$$

Definition Quantity Ω_i of a substance of species i and density ρ_i

$$\Omega_i \equiv \frac{\rho_i}{\rho_c}$$

and

$$\Omega = \sum_i \Omega_i \equiv \sum_i \frac{\rho_i}{\rho_c}$$

$\rightsquigarrow$ Friedmann equation

$$\Omega - 1 = \frac{k}{H^2 a^2}$$

$\rightsquigarrow$ value of Ω determines sign of k

$\rho < \rho_c$	$\Omega < 1$	$k = -1$	open
$\rho = \rho_c$	$\Omega = 1$	$k = 0$	flat
$\rho > \rho_c$	$\Omega > 1$	$k = 1$	closed

$*$ General expression for the expansion rate

$$\frac{H^2(z)}{H_0^2} = [\Omega_X (1+z)^{3(1+\alpha_X)} + \Omega_K (1+z)^2 + \Omega_M (1+z)^3 + \Omega_R (1+z)^4]$$

M, R - labels for matter and radiation

$\Omega_K = \frac{-k}{a_0^2 H_0^2}$; z - redshift;

X - generic substance w/ eq. of state $p_X = \alpha_X \rho_X$

A $\mathcal{B}$rief $\mathcal{H}$istory of the $\mathcal{U}$niverse

Extrapolation of known physics back to *Planck epoch* ($t = 10^{-43}$ s) $\leftarrow$ gravitational interaction becomes strong

$\mathcal{E}$volution of the $\mathcal{U}$niverse

- $T \sim 10^{16}$ GeV Grand unified group G breaks down into SM group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$.

- $T \sim 10^2$ GeV: EWSB: $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \rightarrow SU(3)_C \otimes U(1)_{em}$
could be origin of baryogenesis and possibly of primordial magnetic field.s

- $T \sim 10^1 - 10^3$ GeV Weakly interacting dark matter candidates w/ TeV scale masses *freeze out*.

- $T \sim 0.3$ GeV: QCD phase transition $\rightsquigarrow$ confinement of quarks and gluons into hadrons.

- $T \sim 1$ MeV Neutron freeze-out.

- $T \sim 100$ keV: Nucleosynthesis: protons and neutrons fuse into light elements (D, ^{3}He, ^{4}He, Li).
Big Bang Nucleosynthesis (BBN).

- $T \sim 1$ eV: Matter density becomes equal density of radiation $\rightsquigarrow$ formation of structures can start.

- $T \sim 0.4$ eV: Photon decoupling produces Cosmic background radiation (CMB).

- $T \sim 2.7$ K $\sim 10^{-4}$ eV: Today.

Relic Density

Basics of thermal relic density calculation

Thermal equilibrium: Interaction rate $<$ expansion rate of the Universe $\rightsquigarrow$ particle *decoupled*.

Boltzmann equation $\rightsquigarrow$ equation for the particle number density n:

$$\frac{dn}{dt} + 3Hn = - < \sigma v > (n^2 - (n^{eq})^2) \qquad (1)$$

σv - total annihilation cross section multiplied by velocity, brackets denote thermal average
H - Hubble constant; n^{eq} - number density at thermal equilibrium
Massive particles (non-relativistic limit) & Maxwell Boltzmann approximation $\rightsquigarrow$

$$n^{eq} = g\left(\frac{mT}{2\pi}\right)^{\frac{3}{2}} e^{-m/T} \qquad (2)$$

m - particle mass; T - temperature.
Introduction

$$Y \equiv \frac{n}{s} \quad , \quad Y^{eq} \equiv \frac{n^{eq}}{s} \qquad (3)$$

s - entropy density $s = 2\pi^2 g_* T^3/45$; g_* counts number of relativistic degrees of freedom.
Conservation of entropy per co-moving volume ($sa^3 = const.$) $\rightsquigarrow$

$$\dot{n} + 3Hn = s\dot{Y} \qquad (4)$$

and hence for Eq. (1)

$$s\dot{Y} = - < \sigma v > s^2(Y^2 - (Y^{eq})^2) \, . \qquad (5)$$

Introduction $x \equiv m/T \rightsquigarrow$

$$\frac{dY}{dx} = -\frac{<\sigma v> s}{Hx}(Y^2 - (Y^{eq})^2) \; . \tag{6}$$

Heavy states $\rightsquigarrow$ approximate $<\sigma v>$ with non-relativistic expansion in powers of v^2

$$<\sigma v> = a + b < v^2 > + \mathcal{O}(< v^4 >) \approx a + 6\frac{b}{x} \; . \tag{7}$$

For Eq. (6) this leads in terms of $\Delta = Y - Y^{eq}$ to

$$\Delta' = -Y^{eq'} - f(x)\Delta(2Y^{eq} + \Delta) \tag{8}$$

with $'$ denoting d/dx and

$$f(x) = \sqrt{\frac{\pi g_*}{45}} m M_{Pl}\Big(a + 6\frac{b}{x}\Big)x^{-2} \; . \tag{9}$$

Introduction of $x_F \equiv m/T_F$, where $T_F =$ freeze-out temperature of the relic particle $\rightsquigarrow$ analytic solution of Eq. (8) in 2 extreme regions $x \ll x_F$ and $x \gg x_F$

$$\Delta = -\frac{Y^{eq'}}{2f(x)Y^{eq}} \qquad \text{for } x \ll x_F \tag{10}$$

$$\Delta' = -f(x)\Delta^2 \qquad \text{for } x \gg x_F \; . \tag{11}$$

Regions correspond to long-before and long-after freeze-out. Integrating last equation between x_F and ∞ and using $\Delta_F \gg \Delta_\infty \rightsquigarrow \Delta_\infty$ and

$$Y_\infty^{-1} = \sqrt{\frac{\pi g_*}{45}} M_{Pl} m x_F^{-1}(a + 3b/x_F) \; . \tag{12}$$

Present density of a generic relic X is given by $\rho_X = m_X n_X = m_X s_0 Y_\infty$ with $s_0 = 2889.2$ cm^{-3} (present entropy density).

Relic density can be expressed in terms of the critical density $\rightsquigarrow$

$$\Omega_X h^2 \approx \frac{1.07 \times 10^9 \text{ GeV}^{-1}}{M_{Pl}} \frac{x_F}{\sqrt{g_*}} \frac{1}{a + 3b/x_F} , \tag{13}$$

a, b expressed in GeV^{-2} and g_* evaluated at the freeze-out temperature.

Convention: Relic density written in terms of the Hubble parameter $H_0/100$ km s^{-1} Mpc^{-1}.

Calculation of annihilation cross section (in all possible channels) and extraction of the parameters a and $b \Rightarrow$ estimate of the relic density.

Order-of-estimate with the approximate version

$$\Omega_X h^2 \approx \frac{3 \times 10^{-27} \, cm^3 s^{-1}}{< \sigma v >} . \tag{14}$$

Approximation Eq. (7) not always justified:

Dramatic change in relic density can be introduced by resonance enhancements: *coannihilations.*

$\mathcal{L}$inks with $\mathcal{P}$hysics beyond the $\mathcal{SM}$

Concepts of dark energy and dark matter: no explanation within SM nor astrophysics $\rightsquigarrow$ physics beyond the SM

Two examples of popular extensions of the SM:

• *Supersymmetry:* Complete symmetry between fermions and bosons. Provides an excellent dark matter candidate: its lightest stable particle, the *neutralino.*

• *Extra dimensions:* Extra spatial dimensions. *Unified* extra dimensions: all particles and fields of the SM can propagate in the extra dimensions $\rightsquigarrow$ lightest Kaluza-Klein particle (lightest of all states corresponding to the first excitations of the SM particles) is a viable DM candidate.

• Neutrinos ruled out as DM candidates. Most of DM is non-baryonic.

Strong interplay between particle physics, theoretical physics, cosmology, astrophysics:
▷ Theoretical particle physics: new theories predicting new particles as excellent DM candidates
▷ Cosmological and astrophysical observations constrain properties of such particles and hence the parameters of the new theories.

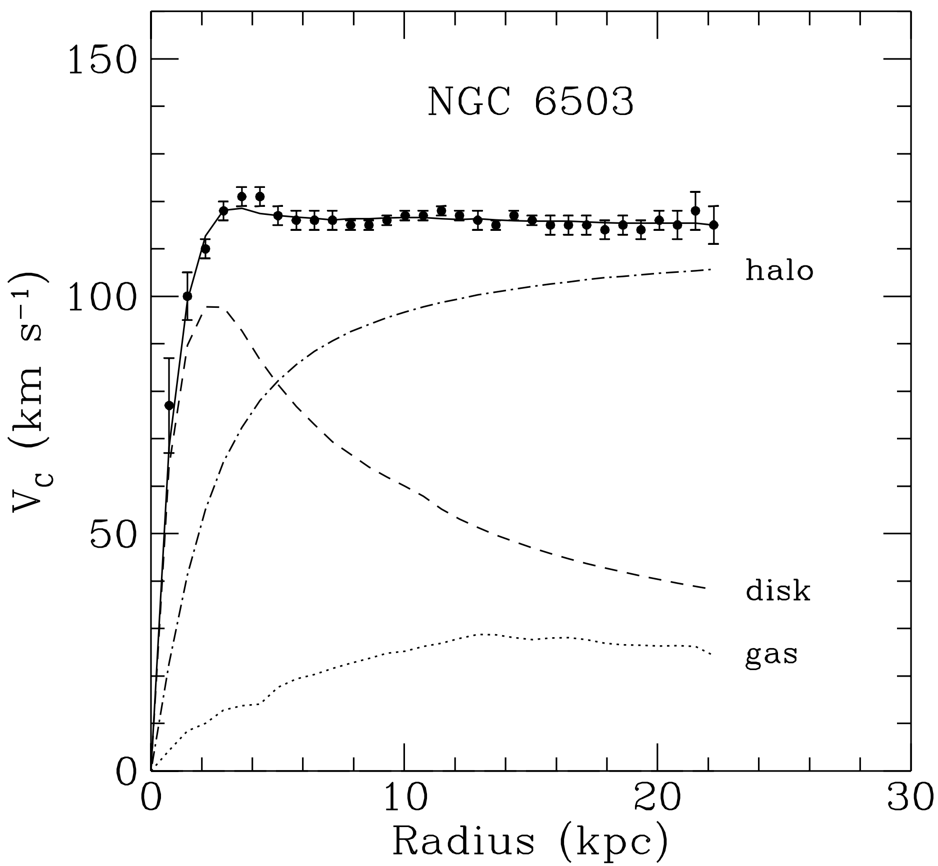
$\mathcal{E}$vidence and $\mathcal{D}$istribution

(1) The galactic scale: *Rotation curves* of galaxies
$\longleftarrow$ combine observations of 21cm line w/ optical surface photometry
observed rotation curves: usually flat behaviour
Newtonian dynamics $\rightsquigarrow$ circular velocity:

$$v(r) = \sqrt{\frac{GM(r)}{r}} \qquad M(r) = 4\pi \int \rho(r) r^2 dr \qquad \rho(r) \text{ mass density profile} \tag{15}$$

$\Rightarrow v(r) \sim 1/\sqrt{r}$ beyond optical disc.

Rotation curve of NGC 6503.



$v(r)$ approximately constant $\rightsquigarrow$ existence of halo w/ $M(r) \sim r$ and $\rho \sim 1/r^2$

- uncertainties of slope in the innermost regions of galaxies
- rotation curves of galaxies $\rightsquigarrow$ strong evidence of existence of spherical DM halo
- total amount of DM difficult to quantify $\longleftarrow$ do not know to what distances halos extend.

Further DM arguments on subgalactic and inter-galactic scales:

- ◇ *Weak modulation of strong lensing* around individual massive elliptical galaxies $\rightsquigarrow$ evidence for substructures on scales of $\sim 10^6$ M_{sol}.
- ◇ *Oort discrepancy* in the disk of the Milky Way: inconsistency between the amount of stars in the solar neighbourhood and the gravitational potential implied by their distribution.
- ◇ *Weak gravitational lensing* of distant galaxies by foreground structure.
- ◇ *The velocity dispersions of dwarf spheroidal galaxies* imply mass-to-light ratios larger than those observed in our "local" neighbourhood.
- ◇ *The velocity dispersions of spiral galaxy satellites* suggest existence of dark halos around spiral galaxies extending well behind the optical disc.

(2) The scale of galaxy clusters:

F. Zwicky (1933): measurements of velocity dispersion of galaxies in the Coma cluster $\Rightarrow$ mass-to-light ratio $\sim$ 400 solar masses per solar luminosity $\sim$ 2 o.m. ratio in the solar neighbourhood.
Today: $\Omega_M \sim 0.2 - 0.3$ on cluster scales. Convenient calibration $\Omega_M = (M/L)/1000$.

For baryonic mass of a typical cluster, the temperature should obey the relation

$$kT \approx (1.3 - 1.8)\ \text{keV} \left(\frac{M_r}{10^{14} M_{sol}}\right)\left(\frac{1\ \text{Mpc}}{r}\right) . \qquad (16)$$

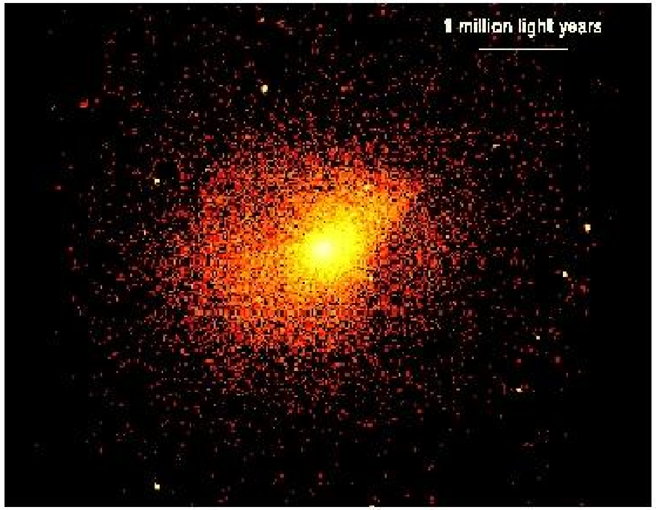
Discrepancy this temperature and observed temperature $T \approx 10$ keV (M_r identified with baryonic mass) $\rightsquigarrow$ existence of substantial amount of DM in clusters.

Gravitational lensing data:

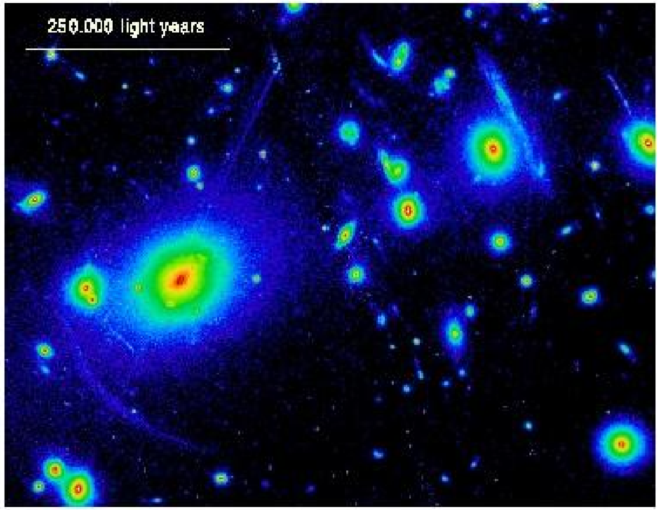
General relativity: Light propagates along geodesics which deviate from straight lines when passing near intense gravitational fields.

Distortion of images of background objects due to gravitational mass of a cluster $\rightsquigarrow$ infer mass of the cluser.

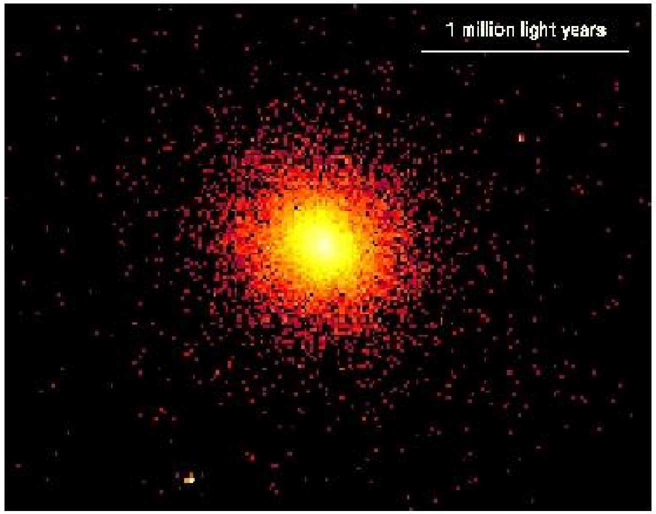
Gravitational arcs



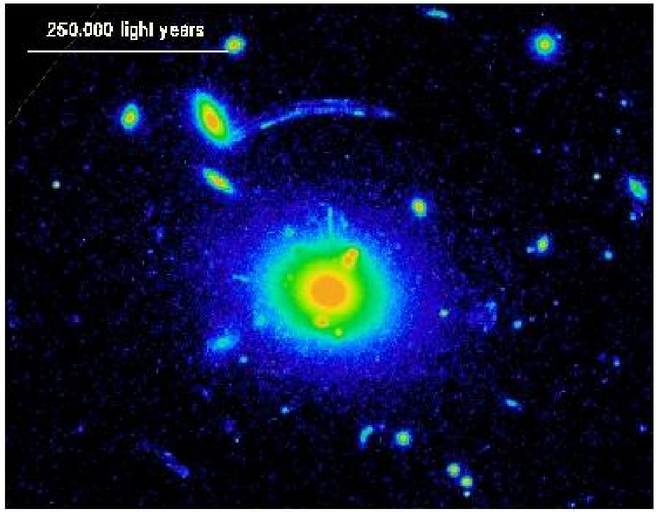
Abell 2390: Chandra (ACIS)



Abell 2390: HST (WFPC2)



MS2137.3-2353: Chandra (ACIS)



MS2137.3-2353: HST (WFPC2)

(3) Cosmological scales:

Analysis of Cosmic Microwave Background (CMB): *Total amount of DM* in the Universe

* George Gamow et al. (1948): Existence of background radiation originating from the propagation of photons in the early Universe (once decoupled from matter).
* Arno Penzias and Robert Wilson (1965): Discovery of cosmic microwave background.
* Today: CMB known to be *isotropic* at 10^{-5} level
  black body spectrum corresponding to $T = 2.726$ K

Analysis of CMB anisotropies $\rightsquigarrow$ accurate testing of cosmological models & put stringent constraints on cosmological parameters

* Analysis of Wilkinson Microwave Anisotropy Probe (WMAP) data:
  abundance of baryons and matter in the Universe:

$$\Omega_b h^2 = 0.024 \pm 0.01 \qquad \Omega_M h^2 = 0.14 \pm 0.02 \tag{17}$$

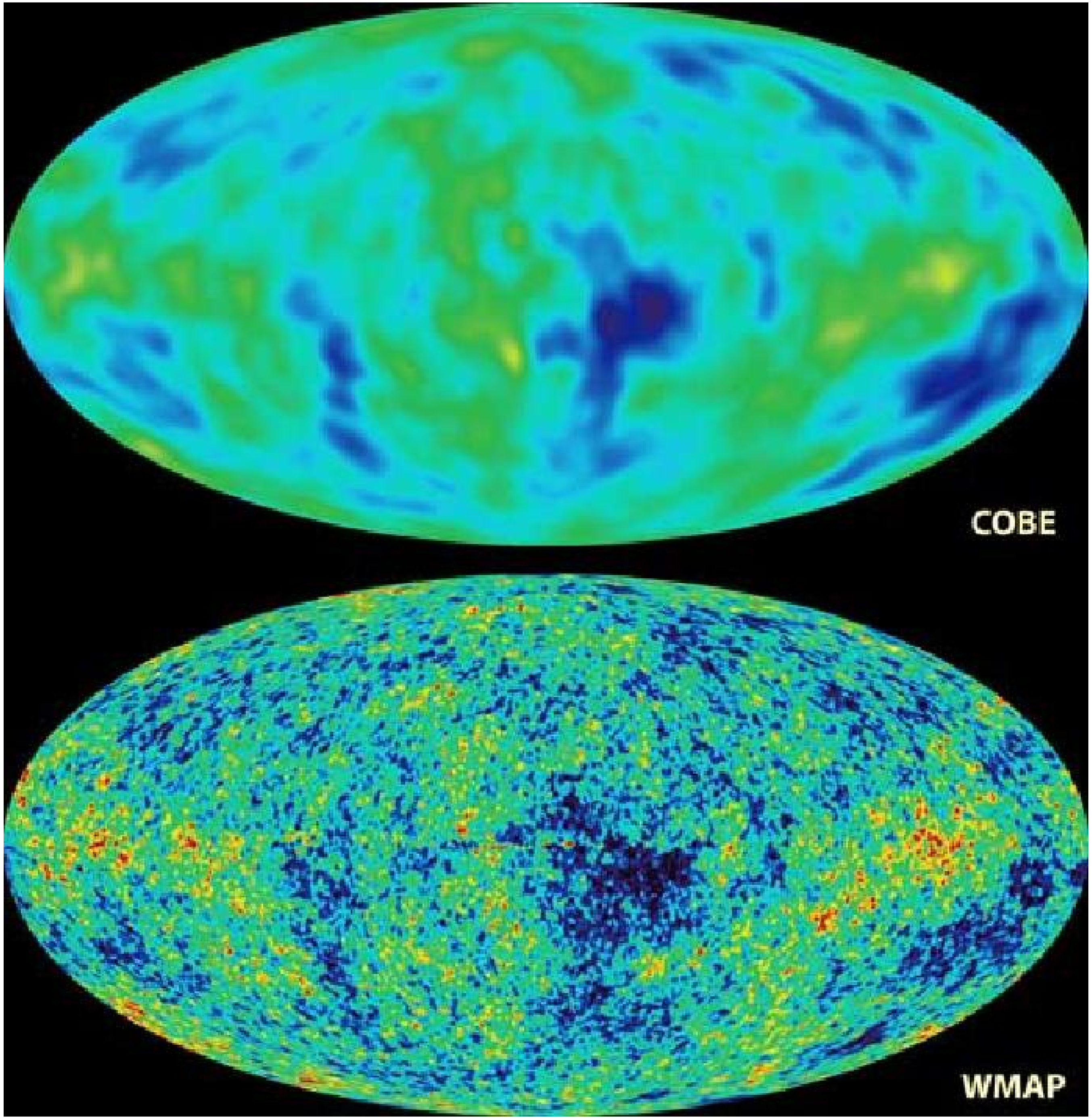
With data from CMB experiments at smaller schalles (ACBAR, CBI) and astronomical measurements of the power spectrum from large scale structure (2dFGRS) $\rightsquigarrow$

$$\Omega_b h^2 = 0.0224 \pm 0.0009 \qquad \Omega_M h^2 = 0.135^{+0.008}_{-0.009} \tag{18}$$

Value consistent with predictions from Big Bang nucleosynthesis

$$0.018 < \Omega_b h^2 < 0.023 \tag{19}$$

CMB temperature fluctuations. Comparison between COBE and WMAP



Local Density

DM density in the region of our solar system: very important for direct and indirect DM detection

Uncertainties

- Determination of local DM density: observation of the rotation curves of the Milky Way
  - ◇ difficult from our location within the galaxy
  - ◇ rotation curves measure total mass within an orbit $\rightsquigarrow$ density distributions of galactic bulge and disk are needed
- DM velocity distribution in local region needed to accurately calculate direct and indirect detection rates. Inferred also from observed rotation curves.

Different groups → somewhat different conclusions for local density and velocity DM distributions (See plot by Bergstrom,Ullio,Buckley on the next page.)

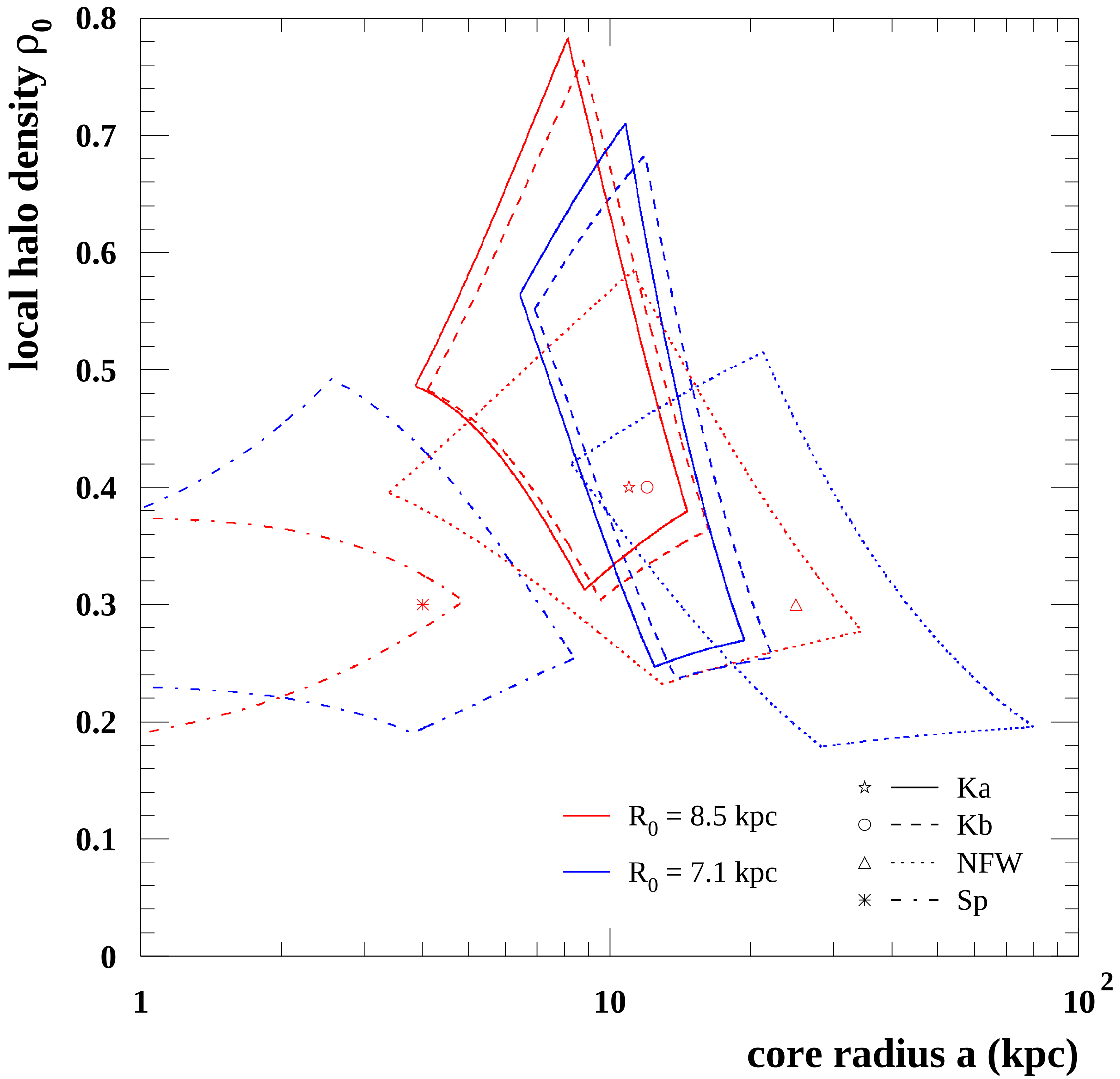
∗ DM velocity distribution typically described by its average velocity

$$\bar{v} = < v^2 >^{1/2} \approx 270 \text{ km/s} \tag{20}$$

∗ Loal DM densities acceptable in range $0.2 - 0.8$ GeV/cm^3

The range of local DM densities acceptable w/ observations of rotation curves for a variety of halo profiles and galactocentric distances.

Bergstrom, Ullio, Buckley



Candidates

▷ Evidence of non-baryonic DM at all observed astrophysical scales $\rightsquigarrow$

What is the Dark Matter made of?

▷ 2 popular candidates

◇ Supersymmetric *neutralino*

◇ $B^{(1)}$ *particle* - first Kaluza-Klein (KK) excitation of the B boson in theories w/ universal Extra Dimensions (ED)

The Non-Baryonic Candidate Zoo

∗ Standard Model Neutrinos

- Formerly considered excellent DM candidates
- With $m_i = i$-th ν mass:

$$\Omega_\nu h^2 = \sum_{i=1}^{3} \frac{m_i}{93eV} \tag{21}$$

upper limit on m_ν $\rightsquigarrow$

$$\Omega_\nu h^2 \lesssim 0.07 \tag{22}$$

$\Rightarrow$ ν's not abundant enough to be the dominant DM component

- ν's are relativistic collisionless particles $\rightsquigarrow$ erase fluctuations below a scale of ~ 40 Mpc (moving from high to low density regions)

$\Rightarrow$ *top-down* formation: big structures form first.

<u>but:</u> our galaxy appears to be older than the Local Group!

$\Rightarrow$ ν's are no viable DM candidate.

* *Sterile Neutrinos*

- Hypothetical particles similar to SM ν's, but w/o SM weak interactions (apart from mixing)
- Introduced by Dodelson and Widrow 1993
- Analysis of their cosmological abundance and study of their decay products $\rightsquigarrow$ stringent cosmological and astrophysical constraints on sterile ν's
- Light ν's with $m_\nu \lesssim$ few keV ruled out.
- Could also be cold DM if small lepton asymmetry exists.

* *Axions*

- Laboratory searches, stellar cooling and dynamics of supernova 1987A $\rightsquigarrow$ axions very light $\lesssim 0.01$ eV.
- Extremely weakly interacting with ordinary particles $\rightsquigarrow$ not in thermal equilibrium in the early universe.
- Calculation of the axion relic density is uncertain, depends on assumptions made regarding the production mechanism.

∗ Supersymmetric candidates

- Neutralinos

In models w/ R-parity conserving SUSY the most widely studied DM candiates (see later)

- Sneutrinos

Have a cosmologically interesting relic density if $550 \lesssim m_{\tilde{\nu}} \lesssim 2300$ GeV. However, calculated scattering cxn of $\tilde{\nu}$ with nucleons much larger than the limits from direct DM detection experiments.

- Gravitinos

Superpartners of the graviton. In GMSB they can be the lightest SUSY particle and stable. $\rightsquigarrow$ Gravitinos very strongly theoretically motivated.
With only gravitational interactions $\rightsquigarrow$ very difficult to observe.

- Axinos

Superpartner of the axion. Can be warm, hot or cold DM candidate. Axinos and gravitinos share similar phenomenological properties.

∗ Dark Matter from Little Higgs (LH) Models

- Alternative mechanism to stabilize the weak scale.
- SM Higgs is a pseudo-Goldstone boson with its mass protected by approximate non-linear global symmetries.
- Divergences to the Higgs mass are only at the 2-loop level $\rightsquigarrow$ weak scale stabilizeable in effective theory up to $\sim$ 10 TeV.

- "Theory space" little Higgs models
  provide a possibly stable scalar particle leading to the measured DM density

- Another variety of little Higgs models

(motivated to solve the problem of the hierarchy between the EW scale and the masses of new particles constrained by EW precision measurements)

$\longrightarrow$ introduction of new symmetry at the TeV scale

$\rightsquigarrow$ existence of stable WIMP candidate with a $\sim$ TeV mass.

Discrete symmetry has to be assumed, which is not broken by the operators in the UV completion, to protect potential DM candidate from decay.

∗ Kaluza-Klein states

- KK excitations of SM fields appear in universal extra dimension models
- another DM candidate in framework of "warped" universal extra dimensions: exotic particle w/ gauge quantum numbers of a right-handed neutrino, but carrying fractional baryon-number.

∗ Superheavy Dark Matter

- sometimes called *Wimpzillas* - superheavy weakly interacting massive particles

∗ Q-balls, mirror particles, CHArged Massive Particles (CHAMPs), self-interacting DM, D-matter, cryptons, superweakly interacting DM, brane world DM, heavy fourth generation neutrinos, etc.

See e.g. reviews of non-baryonic candidates:

- J.R. Ellis, Phys. Scripta **T85** (2000) 221 [astro-ph/9812211].
- L. Bergstrom, Rept. Prog. Phys. **63** (2000) 793 [hep-ph/0002126].

- **Remarks**

- not assured that DM made of a single particle species
- we know, that SM ν's contribute; cannot account for all of DM, however
- $N = 2$ SUSY DM allows the posssibility of 2 stable DM relics
- in following assumed, that
  abundance of candidates satisifes limits of CMB analyis
- upper bound strict, but lower bound can be relaxed, assuming that the discussed candidate is sub-dominant component of DM

$\mathcal{N}$eutralino Dark Matter

▷ Lighest neutralino $\tilde{\chi}_1^0$ as DM candidate in R-parity conserving MSSM

▷ Linear combination of Bino, Wino, Higgsinos

$$\tilde{\chi} = N_{11}\tilde{B} + N_{12}\tilde{W}_3 + N_{13}\tilde{H}_1^0 + N_{14}\tilde{H}_2^0 \tag{23}$$

▷ Most relevant interactions for DM:

◇ self-annihilation

◇ elastic scattering of nucleons

▷ Neutralinos are expected to be extremly non-relativistic in present epoch $\rightsquigarrow$
Can safely keep only a-term in usual expansion of the annihilation cross section for indirect detection.
However, this is not sufficient for calculations of the relic density.

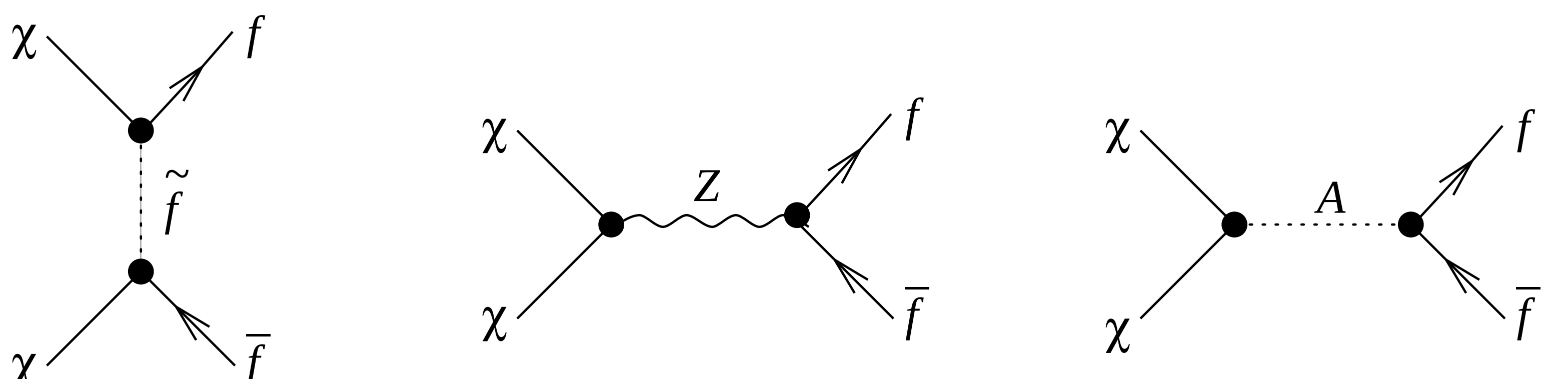
$$\sigma v = a + bv^2 + \mathcal{O}(v^4) \tag{24}$$

▷ At low velocities, leading channels for neutralino annihilation are those into

- (1) fermions (primarily heavy fermions, *i.e.* t, b, c, τ)
- (2) gauge boson pairs (W^+W^-, ZZ)
- (3) final states with Higgs bosons

▷ Complete list of all tree level processes and cross sections:
G. Jungman, M. Kamionkowski and K. Griest, Phys. Rept. **267** (1996) 195 [hep-ph/9506380]

(1) *Annihilation into fermions*



▷ Cross section for the process

$$\sigma v(\tilde{\chi}\tilde{\chi} \to \bar{f}_i f_i)_{v\to 0} = \frac{c_f \beta_f}{128\pi m_{\tilde{\chi}}^2} |\mathcal{A}_A(\tilde{\chi}\tilde{\chi} \to \bar{f}_i f_i) + \mathcal{A}_{\tilde{f}}(\tilde{\chi}\tilde{\chi} \to \bar{f}_i f_i) + \mathcal{A}_Z(\tilde{\chi}\tilde{\chi} \to \bar{f}_i f_i)|^2 \qquad (25)$$

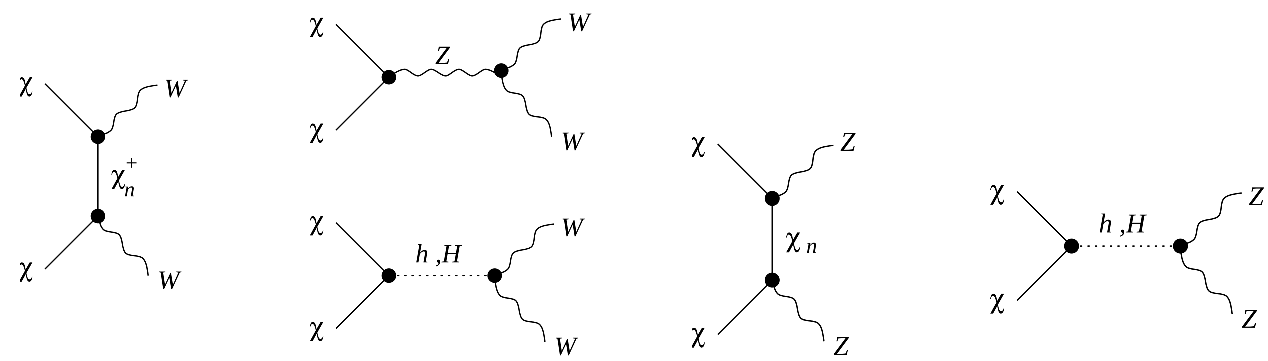
with

$\mathcal{A}_A(\tilde{\chi}\tilde{\chi} \to \bar{f}_i f_i)$	s-channel pseudoscalar Higgs exchange amplitude
$\mathcal{A}_{\tilde{f}}(\tilde{\chi}\tilde{\chi} \to \bar{f}_i f_i$	t-channel sfermion exchange amplitude
$\mathcal{A}_Z(\tilde{\chi}\tilde{\chi} \to \bar{f}_i f_i$	s-channel Z boson exchange amplitude
$\beta_f = \sqrt{1 - m_f^2/m_{\tilde{\chi}}^2}$	
c_f	color factor, $= 3$ for quarks, 1 otherwise

Remarks:

- neutralino annihilation amplitudes to fermion pairs are proportional to the final-state fermion mass:
- $\tilde{f}$ and Z exchange because Zff and $\tilde{\chi}ff$ couplings preserve chirality
- A exchange because of Yukawa coupling
- Yukawa coupling $\sim \tan\beta(\cot\beta)$ for down- (up-) type fermions
- Net result: Neutralino annihilation into fermions is dominated by heavy final states, $b\bar{b}, \tau^+\tau^-$ and $t\bar{t}$ if kinematically allowed
- $\tan\beta$ large $\rightsquigarrow$ down-type fermions may dominate over up-type fermions ($b\bar{b}$ over $t\bar{t}$ even for heavy $\tilde{\chi}$)

(2) *Annihilation into gauge bosons*



▷ Cross section for the process

$$\sigma v(\tilde{\chi}\tilde{\chi} \to VV)_{v\to 0} = \frac{1}{S_V}\frac{\beta_V}{128\pi m_\chi^2}|\mathcal{A}(\tilde{\chi}\tilde{\chi} \to VV)|^2 \quad (26)$$

where

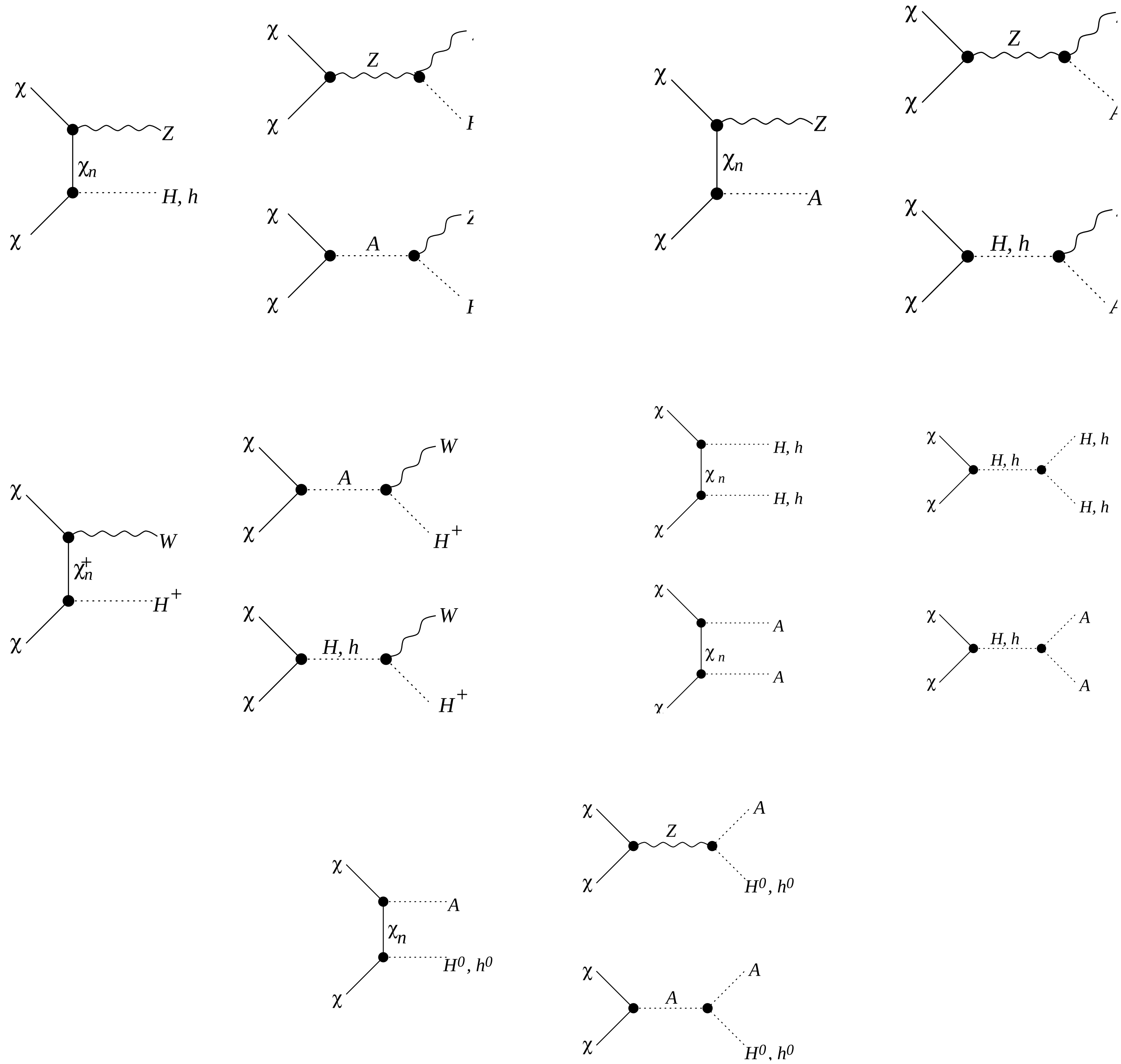
V indicates the gauge boson ($V = W, Z$)

S_V is a statistical factor, $S_V = 1, 2$ for $V = W, Z$

Remarks:

- Pure-gaugino neutralinos have no S-wave amplitude to gauge bosons.
- Pure-higgsinos or mixed higgsino-gauginos can annihilate efficiently via these channels, even at low velocities

(3) *Annihilation into Higgs bosons*



Remark:

In the low velocity limit no amplitude for neutralino annihilations into $H^+H^-, hh, HH, AA, ZA \rightsquigarrow$

▷ Low velocity cross section for any of the various modes

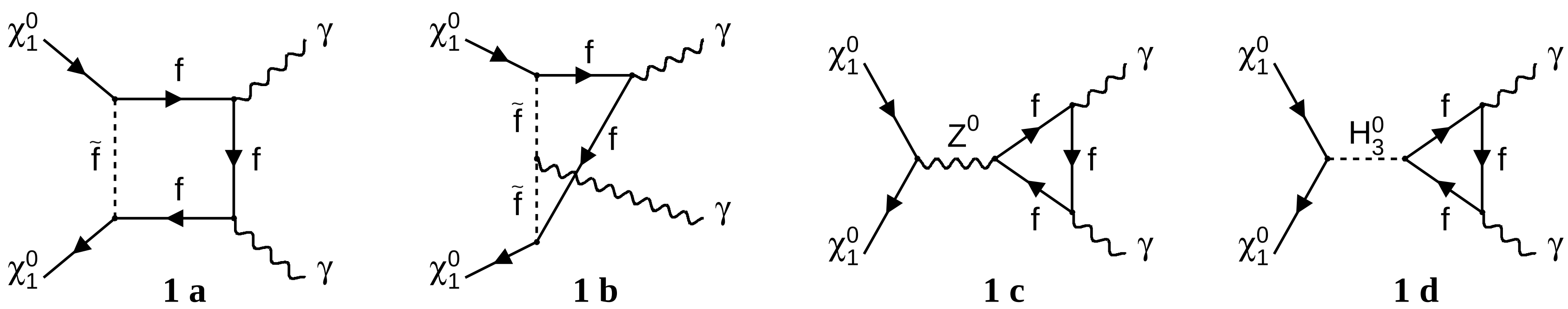
$$\sigma v(\tilde{\chi}\tilde{\chi} \to XY)_{v\to 0} = \frac{\beta_{XY}}{128\pi m_{\tilde{\chi}}^2} |\mathcal{A}(\tilde{\chi}\tilde{\chi} \to XY)_{v\to 0}|^2 \qquad (27)$$

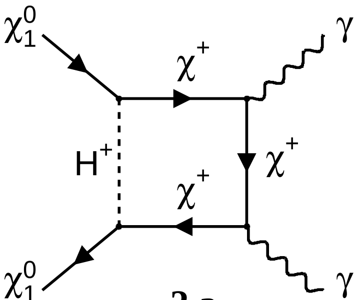
where $XY = Zh, ZH, W^\pm H^\mp, Ah, AH$.

(4) *Annihilation into photons*

Remark:

Annihilation into $\gamma\gamma$ and $Z\gamma$ mediated via loops.





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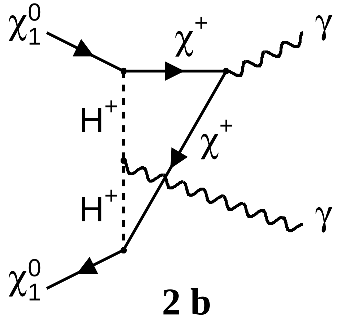

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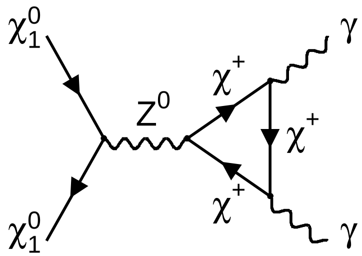

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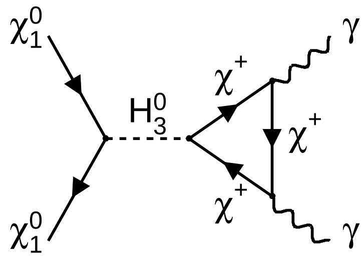

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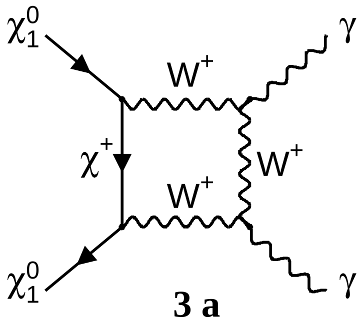

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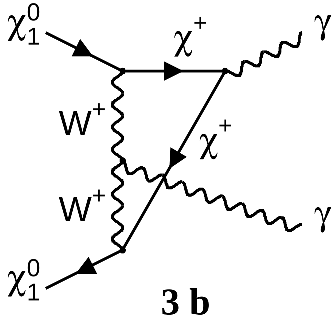

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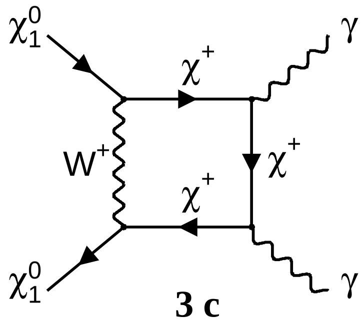

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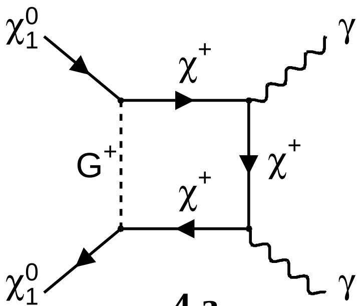

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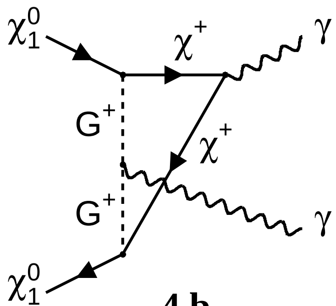

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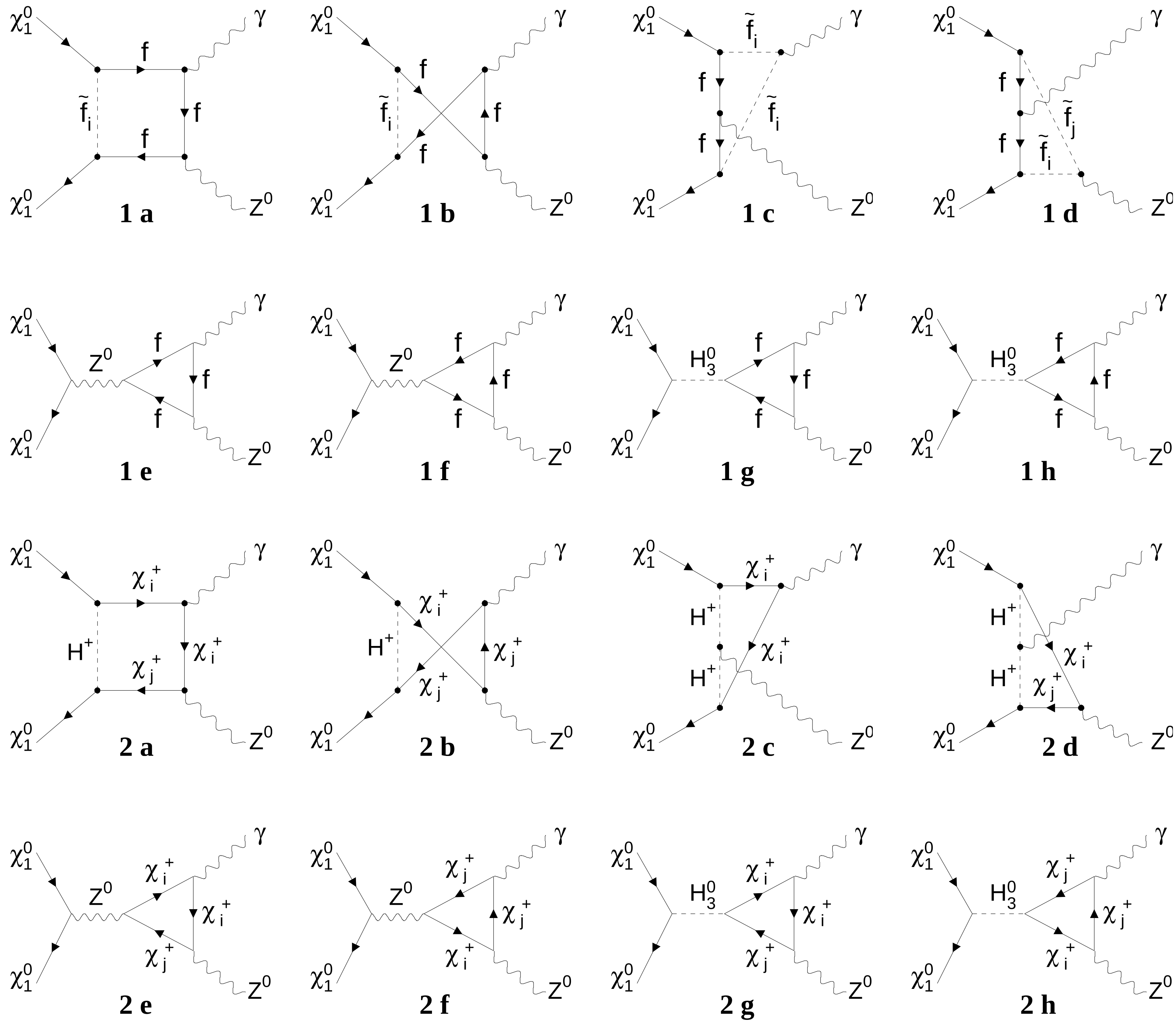
χ₁⁰
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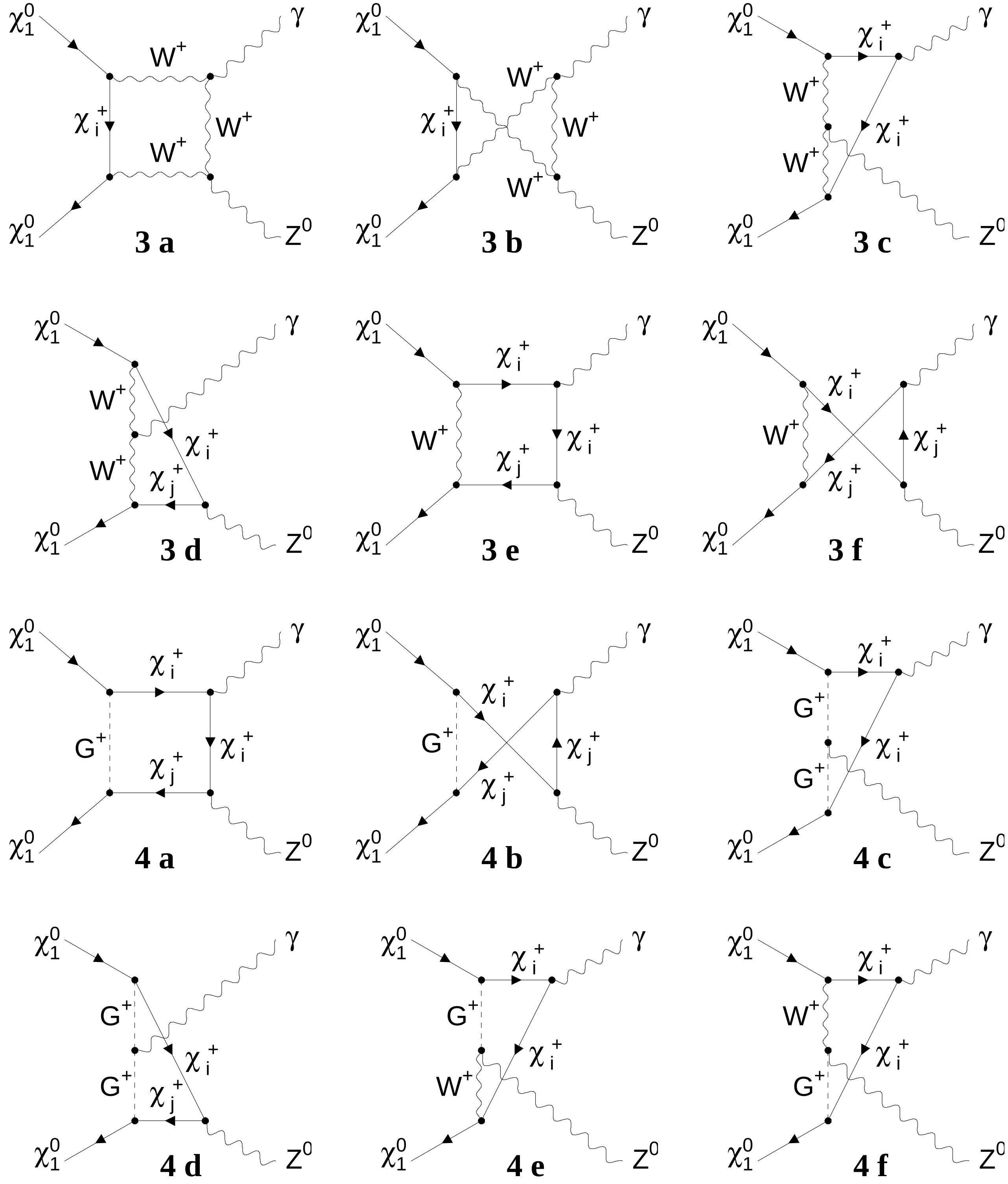
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Supersymmetric Models

◇ MSSM: huge number of parameters

◇ number of parameters must be reduced $\rightsquigarrow$ practical phenomenological study of the MSSM

◇ $\rightarrow$ make theoretically well motivated assumptions $\rightsquigarrow$ reduce free parameters from > 100 to smaller quantity

◇ different assumptions $\rightsquigarrow$ different SUSY models

mSUGRA

▷ assume: MSSM obeys a set of boundary conditions at the GUT scale:

- gauge coupling unification

$$\alpha_1(M_U) = \alpha_2(M_U) = \alpha_2(M_U) \equiv \alpha_U \qquad (28)$$

with $\alpha_i = g_i^2/4\pi$

- Unification of the gaugino masses

$$M_1(M_U) = M_2(M_U) = M_3(M_U) \equiv m_{1/2} \qquad (29)$$

- Universal scalar (sfermion and Higgs boson) masses

$$\begin{aligned} M_{\tilde{Q}}(M_U) &= M_{\tilde{u}_R}(M_U) = M_{\tilde{d}_R}(M_U) = M_{\tilde{L}}(M_U) = M_{\tilde{t}_R}(M_U) \\ &= M_{H_u}(M_U) = M_{H_d}(M_U) \equiv m_0 \end{aligned} \qquad (30)$$

- Universal trilinear couplings

$$A_u(M_U) = A_d(M_U) = A_l(M_U) \equiv A_0 \tag{31}$$

Minimization condition of the Higgs potential $\rightsquigarrow$ 5 free parameters

$$\tan\beta \ , \ m_{1/2} \ , \ m_0 \ , \ A_0 \ , \ \text{sign}(\mu) \tag{32}$$

Constrained MSSM (CMSSM)

Simplified version of the MSSM in which universality at grand unification scale is assumed

Figures

see J.Ellis arXiv:1001.3651 [hep-ph]

- For given $\tan\beta, A_0, sgn(\mu)$ there are regions of the CMSSM parameter space yielding acceptable relic density and satisfying other phenomenological constraints.

◇ Included are the LEP lower limits: $m_{\tilde{\chi}^\pm} > 104$ GeV, $m_{\tilde{e}} > 99$ GeV, $m_h > 114$ GeV.

◇ The former 2 constrain $m_{1/2}$ and m_0 directly via the sparticle masses,

◇ the latter indirectly via sensitivity of radiative corrections to Higgs mass to the sparticle masses, principally $m_{\tilde{t}}, m_{\tilde{b}}$.

◇ $BR(b \to s\gamma)$ must be consistent w/ experimental measurements. These agree with SM $\rightsquigarrow$ bound on MSSM particles, chargino and $H^\pm$ in particular.

◇ $b \to s\gamma$ constraint is more important for $\mu < 0$, but also relevant for $\mu > 0$, in particular for large $\tan\beta$

◇ $b \to s\gamma$ also excludes small values of $m_{1/2}$.

◇ $g_\mu - 2$ (measured in the E821 experiment at Brookhaven National Laboatory) provides further constraints.

◇ Most precise constraint on SUSY may be provided by the density of cold dark matter, determined from astrophysical and cosmological measurements by WMAP and other experiments

$$\Omega_{CDM} = 0.1099 \pm 0.0062 \tag{33}$$

◇ Applied to the relic LSP density $\Omega_{LSP}h^2 \rightsquigarrow$ very tight relation between SUSY model parameters, fixing some combination of them at the % level $\rightsquigarrow$ narrow WMAP strips.

Caution!
SUSY might not be the only contribution to the cold dark matter $\rightsquigarrow$ Ω_{CDM} should be interpreted as an upper liimit on $\Omega_{LSP}h^2$.

However, most of the SUSY parameter space in simple models gives SUSY relic density exceeding the WMAP range.

Regions of the WMAP strips

◇ Low values of $m_{1/2}$ and m_0: simple $\tilde{\chi} - \tilde{\chi}$ annihilations via crossed-channel sfermion exchange are dominant, bulk region, now largely excluded by LEP lower limit on Higgs mass.

◇ Larger $m_{1/2}$, relatively small m_0: close to boundary of region where $m_{\tilde{\tau}_1} < m_{\tilde{\chi}}$, coannihilation between $\tilde{\chi}$ and sleptons important $\rightsquigarrow$ suppression of the neutralino relic density into the WMAP range.

- ◇ Larger $m_{1/2}, m_0$ and $\tan\beta$: neutralino relic density may be reduced by rapid annihilation through direct-channel H, A Higgs bosons.
- ◇ Large m_0, focus-point region: close to boundary where EWSB not possible any more, lightest neutralino acquires a significant higgsino component here. (In most of the mSUGRA parameter space, lightest neutralino is gaugino-like with a mass of a couple hundred GeV or less.)

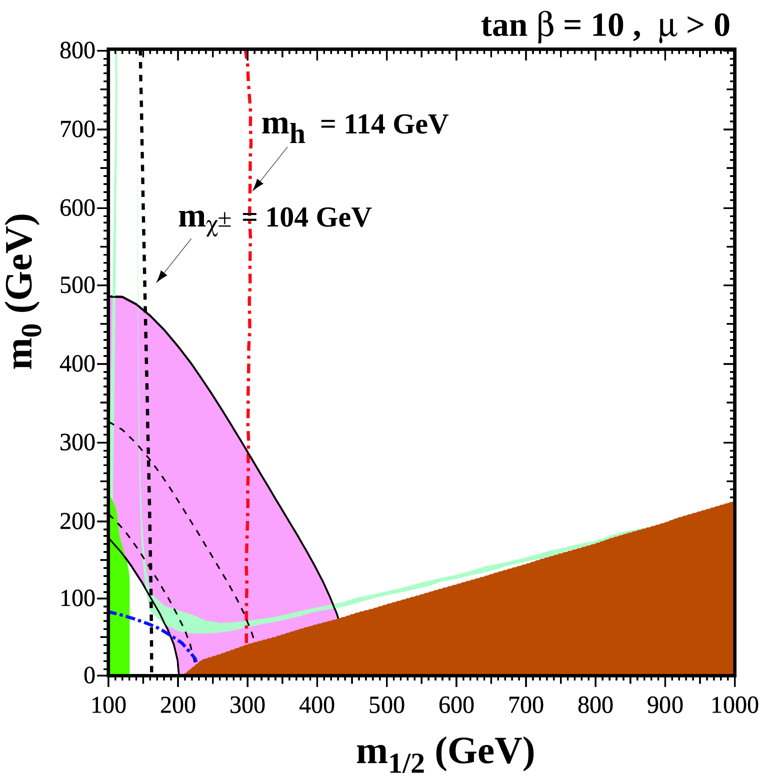
  Can be significantly heavier here.

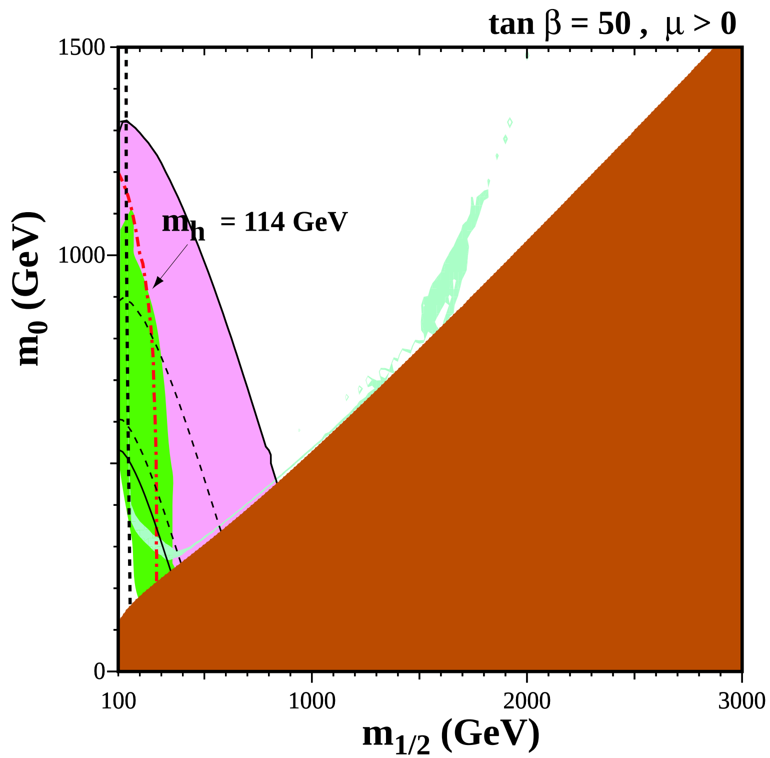
  Very large scalar masses are possible here w/o violating naturalness constraints $\leftarrow m_{H_u}^2$ has pseudo-fixed-point behaviour: can start with a wide range of input values and run to a similar negative value at the low scale. This indicates that in the focus point region EWSB does not require fine-tuning in the high energy input values.

  Typical feature: large scalar masses (usually $\sim$ TeV). Main reason for larger higgsino content in the LSP is the larger input value of the soft scalar mass. The tree level EWSB condition gives

$$\frac{1}{2} m_Z^2 \sim -m_{H_u}^2 - \mu^2 \, . \tag{34}$$

In typical mSUGRA scenarios, $m_{H_u}^2$ is driven to some large negative values due to the running of the RGE. This requires a large value of μ to give the correct Z mass. In the focus point region, however, it is possible that the large input value of the scalar soft mass makes $m_{H_u}^2$ less negative $\rightsquigarrow$ a smaller value of μ is possible $\rightsquigarrow$ a larger higgsino content in the LSP.





- ◇ $\mu > 0, A_0 = 0, m_t = 175$ GeV and $m_b(m_b)^{\overline{\mathsf{MS}}}_{SM} = 4.25$ GeV
- ◇ near-vertical (red) dot-dashed lines: contours for $m_h = 114$ GeV
- ◇ near-vertical (black) dashed line: cotour $m_{\tilde{\chi}^\pm} = 104$ GeV
- ◇ dot-dashed curve in the lower left: region excluded by LEP bound $m_{\tilde{e}} > 99$ GeV
- ◇ medium (dark green) shaded region: excluded by $b \to s\gamma$
- ◇ light (turquoise) shaded area: cosmologically preferred region
- ◇ dark (brick red) shaded region: LSP is the charged $\tilde{\tau}_1$
- ◇ Shaded (pink) region, bounded by solid black lines: region allowed by E821 measurement of a_μ at the 2-σ level (dashed lines: 1-σ ranges)

Extra Dimensions

* Our world appears to consist of $3+1$ dimensions, other dimensions however possible and appear at higher energy scales
* Kaluza, 1921: unify electromagnetism with gravity by identifying the extra components of the metric tensor with the usual gauge fields.
* More recently: hierarchy problem could be addressed and possibly solved by exploiting the geometry of spacetime. $M_{EW} \sim 100$ GeV $\leftrightarrow M_{Planck} \sim 10^{19}$ GeV
* Many ED models: $3+1$ dimensional spacetime: *brane* embedded in a $(3+1+\delta)$ spacetime: *bulk*
* Possible: gravity tested for $d \gtrsim 1$ mm $\rightarrow$ modifications for $d \lesssim 1$ mm possible.
* 2 big classes of models:
  - Arkani-Hamed,Dimopoulos,Dvali (ADD): ED compactified on circles (or other topology) of some size R; large volume of the ED $\leadsto$ lowering of M_{Planck} near EW scale

$$M^2_{Planck} = R^\delta M_D^{2+\delta} \qquad M_D \sim 1 \text{ TeV} \tag{35}$$

  - Randall-Sundrum (RS): ED with large curvature (warped ED). Exponential "warp" factor:

$$\Lambda_{SM} = M_{Planck} \exp(-kr_e\pi) \sim 1 \text{ TeV} \tag{36}$$

* Further motivation from *string theory* and *M-theory*: appear to be best candidates for a consistent theory of quantum gravity and a unified description of all interactions. Such theories may require the presence extra dimensions.

* General feature: Upon compactification of ED all the fields propagating in the bulk have momentum quantified in units of $p^2 \sim 1/R^2$. $\rightsquigarrow$ for each bulk field a set of Fourier expanded modes, Kaluza-Klein (KK) states appears.

* From our 4-dimensional world, KK states appear as a series (tower) of states with masses $m_n = n/R$, n labels the mode number. Each of these states contains the same quantum numbers.

* Many scenarios: SM fields assumed to be confined on the brane, only gravity allowed to propagate in the bulk.

* If ED small: would be possible for all fields to freely propagate in the ED: Universal Extra Dimensions.

Universal Extra Dimensions (UED)

All fields are allowed to propagate in the bulk.

Phenomenological motivation to have all SM fields propagate in the bulk:

- Motivation for 3 families from anomaly cancellation
- Attractive dynamical EWSB
- Prevention of rapid proton decay
- *Provides a viable dark matter candidate*

* 1 extra dimension: constraint on the compactification scale from EW precision measurements

$$R^{-1} \gtrsim 300 \text{ GeV} \ \ (\gtrsim 280 \text{ GeV if } M_H \gtrsim 800 \text{ GeV}) \qquad (37)$$

(Other models with SM bosons propagating in the bulk, fermions localized in 4 dimensions: $R^{-1} \gtrsim$ several TeV.)

$*$ Viable DM candidate: momentum conservation in the compactified dimensions $\rightsquigarrow$ KK-parity.

$*$ KK-parity: all odd-level KK particles are charged under this symmetry $\rightsquigarrow$ lightest (first level) KK state is stable.

The lightest Kaluza-Klein particle

Lightest Kaluza-Klein particle (LKP) in UED likely to be associated with the first KK excitation of the hypercharge gauge boson. Notation: $B^{(1)}$.

$*$ Calculation of $B^{(1)}$ relic density $\rightsquigarrow$ $M_{B^{(1)}} \in 400...1200$ GeV, above current experimental constraint

Plot (see next page):

- relic density of $B^{(1)}$ versus its mass, including coannihilations with the next-to-lightest KK particle (here $e_R^{(1)}$).

Coannihilations: If another particle has mass similar to the relic particle and shares a quantum number with it $\rightsquigarrow$ coannihilations have to be considered in the calculation of the relic density.

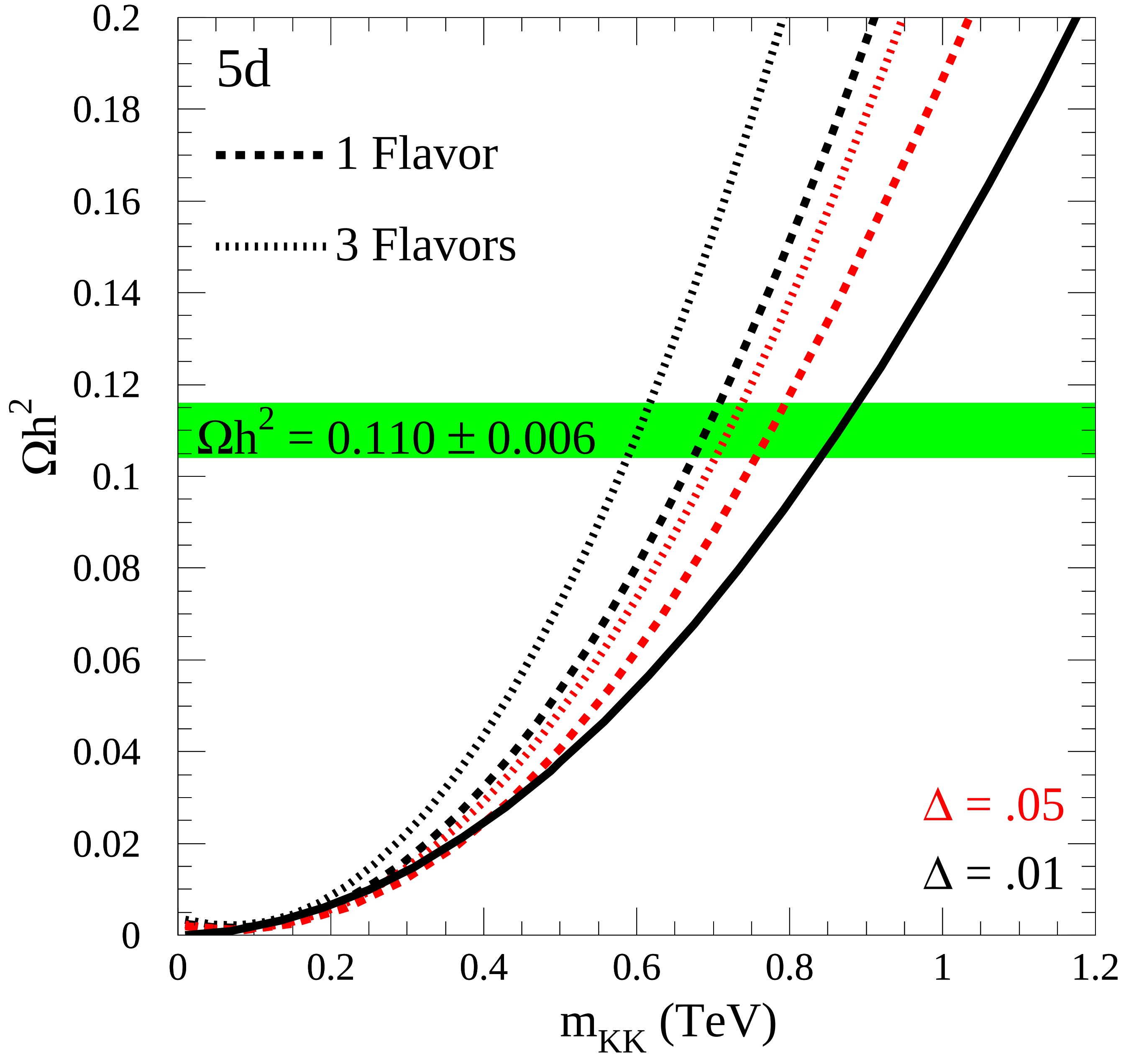
- WMAP constraints on CDM relic density included.
- Results on relic density calculation can vary depending on the spectrum of the other first level KK states.
- Density of KK DM is *increased* through coannihilations with other KK particles. Unlike in SUSY!
- Reason: In case of neutralinos, the cxn for the interaction between neutralinos and the NLSP is much larger than the neutralino self-annihilation cxn $\rightsquigarrow$ DM particles are kept longer in thermodynamic equilibirum $\rightsquigarrow$ decoupling with a lower relic density.

- The interactions between the $B^{(1)}$ and the $e_R^{(1)}$ are comparable to the $B^{(1)}$ self-interaction. $\rightsquigarrow$ Decoupling in presence of coannihilations happens essentialy at the same time as in the case with no coannihilations. The $B^{(1)}$ relic density becomes larger since the $e_R^{(1)}$, after decoupling at the same time, decays in the $B^{(1)}$.
- Spectrum of first level KK states calculated at one loop [Cheng,Matchev,Schmaltz]. Variations in the spectrum $\rightsquigarrow$ variations for the predicted LKP relic abundance.
- $B^{(1)}$ annihilation cross section is given by

$$\sigma v = \frac{95 g_1^4}{324 \pi m_{B^{(1)}}^2} \approx \frac{0.6 \text{ pb}}{m_{B^{(1)}}[\text{TeV}]} \tag{38}$$

- Branching ratios for $B^{(1)}$ annihilations are almost independent of the particle mass.
- Unlike in SUSY the bosonic nature of the LKP $\rightsquigarrow$ no chirality suppression in its annihilations. $\rightsquigarrow$ can annihilate efficiently to fermion-fermion pairs.
- Since annihilation cxn is $\sim$ final state hypercharge $\rightsquigarrow$ large fraction of LKP annihilations produce charged lepton pairs.
- Direct detection of LKP via its elastic scattering with nuclei: one-ton detector needed to probe the expected heavy masses as indicated by the relic densiy calculation of the LKP. $\rightsquigarrow$ next generation of direct detection experiments such as GENIUS or XENON.
- LHC should probe most of the relevant KK mass parameter space (up to $R^{-1} \sim 1.5$ TeV) and confirm or rule out UED at the TeV scale.

Servant, Tait



$\mathcal{E}$xperiments

Direct Detection Experiments

$*$ *Idea:* Galaxy filled with WIMPs $\rightsquigarrow$ many of them should pass through the Earth $\rightsquigarrow$ look for interactions with matter, e.g. recoil energy of nuclei, as WIMPs scatter off them.

$*$ *Key ingredients for the signal calculation:* density and velocity distribution of the WIMPs in the solar neighborhood and the WIMP-nucleon scattering cross section. $\Rightarrow$ evaluate rate of events in an experiment per unit time, per unit detector material mass.

$*$ *The rate (approximately):*

$$R \approx \sum_i N_i n_\chi < \sigma_{i\chi} > , \tag{39}$$

index i runs over nuclei species present in the detector. Number of target nuclei in the detector:

$$N_i = \frac{\text{Detector mass}}{\text{Atomic mass of species } i} . \tag{40}$$

Local WIMP density:

$$n_\chi \equiv \frac{\text{WIMP energy density}}{\text{WIMP mass}} . \tag{41}$$

$< \sigma_{i\chi} >$: cross section for scattering of WIMPs off nuclei of species i, averaged over the relative WIMP velocity with respect to the detector.

Scattering classifications

∗ *Elastic and Inelastic Scattering:*

Elastic scattering: Interaction of the WIMP with a nucleus as a whole, causing it to recoil.
Ideally often enough to measure recoil energy spectrum in the target.
With a Boltzman velocity distribution of WIMPs, centered at 270 km/s, the spectrum of recoils is exponential with typical energies of $< E >\sim 50$ keV.
Current experiments can detect recoils as low as 1-10 keV.

Inelastic scattering: WIMP interacts with orbital electrons in the target either exciting them, or ionizing the target.
Alternatively, the WIMP could interact with the target nuclei leaving it in an excited nuclear state.
Signature of this process: recoil followed by the emission of a photon $\sim 1ns$ later. Has to compete with backgrounds of natural radioactivity.

∗ *Spin-Dependent and Spin-Independent Scattering:*

WIMP scattering off nuclei: 2 classes of couplings.
Spin-Dependent: Axial-vector interactions resulting from couplings to the spin content of a nucleon.
Cross sections are rather proportional to $J(J+1)$ than the number of nucleons. $\rightsquigarrow$
Little is gained by using heavier target nuclei.

Spin-Independent: Scalar scattering. Cross section increases dramatically with the mass of the target nuclei.
Dominates typically over spin-dependent scattering in current experiments ($\leftarrow$ use heavy atoms as targets).

Remark: A WIMP which is not a Majorana particle could also scatter by vector interactions.
Examples: heavy Dirac neutrinos, MSSM sneutrinos. Not: neutralinos, KK DM.

Experimental efforts - Direct detection

Measure nuclear recoil produced by DM scattering

$*$ *Observation of scintillation:*

DAMA, ZEPLIN-I, NAIAD, LIBRA

$*$ *Observation of photons:*

CREST, CUORICINO

$*$ *Observation of ionization:*

HDMS, GENIUS, IGEX, MAJORANA, DRIFT

$*$ *Multiple techniques - ionization and photon:*

CDMS, Edelweiss

$*$ *Multiple techniques - scintillation and photon:*

CRESST-II, ROSEBUD

$*$ *Multiple techniques - scintillation and ionization:*

XENON, ZEPLIN-II, ZEPLIN-II, ZEPLIN-MAX

Use of large array of techniques and technologies $\rightsquigarrow$

- accelerate progress of the field
- vary the systematic errors from experiment to experiment

Some experiments separate WIMP signatures from bkg by looking for an annual modulation in their rate: Earth's annual motion around the Sun $\rightsquigarrow$ relative velocity relative to the galaxy's frame of reference $\rightsquigarrow$ Earth's velocity

$$v_E = 220km/s(1.05 + 0.07\cos(2\pi(t - t_m))) \tag{42}$$

$t_m \sim$ beginning of June, times in units of years $\Rightarrow$
$\approx 7\%$ variation in the WIMP flux and direct detection rate over the course of the year.
Smallness $\rightsquigarrow$ many experiments needed to identify such a signature

Indirect Detection:

Gamma-Ray Experiments

- Observation of products of WIMP annihilations in the galactic halo, the center of the Sun, other regions.
- Annihilation products: neutrinos, positrons, anti-protons, gamma-rays
- Direct observation must be made from space: in this energy range (GeV-TeV) photons interact with matter via e^+e^- pair production $\rightsquigarrow$ interacton length ~ 38 g cm^{-2} $\ll$ thickness of Earth atmosphere (1030 g cm^{-2} $\Rightarrow$ γ's cannot reach ground based telescopes.)
- Ground based experiments: Indirect observation.

Ground-based telescopes

γ's interact with atmosphere $\rightsquigarrow$ produce an electromagnetic cascade $\rightsquigarrow$ shower of secondary particles $\rightsquigarrow$ ground-based telescopes *indirectly* observe γ rays through detection of secondary particles and the Cerenkov light ($\leftarrow$ from their passage through the Earth's atmospere).

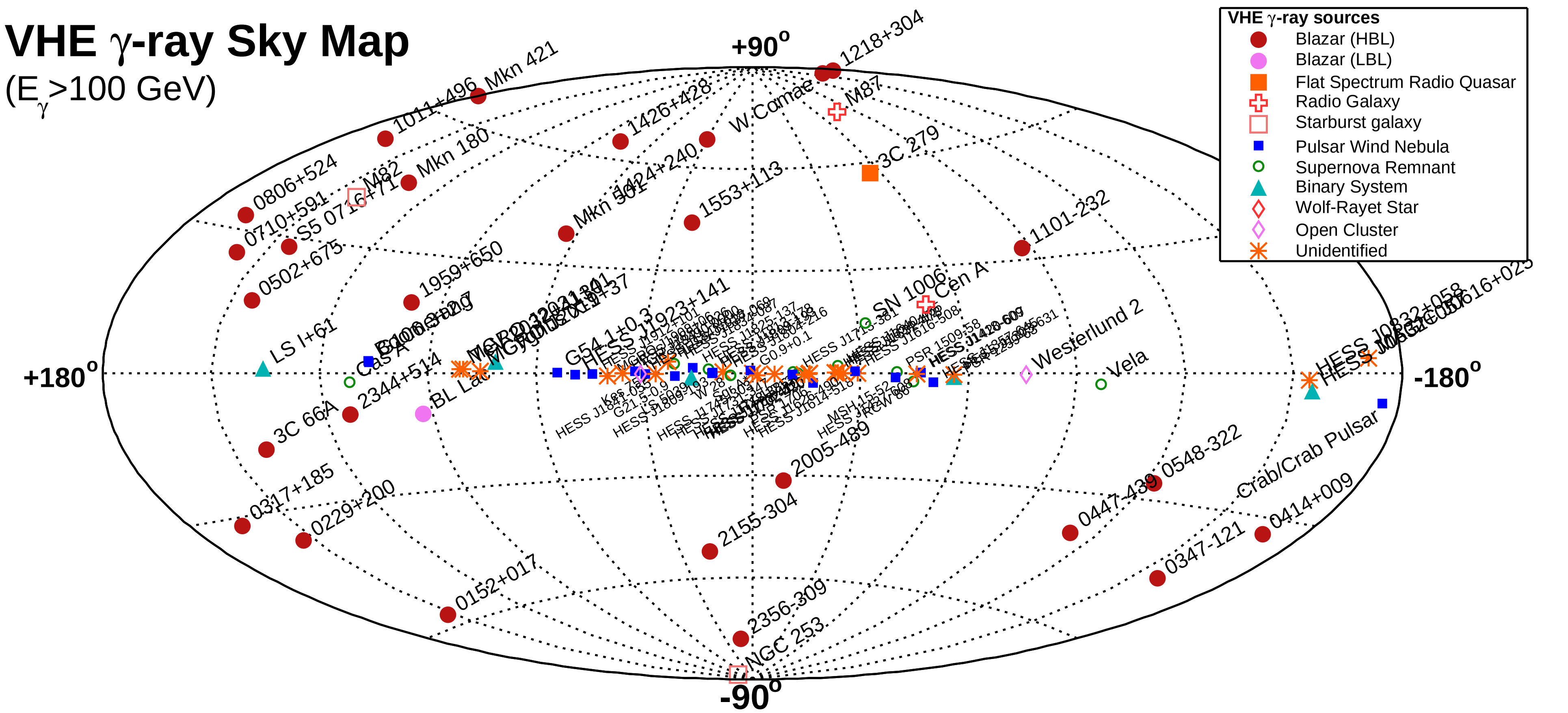
- However: Most of the observed Cerenkov light is due to cosmic-ray induced showers with isotropic arrival directions. $\rightsquigarrow$ detecting gamma-ray showers:
  excess above the isotropic bkg of cosmic rays in the direction of a source
- Distinction between cosmic-ray and gamma-ray induced air showers: observed Cerenkov light compared with numerical simulations of atmospheric showers
- Difficulties: treatment of interactions at very high energies, uncertainties associated with the density profile of the atmosphere and the Earth's magnetic field.
- Methods for collecting Cerenkov light: (array of) telescopes, converted solar arrays, ...
- Experiments: Whipple (Arizona, USA), Crimea (Ukraine), SHALON (Thien Shen, Ru), CANG-II (Woomera, Au), HEGRA (La Palma, Es), CAT (Pyrénées, Fr), TACTIC (Mt. Abu), Durham (Narrabri), 7TA (Utah, USA), Potchefstroom (South Africa), Pachmarhi (India), Beijing (China), STACEE (Albuquerque, USA), CELESTE (Themis, Fr), Solar-2 (Barstow, USA), Milagro (Fenton Hill US), Tibet HD (Tibet)
- Next generation experiments: MAGIC (La Palma), CANGAROO-III (Australia), HESS (Namibia) VERITAS (Arizona, USA)

Space-based telescopes

First high-energy (above GeV) gamma-ray space telescope: EGRET (Energetic Gamma-Ray Experiment Telescope):
Launched 1991, energies up to ~ 30 GeV

Next space-based gamma-ray observatory: GLAST (Gamma-ray Large Area Space Telescope):
Launched June 2008, sensitive to gamma-rays up to several hundred GeV.

Sky map for sources of very high-energy (VHE) gamma-rays (MPI Magic Telescope Group - 11/2009)



2009-12-17 - Up-to-date plot available at http://www.mppmu.mpg.de/~rwagner/sources/

Neutrino telescopes

Annihilations of DM particles: production of neutrinos

High-energy neutrino telescopes, large volume Cerenkov detectors: AMANDA, ANTARES, IceCube

* ν's much more difficult to observe than γ-rays $\leftarrow$ weak interactions with ordinary matter
* However, not easily absorbed $\rightsquigarrow$ observation in low bkg
* GeV-TeV range: ν's easily observed by their "muon tracks" produced in charge current interactions inside or nearby the detector volume.
* Muons travel through detector emitting Cerenkov light $\rightsquigarrow$ reconstruction of trajectory.

AMANDA Antarctic Muon and Neutrino Detector Array-Neutrino-Telescope: 680 optical modules in 1,5-2 km depth under the ice

IceCube: Successor project, under construction, thousands of spherical optical sensors in 1,5-2,5 km depth, look for point sources of ν's in the TeV range

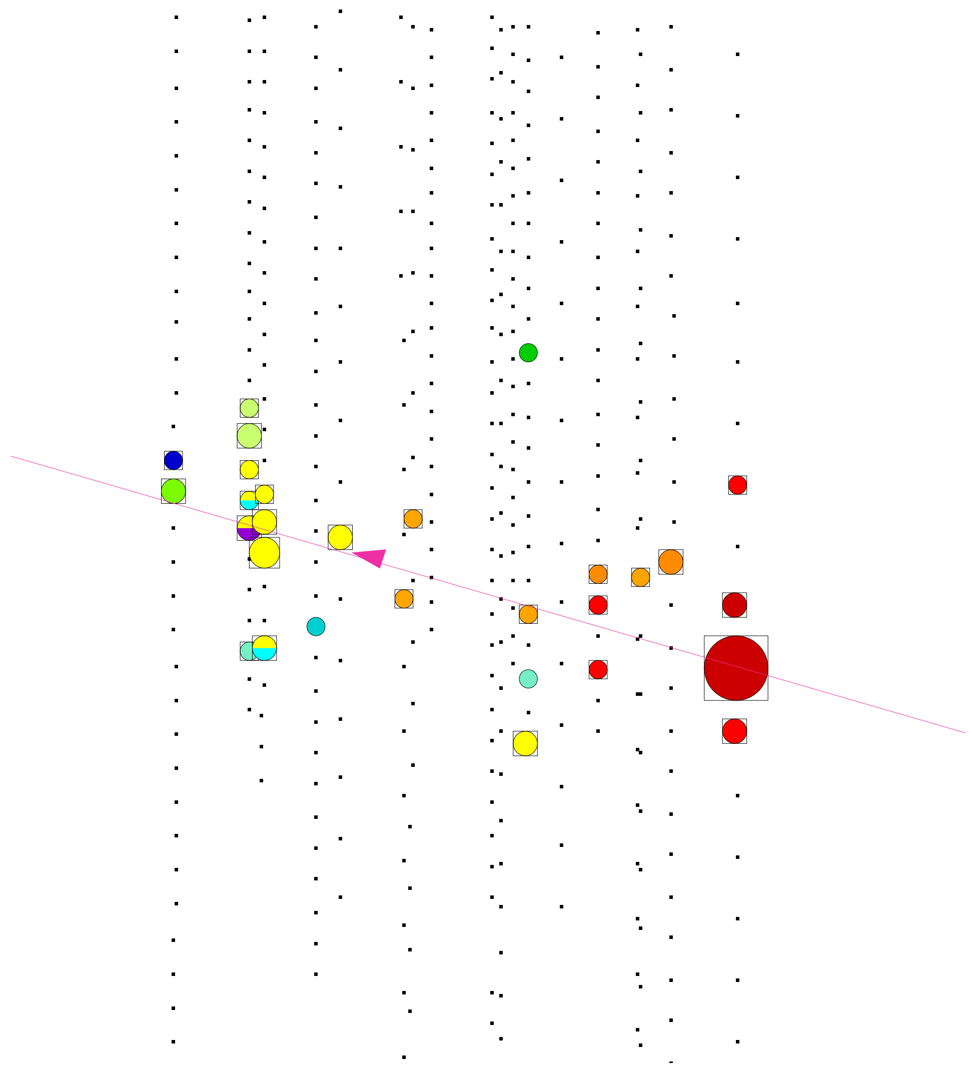
ANTARES: Large area water Cerenkov detector in the deep Mediterranean Sean for the detection of muons

Positron and Anti-Proton Experiments

Evidence for DM in the spectra of cosmic positrons and anti-protons; but: do not point to their source due to the presence of galactic magnetic fields.

HEAT: (High-Energy Antimatter Telescope); first flight 1994-1995; indicated excess in positron flux peaking at ~ 9 GeV and extending to higher energies; could be signature of DM annihilation in the local galactic halo; observation confirmed. 2000.

A muon neutrino event in AMANDA.



Shown: central part of the detector; colorscale and symbol size correspond to hit time and amplitude

BESS: (Ballon borne Experiment Superconducting Solenoidal spectrometer); balloon flights since 1993; cosmic anti-proton spectrum in range 200 MeV to 3 GeV; above to ~ 40 GeV: CAPRICE. Appears to be a mild excess in the anti-proton spectrum in the hundreds of MeV range.

PAMELA: measurement of cosmic positrons (50 MeV to 270 GeV) and anti-protons (80 MeV-190 GeV); carried on board of a Russion satellite, launched June 2006.
2009: “PAMELA finds an anomalous cosmic positron abundance.”
“The PAMELA experiment has published evidence of a cosmic-positron abundance in the 1.5-100 GeV range. This high-energy excess, which they identify with statistics that are better than previous observations, could arise from nearby pulsars or dark-matter annihilation.”

AMS-02: The Alpha Magnetic Spectrometer, mounted on the International Space Station; designed to search for various types of unusual matter by measuring cosmic rays.

Thank you for your attention!