SUSY PHENOMENOLOGY

Sheet 1

Exercice 1: Running coupling constants and unification

The one-loop equations of the running coupling constants are given by

$$\frac{1}{\alpha_j(Q^2)} = \frac{1}{\alpha_j(\mu^2)} + \frac{b_j}{4\pi} ln\left(\frac{Q^2}{\mu^2}\right) \tag{1}$$

In GUTs one finds for the b_j's:

$$\begin{pmatrix} b1 \\ b2 \\ b3 \end{pmatrix} = \begin{pmatrix} 0 \\ 22/3 \\ 11 \end{pmatrix} - N_F \begin{pmatrix} 4/3 \\ 4/3 \\ 4/3 \end{pmatrix} - N_H \begin{pmatrix} 1/10 \\ 1/6 \\ 0 \end{pmatrix} \tag{2}$$

where N_F is the number of particle families and N_H the number of Higgs doublets.

1. Calculate the GUT unification scale from the above equations by comparing α_1 and α_2, and start at the energy of the Z boson mass.

2. Determine the Weinberg angle from the Z boson mass as function of $\alpha(m_Z)$ and $\alpha_s(m_z)$.

Input parameters:

$$\begin{aligned} \sin^2\theta_w(M_{GUT}) &= \frac{3}{8} \\ m_Z &= 91.188\ GeV \\ \alpha(m_Z) &= \frac{1}{127.9} \\ \alpha_s(m_z) &= 0.119 \pm 0.004 \end{aligned}$$